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## BOARD OF GOVERNORS OF THE FEDERAL RESERVE SYSTEM

DIVISION OF RESEARCH AND STATISTICS

**Date:** April 20, 2009

**To:** Federal Open Market Committee

**From:** Eileen Mauskopf and Jae Sim

**Subject:** Optimal Paths for Large-scale Asset Purchases

### Introduction

This memo discusses a number of issues related to the development of “optimal” paths for large-scale asset purchases (LSAP) to achieve the Committee’s objectives for output and inflation. Our approach to this problem is similar to that used to generate the optimal policy paths for the federal funds rate regularly presented in the Bluebook. That is, we ask what path of asset purchases would minimize a policymaker loss function, conditional on the dynamics of the FRB/US model and the assumptions underlying the staff baseline outlook. Although any solution to this general planning problem will be sensitive to many specification issues, two issues are front and center in determining the optimal strategy and quantifying the net benefit from an LSAP program. First, what costs does the Committee associate with an LSAP program? Second, how responsive are long-term interest rates to purchases of long-term securities by the Federal Reserve? This memo begins by exploring these two specification issues and then presents some illustrative simulation results.

### Specifying the Loss Function

In the optimal policy simulations that are regularly featured in the Bluebook, we assume that monetary policymakers want to keep inflation ( $\pi$ ) close to a specified goal ( $\pi^*$ ), keep the unemployment rate ( $U$ ) close to the NAIRU ( $U^*$ ), and avoid quarter-to-quarter changes in the federal funds rate (RFF). Consistent with these assumptions, the Bluebook exercises use an optimal-control routine to find the path of the federal funds rate that minimizes the loss function

$$(1) \quad L = \sum_{i=0}^m \beta^i \left[ \left( \pi_{t+i} - \pi^* \right)^2 + \left( U_{t+i} - U^* \right)^2 + \left( \Delta RFF_{t+i} \right)^2 \right],$$

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subject to the assumptions embedded in the baseline staff projection and the dynamics of the FRB/US model.<sup>1</sup> In this memo, we follow the same basic approach, solving for paths for both the nominal funds rate and asset purchases that jointly minimize a loss function that includes the losses associated with the conduct of an asset purchase program.

Among the costs that we assume that the Committee will consider are those related to the volume of assets being brought onto the Federal Reserve's balance sheet. For example, holding a large portfolio of long-term securities exposes the Federal Reserve (and thus taxpayers) to appreciable capital losses if interest rates rise quickly as the economy recovers.<sup>2</sup> In addition, the larger the accumulation of assets now, the larger will be the eventual disposition of assets; without the ability to issue Fed bills, a large sale of securities—if it proved necessary—might be disruptive to financial markets.<sup>3</sup> Other concerns are less operational and more political in nature. Holding a large volume of agency debt and MBS may create the appearance that the Federal Reserve has tilted the allocation of credit towards housing. Moreover, the Federal Reserve could face pressure to dispose of these assets more slowly than warranted by macroeconomic considerations. And holding so much long-term Treasury debt and agency securities could even lead the public to question the independence of the Federal Reserve or conclude that the federal debt is being monetized.

Aside from being concerned about the volume of long-term Treasury and agency securities held on the balance sheet, policymakers may also be concerned about the volume of Desk purchases or sales of these securities during any particular quarter. If the volume of such transactions is too large, market functioning could be impaired with potentially adverse consequences for the economy.

To address these potential costs, we add two terms to the loss function:

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<sup>1</sup> In the loss function,  $m$  defines the planning horizon over which the optimal path is calculated (typically 48 quarters into the future).  $\beta$  is a discount factor equal to 0.98, implying that near-term economic conditions are weighted more heavily in planning than ones further removed in time. Finally, to ensure that the nominal federal funds rate does not fall below zero, the Bluebook loss function also includes a nonlinear term (not shown) that penalizes further downward movements in the federal funds rate once it falls to a level just above zero.

<sup>2</sup> A rough estimate of capital loss can be computed as follows: Assume that the interest rate climbs 300 basis points between the purchase and sale of a security and that the remaining term-to-maturity on the security being sold is about 6 years. This gives roughly a 13 percent capital loss on that security. We note that it is not necessarily clear that the Committee would want to penalize capital losses; after all, the goal of this program is to reduce interest rates now to restore output to potential. To the extent the program is successful, long-term rates will rise over time.

<sup>3</sup> As pointed out by Seth Carpenter, the disposition of securities held by the Fed will occur through both active sales of securities and as securities mature or—in the case of MBS—as mortgages are pre-paid. The cost associated with the disposition of securities is likely considerably smaller in the latter instances.

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$$(2) \quad L = \sum_{i=0}^m \beta^i \left[ \left( \pi_{t+i} - \pi^* \right)^2 + \left( U_{t+i} - U^* \right)^2 + \left( \Delta RFF_{t+i} \right)^2 + \theta_1 (A)^2 + \theta_2 (\Delta A)^2 \right]$$

In this expression,  $A$  is the total volume of long-term Treasury securities, agency debt, and agency MBS held by the Federal Reserve, scaled by the outstanding stock of these long-term securities;  $\Delta A$  is the quarterly purchase/sale of these securities by the Federal Reserve, scaled by the quarterly gross issuance of long-term Treasuries and MBS. The quadratic form of the additional terms indicates that doubling either the volume of long-term assets on the Federal Reserve's balance sheet or the amount of purchases/sales in any quarter—holding the scale variables constant—quadruples the implicit cost of the LSAP program to policymakers. Parameters  $\theta_1$  and  $\theta_2$  determine the relative weight placed by policymakers on deviations of these terms from zero compared to their desire to keep inflation near its target, to keep unemployment near the NAIRU, and to avoid changes in the federal funds rate.

Because we do not know how FOMC participants might weigh the costs of engaging in an LSAP program against the costs of not meeting their macroeconomic goals, we face the practical difficulty of assigning the appropriate weights to  $\theta_1$  and  $\theta_2$  in equation 2. Accordingly, we allow for different settings of these weights in our simulations.

We note that the optimal-control analysis is almost certainly more complicated than we have allowed for. For example, the Desk has indicated a concern that dealers on the other side of the transaction not be surprised by the Federal Reserve's actions. One possible way to handle this concern would be to assign a cost to quarterly variations in the volume of Fed transactions, and thus the addition of a third term to the loss function. In addition, the costs associated with purchases of agency MBS may differ from those associated with buying long-term Treasury securities or agency debt. If these differences are sufficiently important, then the optimal-control procedure might be better framed in terms of jointly finding three optimal paths—one for the federal funds rate, one for holding of Treasury securities and agency debt, and one for holdings of agency MBS. We ignore these additional complications in this note, in part because the simulation results presented below are already computationally difficult to produce, and in part because taking account of them would probably be asking too much of this model and our knowledge of the quantitative distinction between the effects of the different asset purchases.

## **Specifying the Link between Large-Scale Asset Purchases and Interest Rates**

The rationale for the LSAP program rests on the assumption that, by reducing the supply of longer-term securities available to private investors, Federal Reserve purchases of such securities can lower long-term interest rates, including mortgage rates. This assumption follows from the “preferred-habitat” theory of the term structure, which maintains that investors hold distinct preferences about investment horizons and thus require a premium to buy a security whose remaining maturity is outside the preferred horizon. As a result, yield differentials across securities of different maturities reflect both the expected path of future interest rates as well as the relative supply of these securities. For a given state of preferences, a decrease in the relative supply of a security of a particular maturity lowers its yield.

The standard FRB/US equations for long-term interest rates do not reflect this notion of investor preference for particular investment horizons. Instead, the risk premiums on long-term instruments depend only on expected future output gaps and not on the relative supply of securities of different maturities. Thus, we need to modify the FRB/US equations to include this influence on yields. To do this, we assume that the influence of the maturity distribution on yields is captured through the net supply of long-term debt by the federal government to the private sector (the total issuance of long-term Treasuries less the holdings of long-term Treasuries, agency debt, and agency MBS by the federal government including the Federal Reserve). More specifically, this measure of net supply, scaled by Treasury bills and bonds across the maturity spectrum, is added to the FRB/US equations for long-term yields.<sup>4</sup>

Considerable uncertainty attends the nature of the link between the net supply of these assets and long-term interest rates. Accordingly, we consider two alternative representations of the way variations in our measure of net supply affect yields. In the first, we view the effect on long-term interest rates as deriving from the communication of the Federal Reserve regarding the size of its intended holdings of long-term securities. Thus investors respond not to the current holdings but to what they expect the holdings of the Federal Reserve will be sometime in the near future. In our baseline calibration of the magnitude of this effect, we assume that a reduction of 10 percentage points in the net supply of long-term debt relative to total government debt (without regard to maturity)—say, from 60 percent to 50 percent—lowers the yield on Treasury securities, agency debt, and agency MBS 40 basis points, all else equal. This calibration is in the range of results reported in past research and is consistent with the response of long-term interest rates to recent Federal Reserve communications about large-scale asset purchases.<sup>5</sup> Under this

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<sup>4</sup> Because the Federal Reserve essentially has balanced assets and liabilities, this measure is equivalent to the supply of all federal government liabilities to the private sector net of federal government claims on the private sector.

<sup>5</sup> See the memo by Joe Gagnon, “A Preliminary Assessment of the Effects of the Federal Reserve’s Large-Scale Asset Purchases on Interest Rates”. As discussed by Gagnon, this assumption implies smaller interest

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specification, a \$1.75 trillion expansion in Federal Reserve holdings of long-term assets (the volume of LSAPs authorized to date) reduces long-term interest rates by roughly 100 basis points.

In the alternative specification, we assume that long-term interest rates are influenced not by the stock of long-term Treasury and agency securities held by the Federal Reserve, but by the quarterly flow of Federal Reserve purchases or sales of such assets relative to total government debt without regard to maturity. Under this specification, long-term rates are reduced only as long as the Desk continues to increase the System's holdings; when purchases stop and the holdings of the Federal Reserve stabilize at a constant level, interest rates return to normal. Based on recent experience, we assume that purchasing \$600 billion in long-term assets in a quarter reduces long-term rates in that quarter 100 basis points; symmetrically, selling \$600 billion in a quarter would increase long-term rates by the same amount.

Note that this approach assumes that private investors view Treasuries and agency MBS of a given maturity as perfect substitutes. As a result, the market in which the Federal Reserve conducts its operations does not matter—a billion-dollar purchase of either long-term Treasury securities or agency MBS is assumed to reduce yields on both securities by the same amount. This assumption differs from that used in previous analyses presented to the Committee, in which purchases of a given security had a larger effect on own yields than on the yields of other securities.<sup>6</sup> We also assume that yields on corporate bonds and conventional fixed-rate mortgages move one for one with yields on 10-year Treasury securities, excluding endogenous changes in risk premiums related to the expected state of the economy.

### **Linking the Changes in Interest Rates to Economic Activity: A Word of Caution**

Given the changes in yields that are assumed to follow from the large scale asset purchases, the effects on the economy follow the usual transmission mechanisms in FRB/US. It is worth noting, however, that FRB/US does not include a model of the banking sector, so these projections implicitly assume that the large scale asset purchases have no direct effect on banks' lending behavior. In practice, because the purchases would cause a very significant increase in the reserves holdings of banks relative to total bank assets, it is possible that they would cause some banks' leverage ratios to bind tightly, leading the banks to pull back on securities purchases and new lending. In that case, the effects of the asset purchases on the economy would be smaller than assumed

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rate effects than assumed in recent staff analyses presented to the FOMC, in part because yield spreads have since returned to more normal levels.

<sup>6</sup> The earlier specification, which allowed for greater "own-yield" effects (particularly for MBS), largely reflected heightened strains in conventional mortgage markets. These strains have substantially abated in recent months, in part because of the Federal Reserve's intervention in this market.

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here. This effect could be reduced if the Federal Reserve had the authority to issue Fed bills (or the Treasury issued more bills under the Supplementary Financing Program and deposited the funds with the Federal Reserve) and so offset the effects of the asset purchases on reserves. Alternatively, the regulations regarding the calculation of the leverage ratio could be changed to exclude holdings of reserves.<sup>7</sup>

### **Simulation Results when the Expected Size of Federal Reserve Purchases Affects Yields.<sup>8</sup>**

Figures 1, 2, and 3 illustrate the optimal size of Federal Reserve holdings of long-term securities when long-term interest rates decline in anticipation of a buildup of Federal Reserve holdings. In these figures, we vary the assumptions about the costs associated with the volume of Federal Reserve holdings of long-term securities (figure 1) and the costs associated with purchases and sales of these securities (figure 2). We also show the extent to which optimal Federal Reserve holdings depend on the sensitivity of long-term yields to the relative maturity structure of securities owned by the private sector (figure 3).

Figure 1 shows the optimal path of LSAPs under three alternative parameterizations of  $\theta_1$ , the weight policymakers place on the ratio of the Federal Reserve's holdings of long-term assets relative to the total supply of such long-term assets. These alternative weights represent different answers to the following question: From the Committee's point of view, *what size deviation in the unemployment rate from the NAIRU would generate a cost equivalent to the cost incurred were the Federal Reserve to own 100 percent of long-term Treasury and agency debt and agency MBS?* As noted above, this question could alternatively be framed in terms of the comparative loss with respect to the deviation of inflation from its desired rate or with respect to changes in the federal funds rate.<sup>9</sup>

The black solid line in figure 1 shows the baseline forecast as of the March Greenbook (which was completed prior to the FOMC announcement of an additional \$1.1 trillion in LSAPs by year-end). The green solid line, showing the largest volume of Federal Reserve holdings, represents the case in which the Committee equates the cost of cornering the long-term securities market to that associated with a deviation of the unemployment rate from the NAIRU of 1¾ percentage point. Under this calibration, the

<sup>7</sup> It is possible that even if the leverage ratio calculation were changed, banks would still limit the growth in their assets in response to pressures from creditors and counterparties to strengthen their balance sheets.

<sup>8</sup> In all simulations shown below, we implicitly assume that no securities reach maturity prior to the desirable end of the LSAP program; that is, all disposition of securities occurs through sales of such securities by the Fed.

<sup>9</sup> Because the loss function in LSAPs is quadratic, were the Fed to hold 50 percent of the debt outstanding, the associated cost would be ¼ as large as the cost of holding 100 percent of the debt.

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Fed would acquire nearly \$6 trillion (out of about \$10 trillion outstanding) in assets by late this year. By contrast, the green dashed line shows the optimal holdings by the Fed when the Committee judges the costs associated with cornering the market to be much more severe. In this case, the costs are equivalent to a deviation of the unemployment rate from the NAIRU of 7½ percentage points; Federal Reserve holdings peak at about \$2 trillion, close to the Committee's current commitment to purchase \$1.7 trillion in assets by the end of this year. The third case (the black dashed line) is an intermediate one in which the comparative cost is assumed to be that associated with a deviation of the unemployment rate from the NAIRU of 4 percentage points; under this assumption, Federal Reserve holdings top out at \$4 trillion.<sup>10</sup>

Under all these parameterizations of the cost function, mortgage rates and corporate bond yields fall substantially relative to the baseline. Federal Reserve holdings of assets return to baseline by the end of 2013 and the federal funds rate pulls away from the zero bound considerably sooner than in the baseline case (in the earliest instance, at the start of 2013; at the latest, the end of 2014).

In all the simulations shown so far, optimal policy results in a rapid run-up in Federal Reserve holdings. Because the cost of purchasing securities likely increases in a nonlinear manner with the size of the purchase, the Committee may wish to avoid large purchases in any one quarter. In addition, the scale of purchases during a quarter—particularly of agency MBS—could potentially be greater than the new issuance of the respective securities.<sup>11</sup> Under these circumstances, the Federal Reserve would end up buying MBS of some earlier vintage while shutting out other investors from the market for new issuance.<sup>12</sup> Such an outcome would not be optimal if the Committee not only cared about the overall size of its holdings of long-term securities, but also wished to avoid crowding out private ownership of new (“on-the-run”) securities.

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<sup>10</sup> In all three cases, we need to assume that, in addition to the penalty placed on the stock of holdings, the costs include a non-trivial penalty associated with Federal Reserve purchases or sales of securities. This is required by the assumption that interest rates react, not to current Federal Reserve holdings, but to expected future holdings. For example, suppose yields respond to expected Federal Reserve holdings two quarters in the future. Then to effect a change in long-term rates in 2009:Q2, the Federal Reserve need only announce that it intends to acquire assets in 2009:Q4. Absent a (quadratic) cost associated with purchases, the optimal path would then have the Fed hold nothing until 2009:Q4 (because it is costly to hold assets)—so that it would sell off the \$600 billion accumulation of assets over 2009:Q2 and Q3 that is part of the March baseline and then repurchase those and more in 2009:Q4. To avoid the jagged path associated with the “startup” of the program, we put a small weight on  $\theta_2$ .

<sup>11</sup> This issue is less relevant to the purchase of Treasury securities, where the Desk routinely buys off-the-run securities.

<sup>12</sup> The new issuance of long-term Treasury securities and MBS are exogenous in these simulations and are based on some simple assumptions. The Treasury series is based on a crude reading of the historical relationship between Treasury issuance of long-term securities to gross issuance of securities. The MBS projection was based on the expected path of refis and home sales in the staff baseline. An obvious shortcoming is that neither refis nor home sales were endogenous in these simulations.

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Figure 2 compares results for three different parameterizations of  $\theta_2$ , while  $\theta_1$  is fixed at its intermediate value in figure 1 (in which the penalty for the Fed's ownership of all long-term securities is equated to a deviation in the unemployment rate from the NAIRU of 4 percentage points). The black dashed line in figure 2 reproduces the black dashed line in figure 1. In both cases,  $\theta_2$  is 1, so that the cost associated with the Federal Reserve cornering the new issuance market is equated to the cost arising from a deviation in the unemployment rate from the NAIRU of 1 percentage point. At this penalty setting for  $\theta_2$ , Federal Reserve asset purchases exceed gross issuance in at least one quarter. Raising  $\theta_2$  to a value equivalent to an unemployment rate gap of  $2\frac{1}{4}$  percentage points (the green dashed line) leads to a considerably more restrained path in building up the balance sheet, along which the Fed never purchases more than the gross issuance in that quarter. Lowering this penalty, so that it is equivalent to an unemployment rate gap of 0.1 percentage point (the green solid line), has the Federal Reserve swamping the market for new issues in several quarters.

Figure 3 presents the LSAP path under different assumptions about the sensitivity of interest rates to changes in the maturity distribution of debt engineered by Federal Reserve asset purchases. Parameter settings for the loss function are those used to generate the black dashed lines in figures 1 and 2. The black dashed line in figure 3 uses the baseline calibration for the response of the long-term yield to changes in the maturity distribution of securities. The green dashed line is associated with a response of yields to changes in the maturity distribution that is twice as large as that associated with the black dashed line, while the green solid line is associated with a response that is half as large. The interesting point in this graph is that, in both of these instances, the optimal LSAP holdings never get as large as they do for the intermediate case. When interest rates are not as responsive to changes in the maturity distribution, the optimal response is to use such purchases less intensively given the cost of holding and purchasing assets. But, when interest rates are more responsive, the figure shows that a feasible outcome may also involve less intensive use of this margin—that is, it takes a smaller volume on the balance sheet of the Federal Reserve to effect a given reduction in the unemployment rate because long-term rates are so sensitive to changes in the maturity distribution. In this more-responsive case, the federal funds rate lifts off from the zero bound in 2012, two years before it does in the intermediate case.

### **Simulation Results when Federal Reserve Purchases or Sales Affect Yields.**

Figure 4 illustrates the optimal path for Federal Reserve holdings of long-term assets when it is the quarterly flow of Federal Reserve purchases and sales, and not the stock of assets held, that influences long-term yields. To get a meaningful solution in this case, we must assume that the purchase or sale of assets is costly. Otherwise, with yields responding only to the change in the net supply of long-term securities, the optimal solution would be for the Federal Reserve to immediately buy whatever is necessary to



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push down long-term yields to the level that would eliminate the output gap (respecting of course the zero lower bound on long-term interest rates). In figure 4, we vary the size of the penalty associated with Federal Reserve purchases or sales of securities, but we hold fixed the cost associated with Fed asset holdings and the response of yields to Fed purchases. The former,  $\theta_1$ , is set to the same value used in figures 2 and 3 and represents the intermediate setting in figure 1. The flow calibration, noted previously, implies that each \$1 trillion of long-term securities purchased by the Federal Reserve, holding the stock of federal government debt constant, lowers long-term yields by 166 basis points in that quarter.

In figure 4, the paths taken by long-term rates are, not surprisingly, very choppy. This reflects the assumption that yields respond only to that quarter's purchase or sale of long-term assets. If this is an accurate specification of the transmission channel between Federal Reserve asset purchases and interest-rate differentials, the Committee would potentially want to include in the loss function some penalty on quarterly changes in long-term rates in order to avoid escalating the public's uncertainty about the evolution of long-term interest rates.

## Discussion

These simulation results should be taken only as illustrative examples of the "optimal" paths for asset purchases by the Federal Reserve. Clearly, Committee concerns about asset purchases are much broader than can be expressed by the relatively simple loss function used in these optimal control exercises. We have parameterized as best we can the possible costs that lend themselves to the specification of a loss function in an optimal control analysis. But these parameterizations and functional forms are likely an imperfect representation of the preferences of policymakers. This inadequacy is perhaps greatest with respect to the costs associated with an exit strategy from an LSAP program, something we have considered only briefly in this analysis.

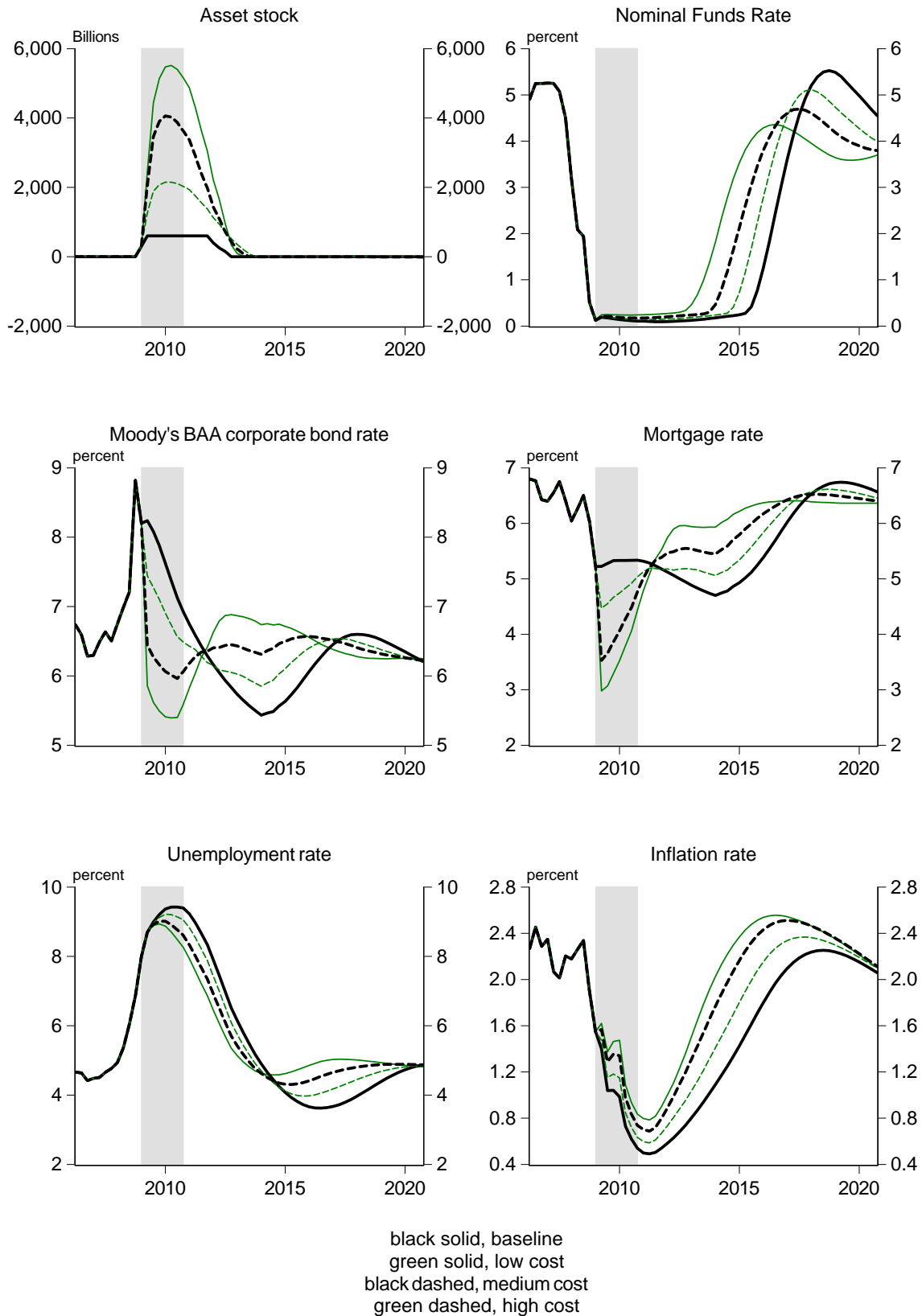
We have used the behavior of long-term yields since the November announcement of the LSAP program to assess the interest-rate responses to changes in the maturity distribution of securities. But, as indicated in the memo by Gagnon, obtaining a reliable reading is complicated by other events that have likely affected movements in yields, such as the weaker than expected economy and the larger than expected path of future deficits. Moreover, we cannot be certain that, going forward, the sensitivity of yields to changes in the maturity distribution will be close to what we have assumed. For example, in this exercise, we modified the equation for long-term interest rates by assuming that yields decline by a constant amount for each percentage-point reduction in the ratio of the net supply by the government of long-term debt to total government debt. If private investors (like pension funds) have strong preferences for long-term government or

agency debt securities, then reducing the supply of these securities may result in a nonlinear response in prices and yields.

So what should be taken away from this analysis? Despite all the uncertainties associated with the operation of LSAPs and despite the numerous simplifications made in this analysis, we draw several conclusions. First, this additional instrument for monetary policy seems likely to help the Federal Reserve better achieve its objectives in the current environment. Second, the desirable path for asset holdings is hump-shaped, rising noticeably over the next year or so, and then declining in subsequent years back to zero. Finally, asset purchases allow monetary policy to be conducted through changes in the federal funds rate sooner than would occur without the LSAP program.

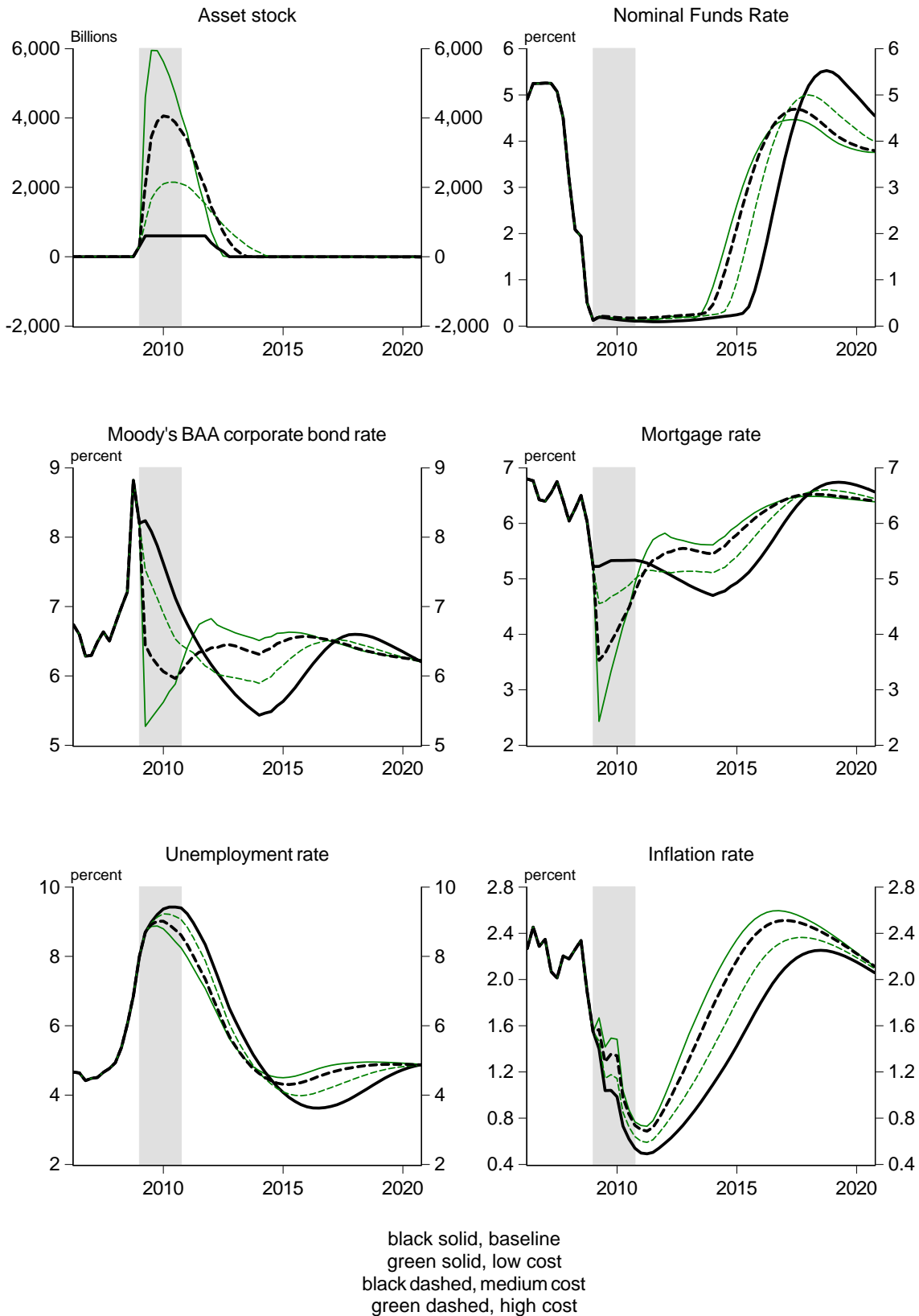
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**Figure 1**  
 LSAP Optimal Policy: alternative costs associated with the volume of assets held by the Federal Reserve (when yields respond to volume of assets held)



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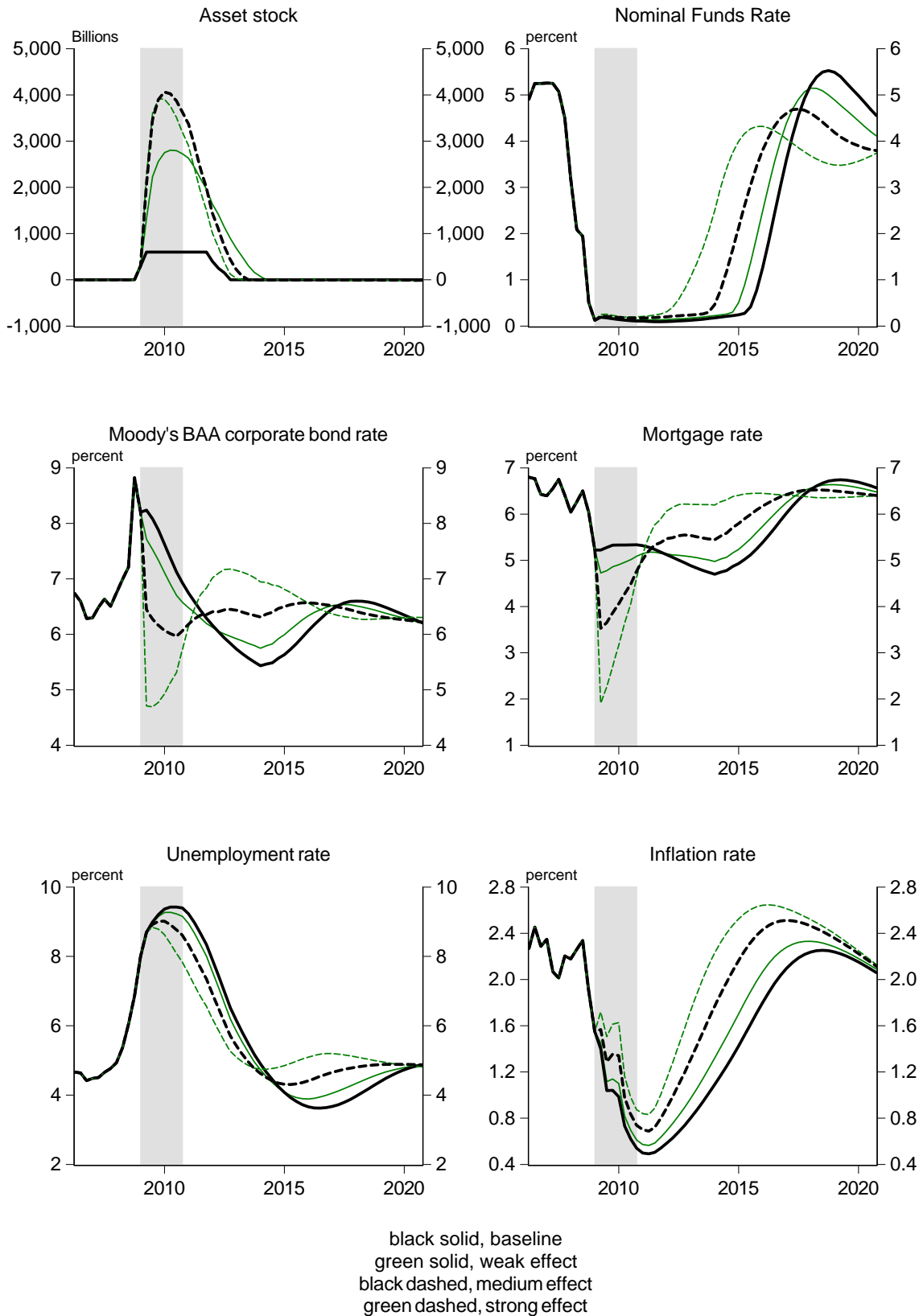
**Figure 2**  
 LSAP Optimal Policy: alternative costs associated with the purchase/sale of assets by the Federal Reserve (when yields respond to volume of assets held)



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Figure 3

LSAP Optimal Policy: alternative effects on long-term yields associated with the volume of assets held by the Federal Reserve (when yields respond to volume of assets held)



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Figure 4  
LSAP Optimal Policy: alternative costs associated with the purchase/sale of assets by the Federal Reserve (when yields respond to sales and purchases of assets)

