Federal Reserve Bank of Chicago

Subject: Summary of Chicago Fed DSGE Model for Federal Reserve System Researchers

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Date: June 8, 2011

Overview

In this memo, we describe the Chicago Fed's estimated dynamic stochastic general equilibrium model. This framework yields a history of identified structural shocks, which we apply to illuminate recent macroeconomic developments. To aid the understanding of these results, we follow them with summaries of the model's structure, the data and methodology employed for estimation, and the estimated model's dynamic properties.

In several respects the Chicago Fed DSGE model resembles many other New Keynesian frameworks. There is a single representative household that owns all firms and provides the economy's labor. Production uses capital, differentiated labor inputs, and differentiated intermediate goods. The prices of all differentiated inputs are "sticky", so standard forward-looking Phillips curves connect wage and price inflation with the marginal rate of substitution between consumption and leisure and marginal cost, respectively. Other frictions include investment adjustment costs and habit-based preferences. Monetary policy follows a Taylor rule with shocks.

The model's decomposition of the price of investment goods with respect to consumption into two components distinguishes it from other similar frameworks. We empirically identify the *technological* component with the investment-consumption relative deflator from the NIPA, and we tie the *financial* component to the High Yield-AAA corporate bond spread.

Another distinguishing feature of the Chicago model is the use of multiple price indices. For this, alternative available indices of inflation are decomposed into a

Table 1. Model Forecasts Q4 over Q4

	2010	2011	2012	2013	2014
Real GDP	2.71	3.22	4.40	4.24	4.17
Federal Funds Rate	0.19	0.11	1.24	2.73	3.44
PCE Core	0.80	1.46	0.68	0.34	0.54
Consumption	1.63	2.52	2.68	2.56	2.68
Investment	10.40	7.63	11.25	11.27	10.34
Marginal Cost	0.93	-1.46	-0.81	0.15	0.56

single model-based measure of consumption goods and services inflation and idiosyncratic (series specific) disturbances that allow for persistent deviations from this common component. Estimation uses a factor model with the common factor derived from the DSGE framework.

Forecasting Methodology

Constructing forecasts based on this model requires us to assign values to its many parameters. We do so using Bayesian methods to update an uninformative prior with data from 1987:Q1 through 2008:Q4. All of our forecasts condition on the parameters equaling their values at the resulting posterior's mode. These parameter values together with the data yield a posterior distribution of the economy's state in the final sample quarter. For the calculation of this initial state's distribution, we add a sequence of *forward guidance shocks* that signal the future path of the Federal Funds rate. These shocks begin arriving in 2009:Q1 and continue to the present. We construct them so that model-based expectations of the policy rate equal actual market-based expectations for the first five quarters of each quarter's forecast horizon. The forecasts begin with 2011:Q3 and extend through 2014:Q4. Our plug for 2011:Q2 GDP growth was set at 2.4 percent.

Figure 1. Quarterly Model Forecasts

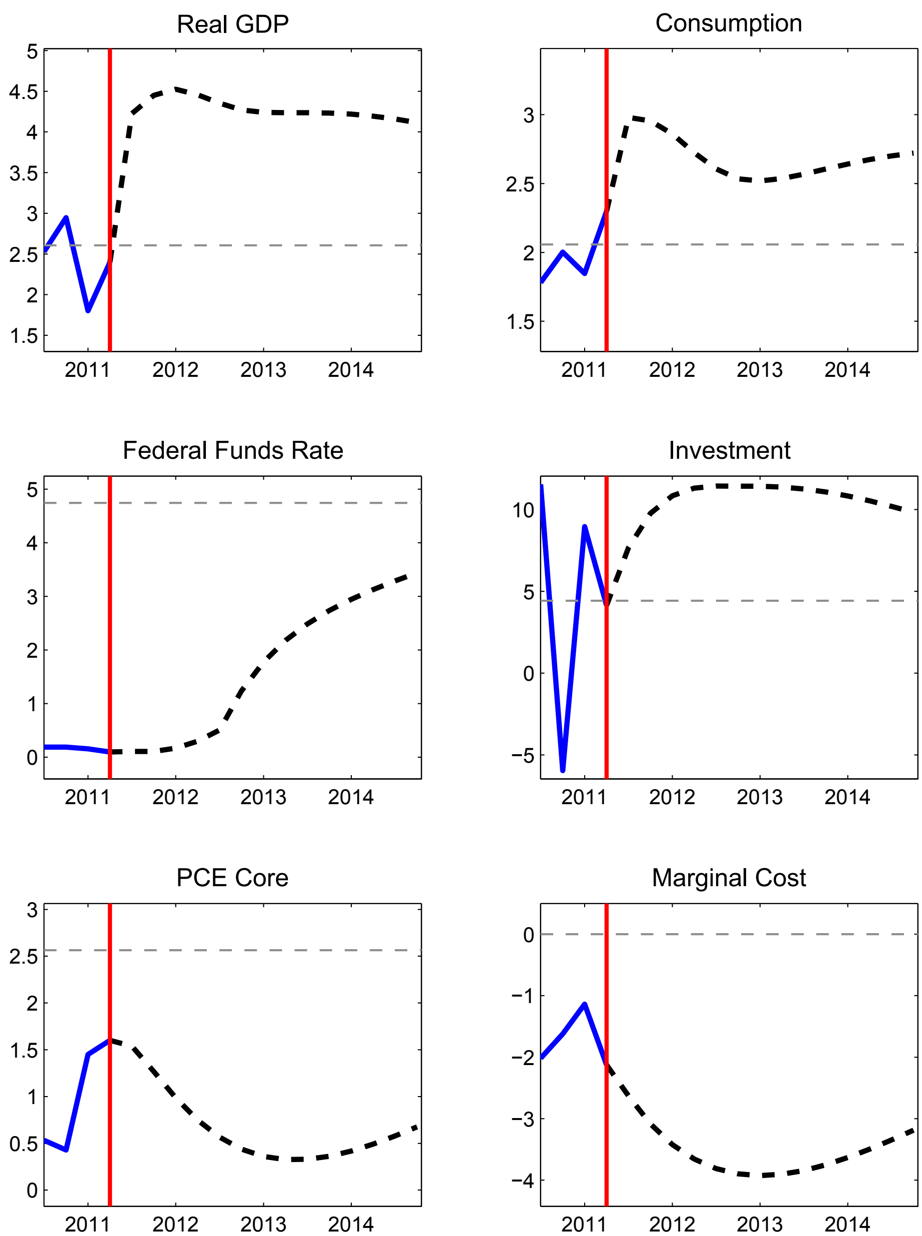
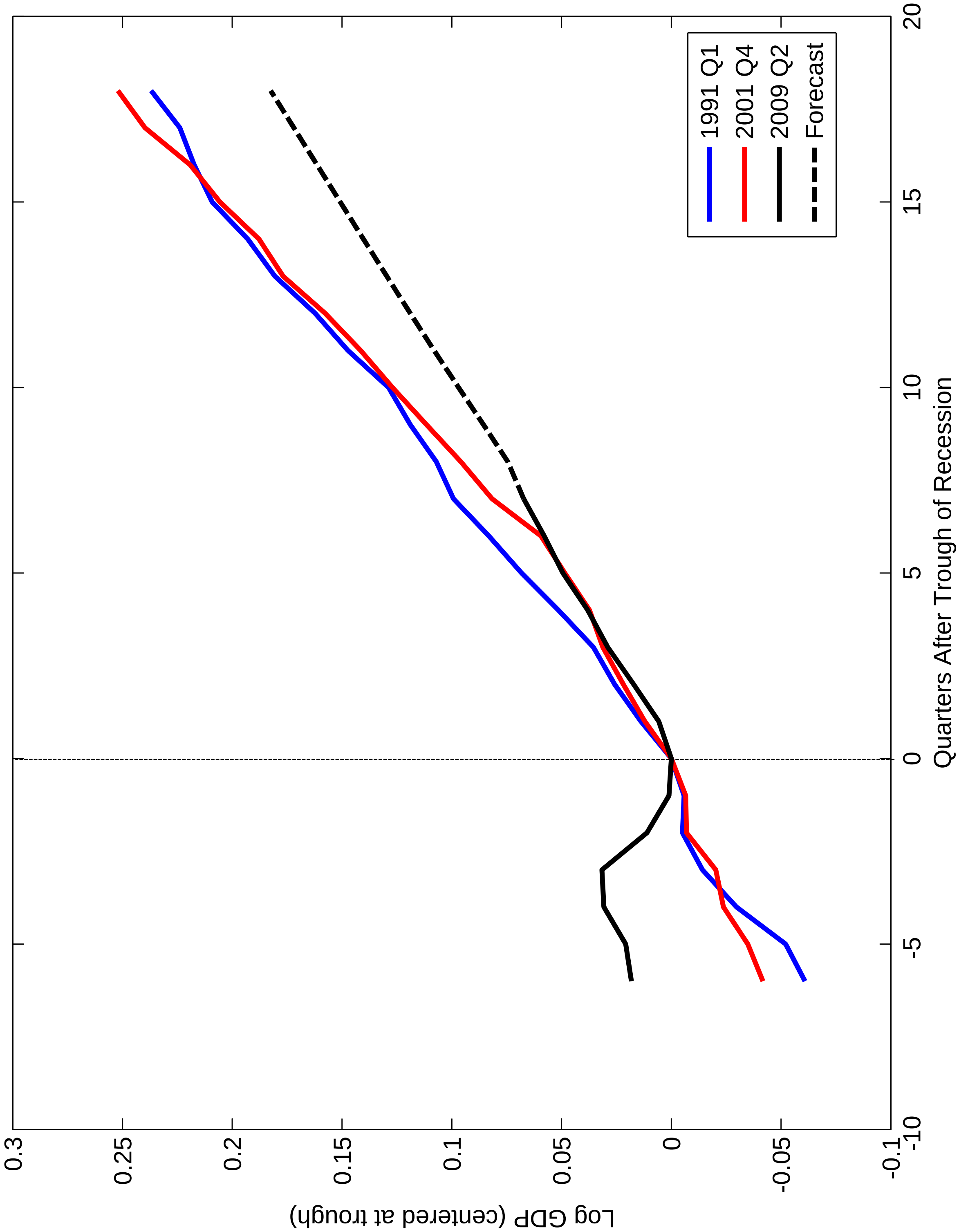


Table 1 presents data from 2010 and forecasts for the following four years. The first three rows correspond to three key macroeconomic observables, Real GDP growth (Q4-over-Q4), the Federal Funds Rate (Q4 average), and growth of the Core PCE deflator (Q4-over-Q4). The following rows report forecasts of Q4-over-Q4 growth for three model-defined aggregates of importance: Consumption of nondurable goods and non housing services, Investment in durable goods, residential housing, and business equipment and structures, and the marginal cost of production. Figure 1 complements this with quarter-by-quarter data and forecasts of these series. The plots' dashed grey lines indicate the series' long-run values.

The economy's long-run GDP growth rate – which we identify with potential growth – equals 2.6 percent. Our forecast for 2011 only exceeds this by 62 basis points, so an unfortunate series of negative shocks could easily send the economy below potential. To place our forecasts from 2012 onward into context, Figure 2 plots the level of GDP for the three recoveries in our sample. All have been normalized so that their values equal zero at the NBER trough, and we extend the current recovery with our forecast, given by the dashed line. It is well-known that the ongoing recovery has been tepid when one considers the depth of the most recent recession. The plot indicates that this comparison will become less favorable over time.

Recall that we hard-wire the current values of monetary-policy news shocks to match current market expectations. Currently, these date the tightening of monetary policy at 2012:Q2. Thereafter, the forecast rate begins to rise as the conventional Taylor rule dynamics take over. Although the forecasted path for core PCE is nearly deflationary from 2012 through 2014, the Taylor rule sees expected output growth as strong enough to merit the removal of the extraordinary accommodation in place since 2008.

Figure 2. Comparison of Recent Recoveries



Shock Decompositions

Our analysis identifies the structural shocks responsible for past fluctuations. To summarize this information, we follow a suggestion of Charlie Evans: Fix an object to be forecast, such as Q4-over-Q4 real GDP growth. Then, pick a date in the past and forecast the object conditional on the information as of that date. This is *not* a real-time forecast, because it uses revised data and model parameters estimated with the complete sample. The model can be used to decompose the forecast error into structural shocks. (A detailed explanation of the forecast error decomposition procedure begins below on page 16) We repeatedly advance the forecast date, decompose the forecast error, and finally plot the results. In total, the model features nine structural shocks and four idiosyncratic disturbances without structural interpretations. For parsimony's sake, we group the shocks according to a simple taxonomy.

Demand These are the structural non-policy shocks that move output and inflation in the same direction. The model features two of them. One changes the households rate of time discount, and the other alters the rate at which consumption goods can be transformed into capital goods. We call these the Time Discount and Marginal Efficiency of Investment shocks.

Supply Five shocks move real GDP and inflation in opposite directions on impact. These *supply shocks* directly change

- Neutral Technology,
- Investment-Specific/Capital-Embodied Technology,
- Markups of Intermediate Goods Producers,
- Markups of Labor Unions, and
- Households' Disutility from Labor

Policy The model's monetary policy follows a Taylor rule with interest-rate smoothing and an *i.i.d.* policy shock. Additionally, we have incorporated

Table 2. The Model's Decomposition of Business-Cycle Variance

	Demand	Supply	Monetary Policy	Residual
Real GDP Growth	0.72	0.21	0.03	0.05
Federal Funds Rate	0.83	0.07	0.07	0.02
Core PCE Inflation	0.10	0.81	0.01	0.08

Note: For each variable, the table lists the fraction of variance at frequencies between 6 and 32 quarters attributable to shocks in the listed categories.

forward guidance shocks since 2008:Q4. These are revealed to the model's agents one to five quarters before they effect the federal funds rate, and they allow our forecasting exercise to match model-based expectations with information from futures markets.

Residual We group other shocks that are usually of small importance into a residual category. These include a shock to the sum of government spending and net exports.

Table 2 reports the fraction of business-cycle variance attributable to shocks in each category for three key variables, the growth rate of Real GDP, the Federal Funds Rate, and Core PCE Inflation. Three facts stand out here. First, demand shocks dominate business cycles. Supply shocks account for only 21 percent of the GDP growth rate's total business-cycle variance, and the non-systematic part of monetary policy shocks makes only a minor contribution. The accounting for the Federal Funds Rate's variance is similar. Perhaps this is unsurprising, because we classify the shock that directly moves households' rate of time preference as "demand." Nevertheless, supply shocks' unimportance for the Federal Funds Rate contrasts sharply with their dominance of inflation fluctuations. The shock to intermediate goods' firms optimal markup accounts for about half of supply shocks 81 percent contribution, and the Hicks-neutral technology shock accounts for another quarter of this.

The Model's Specification and Estimation

Our empirical work uses eleven variables, measured from 1987:Q1 through the present:

- Growth of nominal per capita GDP,
- Growth of nominal per capita consumption, which sums Personal Consumption Expenditures on Nondurable Goods and Services;
- Growth of nominal per capita investment, which sums Business Fixed Investment, Residential Investment, Personal Consumption Expenditures on Durable Goods, and Inventory Investment;
- Per capita hours worked in Nonfarm Business,
- Growth of nominal compensation per hour worked in Nonfarm Business,
- Growth of the implicit deflator for GDP,
- Growth of the implicit deflator for consumption, as defined above,
- Growth of the implicit deflator for investment, as defined above,
- Growth of the implicit deflator for core PCE,
- The interest rate on Federal Funds, and
- High Yield-AAA Corporate Bond Spread.

We do not directly use data on either government spending or net exports. Their sum serves as a residual in the national income accounting identity. To construct series measured per capita, we used the civilian non-institutional population 16 years and older. To eliminate level shifts associated with the decennial census, we project that series onto a fourth-order polynomial in time.

Our model confronts these data within the arena of a standard linear state-space model. Given a vector of parameter values, θ, log-linearized equilibrium conditions yield a first-order autoregression for the vector of model state variables, ζ_t.

$$\begin{aligned} \zeta_t &= F(\theta)\zeta_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim N(0, \Sigma(\theta)) \end{aligned}$$

Here, ε_t is a vector-valued innovation built from the model innovations described above. Many of its elements identically equal zero. Table 3 lists the elements of ζ_t. Habit puts lagged nondurable consumption into the list, and investment adjustment costs place lagged investment there. Rules for indexing prices and wages that cannot adjust freely require the state to include lags of inflation and technology growth. The list includes the lagged policy rate because it appears in the Taylor rule.

Gather the date t values of the eleven observable variables into the vector y_t. The model analogues to its elements can be calculated as linear functions of ζ_t and ζ_{t-1}. We suppose that the data equal these model series plus a vector of "errors" v_t.

$$\begin{aligned} y_t &= G(\theta)\zeta_t + H(\theta)\zeta_{t-1} + v_t \\ v_t &= \Lambda(\varphi)v_{t-1} + e_t \\ e_t &\sim N(0, D(\varphi)) \end{aligned}$$

Here, the vector φ parameterizes the stochastic process for v_t. In our application, the only non-zero elements of v_t correspond to the observation equations for the two consumption-based measures of inflation, the GDP deflator, and the High Yield-AAA spread. The idiosyncratic disturbances in inflation fit the high-frequency fluctuations in prices and thereby allow the price markup shocks to fluctuate more persistently. These errors evolve independently of each other. In this sense, we follow Boivin and Giannoni (2006) by making the model errors "idiosyncratic". The other notable feature of the observation equations concerns

Table 3. Model State Variables

Symbol	Description	Disappears without
C_{t-1}	Lagged Consumption	Habit-based Preferences
I_{t-1}	Lagged Investment	Investment Adjustment Costs
π^p_{t-1}	Lagged Price Inflation	Indexing "stuck" prices to lagged inflation
K_t	Stock of Installed Capital	
A_t	Hicks-Neutral Technology	
a_t	Growth rate of A_t	Autoregressive growth of A_t
a_{t-1}	Lagged Growth Rate of A_t	Indexing "stuck" wages to lagged labor productivity growth
Z_t	Investment-Specific Technology	
z_t	Growth rate of Z_t	Autoregressive growth of Z_t
z_{t-1}	Lagged Growth Rate of Z_t	Indexing "stuck" wages to lagged labor productivity growth
ϕ_t	Labor-Supply Shock	
b_t	Discount Rate Shock	
$\lambda_{w,t}$	Employment Aggregator's Elasticity of Substitution	Time-varying Wage Markups
$\lambda_{p,t}$	Intermediate Good Aggregator's Elasticity of Substitution	Time-varying Price Markups
μ_t	Marginal Efficiency of Investment Shock	
g_t	Government Spending Share Shock	
R_{t-1}	Lagged Nominal Interest Rate	Interest-rate Smoothing
$\varepsilon_{R,t}$	Monetary Policy Shock	

the GDP deflator. We model its growth as a share-weighted average of the model's consumption and investment deflators.

We denote the sample of all data observed with Y and the parameters governing data generation with $\Theta = (\theta, \varphi)$. The prior density for Θ is $\Pi(\Theta)$, which we specify to be similar to that employed by Justiniano, Primiceri, and Tambalotti (2011). Given Θ and a prior distribution for ζ_0, we can use the model solution and the observation equations to calculate the conditional density of Y, $F(Y|\Theta)$. To form the prior density of ζ_0, we apply the Kalman filter. The actual estimation begins with 1987:Q1. Bayes rule then yields the posterior density up to a factor of proportionality.

$$P(\Theta|Y) \propto F(Y|\Theta)\Pi(\Theta)$$

We calculate our forecasts with the model's parameter values set to this posterior distribution's mode.

Three Key Equations

This section summarizes the inferred parameters by reporting the estimates of three key log-linearized equations: the Taylor Rule, the Price Phillips Curve, and the Wage Phillips Curve.

Taylor Rule

$$R_t = 0.84R_{t-1} + (1-0.84)\left[1.63\left(\frac{1}{4}\sum_{j=0}^{3}\pi_{t-3}\right) + \frac{1.77}{4}(y_t - y_{t-4})\right] + \epsilon_t^{mp} + \sum_{s=1}^{5}\xi_{t-s}^{s},$$

Besides the lagged interest rate, the variables appearing on the right-hand side are the four-quarter average of consumption inflation, the most recent four-quarter output growth rate, the current monetary policy shock (ϵ_t^{mp}), and the five previous quarters' signals of the current monetary policy stance, ξ_{t-s}^{s} for $s = 1, \ldots, 4$. (These signals play a prominent role in forecasting, but we do not yet use them during estimation.) Note that

- Holding the economy's growth rate fixed, the long-run response of R_t to a permanent one-percent increase in inflation is 1.63 percent. Thus, the model satisfies the Taylor principle.

- Since the four-quarter growth rate of output replaces the usual output gap in the rule, it is difficult to compare the estimated coefficient of 0.44 with the typical calibrated output response of 0.5.

Price Phillips Curve

$$\pi_t^p = 0.80 E_t \pi_{t+1}^p + 0.20 \pi_{t-1}^p + 0.019 s_t + \epsilon_t^p$$

Here, s_t represents intermediate goods producers' common marginal cost.

- The associated Calvo probablity of an individual firm not updating its price in a given quarter equals 0.86, which is well in line with other calibrations.

- Producers unable to update their price with all current information are allowed to index their prices to a convex combination of last quarter's inflation rate with the steady-state inflation rate. This places π_{t-1}^p in the Phillips curve. The estimated weight on steady-state inflation is 0.76.

Wage Phillips Curve

The Wage Phillips curve can be written as

$$\pi_t^w + \pi_t^p + j_t - \iota_w \left(\pi_{t-1}^p + j_{t-1} \right) = \beta E_t \left[\pi_{t+1}^w + \pi_{t+1}^p + j_{t+1} - \iota_w \left(\pi_t^p + j_t \right) \right] + \kappa_w x_t + \epsilon_t^w,$$

where π_t^w and π_t^p correspond to inflation in real wages and consumption prices respectively, $j_t = z_t + \frac{\alpha}{1-\alpha} \mu_t$ is the economy's technologically determined stochastic trend growth rate, with α equal to capital's share in the production function, z_t the growth rate of neutral technology, and μ_t the growth rate of investment-specific technical change. The term $\pi^p_{t-1} + z_{t-1} + j_t$ arises from indexation of wages to a weighted average of last quarter's productivity-adjusted good's price inflation and its steady state value. The estimated weight on the steady state equals 0.64. The log-linearized expression for the ratio of the marginal disutility of labor, expressed in consumption units, to the real wage is

$$x_t = b_t + \psi_t + \nu l_t - \lambda_t - w_t,$$

where b_t and ψ_t are disturbances to the discount factor and the disutility of working, respectively, l_t hours,λ_t the marginal utility of consumption and w_t the real wage. Finally, ϵ^w_t is a white noise wage markup shock.

Note that without indexation of wages to trend productivity, this equation says that nominal wage inflation (adjusted by trend growth) depends positively on future nominal wage inflation (also appropriately trend-adjusted), and increases in the disutility of labor-real wage gap.

The estimated equation is given by

$$\pi^w_t+\pi^p_t+j_t-0.32\left(\pi^p_{t-1}+j_{t-1}\right) = 0.9988\times E_t[\pi^w_{t+1}+\pi^p_{t+1}+j_{t+1}-0.32\left(\pi^p_t+j_t\right)]+0.0055x_t+\epsilon^w_t,$$

The estimated Calvo probability of a wage remaining unadjusted in a given quarter underlying the estimate of $\kappa_w = 0.0055$ equals 0.73.

The Model's Shocks

Our discussion of recent macroeconomic developments above featured the following shocks prominently: The discount rate shock, the "financial" shock to the marginal efficiency of investment, and both anticipated and unanticipated monetary policy shocks. In this section, we provide greater detail on the model's responses to these four shocks.

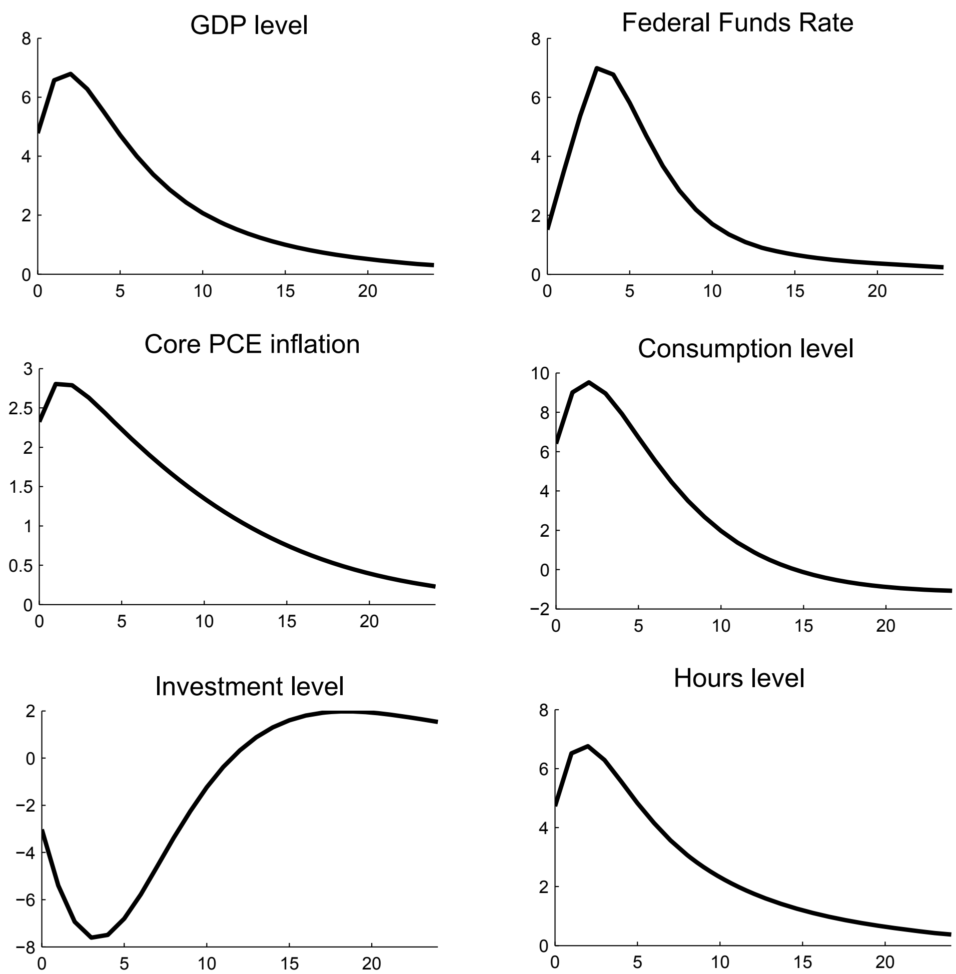
Figure 3 plots responses to a discount rate shock that increases impatience and tilts desired consumption profiles towards the present. The variables examined are real GDP, the Federal Funds Rate, inflation, consumption, investment, and hours worked. The responses are scaled so that the change in GDP after 16 quarters equals one percent.

In a neoclassical economy, this shock would be contractionary on impact. Upon becoming more impatient, the representative household would increase consumption and decrease hours worked. To the extent that the production technology is concave, interest rates and real wages would rise; and regardless of the production technology both real GDP and investment would drop.

Increasing impatience instead *expands* activity in this New Keynesian economy. As in the neoclassical case, consumption rises on impact. Habit causes the consumption growth to persist for two more quarters. Adjustment costs penalize the sharp contraction and recovery of investment from the neoclassical model, so instead investment remains unchanged on impact. Market clearing requires either a rise of the interest rate (to choke off the desired consumption expansion) or an expansion of GDP. By construction, the Taylor rule prevents the interest rate from rising unless the shock is inflationary or expansionary. Therefore, GDP must rise. This in turn requires hours worked to increase. Two model features overcome the neoclassical desire for more leisure. First, some of the labor variants' wages are sticky. For those, the household is obligated to supply whatever hours firms demand. Second, the additional labor demand raises the wages of labor variants with wage-setting opportunities. This rise in wages pushes marginal cost up and lies behind the short-run increase in inflation. After inflation has persisted for a few quarters, monetary policy tightens and real rates rise.

Since the discount rate shock moves output and prices in the same direction, a Keynesian analysis would label it a shift in "demand." In the neoclassical sense, it is also a demand shock, albeit a reduction in the demand for future goods. The matching neoclassical supply shock in our model is to the marginal efficiency of investment. A positive shock to it increases the supply of future goods. Figure 4 plots the responses to such a shock. Barro and King (1984) and Greenwood,

Figure 3. Responses to a Discount Rate Shock



Note: The impulse was scaled to yield a one percent increase in GDP after 16 quarters.

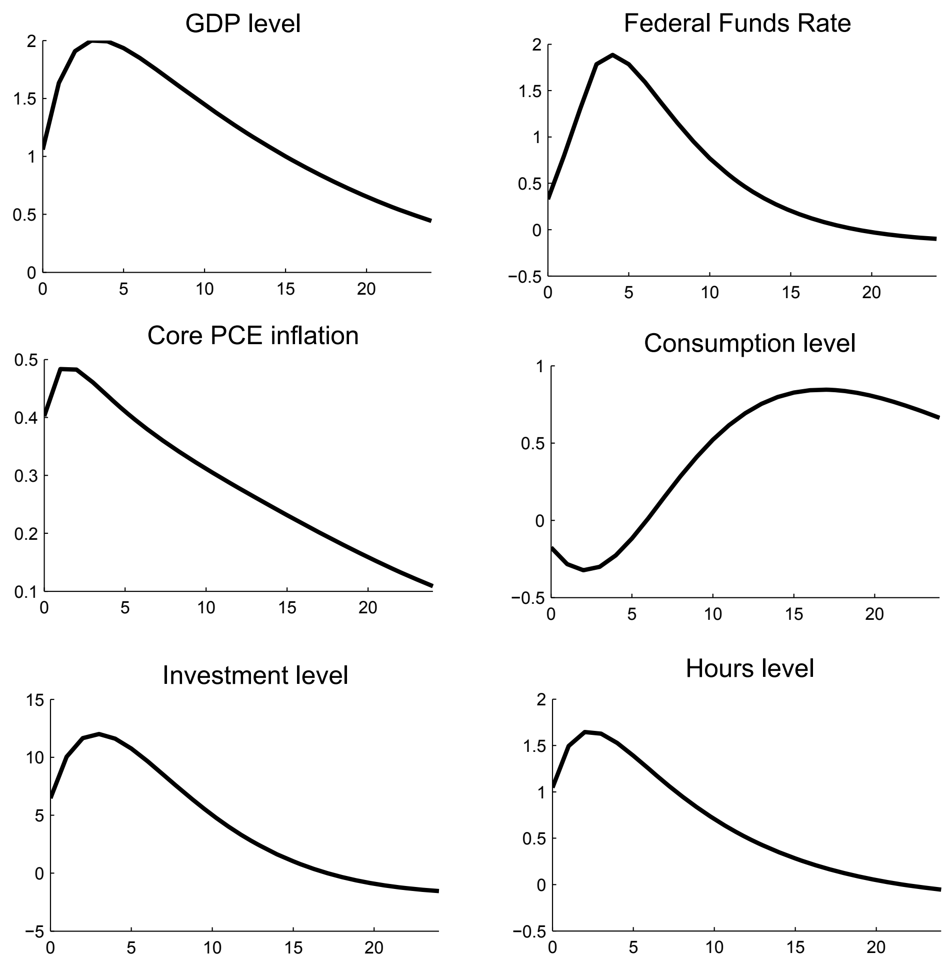
Hercowitz, and Huffman (1988) consider the analogous responses from a neoclassical model. Increasing the marginal efficiency of investment raises investment, hours worked, GDP, and the real interest rate but decreases consumption. Two aspects of our model stop consumption from falling on the same shock's impact. First, habit-based preferences penalize an immediate decrease in consumption. Second, monetary policy responds to the shock only slowly, so real interest rates actually *fall* slightly on impact. The investment adjustment costs give the responses of GDP, hours, and investment their hump shape. Although this shock changes the economy's technology for intertemporal substitution – and therefore deserves the neoclassical label "supply" – it makes prices and output move in the same direction. For this reason, it falls into our Keynesian taxonomy's "demand" category.

As noted above, monetary policy shocks partially offset negative demand shocks in 2010. Figures 6 and 5 present the impulse response functions for two of these, the unanticipated "contemporaneous" shock and the shock revealed to all agents five quarters in advance. The responses to the unanticipated shock are standard, but those following an anticipated shock require more explanation. At the announcement date, the expected value of the policy rate five quarters hence increases by 300 basis points. Because both Phillips curves are forward looking, this expected contraction causes both prices and quantities to fall. This anticipated weakness then feeds through the Taylor rule to create a gradual *easing* of policy. When the anticipated tightening arrives, it mostly offsets the prior endogenous easing. *The policy rate rises only 85 basis points above its pre-shock value.* These responses are clearly worthy of further study.

Shock Decomposition Methodology

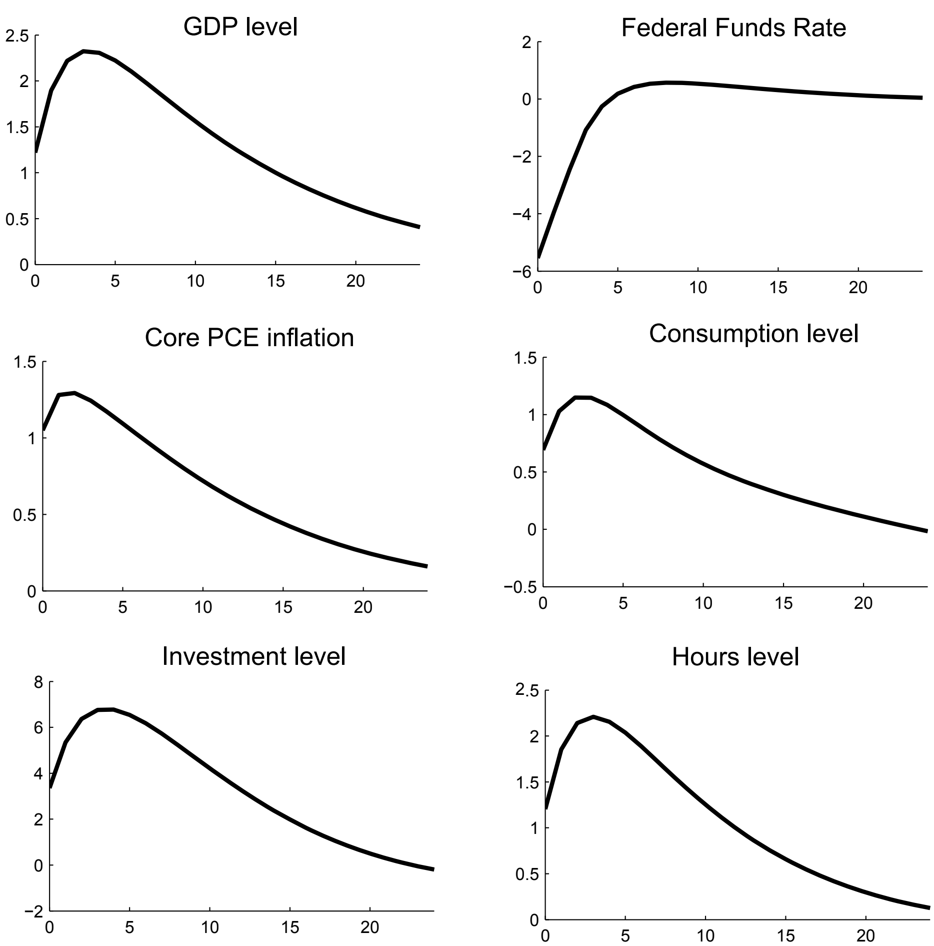
We credit Charles Evans with the original ideas behind this decomposition. For the shock decomposition, we set the model's parameters to their values at the posterior distribution's mode, $\hat{\theta}$. Using *all* available data we use the Kalman smoother to extract sequences of estimated states $\{\hat{\zeta}_t\}_{t=1}^{T}$ and a innovations

Figure 4. Responses to a Marginal Efficiency of Investment Shock



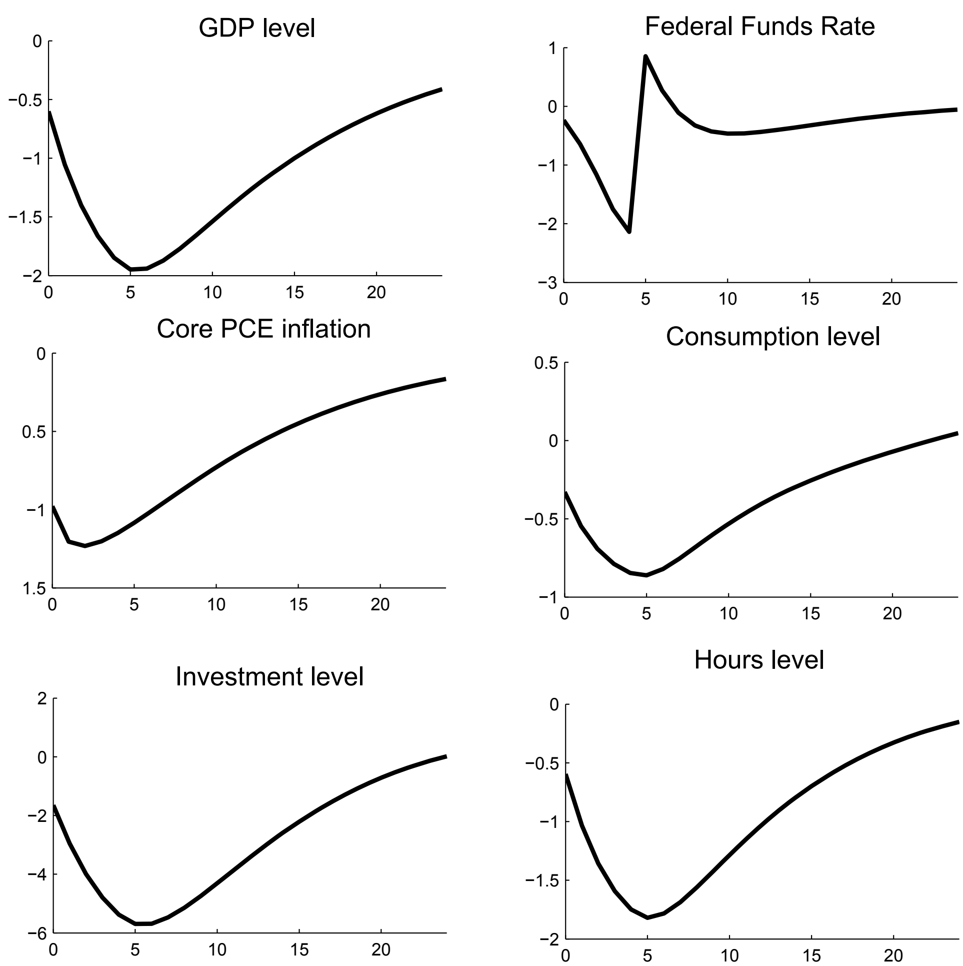
Note: The impulse was scaled to yield a one percent increase in GDP after 16 quarters.

Figure 5. Responses to an Unanticipated Monetary Policy Shock



Note: The impulse was scaled to yield a one percent decrease in GDP 16 quarters after that date.

Figure 6. Responses to an Anticipated Monetary Policy Shock



Note: The monetary policy shock is revealed to all agents five quarters before its realization, and the impulse was scaled to yield a one percent decrease in GDP 16 quarters after that date.

$\{\hat{\varepsilon}_t\}_{t=1}^{T}$. By construction, these satisfy the estimated transition equation for the state.

$$\hat{\zeta}_t = F(\hat{\theta})\hat{\zeta}_{t-1} + \hat{\varepsilon}_t,$$

To keep this discussion simple, we henceforth suppose that the "error" shocks in v_t equal zero. Incorporating them into the analysis changes the actual calculations only little.

For concreteness, suppose that the forecasted object of interest is Q4-over-Q4 GDP growth for 2010. We position ourselves in 2009:Q4 and calculate

$$\begin{aligned}
\hat{\zeta}_{\text{2010:Q1}}^{\text{2009:Q4}} &\equiv F(\hat{\theta})\hat{\zeta}_{\text{2009:Q4}} \\
\hat{\zeta}_{\text{2010:Q2}}^{\text{2009:Q4}} &\equiv F(\hat{\theta})\hat{\zeta}_{\text{2010:Q1}}^{\text{2009:Q4}} \\
&= F^2(\hat{\theta})\hat{\zeta}_{\text{2009:Q4}} \\
&\vdots \\
\hat{\zeta}_{\text{2010:Q4}}^{\text{2009:Q4}} &\equiv F(\hat{\theta})\hat{\zeta}_{\text{2010:Q3}}^{\text{2009:Q4}}
\end{aligned}$$

These are the "expectations" of the model's states in each quarter of 2010 conditional on the state at the end of 2009 equalling its estimated value.

With these "state forecasts" in hand, we can construct corresponding forecast errors by comparing them with their "realized values" from the Kalman smoother. For the period t state forecasted in 2009:Q4, we denote these with

$$\hat{\eta}_t^{\text{2009:Q4}} = \hat{\zeta}_t - \hat{\zeta}_t^{\text{2009:Q4}}.$$

These forecast errors are related to the structural shocks by

$$\hat{\eta}_t^{\text{2009:Q4}} = \sum_{j=1}^{t-\text{2009:Q4}} F^{j-1}(\hat{\theta})\hat{\varepsilon}_{\text{2009:Q4}+j}.$$

The shock decomposition is based on four alternative forecasts, $\hat{\zeta}(\iota)_t^{\text{2009:Q4}}$ for $t = \text{2010:Q1}, \ldots, \text{2010:Q4}$ and $\iota \in \{\mathcal{D}, \mathcal{S}, \mathcal{M}, \mathcal{R}\}$. Here, ι indexes one of the four groups of structural shocks. For these, let $\hat{\varepsilon}(\iota)_t$ denote a version of $\hat{\varepsilon}_t$ with all shocks except those in group ι set to zero. With these, we construct

$$\begin{aligned}
\hat{\zeta}(\iota)^{2009:\text{Q4}}_{2010:\text{Q1}} &\equiv F(\hat{\theta})\hat{\zeta}_{2009:\text{Q4}} + \hat{\varepsilon}(\iota)_{2010:\text{Q1}}, \\
&\vdots \\
\hat{\zeta}^{2009:\text{Q4}}_{2010:\text{Q4}} &\equiv F(\hat{\theta})\hat{\zeta}^{2009:\text{Q4}}_{2010:\text{Q3}} + \hat{\varepsilon}(\iota)_{2010:\text{Q4}},
\end{aligned}$$

and

$$\hat{\eta}(\iota)^{2009:\text{Q4}}_t \equiv \hat{\zeta}_t - \hat{\zeta}(\iota)^{2009:\text{Q4}}_t.$$

By construction,

$$\hat{\eta}^{2009:\text{Q4}}_t = \sum_{\iota \in \{\mathcal{D},\mathcal{S},\mathcal{M},\mathcal{R}\}} \hat{\eta}(\iota)^{2009:\text{Q4}}_t.$$

That is, each forecast error can be written as the sum of contributions from each of the shock groups. Using the observation equations, we transform these into components of the forecast error for observable variables.

With this completed, we can then move the forecast date forward to 2010:Q1. The decomposition for that date proceeds similarly, except that we treat growth in 2010:Q1 as data.

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