BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM

DIVISION OF RESEARCH AND STATISTICS

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To: Federal Open Market Committee

From: Flint Brayton and Dave Reifschneider

Subject: Revised Bluebook Estimates of the Equilibrium Real Rate — Technical Documentation

Strictly Confidential (FR)
Class II – FOMC

Introduction

This memo provides background information on the econometric models used to produce the estimates of the equilibrium real rate of interest (R*) featured in the new Bluebook exhibit; it also describes our method of calculating the related estimates of uncertainty. As discussed in the accompanying overview memo, the new exhibit presents estimates for two different concepts of R*.[1] The first, which is short-run in nature, is the value of the real funds rate that, if sustained, would be projected to close the output gap twelve quarters in the future. The second concept of the equilibrium real rate (R^*_{MR}), which is medium-term in nature, is the value of the real funds rate projected to prevail in seven years, under the assumption that monetary policy closes the output gap in three years and thereafter adjusts the stance of policy to keep the gap closed. This measure is conceptually similar to the R* term that appears in the Taylor rule.

Both concepts of R* are defined explicitly as answers to forecast-related questions; accordingly, estimates of R* can be derived with any econometric model that links the level of real activity to the level of the real funds rate. In constructing the new

[1] See Flint Brayton and Dave Reifschneider, "Revised Bluebook Estimates of the Equilibrium Real Rate — Overview," memo to the Federal Open Market Committee (December 7, 2004).

Bluebook exhibit, we have elected to use three forecasting models for this purpose: a simple equation for the output gap; a small economic model; and the staff's large-scale model, FRB/US. The next three sections of this memo document the relevant properties of these models. We then provide background information on the procedures used to produce estimates of R* that are consistent with the staff Greenbook projection. We conclude by outlining the statistical methods used to compute confidence intervals for the various estimates of R* reported in the Bluebook.

Simple output gap equation

In this single-equation model, the output gap depends on a constant and three lags each of the output gap and the real funds rate (table 1). The lag length was chosen on the basis of tests of the hypothesis that, in an equation with *n* lags on each explanatory variable, the coefficients on the *nth* lag are zero. In particular, the p-value of the restriction that the coefficients on the fourth lag are zero is 0.23, whereas the p-value associated with the coefficients on the third lag is 0.002. In the equation, the coefficient estimates imply that a sustained rise in the real funds rate of 1 percentage point eventually reduces output relative to potential 1.4 percent.

Table 1
Specification of the Simple Output Gap Equation
(estimation period 1966:Q1 to 2004:Q3; t-statistics in parentheses)

Output gap:

$$gap(t) = .1934(2.0) + 1.1035(13.7)\ gap(t\text{-}1) + .06259(.5)\ gap(t\text{-}2) - .2260(2.9)\ gap(t\text{-}3)$$
$$+ .0388(.7)\ rff(t\text{-}1) - .2849(\text{-}3.7)\ rff(t\text{-}2) + .1633(2.9)\ rff(t\text{-}3)$$

adjusted $R^2 = .94416$
equation standard error = .71858

Variable definitions

gap	output gap (percentage difference between actual GDP and staff estimate of potential)
rff	real funds rate (nominal funds rate less 4-quarter moving average of core PCE inflation)

This equation is similar in spirit to the statistical filter that was previously employed to derive estimates of R* for the Bluebook, in that both rely on a very simple characterization of the relationship between output and the real funds rate. But the two approaches differ in the details of this characterization. One difference is that the new equation includes additional lags of the output gap and the real funds rate. These lags are statistically significant and affect the fundamental nature of the *long-run* value of the equilibrium real rate, R^*_{LR}. In the statistical filter, the relationship between the output gap and the real funds rate is subject to both transitory and permanent shocks. In this model, we define R^*_{LR} to be the permanent component under the assumption that it is a random walk. With the added lags in the new output gap equation, however, the estimated variance of permanent shocks falls to zero, indicating that R^*_{LR} is a constant; we estimate that constant to be about 2-1/4 percent.[2]

The conclusion that permanent shifts are absent from the long-run value of R* is plausible in this context in that statistical tests do not indicate the presence of permanent shifts in the real funds rate and such shifts do not characterize the output gap by construction.[3] However, standard economic theories do not prescribe that the long-run value of the equilibrium real rate R^*_{LR} is constant. The long-run value of R* may well vary over time, but these movements may be either difficult to detect using the framework of the simple gap equation or poorly characterized by a random walk.

Other researchers have found significant variation in the permanent component of R* when it is assumed to follow a random walk.[4] The estimates in these papers are based on models that, though similar to ours, differ in some respects. One difference is that the equations in these papers include fewer lagged values of the explanatory variables than we do. As discussed above, the properties of R^*_{LR} seem to be sensitive to this aspect of specification. Another difference is that some of the estimates

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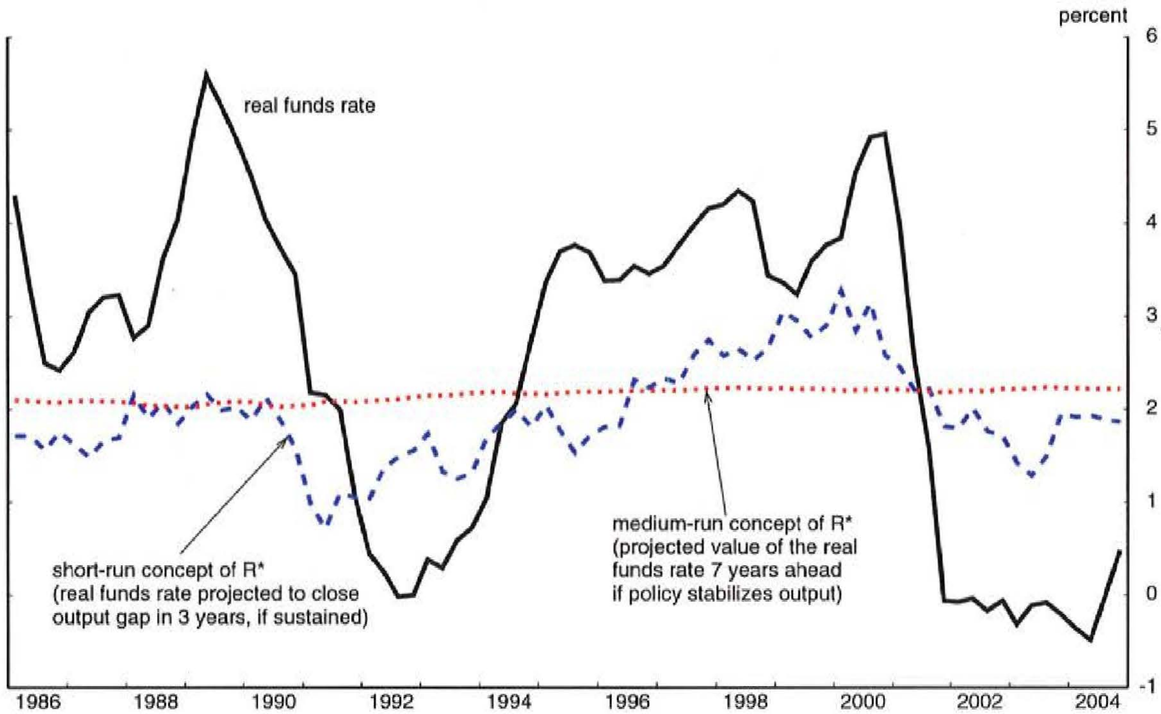
[2] When the simple output gap equation reported in table 1 is modified to allow R^*_{LR} to follow a random walk, the maximum likelihood estimate of the standard deviation of the random walk innovations is zero. The Stock-Watson median-unbiased estimate of the standard deviation, which might be preferred in this case because of a possible bias of the maximum likelihood value toward zero, is positive. However, the estimated value (.014 percentage points) is insignificantly different from zero in both statistical and economic terms.

[3] The null that the real funds rate is nonstationarity is rejected at the 3 percent level using an ADF test.

[4] For example, see Thomas Laubach and John C. Williams (2003),"Measuring the Natural Rate of Interest," *Review of Economic and Statistics*, 85 (4), 1063-1070; and Todd Clark and Sharon Kozicki (2004), "Estimating Equilibrium Real Interest Rates in Real Time," Federal Reserve Bank of Kansas City Research Working Paper 04-08.

Figure 1
Estimates of R* Generated Using the Simple Output Gap Equation



reported in these papers are based on models which estimate R^*_{LR} and potential output simultaneously, whereas we use the Board staff's pre-determined estimate of potential.

From a policy perspective, we are most interested in the short-run and medium-run measures of R*, which are presented in figure 1. Historical values of R^*_{SR} (dashed blue line) have ranged between 3/4 percent and 3-1/4 percent since the mid-1980s and recently have been a bit less than 2 percent; over the full estimation sample — 1966:Q1 to 2004:Q3 — R^*_{SR} ranges from -3/4 percent to 4-1/2 percent. In light of the very simple structure of this equation, the primary source of variation over time in this concept of R* is variation in the output gap, and indeed the correlation between the two series is 0.95. In general, when output is above potential, R^*_{SR} exceeds R^*_{LR}, and the opposite typically holds when output is below potential. This positive correlation arises because the output gap would be expected to close only partly during the twelve-quarter period associated with the definition of R^*_{SR} if the real funds rate were held constant at R^*_{LR}.

The dotted red line in figure 1 shows the simple equation's estimate of R^*_{MR}. Because this concept of R* is the value of the real funds rate consistent with maintaining output at potential once such a state has been achieved, the simple structure of this equation results in R^*_{MR} being the same as the long-run predicted values of the equilibrium real rate, and thus it is constant at 2-1/4 percent.[5]

Small economic model

In the simple output gap equation, such fundamental economic factors as trend GDP growth and fiscal policy have no role in determining R*. In contrast, the FRB/US model (discussed in the next section) has a detailed treatment of the components of aggregate demand, the supply side of the economy, financial markets, and fiscal policy, and so its R* estimates are influenced by a large number of economic factors. But this complexity is also a drawback because the scale of FRB/US makes it difficult to disentangle the contributions of various fundamental forces to movements in estimates of R*. To chart a middle course between these extremes, we have developed a small-scale model of the economy with which to compute estimates of the equilibrium real rate (see summary of its structure in table 2). The following is a brief discussion of the individual equations and the estimates of R* produced by the system as a whole.

Output gap equation

Output relative to the staff estimate of potential responds gradually to movements in the real bond rate, trend GDP growth, the ratio of the high-employment federal budget surplus to potential GDP, and the equity premium. The structure of the equation implies that the long-run equilibrium real rate moves one-for-one with changes in the trend rate of GDP growth; a sustained increase of 1 percentage point in the ratio of the federal surplus to GDP reduces the long-run value of R* about 30 basis points; and an

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[5] Strictly speaking, the historical values of R^*_{MR} vary slightly over time, but this variation is an artifact of defining the real funds rate as a compounded annual yield in estimation but converting it back to conventional units of measurement for tables and graphs.

Table 2
Specification of the Small Economic Model
(estimation period 1966:Q1 to 2004:Q2; t-statistics in parentheses)

1. Output gap

$$gap(t) = 2.3277\ (5.6) + .7672\ (21.9)\ gap(t\text{-}1) - .5160\ (-6.8)\ [\,rbaa(t\text{-}1) - g^*(t\text{-}1)]$$
$$- .3951\ (-5.6)\ eqprem(t\text{-}1) - .1431\ (-3.1)\ MA(govsrp(t\text{-}1),4)$$

adjusted R^2 = .94492
equation standard error = .71588

2. Federal high-employment government surplus

$$govsrp(t) = .4134\ (5.5)\ govsrp^*(t\text{-}1) + .5408\ (7.0)\ govsrp(t\text{-}1)$$
$$+ .3475\ (4.0)\ govsrp(t\text{-}2) - MA(govsrp(t\text{-}3),4)$$

adjusted R^2 = .86095
equation standard error = .50143

3. Real BAA bond rate

$$rbaa(t) = .8407\ (5.7) + .7176\ (15.2)\ rbaa(t\text{-}1) + .1378\ (6.8)\ rff(t)$$
$$+ .1446\ (3.6)\ R^*_{MR}(t) - .0678\ (-3.1)\ gap(t\text{-}1)$$

adjusted R^2 = .9261
equation standard error = .40077

4. Equity premium

$$eqprem(t) = .3376\ (2.6) + .9002\ (25.7)\ eqprem(t\text{-}1)$$

adjusted R^2 = .81289
equation standard error = .55916

5. Potential GDP growth

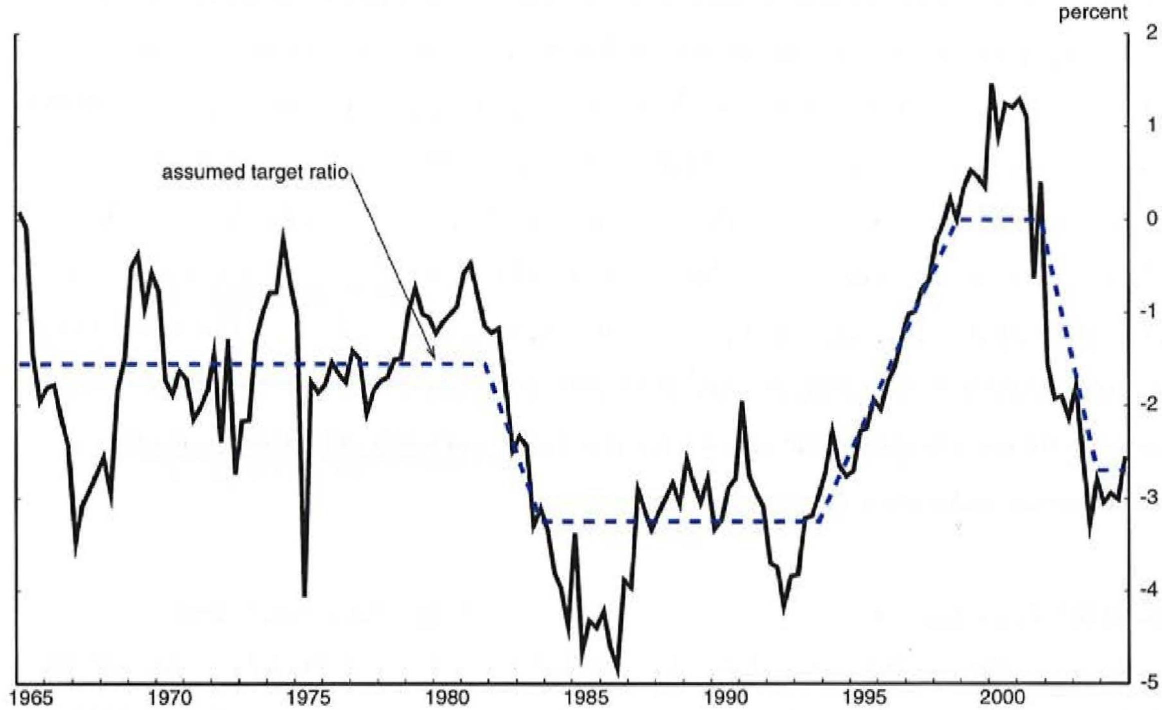
$$g^*(t) = g^*(t\text{-}1)$$

Variable definitions

Eqprem	equity premium (equal to 100*D/W – r + g*, where D/W is the ratio of NIPA dividends to Flow-of-Funds stock market wealth, and r is the real yield on 10-yr Treasury bonds)
gap	output gap (percentage difference between actual GDP and staff estimate of potential)
g*	growth rate of the staff estimate of potential GDP
govsrp	high-employment federal budget surplus as a ratio to (nominal) potential GDP
govsrp*	assumed target ratio of federal budget surplus to potential GDP (see text for details)
rbaa	real BAA rate (nominal BAA rate less long-run expectation for inflation, where the latter is a FRB/US measure derived from survey data)
rff	real funds rate (nominal funds rate less 4-quarter moving average of core PCE inflation)
R^*_{MR}	value of the real funds rate 28 quarters ahead, as predicted by the small model

Note: MA(x(t),n) denotes the n-quarter moving average of x(t).

Figure 2
High-Employment Federal Budget Surplus as a Ratio to Potential GDP
(surplus deflated by GDP chain-weight price index)



increase of 1 percentage point in the equity premium boosts its long-run value about 3/4 percentage point. The latter estimate strikes us as surprisingly large; the same effect is only half as big in the FRB/US model. However, part of this equity premium effect may actually be a growth rate effect. By construction, errors in the measurement of trend growth produce offsetting errors in the measured equity premium. With the estimated long-run effect of trend growth on the equilibrium real rate constrained to unity, such measurement error would bias up the estimated coefficient on the equity premium in the output gap equation and so overstate the true influence of this factor on R*. For this reason, we are inclined to regard the surprising magnitude of the equity premium effect in this model as partly a growth rate effect.

Federal budget surplus equation

Before the 1980s, the ratio of the high-employment federal budget surplus (deflated by the GDP price index) to potential GDP appeared to follow a stationary autoregressive process. In particular, the budget surplus ratio seemed to display gradual reversion to a fixed long-run average of about -1.5 percent (figure 2). In our specification of this equation, we have assumed that such mean reversion to a "target" continued to characterize the budget balance after 1980, with the difference that the target ratio began to shift from Administration to Administration. Specifically, we have assumed that the target ratio moved down to -3.2 percent during the Reagan and George H. W. Bush Administrations and then rose gradually and peaked at zero during the second Clinton Administration. We assume that the current long-run target is about -2.7 percent of GDP, which is similar to the Congressional Budget Office's projections for the budget balance early in the next decade if the various tax provisions currently scheduled to expire in 2010 are instead continued.

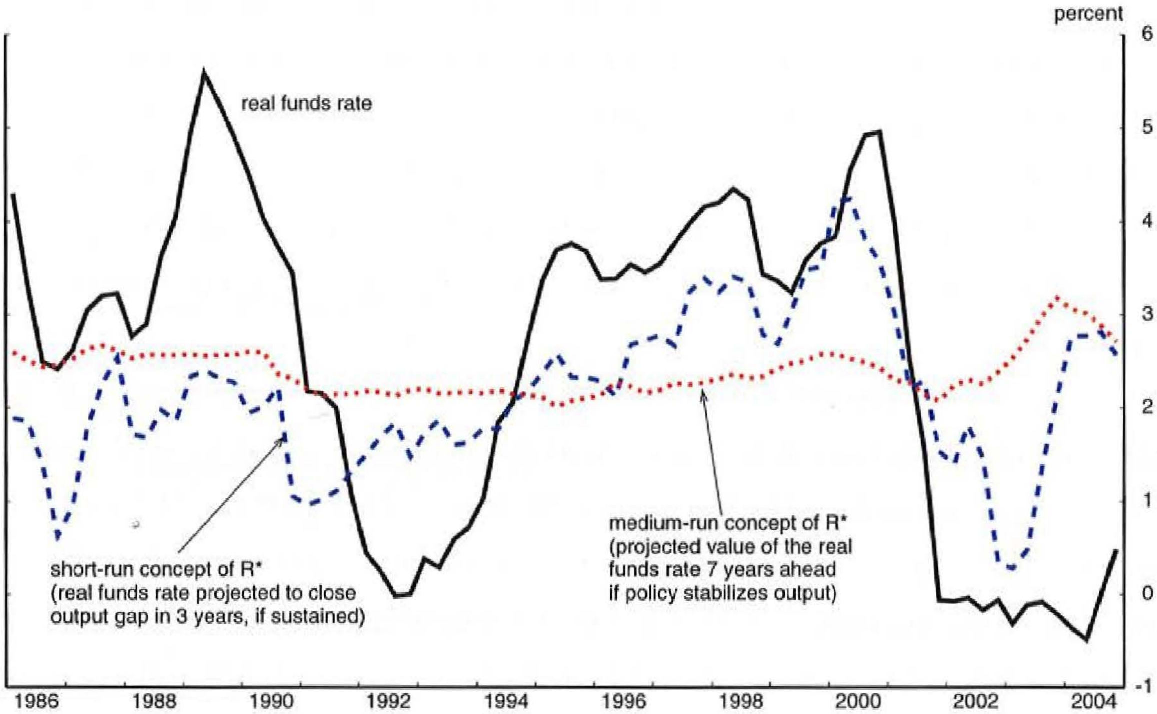
Real bond rate equation

The real bond rate, defined as the difference between the BAA bond rate and a survey-based measure of expected long-run inflation, is modeled using a simple reduced-form specification, rather than a formal asset valuation model that relates the bond rate to explicit expectations about the future path of short-term rates. However, the specification is partly forward-looking in spirit, in that the current real bond rate depends in part on the model's projection for the value of the real funds rate seven years ahead — that is to say, R^*_{MR}. The bond rate also depends on the current level of the real funds rate and the lagged bond rate, as well as a constant term and the lagged output gap (which is intended to capture the effect of cyclical changes in risk). Because the coefficients on the three interest rate terms are constrained to sum to unity, the model always predicts the real bond rate (less a risk premium) to converge over time to that value of the real funds rate consistent with a zero output gap in the medium to long run.

Other equations and variables

The remaining two formal equations have simple specifications — the equity premium follows an autoregressive process that displays gradual reversion to a fixed long-run mean, whereas trend GDP growth is assumed to follow a random walk. In

Figure 3
Estimates of R* Generated Using the Small Economic Model



addition, the model contains two "exogenous" variables: the target ratio of the high-employment budget surplus to potential GDP and the real funds rate. In the forecasts that underlie the estimates of R*, the former variable is always projected at its last value, implying that any calculation of the medium-run value of R* in, say, the 1980s incorporates the assumption that the large deficits of that time persist indefinitely; projections made during the second Clinton Administration instead assume that the budget is balanced in the long run. As for the real funds rate, in forecasting it is set at whatever fixed value closes the output gap in twelve quarters; beyond twelve quarters, the real funds rate is adjusted to keep the output gap closed.

R estimates from the small model*

Figure 3 shows the historical predictions of the small model for both R^*_{SR} and R^*_{MR}. Relative to the predictions of the simple output gap equation, small-model estimates for either concept of the equilibrium real rate display greater variation over time. R^*_{SR} (the dashed blue line) was quite low at the start of the 1990s when the economy was in recession, but as the economy boomed over the second half of the 1990s, the short-run equilibrium real rate rose markedly, peaking at a little over 4 percent in early 2000. After the stock market slumped and the economy fell into recession, R^*_{SR} fell almost to zero, but it has since rebounded to an average of 2-3/4 percent this year. In contrast, the estimated value of R^*_{MR} has been relatively stable over the past twenty years; even so, at 2-3/4 percent, it now is almost 3/4 percentage point above its level in the early 1990s.

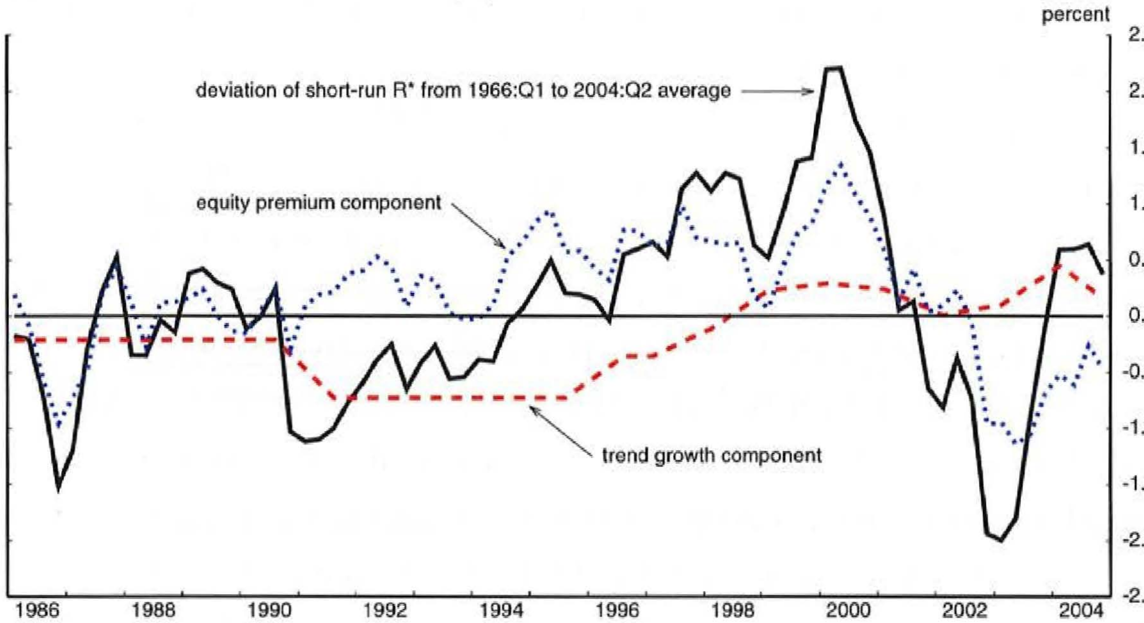
Figure 4 provides some perspective on the economic causes of these swings in the short-run equilibrium real funds rate. According to the small model, the combination of a low equity premium and rapid potential GDP growth combined to boost R^*_{SR} above its long-run average by about 1-1/2 percentage points in early 2000 (dashed blue and dotted red lines, upper panel). However, with the subsequent return of more normal equity valuations, the combined effect of these two factors on R^*_{SR} is now slightly negative. Initial conditions — defined as the starting level of the output gap plus that portion of the starting level of the real bond rate uncorrelated with current and lagged values of the output gap, trend GDP growth, the equity premium, and the high-employment budget surplus — also pushed up the short-run equilibrium real funds rate over the late 1990s and into 2000 (dashed blue line, lower panel). However, much of this effect was offset by the swing of the cyclically adjusted federal budget balance into surplus during this same period.

The FRB/US model

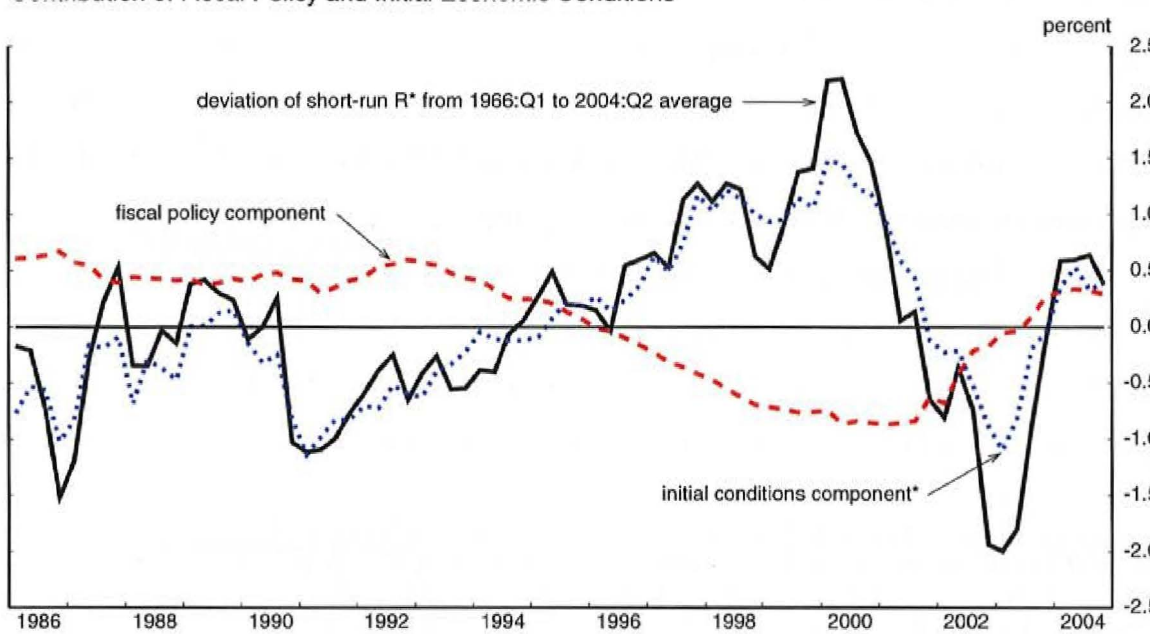
The third model used to estimate R* is FRB/US. Although FRB/US is much larger than the two other models, size *per se* is not the feature that most complicates its use in computing R*; instead, it is the presence of many exogenous variables. Some examples are tax rates and other factors related to the federal tax code, rates of depreciation for different types of capital, and population. Other less-familiar examples of exogenous variables are translation factors needed to link various components of national income.

Figure 4
Accounting for Deviations in Short-Run R* From Its Historical Mean (results derived from the small economic model)

Contribution of Trend Growth and the Equity Premium



Contribution of Fiscal Policy and Initial Economic Conditions



* Amount attributable to the starting output gap and that portion of the initial real bond unrelated to the trend growth, the equity premium, or fiscal policy.

For each date at which R* is to be estimated, our methodology requires projections of all exogenous variables that start at that date and extend to the end of the required simulation period. Most of these projections of exogenous variables are derived with the same mechanical procedure used in preparing the FRB/US forecast prior to each meeting of the FOMC.[6] In the procedure, each exogenous variable is projected with one of several simple forecast rules, such as repeating its most recent historical value (as of the date that R* is computed) or extrapolating its recent rate of growth. These rules are well suited for making forecasts over the twelve-quarter interval needed for computing the short-run measure of R*. Over longer periods, however, these rules are less likely to yield sensible results because their use is not necessarily consistent with the eventual convergence of FRB/US simulations to a balanced growth path (a necessary condition for stability in the real funds rate over the longer run). For this reason, we do not use FRB/US to estimate the medium-run concept of R*.

A second feature specific to the FRB/US is that it incorporates a measure of the output gap that is different from the measure in the Greenbook. Although the staff and FRB/US estimate potential output using a broadly similar approach, the specifics of the two estimation procedures differ in some key details.[7] Moreover, forcing FRB/US to adopt the staff's version of the output gap would violate some fundamental aspects of the design of the model. Thus, in contrast to the two other models, the FRB/US estimate of R^*_{SR} is the value of the real funds rate that closes the FRB/US measure of the output gap. This series is shown as the dashed blue line in figure 5.

This estimate of R^*_{SR} is based on the assumption that projected equation residuals are zero, as are the estimates from the other models. Such an assumption is likely to be the best choice for most equations within their estimation period and over the period that immediately follows. But the sample periods for most of the equations in

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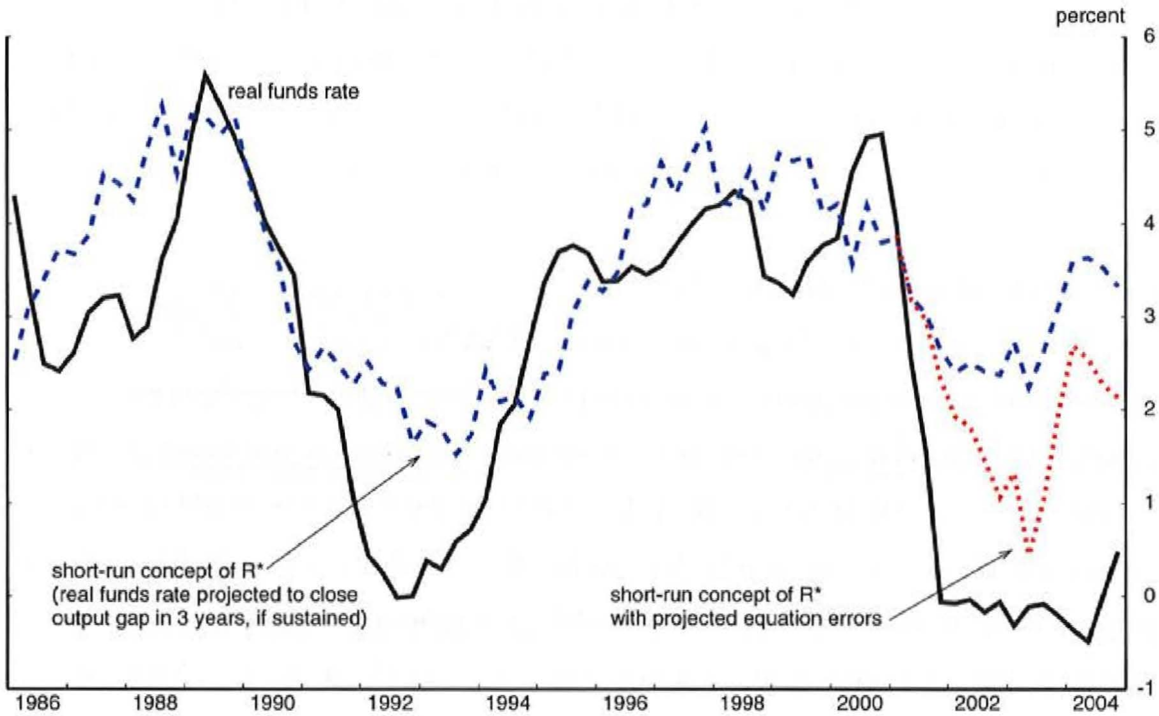
[6] In a standard FRB/US forecast, however, the automated projections of some exogenous variables, especially in the government and foreign sectors, are replaced with specific judgmental forecasts.

[7] There are two primary differences between the approaches. The FRB/US estimates of structural multifactor productivity and most of the other trends on which potential output depends are derived from a specific statistical model; the staff's estimates of these trends are chosen judgmentally based on evidence from a range of statistical models. In addition, the FRB/US estimates do not use future information

Figure 5
Estimates of R* Generated Using the FRB/US Model



FRB/US end in 2002, so there is a risk that the model's equations may have drifted off track over the last few quarters. (This risk is less of a problem for the other two models because they are estimated through the last quarter for which data are available.) We address the possibility of such drift by modifying the projected residuals for each equation starting in 2001, setting them equal to a trailing average of the equation's recent errors multiplied by a weighting factor. This factor gradually rises from zero at the end of 2000 to 1 by the end of 2003, and it is held at 1 thereafter.[8] This modification tends to pull R^*_{SR} down in recent years, as shown by the dotted red line in the figure. This is the FRB/US estimate we prefer for the most recent period.

In principle, movements in the FRB/US estimates of the equilibrium real rate can be linked to changes in fundamental determinants, as was done with estimates

whereas the staff's estimates do — statistically speaking, the FRB/US estimates are one-sided, but the staff estimates are two-sided.

derived from the small economic model. From a practical standpoint, however, such a decomposition is much more complicated in FRB/US because the model contains a very large number of factors that can shift R*. As a result, the regression methods used to decompose R* in the small economic model cannot be employed with FRB/US. Rather, simulation techniques must be used, and these are currently under development.

Greenbook-consistent estimates of R*

Archived datasets allow us to create FRB/US baselines that match all historical Greenbook projections made over the past seven years, using the specific version of the model in place at the time. With these baselines and real-time versions of the model, we are able to estimate the value of the real funds rate that would have been projected to close the output gap twelve quarters from the date at which each Greenbook projection was made. Of course, this methodology is subject to the criticism that the empirical characterization of the monetary transmission mechanism embedded in past versions of FRB/US may not have accurately reflected the staff's thinking at the time. However, because FRB/US is routinely used as an input into the Greenbook projection to help calibrate how the forecast should be revised to take account of changes in the funds rate and other factors, we believe that the model provides a reasonable approximation to the staff's views on this subject.

That said, this approach to computing Greenbook-consistent estimates of R* has its limitations. Usable real-time versions of FRB/US with matching Greenbook baselines are currently available only since 1997. Further, the fact that the Greenbook projection extends only six to ten quarters ahead precludes the computation of a medium-run concept of R*. And even to compute the short-run measure, adaptations have to be made to adjust for the fact that the Greenbook projection does not extend to the end of the required twelve-quarter interval. For Greenbook forecasts published since the middle of 2002, we fill out the twelve-quarter span with an extended version of the Greenbook projection that was prepared at the time. Prior to that, a real-time extended projection is

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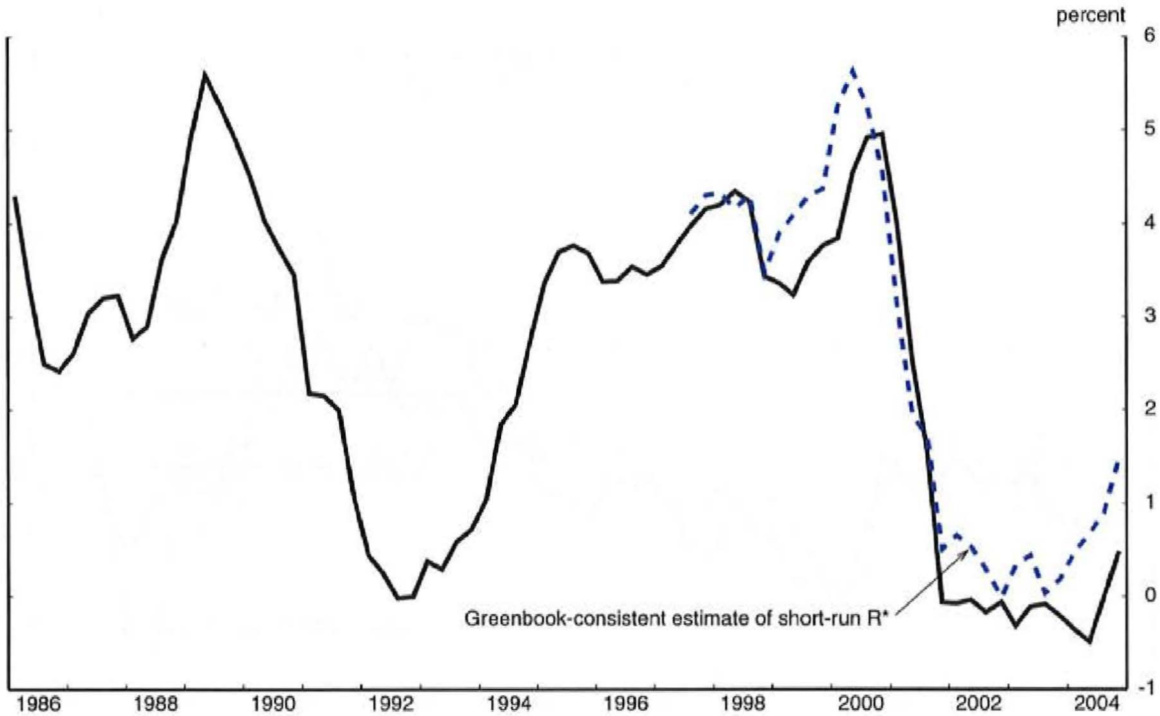
[8] Once the weighting factor reaches 1, this extrapolation method is identical to the automated rule used to project many equation residuals in the FRB/US forecast prepared before each FOMC meeting.

Figure 6
Real-Time Greenbook-Consistent Estimates of the Short-Run Concept of R*

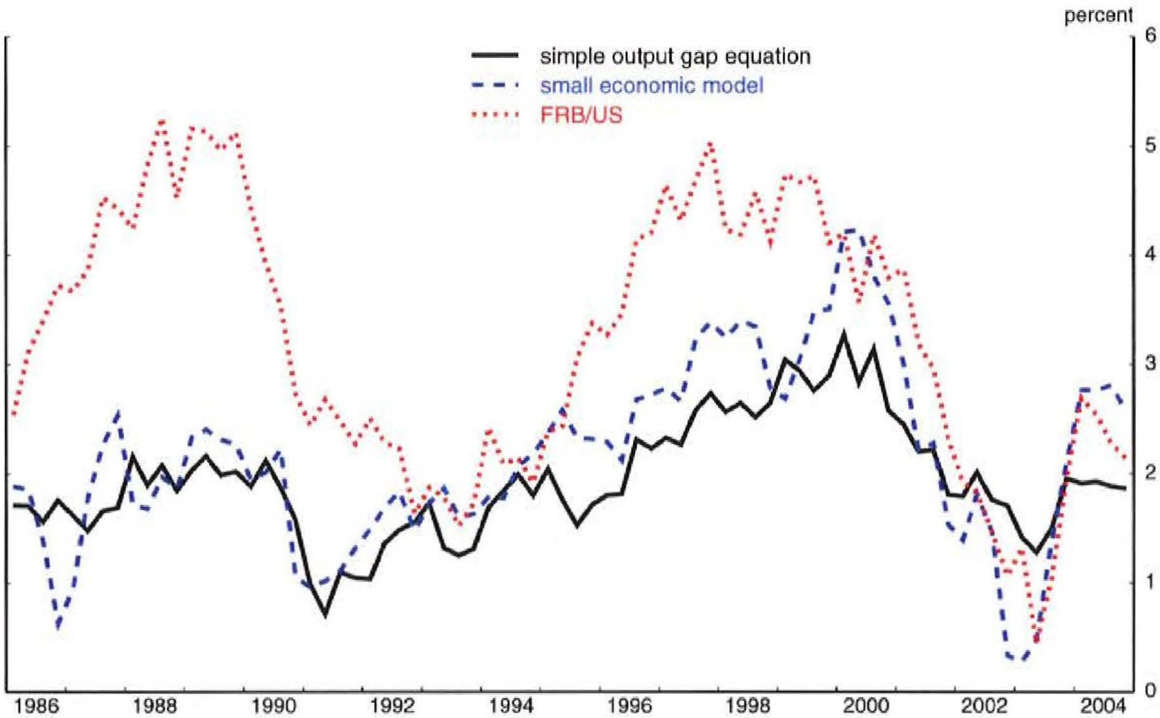


not available, and the analysis is modified to so that R^*_{SR} closes a specified fraction of the initial output gap by the end of the Greenbook forecast period. (The degree of closure is based on the specific length of each Greenbook forecast and the time profile in FRB/US of the effect of a change in the real funds rate on the output gap.) Results from this procedure are shown in figure 6.

Estimating uncertainty about R*

Our analysis focuses on three key, quantifiable sources of uncertainty associated with the various estimates of the equilibrium real rate. One is the availability of more than one plausible model with which to estimate R*. A second source is uncertainty about the true coefficients of any model. The third source is imprecision in our estimates of potential GDP and the output gap. Broadly speaking, these three sources concern uncertainty about the current state of the economy. We also describe a fourth source of uncertainty, one that is associated with the range of shocks that might hit the

Figure 7
Alternative Model-Based Estimates of the Short-Run Concept of R*



economy in the future. As discussed in the overview memo, this last source of uncertainty is excluded from calculations of confidence intervals for the estimates of R* reported in the revised Bluebook exhibit.

Our discussion of uncertainty starts with the short-run concept of R*. One element of such uncertainty is the fact that the three models we employ deliver different point estimates, as shown in figure 7. The series from the simple equation (solid black line) and the small model (dashed blue line) tend to move together fairly closely; the average absolute difference between the two is 45 basis points since 1986. The FRB/US series tends to be more variable than the other two. The width of the range spanned by the three models averages 148 basis points. At present, the three models place the value of R^*_{SR} between about 1-3/4 percent and 2-1/2 percent, up from the range of 1/4 percent to 1-1/2 percent that prevailed from late 2002 through the middle of 2003.

Table 3 reports the degree to which uncertainty about coefficients, potential output, and future shocks contributes to uncertainty about R^*_{SR}; results are reported as standard errors. The estimates of the standard error of R^*_{SR} associated with coefficient uncertainty are bunched around 3/4 percentage point. Those corresponding to the expected range of future economic shocks are substantially larger and lie between 1.7 and 2.5 percentage points. These statistics are calculated using Monte Carlo methods. In particular, the forecasts that measure R^*_{SR} are repeated many times for each model, first, with random perturbations to coefficients based on their estimated distribution and, second, with random values of equation residuals based on historical equation errors (a proxy for future shocks) also included.[9] Standard errors are measured as half the width of the central 70 percent of outcomes for R^*_{SR}. The reported standard errors associated with future shocks by themselves are calculated under the assumption that the effects of coefficient perturbations and future shocks are uncorrelated.

Table 3
Uncertainty about R^*_{SR}
(standard errors, percentage points)

	Simple Equation	Small Model	FRB/US
Sources of uncertainty			
Coefficients	.81	.73	.73
potential output	1.19	1.03	1.11
future shocks	2.04	2.46	1.66
Model-specific aggregates			
current conditions[a]	1.44	1.26	1.33
all sources	2.50	2.77	2.13
Across-model aggregates			
current conditions[a]	1.67		
all sources	2.72		

a. Aggregate of the coefficient and potential output effects.

[9] For each new coefficient draw, the historical equation residuals are recomputed, thereby ensuring that the random sampling of coefficients and equation errors are mutually consistent.

Coefficient distributions are not readily available for the FRB/US model because of the complexity of the procedures used to estimate its structural equations. However, we can compute the effects of coefficient uncertainty on R^*_{SR} for a related large-scale model of the economy that we have constructed; compared to FRB/US, this model has a structure that is more reduced-form in nature and thus its coefficient distributions are directly available. We approximate the effects of coefficient uncertainty in FRB/US with results from this simpler model.

Estimates of the standard error of R^*_{SR} associated with uncertainty about the level of potential output range from 1.0 percentage points to 1.2 percentage points. Estimates of this type of uncertainty have two components. One is associated with the degree to which the average measured level of potential output might be higher or lower than the true level, as evaluated with the benefit of hindsight. This component introduces uncertainty about R* to the extent that its point estimate depends on the average value of the output gap.[10] The other component depends on the (even larger) degree to which the real-time estimate of potential at any date might differ from the true level. It introduces uncertainty about R* to the extent that its point estimate depends on the initial value of the output gap.

Based on this discussion, the calculation of the effect of mismeasurement of the output gap on uncertainty about R^*_{SR} relies on the following formula:

$$R^*_{SR} = \alpha\ gap_{ave} + \beta\ gap_{rt} + (\text{other factors unrelated to the output gap})\,.$$

In the formula, α and β are model-specific coefficients, gap_{ave} is the sample average of the output gap, and gap_{rt} is the real-time estimate of the current output gap. The "other factors" consist of a constant in the simple output gap equation and a constant along with such variables as the budget surplus ratio in the small economic model. In the simple

[10] Consider a very simple output gap equation, $X = \alpha - \beta R$, where X is the output gap, R the real rate of interest, and α and β are coefficients. In this case, the value of R* (at any horizon) is α/β, and the point estimate of α equals the average value of X plus the point estimate of β times the average value of R. Thus, uncertainty about the average value of X leads to uncertainty about α (in addition to that related to conventional coefficient uncertainty, which takes X as given), and this in turn contributes to uncertainty about R*.

output gap equation, regressions and simulations show that α is .730 and β is .296; in the small economic model, α is .669 and β is .209.[11]

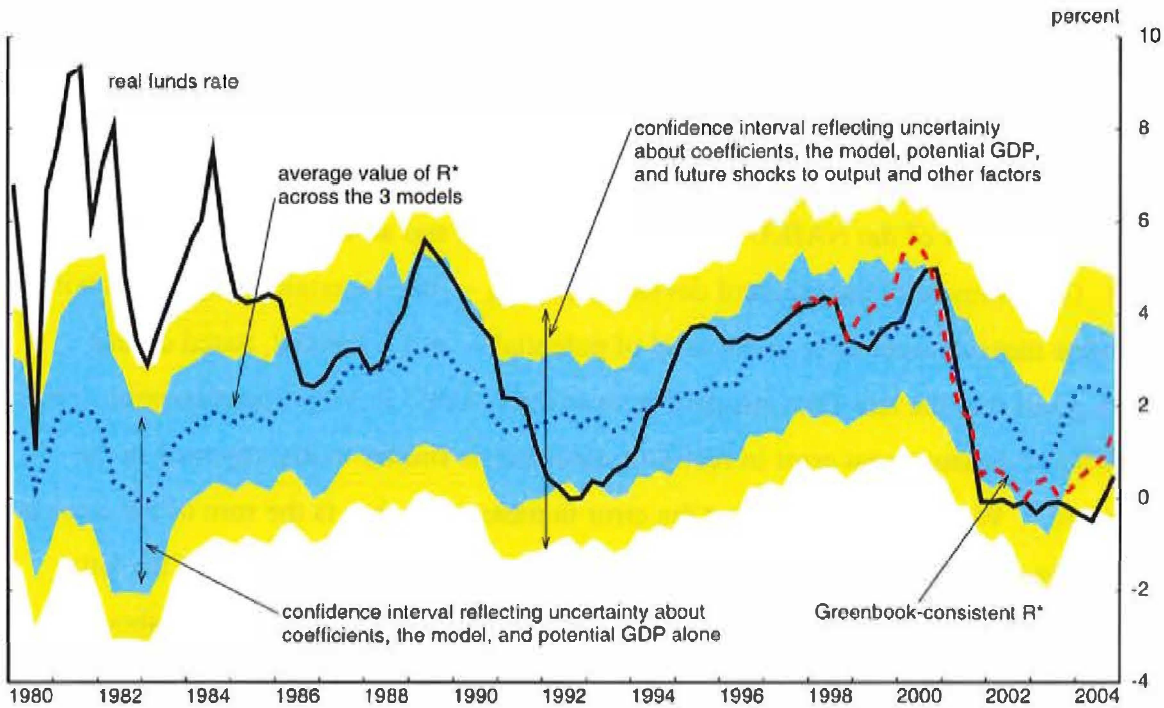
To apply the formula, we assume that the standard error of gap_{ave} — the uncertainty associated with the persistent component of measurement error in the level of potential output — is 1.1 percent. This value is the product of conventional estimates of the standard error of the NAIRU (0.5 percentage point) and an Okun's Law coefficient (2.2). Our estimate of the standard deviation of gap_{rt} — the uncertainty associated with real-time measurement error in the level of potential — is 1.7 percent, based on our rough guess that the real-time standard error of the NAIRU is 3/4 percentage point. This estimate of measurement error in real-time estimates is similar to that reported in the literature.[12] We further assume that the error in measuring gap_{rt} is the sum of the error in measuring gap_{ave} and a second uncorrelated error. The standard deviation of the latter must be 1.3 percentage points, given our assumptions concerning uncertainty about average and real-time potential output.

We construct several types of aggregate estimates of uncertainty about R^*_{SR}. The middle rows of table 4 present estimates of two model-specific measures. The standard errors of the short-run concept of R* associated with uncertainty about the current state of the economy, which combine the effects of imprecise knowledge about coefficients and the output gap, average 1.34 percentage points. When the effects of future shocks are included (the "all sources" line), the average of the model-specific standard errors rises to nearly 2.47 percentage points.

Aggregation of the model-specific measures across the three models leads to estimates that include the effect of model uncertainty. For these calculations, each of the three models is assumed to be equally likely. For each model and each date, a sample of 10,000 estimates of R^*_{SR} is drawn from a normal distribution whose mean is the model's

[11] The large size and nonlinear structure of FRB/US does not lend itself to the derivation of the type of identity used for the other two models. This and other computational reasons have prevented us from directly computing the effect of uncertainty about potential output on the FRB/US estimate of R^*_{SR}. The value in table 3 for FRB/US is an average of the effects in the other two models.

Figure 8
70-Percent Confidence Intervals for the Short-Run Concept of R*



point estimate of R^*_{SR} and whose standard deviation is one of the entries from table 3. Then, for each date the estimates from each of the models are combined to form a sample of size 30,000. An across-model standard error is half the width of the central 70 percent of these estimates, averaged over all quarters since 1980. As shown in the bottom part of the table, the inclusion of model uncertainty boosts the "current conditions" standard error to 1.67 percentage points and the "all sources" standard error to 2.72 percentage points. Figure 8 displays the two 70 percent confidence ranges associated with these estimates.

Estimates of uncertainty about the medium-run concept of R*, shown in table 4, are based on fewer models and fewer sources than are the estimates associated with the short-run concept of R*. As discussed earlier, R^*_{MR} is not calculated for FRB/US. And

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[12] See Athanasios Orphanides, Richard Porter, David Reifschneider, Robert Tetlow, and Frederico Finan (2000), "Errors in the Measurement of the Output Gap and the Design of Monetary Policy," Journal of Economics and Business, 52 (1/2), 117-141.

effects of future shocks are not included because the definition of R^*_{MR} as the rate of interest that would "hold output at potential thereafter" makes sense only when the effects of such shocks are not taken into account. Thus, the "current conditions" category is the broadest one for which R^*_{MR} standard errors are computed. Aggregated across models, this standard error is 0.77 percentage points, a value that is less than half as large as the corresponding estimate for R^*_{SR}. Much of the difference occurs because the medium-run standard error includes only the first of the two components associated with uncertainty about potential output that enter the short-run standard error.

Table 4
Uncertainty about R^*_{MR}
(standard deviations, percentage points)

	Simple Equation	Small Model
Sources of uncertainty		
Coefficients	.75	.19
potential output	.72	.45
Model-specific aggregates		
current conditions	1.04	.49
Across-model aggregates		
current conditions	.77	