Detailed Philadelphia (Prism) Forecast Overview

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Michael Dotsey
Keith Sill

Forecast Summary

The FRB Philadelphia DSGE model denoted Prism, projects that real GDP growth will rebound strongly over the forecast horizon, with output growth approaching 7 percent by mid 2011. Even with such strong output growth, inflation is well contained, reaching 2.2 percent by the end of the forecast horizon. Because inflation is the dominant determinant of the federal funds rate in the model, the forecast is for a gradual increase in the funds rate. Policy begins to tighten by the third quarter of 2011, and the funds rate reaches 3.2 percent by the fourth quarter of 2013. Currently, many of the model's state variables are well below their steady-state values (see figure 8). In particular, consumption, investment, and the capital stock are low relative to steady state, and absent any shocks, the model would predict a swift recovery. Further, the wage rate, and hence marginal cost, is below steady state, implying that, absent any shocks, wages and marginal cost are expected to increase, leading to an increase in inflation. Also, these state variables have been below steady state since the end of the recession. The relatively slow recovery to date and the low inflation that has recently characterized U.S. economic activity require the presence of shocks to offset the strength of the model's internal propagation channels.

Overview of the Great Recession and Early Part of the Recovery

Before proceeding to the forecasts, it may be useful to describe how the model accounts for the Great Recession and what features drove the early stages of the recovery. Because the model does not contain a financial sector, the past recession is a challenge for the model to explain. The recession was marked by a severe impairment in financial intermediation that affected many countries. As a result, risk spreads increased and the cost of funds for both firms and consumers rose. In addition, many financial markets seized up. The deterioration in the economy's ability to allocate resources led to a rapid decline in both investment and consumption, which was accompanied by a large drop in employment. To capture these phenomena, the model must identify various shocks that allow it to match the data. Without a financial sector that endogenously feeds in to the other behavioral relationships, the model is forced to find ways of reducing consumption, investment, and hours worked.

To match the weakness in desired consumption, the model identifies a shock to the parameter that governs the way individuals intertemporally allocate their consumption. Thus, the model identifies an increase in consumers' rate of time preference, or a decrease in their discount factor, causing current consumption growth to be weak relative to future consumption growth. To account for the weakness in investment, the model identifies a negative shock to the efficiency of investment. In particular, a unit of investment produces less capital than it normally would, making investing less desirable. We interpret this shock as one that reflects the efficiency with which investment funding is allocated across firms. Inefficient allocations of capital lead to a less productive economy and less desired investment. Also, the extreme fall in employment cannot be accounted for solely by the discount factor shock and the negative shock to the efficiency of investment, although both shocks work in the right direction. The model also requires that individuals desire more leisure. This is a somewhat unattractive feature of the model, and it is fair to say that additional research needs to be done regarding the way labor markets are modeled. Finally, a negative shock to productivity is also required in order for the model to generate sufficient economic weakness. A recent history of the model's key identified shocks is presented in figure 6.

The shock to the marginal efficiency of investment and the discount factor shock are the shocks most closely aligned with the financial crisis. The discount factor shock directly affects asset prices, and we believe that in our one-sector model, the estimated efficiency of investment shock is in part the result of an inefficient allocation of investment in the economy. At the aggregate level, the impediments caused by adverse financial factors probably resulted in an inefficient allocation of capital, implying that investment was less productive. Also, lack of consumer credit and relatively unfavorable financing terms were factors in weak consumption growth, and the negative shock to the discount factor reduces current consumption growth. It would certainly be desirable to have a structural model of the financial system, but absent that, it is at least reassuring that some of the key shocks the model identifies as important for causing the recession are ones that have the tightest association with financial factors.

Inflation did not fall precipitously during the recession period. The primary factor that contributed to declining inflation was the discount factor shock. The negative discount factor shock also results in declining wages and marginal cost, which tend to reduce inflation pressures. Further, negative shocks to firm markups contributed to the fall in inflation in late 2008 and early 2009. Somewhat offsetting the effects of these shocks was the labor supply shock, which raised wages, marginal cost, and hence inflation as well. Because inflation is the most important variable in the model's estimated policy reaction function, these same shocks are the primary drivers of the funds rate path.

The early stages of the recovery were driven by strong investment and consumption growth. Replicating this behavior required a waning of the shocks that accounted for the recession. Thus, the early stages of the recovery were driven by a decline in the magnitude of both the discount factor shock and the marginal efficiency of investment shock coupled with strong productivity growth. Productivity grows strongly because in the data output is increasing while employment remains weak, requiring significant growth in total factor productivity. The weak employment growth once again is produced through an increased desire for leisure on the part of individuals in the model. The decline in inflation that accompanies the economy's transition into recovery is accounted for by the markup shock turning negative, implying that firms in the model are facing a reduction in pricing power. Further, the continued presence of negative shocks to the discount factor, which helps hold back consumption growth, also helps to restrain inflation through the usual demand channels. Countering somewhat the negative effects that these two shocks have on inflation is the positive shock to leisure, which is responsible for raising marginal cost and inflation, but the magnitude of this shock is falling as employment picks up. The lowering of inflation is what keeps the funds rate near zero even as the economy recovers.

The Current Forecast and Shock Identification

The current forecast is shown in figures 7a-7c, which also displays the shock decomposition. The shock decomposition is a fairly complicated object and will be discussed in detail in the next section. The key identified shocks are shown in figure 6. In the current forecast, output is projected to grow robustly, well above the model's long-run average of 2.7 percent. By the second half of 2011, output is predicted to grow at about 7 percent. Part of this strength is due to the model's state variables having been well below their estimated steady state for a considerable time, which implies strong growth as the model economy returns to steady state. A negative government spending shock was a big factor holding down 2011Q1 growth, and since this shock has low persistence for real output growth, it implies a strong bounce-back in 2011Q2. The model predicts consumption growth (nondurables+services) will run at close to 4 percent in 2011Q2 driven primarily by discount factor shocks that make current consumption relatively attractive. As well, investment continues its strong rebound from its 2010Q4 decline. Over the medium term, the model predicts very strong investment growth that peaks in early 2012. Consumption growth peaks in 2011Q2 and then declines slowly toward steady state over the forecast horizon. Essentially, the extremely strong Prism growth forecast is driven by moderately strong consumption growth and very strong investment growth.

Turning to inflation, the recent ramping up of employment growth led to an estimated shift away from leisure and toward hours worked on the part of individuals. A positive labor supply shock tends to push down inflation in the model because it lowers marginal cost. As the labor market continues to strengthen, it implies less of a cumulative upward pressure on inflation. However, the labor shock effect is more than offset by the persistent negative effects that past negative discount factor shocks have on projected inflation. Negative shocks to price markups also help explain the weakness of inflation over the second half of 2010, but their effects are not very persistent. On net, inflation is predicted to pick up modestly, rising to 2.0 percent at the beginning of 2012. The slow growth in inflation implies that the funds rate will be raised only gradually over the forecast horizon.

The Current Forecast in More Detail

To understand the forecast in more detail requires a brief look into the model and the impulse response functions associated with the key shocks (a detailed description of the model and its equations are contained in supporting appendices). One also needs to understand why these particular shocks are being identified as the primary drivers in the current forecast, as well as what shocks were important in the past. The need to know past shocks is driven by the fact that the model's response to most shocks is persistent. In this document, we display only the most important shocks. For a description of how every shock affects economic activity in the model, please refer to the supporting appendix.

The model is a fairly standard New Keynesian model with sticky prices and sticky wages. Firms and workers are able to reset their prices and wages at random intervals, and those that do not reset can index their current price (wage) to steady-state inflation. The important implication of these nominal rigidities is that prices will be a markup over marginal cost, and wages will be a markup over the marginal disutility of working. The model contains an AR(1) shock to the price markup, and this shock affects inflation directly. The shock also acts as an inefficient wedge in the model and therefore affects economic activity — an increase in either markup lowers output.

Production in the model is fairly standard — intermediate goods firms use capital and labor to produce intermediate goods that are then aggregated via a Dixit-Stiglitz aggregator into a final good that in turn is used for investment, consumption, and government spending. Multifactor productivity, or technology shocks, is nonstationary and its growth rate is modeled as an AR(1). The shock affects output directly and inflation indirectly through its influence on markups. Thus, increases in productivity increase output growth and reduce inflation.

Investment is subject to adjustment costs that are also influenced by a disturbance, μ_t. Specifically,

$$\hat{K}_t(j) = (1-\delta)\hat{K}_{t-1}(j) + \mu_t\left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right)I_t(j),$$

where μ_t is a shock to investment efficiency, which may be caused by financial frictions. Basically, a positive shock implies that fewer resources must be devoted to investment in order to build new capital. This shock is also specified as an AR(1).

Individuals get utility from consumption, which enters via internal habit formation, and get disutility from working. Individuals also receive utility from money balances, but this part of the model is not essential and can be ignored. Preferences are subject to two important shocks: one to time preference, b_t and the other to the labor supply, φ_t. Thus, momentary utility is given by

$$b_t\left[\ln(C_t(j) - hC_{t-1}(j)) - \frac{\phi_t}{1+\nu_l}L_t(j)^{1+\nu_l} + \frac{\chi_t}{1-\nu_m}\left(\frac{M_t(j)}{Z_t P_t}\right)^{1-\nu_m}\right].$$

The preference shocks are composed of a shock to the preference for leisure and a shock to the discount factor. Both shocks are AR(1) as well.

Inflation's relation to output can be seen through the model's derived Phillips curve. It is not a structural equation of the model, but it is useful to compare this equation across models to get an idea of the inflation dynamics in the model. Note that there is no lagged inflation term in the equation, since we do not index current prices to past inflation. The estimates indicate that the Phillips curve is quite flat for this model over the sample period 1984Q1 to 2010Q3, and marginal cost increases are not aggressively passed through to prices.

$$\pi_t = 0.997 E_t \pi_{t+1} + 0.011 mc_t + \upsilon_t$$

Also of importance, especially when analyzing the effects that various shocks have on the funds rate, is the estimated policy rule. As one can see, the movement of inflation relative to target is the primary driver of monetary policy.

$$R_t = .81 R_{t-1} + (1-.81)(2.25\pi_t + 0.06\tilde{y}_t)$$

In order to understand the contributions of the various shocks to output growth, inflation, and nominal interest rates, it is helpful to look at the impulse responses of the variables in the measurement equation (see equation (22) in the technical appendix)**.** The impulse responses to the currently most relevant shocks are shown in figures 1 through 4. We then attempt to link these responses and the recent behavior of these variables (figure 5) to form an idea of why the model is identifying certain shocks as important. While far from perfect, this exercise sheds some light on why the model is identifying a particular state of the economy at a particular time. The estimated shocks are shows in figure 6. Further, the varying persistence of each shock's effect on output growth, inflation, and the interest rate helps to understand the shock decompositions that are given in figures 7a, 7b, and 7c.

Toward the end of 2009, output grew fairly strongly, but employment did not. To capture this behavior, the model placed substantial weight on positive technology shocks. However, weak growth in 2010Q2 was accounted for by increased disutility from work together with negative contributions from the discount factor shock, government spending, and monetary policy and an easing of the TFP shock. The model attributes the stronger growth in the third and fourth quarters in part to an easing of the financial (discount factor) shock. To help the model fit the weak growth in hours worked also required positive shocks to leisure over the last few quarters. Further, the recent strong growth in investment relative to consumption caused the model to identify negative shocks to the rate of time preference (financial shocks) in the first three quarters of 2010, but the sign of the shock is reversed by the Q4 strength in consumption. Although investment grew strongly in 2010, the growth slowdown toward the end of the year is in part accounted for by negative shocks to the marginal efficiency of investment, which helped to counter the positive effects of negative shocks to the discount rate. In addition, although the negative discount rate shock and the positive leisure shock move inflation in opposite directions, near term, the net effect is negative. By 2011Q1, the mei shock is positive as investment growth showed a rebound and the financial shock has been running positive over the last couple quarters as well. To help account for the falling rate of inflation, the model attaches some importance to negative markup shocks. Finally, to dampen somewhat the positive effect of the (negative) financial shock on investment, the model attaches some weight to a negative efficiency of investment shock, which also has a negative effect on inflation.

The forecast is then a product of past shocks working through the model and the fact that the model's state variables are below their steady-state values. At first glance, it is hard to see why output is predicted to grow so robustly. The reason is that the response of output growth to both the discount factor shock and the marginal efficiency of investment shock involves some overshooting. Thus, much of the strong output growth in the projection is due to negative shocks to the discount factor and investment efficiency in 2009 and 2010. Thus, although the initial response to these two shocks is quite negative, about four or five quarters in the future the economy undergoes significant bounce-back in response to these two shocks (see figures 2 and 4).

Regarding inflation, the impulse response functions for the TFP and MEI shocks indicate that these shocks have a small quantitative effect on inflation for a typical-sized shock. Positive productivity shocks lower marginal cost somewhat and negative MEI shocks also lower inflation through their negative effect on demand. But these effects are small. Positive labor supply shocks have a more significant effect. They push up marginal cost and inflation because higher wages are needed to attract labor, while the negative discount rate shocks and the negative markup shock act to offset these pressures. On net, the model, therefore, forecasts only a slight increase in inflation.

The shock decomposition for the funds rate is similar to that of the discount rate in that the discount factor shock is a key driver of the forecast – pulling down the funds rate below its steady-state value for the next 3 years. Note, though, the effect of the efficiency of investment shock. That shock generates overshooting in both output growth and inflation, although the overshooting occurs after about 5 quarters in the case of inflation and about 17 quarters in the case of the funds rate. Consequently, the sequence of large negative MEI shocks over 2007-2009 push up inflation over the forecast horizon but are still pulling down the funds rate. The markup shock impulses indicate a bit of near-term overshooting on inflation and a steady decline toward steady state for the funds rate. The negative markup shocks at the end of 2010 lead to a slightly positive impact on inflation in 2011 and a steady negative effect on the funds rate.

Philadelphia DSGE Forecast Model Impulse Responses

(one-standard-deviation shocks)

Figure 1: Response to technology shock

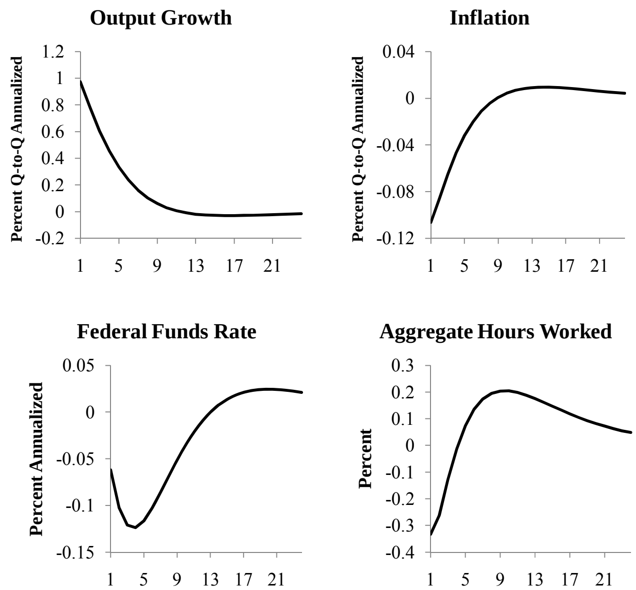


Figure 1 (continued): Response to technology shock

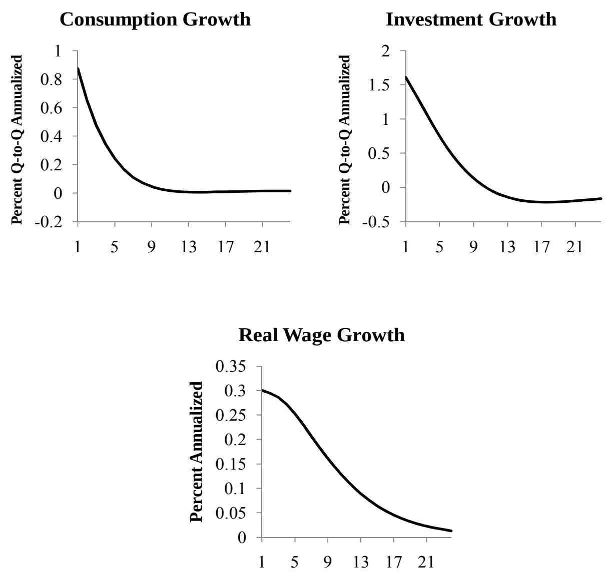


Figure 2: Response to investment efficiency shock

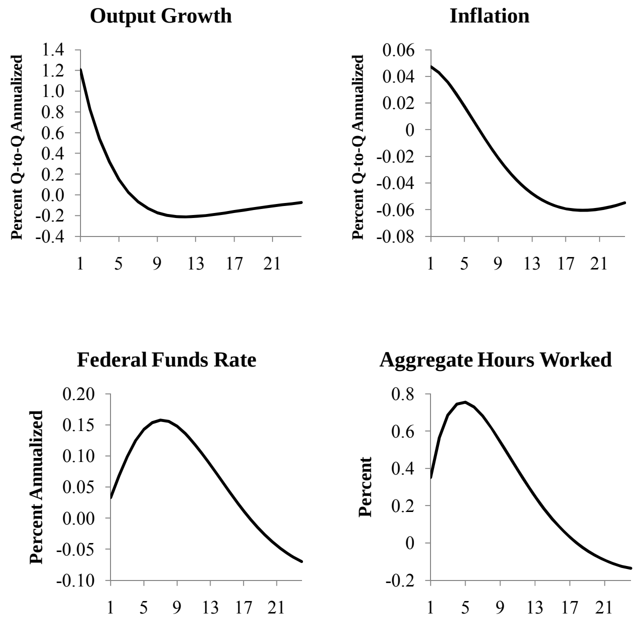


Figure 2 continued: Response to mei shock

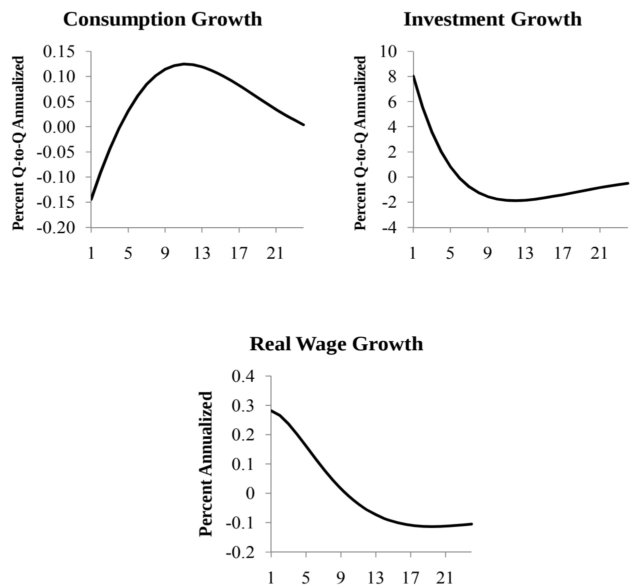


Figure 3: Response to leisure shock

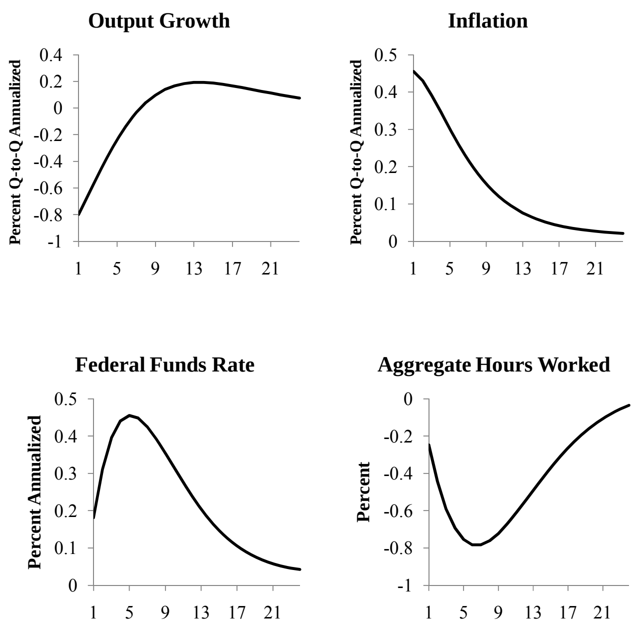


Figure 3 continued: Response to leisure shock

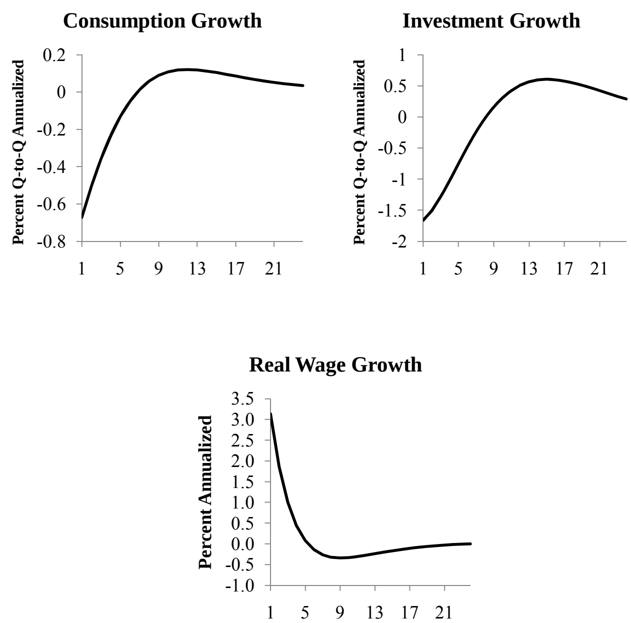


Figure 4: Response to discount factor shock

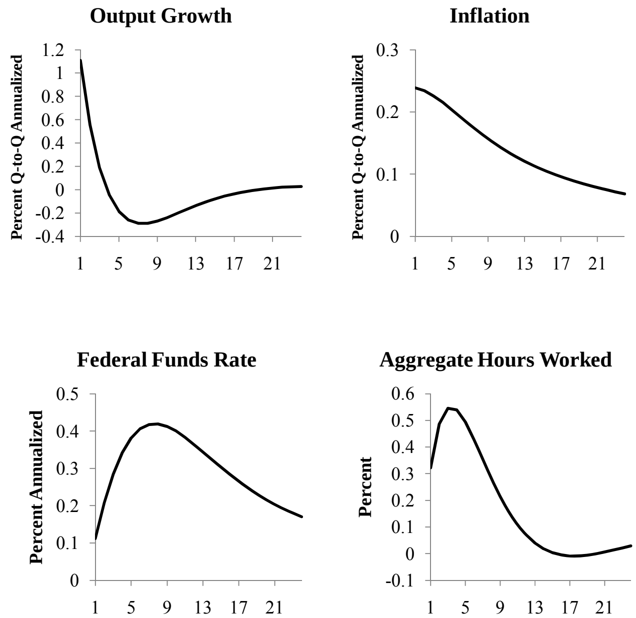


Figure 4 continued: Response to financial shock

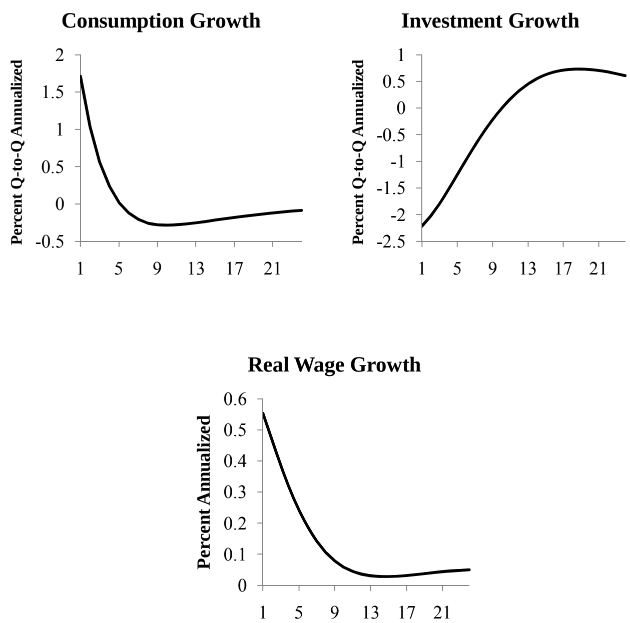


Figure 5a

Series used to estimate model

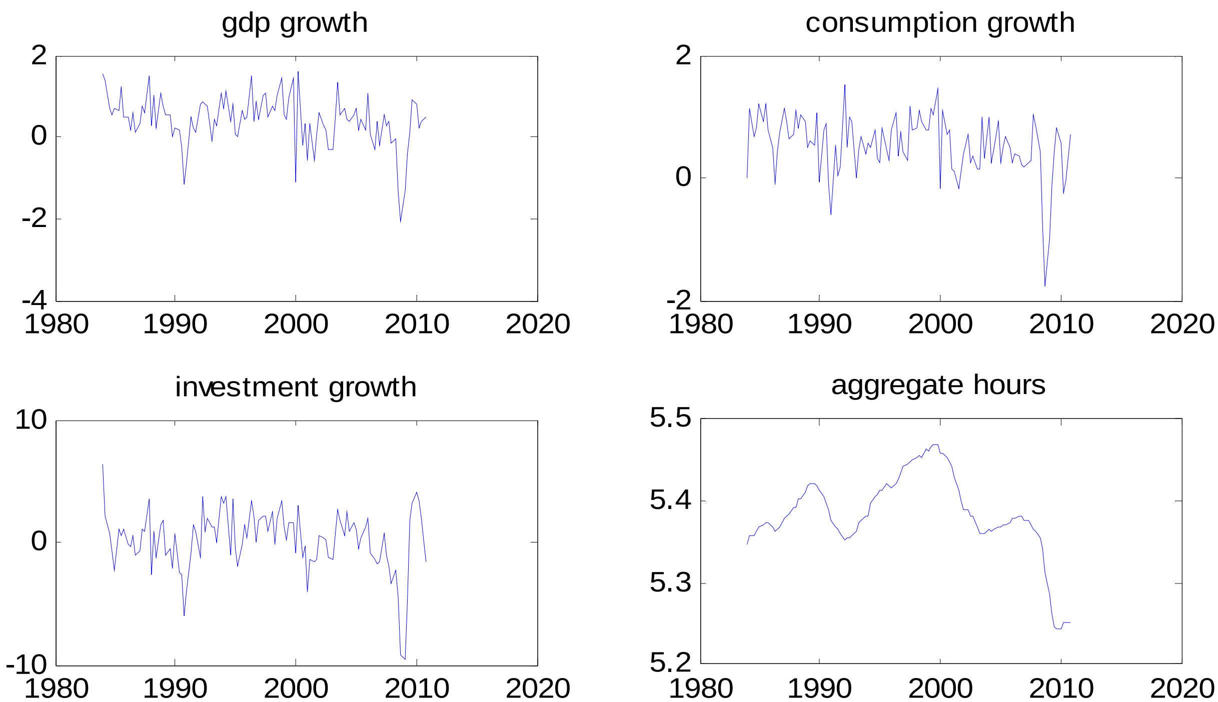


Figure 5b

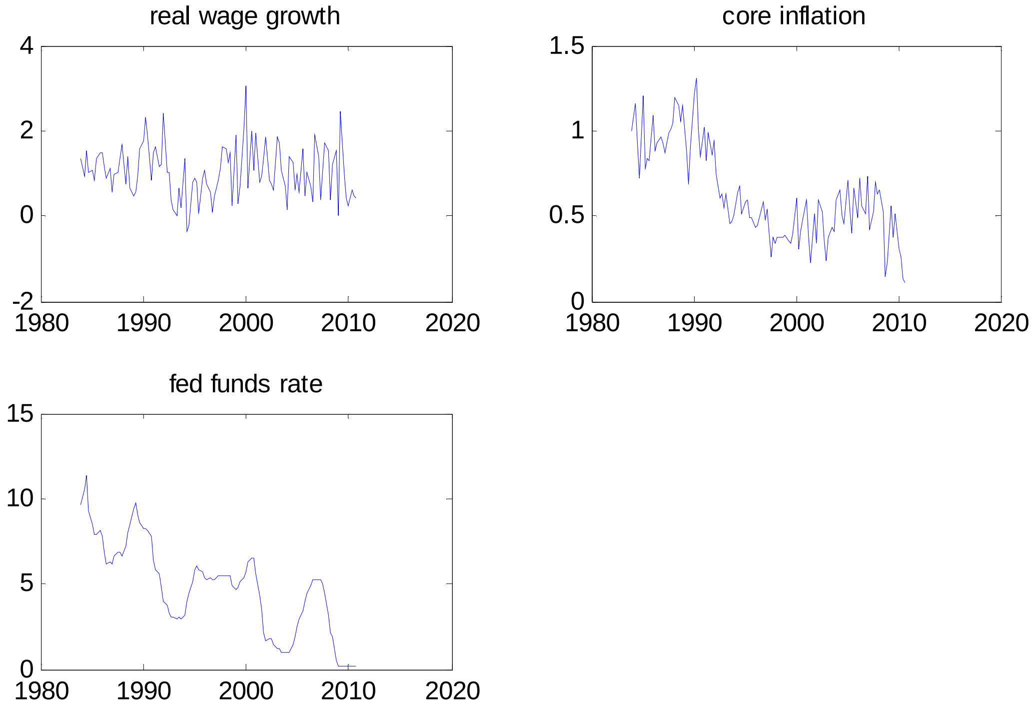

real wage growth
core inflation
fed funds rate
4
2
0
-2
1.5
1
0.5
15
10
5
1980
1990
2000
2010
2020

Figure 6

Estimated Shock History (in standard deviations)

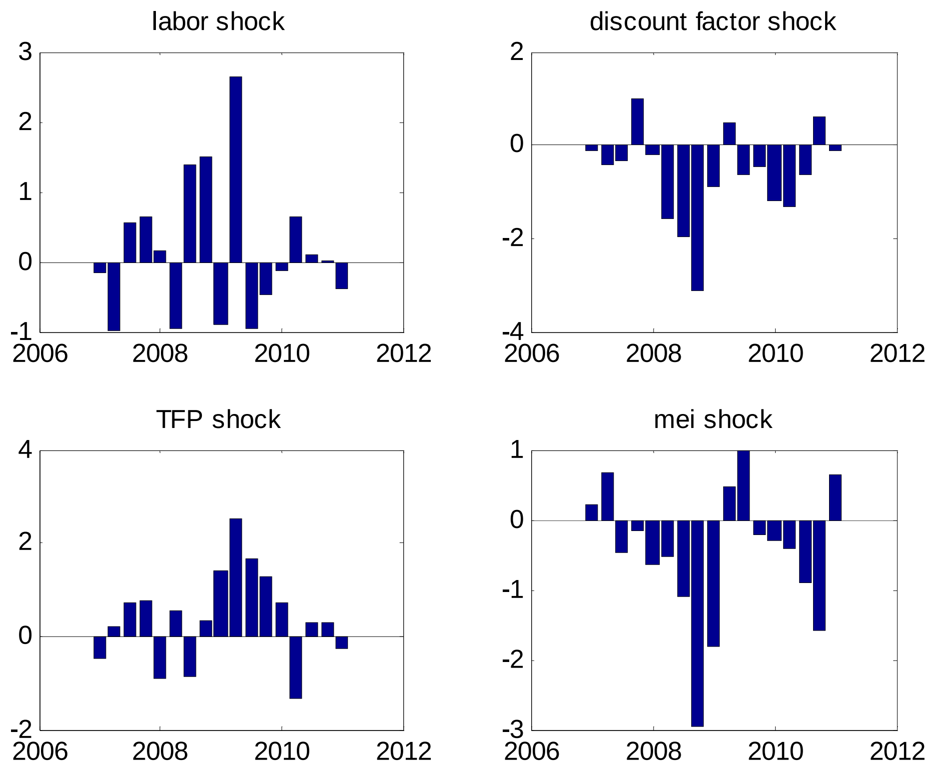


Figure 7a

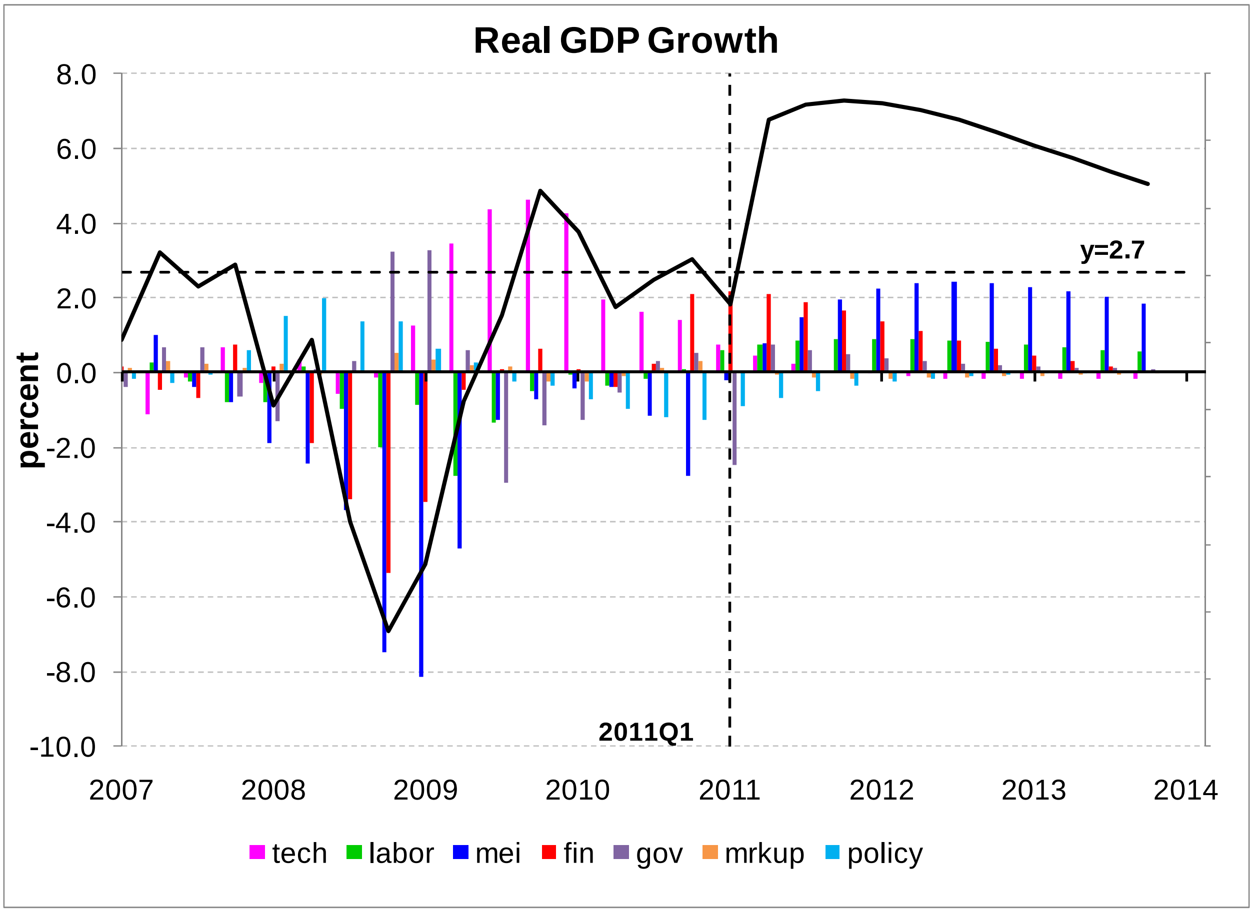

Real GDP Growth
8.0
6.0
4.0
2.0
0.0
-2.0
-4.0
-6.0
-8.0
-10.0
percent
y=2.7
2011Q1
2007
2008
2009
2010
2011
2012
2013
2014
tech
labor
mei
fin
gov
mrkup
policy

Figure 7b

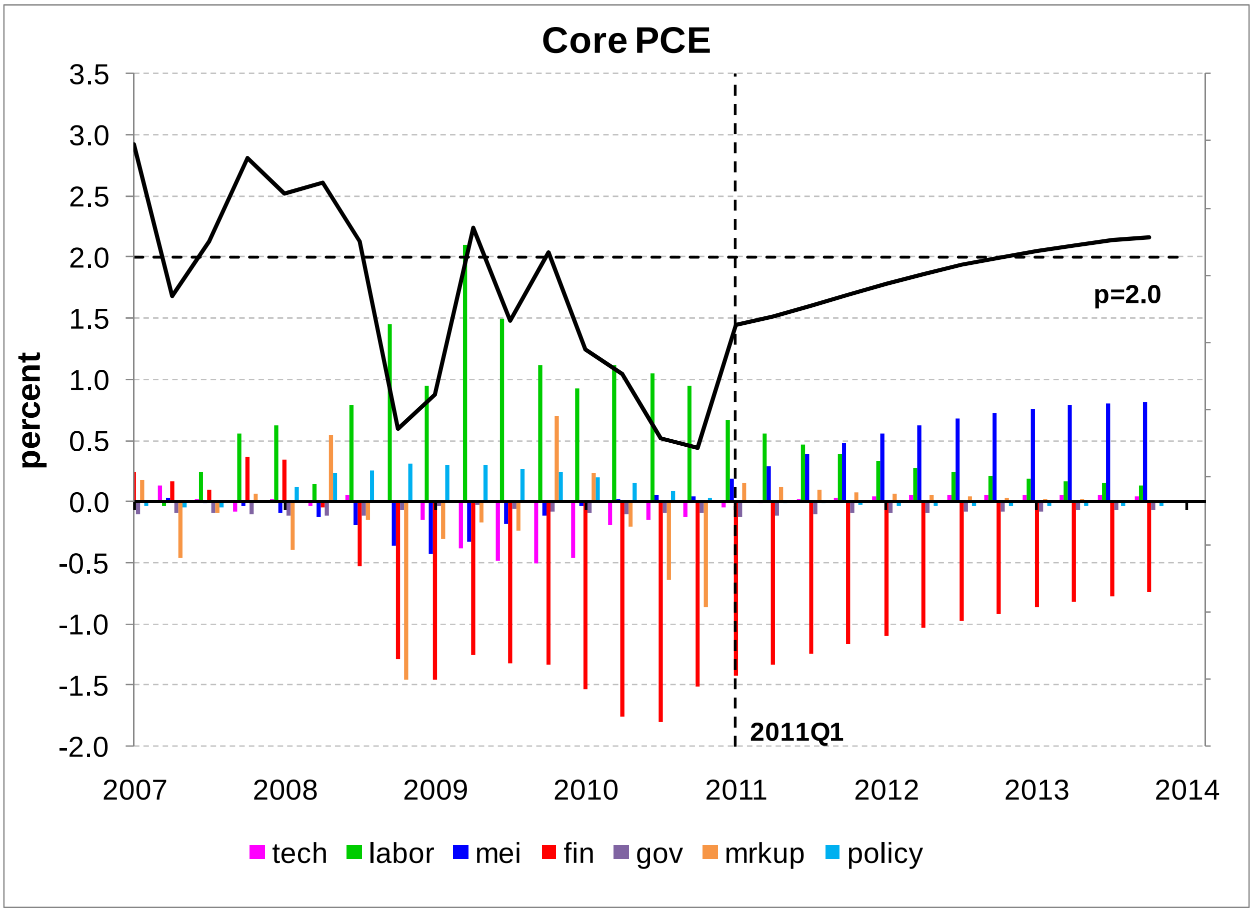

Core PCE
percent
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
2007
2008
2009
2010
2011
2012
2013
2014
p=2.0
2011Q1
tech
labor
mei
fin
gov
mrkup
policy

Figure 7c

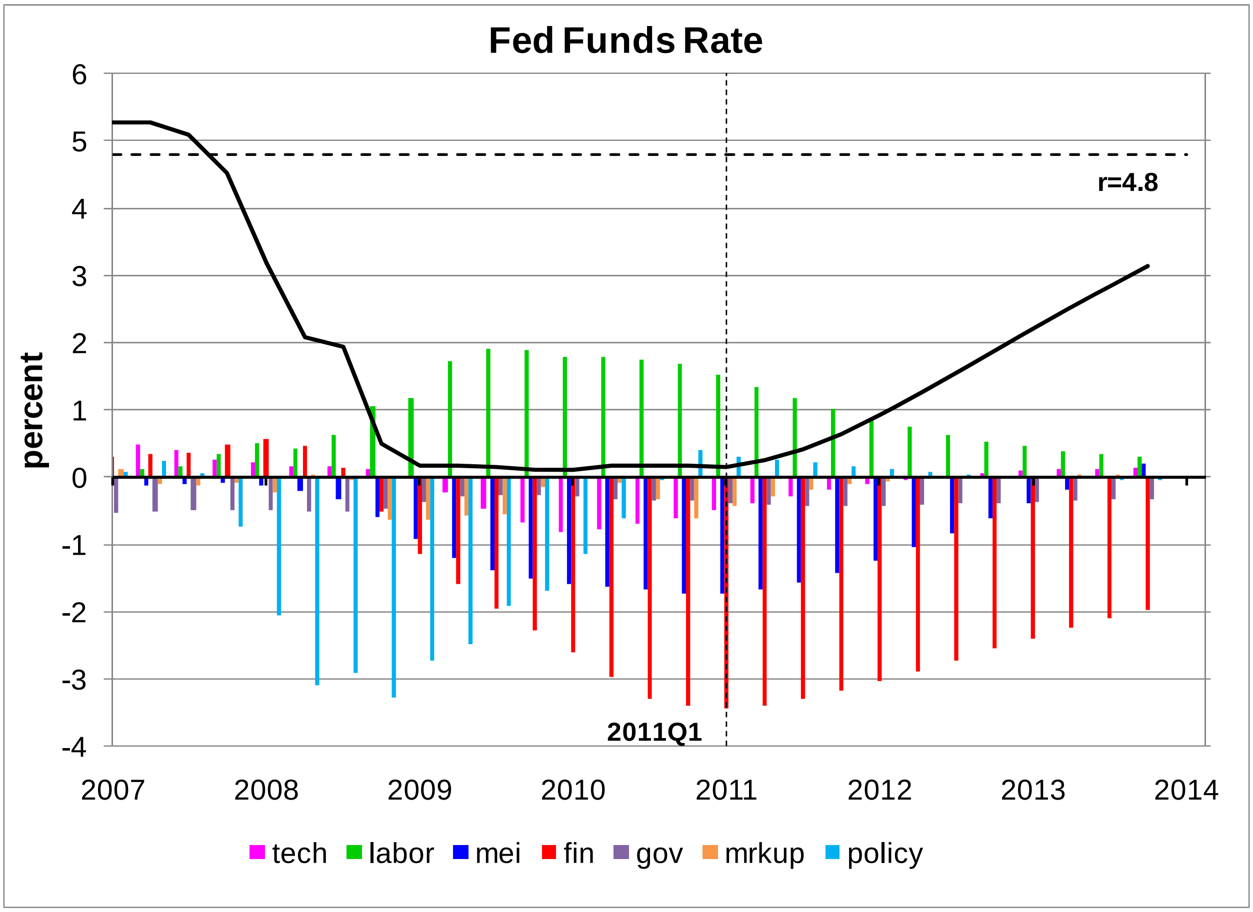

Fed Funds Rate
percent
6
5
4
3
2
1
0
-1
-2
-3
-4
r=4.8
2011Q1
2007
2008
2009
2010
2011
2012
2013
2014
tech
labor
mei
fin
gov
mrkup
policy

Figure 8

Endogenous State Variables

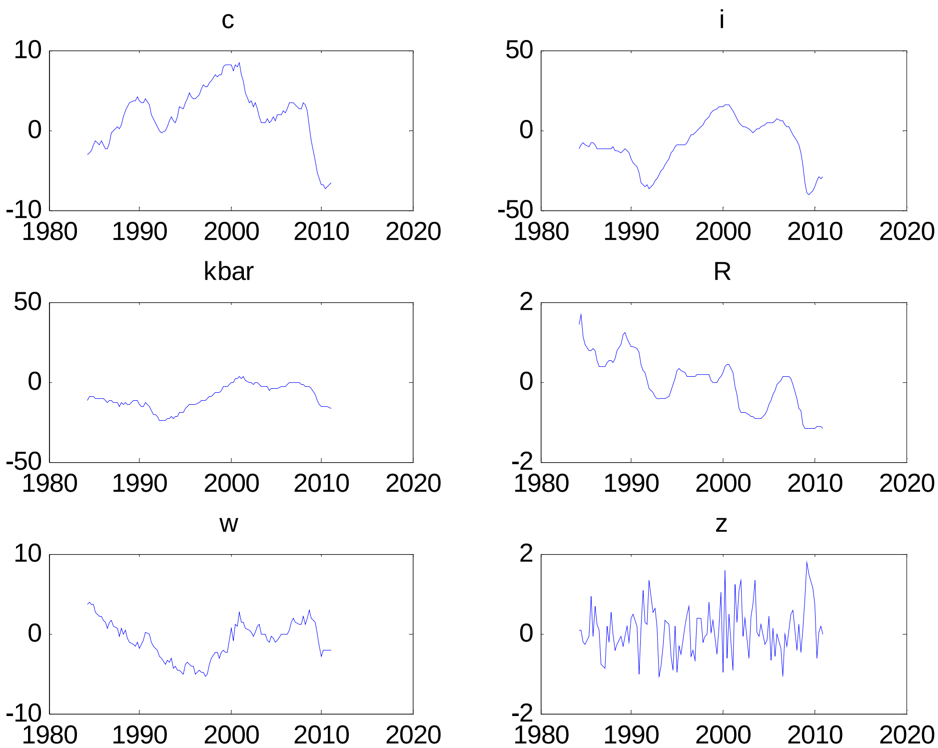
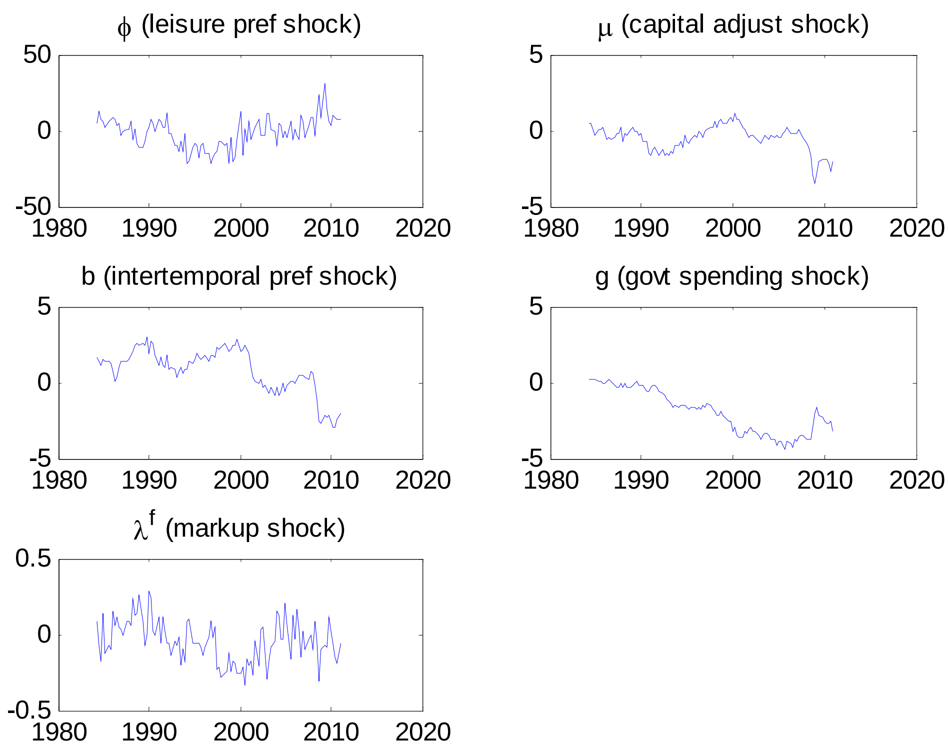


Figure 8

Endogenous State Variables



Technical Appendix: PRISM Model Documentation

Keith Sill

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1 Model Structure

The FRBPHIL DSGE forecasting model (PRISM) is developed and maintained by the Real Time Data Research Center (RTDRC) and by the Research Department of the Federal Reserve Bank of Philadelphia. The model is medium-scale and features nominal and real frictions that include wage and price stickiness, habit formation, and capital adjustment costs. This section of the model documentation describes the DSGE model, which is essentially the model in Del Negro, Schorfheide, Smets, and Wouters (2007).

1.1 Final Goods Producers

There is a final good $Y_t$that is produced as a composite of a continuum of intermediate goods $Y_t(i)$using the technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{f,t}}} \right]^{1+\lambda_{f,t}} \tag{1}$$

with $\lambda_{f,t} \in (0, \infty)$ following the exogenous process:

$$\ln \lambda_{f,t} = (1 - \rho_{\lambda_f}) \ln \lambda_f + \rho_{\lambda_f} \ln \lambda_{f,t-1} + \sigma_{\lambda_f} ?_{\lambda,t} \tag{2}$$

The variable $\lambda_{f,t}$is the desired markup over marginal cost that intermediate goods producers would like to charge. From the first-order conditions for profit maximization and the zero-profit condition (final goods producers are perfectly competitive firms) the demand for intermediate goods is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1+\lambda_{f,t}}{\lambda_{f,t}}} Y_t \tag{3}$$

with the composite good price given by:

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{f,t}}} di \right]^{-\lambda_{f,t}} \tag{4}$$

1.2 Intermediate Goods Producers

There is a continuum of intermediate goods indexed by i. They are produced using the technology:

$$Y_t(i) = \max\{Z_t^{1-\alpha} K_t(i)^{\alpha} L_t(i)^{1-\alpha} - Z_t\Phi, 0\} \tag{5}$$

where $Z_t$is exogenous technological progress that is assumed non-stationary. We define $z_t = \ln(Z_t/Z_{t-1})$and assume that it follows the process:

$$(z_t - \gamma) = \rho_z(z_{t-1} - \gamma) + \varepsilon_{z,t}.$$

Prices are assumed to be sticky and adjust following Calvo (1983). Each firm can readjust prices optimally with probability $1-\zeta_p$in each period. Firms that are unable to reoptimize their prices $P_t(i)$adjust prices mechanically according to:

$$P_t(i) = (\pi_{t-1})^{\iota_p} (\pi_*)^{1-\iota_p} \tag{6}$$

where $\pi_t = P_t/P_{t-1}$and $\pi_*$is the steady state inflation rate of the final good. Those firms that re-optimize price choose a price level $\tilde{P}_t(i)$that maximizes the expected present discounted value profits in all states of nature in which the firm maintains that price in the future:

$$\begin{aligned} &max_{\tilde{P}_t(i)} \quad \Xi_r^p \left(\tilde{P}_t(i) - MC_t\right) Y_t(i) + \\ &E_t \sum_{s=1}^{\infty} \zeta_p^s \beta^s \Xi_{t+s}^p \left(\tilde{P}_t(i)(\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right) - MC_{t+s})Y_{t+s} \end{aligned} \tag{7}$$

subject to

$$Y_{t+s}(i) = \left(\frac{\tilde{P}_t(i)\left(\Pi_{l=1}^s \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p}\right)}{P_{t+s}}\right)^{-\frac{1+\lambda_{f,t}}{\lambda_{f,t}}} Y_{t+s}$$

where $\pi_t \equiv P_t/P_{t-1}$, $\beta^s \Xi_{t+s}^p$ is the household's discount factor and $MC_t$is the firm's marginal cost. Markets are assumed to be complete so all households face the same discount factor. All firms that can re-adjust price face an identical problem. We will consider only a symmetric equilibrium in which all adjusting firms choose the same price – which means that we can drop the $i$index. It then follows that the aggregate price level can be expressed as:

$$P_t = \left[(1-\zeta_p)\tilde{P}^{-\frac{1}{\lambda_f}} + \zeta_p \left(\pi_{t-1}^{\iota_p} \pi_*^{1-\iota_p} P_{t-1}\right)^{-\frac{1}{\lambda_f}}\right]^{-\lambda_f}.$$

In the estimation, we shut down inflation indexation by setting $\iota_p = 0$.

1.3 Households

The objective function for household j is given by:

$$E_t \sum_{s=0}^{\infty} b_{t+s} \left[\ln(C_{t+s}(j) - hC_{t+s-1}(j)) - \frac{\varphi_{t+s}}{1+\nu_l} L_{t+s}(j)^{1+\nu_l} + \frac{\chi_{t+s}}{1-\nu_m}\left(\frac{M_{t+s}(j)}{Z_{t+s}P_{t+s}}\right)^{1-\nu_m}\right]$$

where $C_t(i)$is consumption, $L_t(i)$is labor supply, and $M_t(j)$is money holdings. Household preferences are subject to three shocks: an intertemporal shifter b_t, a labor supply shock φ_t, and a money demand shock χ_t. All preference shocks are assumed to follow an AR(1) process in logs. The household budget constraint, written in nominal terms, is given by:

$$P_{t+s}C_{t+s}(j) + P_{t+s}I_{t+s}(j) + B_{t+s}(j) \leq R_{t+s}B_{t+s-1}(j) + M_{t+s-1}(j) + \\ \Pi_{t+s} + W_{t+s}(j)L_{t+s}(j) + R^k_{t+s}u_{t+s}(j)\hat{K}_{t+s-1}(j) - P_{t+s}a(u_{t+s}(j))\hat{K}_{t+s-1}(j)$$

where $I_t(j)$is investment, $\hat{K}_t(j)$is capital holdings, $u_t(j)$is the rate of capital utilization, and $B_t(j)$is holdings of government bonds. The gross nominal interest rate paid on government bonds is $R_t$and $\Pi_t$is the per-capita profit the household gets from owning firms. Household labor is paid wage $W_t(j)$and households rent an "effective" amount of capital to firms $K_t(j) = u_t(j)\hat{K}_{t-1}(j)$. In return, they receive $R^k_t u_t(j)\hat{K}_{t-1}(j)$. Households pay a consumption cost associated with capital utilization given by $a(u_t(j))\hat{K}_{t-1}(j)$. Capital accumulation is governed by:

$$\hat{K}_t(j) = (1-\delta)\hat{K}_{t-1}(j) + \mu_t\left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right)I_t(j)$$

where $\delta$is the rate of depreciation, $S(\cdot)$is the cost of adjusting investment ($S' > 0,\ S'' > 0$), and $\mu_t$is a stochastic shock to the price of investment relative to consumption, assumed to follow an AR(1) process in logs.

1.4 The Labor Market

The labor market has labor packers that buy labor from households, combine it, and resell it to the intermediate goods producing firms. Labor used by the intermediate goods producers is a composite:

$$L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}}\right]^{1+\lambda_{w,t}}$$

The labor packers maximize profits in a perfectly competitive environment, which leads to the labor demand:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}}.$$

Combining labor demand with the zero-profit condition leads to the aggregate wage expression:

$$W_t = \left[\int_0^1 W_t(j)^{\frac{1}{\lambda_{w,t}}}di\right]^{\lambda_{w,t}}.$$

In the estimation, we fix $\lambda_{w,t} = \lambda_w \in (0,\infty)$. Households have market power, but wage adjustment is subject to a rigidity as in Calvo (1983). Each period, a fraction $1-\zeta_w$of households re-optimize their wage. For those that are unable to re-optimize, $W_t(j)$adjusts as a geometric average of the steady state rate increase in wages and last period's productivity times last period's inflation. For those households that can re-optimize, the problem is to choose a wage $\tilde{W}_t(j)$that maximizes utility in all states of nature in which the household wage is to be held at its chosen value:

$$max_{\tilde{W}_t(j)} E_t \sum_{s=0}^{\infty} (\zeta_w \beta)^s b_{t+s} \left[-\frac{\varphi_{t+s}}{1+\nu_l} L_{t+s}(j)^{1+\nu_l} + \dots \right]$$

subject to

$$W_{t+s}(j) = \left(\Pi_{l=1}^{s} (\pi_*)^{1-\iota_w} (\pi_{t+l-1} e^{z^*_{t+l-1}})^{\iota_w} \right) \tilde{W}_t(j)$$

for $s = 1, \dots, \infty$ as well as to the household budget constraint and the labor demand condition. In the estimation, we shut down nominal wage indexation by setting $\iota_w = 0$.

1.5 Government Policies

The government consists of a fiscal authority and a monetary authority. The monetary authority sets the nominal interest rate according to the feedback rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_*} \right)^{\psi_R} \left(\frac{Y_t}{Y} \right)^{\psi_Y} \right]^{1-\rho_R} \epsilon_{R,t}.$$

The fiscal authority balances its budget by issuing short-term bonds. Government spending is exogenous and given by:

$$G_t = (1 - 1/g_t) Y_t$$

where the government spending shock $g_t$is assumed to follow an AR(1) process.

1.6 Exogenous Processes

There are seven exogenous shocks in the model. The follow the processes:

- Technology process. Let $z_t = \ln(Z_t/Z_{t-1})$

$$(z_t - \gamma) = \rho_z (z_{t-1} - \gamma) + \sigma_z \epsilon_{z,t}$$

- Preference for leisure:

$$\ln \phi_t = (1 - \rho_\phi) \ln \phi + \rho_\phi \ln \phi_{t-1} + \sigma_\phi \epsilon_{\phi,t}$$

- Money demand (this shock is shut down in the estimation and the model is not estimated using a monetary aggregate):

$$\ln \chi_t = (1 - \rho_\chi) \ln \chi + \rho_\chi \ln \chi_{t-1} + \sigma_\chi \epsilon_{\chi,t}$$

- Price-markup shock:

$$\ln \lambda_t = (1 - \rho_\lambda) \ln \lambda + \rho_\lambda \ln \lambda_{t-1} + \sigma_\lambda \epsilon_{\lambda,t}$$

- Capital adjustment cost (marginal efficiency of investment):

$$\ln \mu_t = (1 - \rho_\mu) \ln \mu + \rho_\mu \ln \mu_{t-1} + \sigma_\mu \epsilon_{\mu,t}$$

- Intertemporal preference shifter:

$$\ln b_t = \rho_b \ln b_{t-1} + \sigma_b \epsilon_{b,t}$$

- Government spending shock:

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \epsilon_{g,t}$$

- Monetary policy shock:

$$\epsilon_{R,t}$$

1.7 Log-Linearized Model

Variables are detrended where appropriate and expressed as deviations from steady state.

- Detrending:

$$\begin{aligned}
&y_t = Y_t/Z_t,\ c_t = C_t/Z_t,\ i_t = I_t/Z_t,\ k_t = K_t/Z_t,\ \bar{k}_t = \bar{K}_t/Z_t,\\
&r_t^k = R_t^k/P_t,\ w_t = W_t/(P_t Z_t),\ \tilde{w}_t = \tilde{W}_t/W_t,\ \xi_t = \Xi_t Z_t,\\
&\xi_t^k = \Xi_t^k Z_t,\ z_t = \log(Z_t/Z_{t-1})
\end{aligned}$$

- Marginal cost:

$$mc_t = (1 - \alpha) w_t + \alpha r_t^k. \tag{8}$$

- Phillips curve:

$$\pi_t = \beta \, \mathrm{E}_t \left[\pi_{t+1}\right] + \frac{(1 - \zeta_p \beta)(1 - \zeta_p)}{\zeta_p} mc_t + \frac{1}{\zeta_p} \lambda_{f,t}, \tag{9}$$

with normalization:

$$\lambda_{f,t} = [(1 - \zeta_p \beta)(1 - \zeta_p) \lambda_f / (1 + \lambda_f)] \tilde{\lambda}_{f,t}$$

and $\lambda_f$is the steady state of $\tilde{\lambda}_{f,t}$.

- Capital-labor ratio:

$$k_t - L_t = w_t - r_t^k \tag{10}$$

- Marginal utility of consumption:

$$\begin{aligned}(e^\gamma - h\beta)(e^\gamma - h)\xi_t = -(e^{2\gamma} + \beta h^2)c_t + \beta h e^\gamma E_t[c_{t+1} + z_{t+1}] + \\ he^\gamma(c_{t-1} - z_t) + e^\gamma(e^\gamma - h)\tilde{b}_t - \beta h(e^\gamma - h)\mathrm{E}_t[\tilde{b}_{t+1}]\end{aligned} \tag{11}$$

with the normalization:

$$\bar{b}_t = e^\gamma(e^\gamma - h)/(e^{2\gamma} + \beta h^2)b_t.$$

- Consumption euler equation:

$$\xi_t = \mathrm{E}_t[\xi_{t+1}] + R_t - \mathrm{E}_t[\pi_{t+1}] - \mathrm{E}_t[z_{t+1}]. \tag{12}$$

- Capital accumulation:

$$\begin{aligned}k_t &= u_t - z_t + \bar{k}_{t-1} \\ \bar{k}_t &= (2 - e^\gamma - \delta)[\bar{k}_{t-1} - a_t] + (e^\gamma + \delta - 1)[i_t + (1+\beta)S'' e^{2\gamma}\mu_t]\end{aligned} \tag{13}$$

- Investment:

$$i_t = \frac{1}{1+\beta}[i_{t-1} - z_t] + \frac{\beta}{1+\beta}\mathrm{E}_t[i_{t+1} + z_{t+1}] + \frac{1}{(1+\beta)S'' e^{2\gamma}}(\xi_t^k - \xi_t) + \mu_t \tag{14}$$

where $\xi_t^k$is the value of installed capital, evolving according to:

$$\xi_t^k - \xi_t = \beta e^{-\gamma}(1-\delta)\mathrm{E}_t[\xi_{t+1}^k - \xi_{t+1}] + \mathrm{E}_t[(1 - (1-\delta)\beta e^{-\gamma})r_{t+1}^k - (R_t - \pi_{t+1})]$$

- Capital utilization:

$$u_t = \frac{r_*^k}{a''} r_t^k. \tag{15}$$

- Optimal real wage:

$$\begin{aligned}\tilde{w}_t = \zeta_w \beta E_t[\tilde{w}_{t+1} + \Delta w_{t+1} + \pi_{t+1} + z_{t+1}] + \\ \tfrac{1-\zeta_w\beta}{1+\nu_l(1+\lambda_w)/\lambda_w}(\nu_l L_t - w_t - \xi_t + \tilde{b}_t + \tfrac{1}{1-\zeta_w\beta}\varphi_t)\end{aligned} \tag{16}$$

- Real wage:

$$w_t = w_{t-1} - \pi_t - z_t + \frac{1-\zeta_w}{\zeta_w}\tilde{w}_t. \tag{17}$$

- Production function:

$$y_t = (1-\alpha)L_t + \alpha k_t \tag{18}$$

- Resource constraint:

$$y_t = (1+g_*)[\frac{c_*}{y_*}c_t + \frac{i_*}{y_*}(i_t + \frac{r_*^k}{e^\gamma - 1 + \delta}u_t)] + g_t \tag{19}$$

- Monetary policy rule:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)(\psi_1 \pi_t + \psi_2 y_t) + \sigma_R \epsilon_{R,t}. \tag{20}$$

2 Empirical Application

We use post-1983 U.S. data to estimate the DSGE model. We begin with a description of our data set and the prior distribution for the DSGE model parameters.

2.1 Data and Priors

Seven series are included in the vector of core variables $y_t$that is used for the estimation of the DSGE model: the growth rates of output, consumption, investment, and nominal wages, as well as the levels of hours worked, inflation, and the nominal interest rate. These series are obtained from Haver Analytics (Haver mnemonics are in italics). Real output is computed by dividing the nominal series (*GDP*) by population 16 years and older (*LN16N*) as well as the chained-price GDP deflator (*JGDP*). Consumption is defined as nominal personal consumption expenditures (C) less consumption of durables (*CD*). We divide by *LN16N* and deflate using *JGDP*. Investment is defined as *CD* plus nominal gross private domestic investment (I). It is similarly converted to real per-capita terms. We compute quarter-to-quarter growth rates as log difference of real per capita variables and multiply the growth rates by 100 to convert them into percentages.

Our measure of hours worked is computed by taking non-farm business sector hours of all persons (*LXNFH*), dividing it by *LN16N*, and then scaling to get mean quarterly average hours to about 257. We then take the log of the series multiplied by 100 so that all figures can be interpreted as percentage deviations from the mean. Nominal wages are computed by dividing total compensation of employees (*YCOMP*) by the product of *LN16N* and our measure of average hours. Inflation rates are defined as log differences of the core PCE deflator index (*JCXFE*) and converted into percentages. The nominal interest rate corresponds to the average effective federal funds rate (*FFED*) over the quarter and is annualized.

Our choice of prior distribution for the DSGE model parameters follows DSSW and the specification of what is called a "standard" prior in Del Negro and Schorfheide (2008). The prior is summarized in the first four columns of Table 1. To make this paper self-contained we briefly review some of the details of the prior elicitation. Priors for parameters that affect the steady state relationships, e.g., the capital share α in the Cobb-Douglas production function or the capital depreciation rate are chosen to be commensurable with pre-sample (1955 to 1983) averages in U.S. data. Priors for the parameters of the exogenous shock processes are chosen such that the implied variance and persistence of the endogenous model variables is broadly consistent with the corresponding pre-sample moments. Our prior for the Calvo parameters that control the degree of nominal rigidity are fairly agnostic and span values that imply fairly flexible as well as fairly rigid prices and wages. Our prior for the central bank's responses to inflation and output movements is roughly centered at Taylor's (1993) values. The prior for the interest rate smoothing parameter

ρ_R is almost uniform on the unit interval.

The 90% interval for the prior distribution on υ_l implies that the Frisch labor supply elasticity lies between 0.3 and 1.3, reflecting the micro-level estimates at the lower end, and the estimates of Kimball and Shapiro (2003) and Chang and Kim (2006) at the upper end. The density for the adjustment cost parameter S''spans values that Christiano, Eichenbaum, and Evans (2005) find when matching DSGE and vector autoregression (VAR) impulse response functions. The density for the habit persistence parameter $h$is centered at 0.7, which is the value used by Boldrin, Christiano, and Fisher (2001). These authors find that $h = 0.7$enhances the ability of a standard DSGE model to account for key asset market statistics. The density for a'' implies that in response to a 1% increase in the return to capital, utilization rates rise by 0.1 to 0.3%.

2.2 State Space Representation

The state space representation for the model estimation is given by:

$$S_t = T\ S_{t-1} + R\ e_t \tag{21}$$

with measurement equation:

$$\begin{bmatrix} \Delta \ln(y_t) \\ \Delta \ln(c_t) \\ \Delta \ln(I_t) \\ \ln(H_t) \\ \Delta \ln(W_t) \\ \pi_t \\ R_t \end{bmatrix} = D + Z * S_t \tag{22}$$

Note that we do not allow for measurement error in the estimation.

3 Parameter Estimates

Table 1: Prior and Posterior of DSGE Model Parameters (Part 1)

		Prior		Posterior	
Name	Density	Para (1)	Para (2)	Mean	90% Intv.
		Household			
h	$\mathcal{B}$	0.70	0.05	0.76	[0.71 , 0.81]
a''	$\mathcal{G}$	0.20	0.10	0.26	[0.10 , 0.43]
ν_l	$\mathcal{G}$	2.00	0.75	1.91	[1.07 , 2.69]
ζ_w	$\mathcal{B}$	0.60	0.20	0.74	[0.58 , 0.87]
$400 * (1/\beta - 1)$	$\mathcal{G}$	2.00	1.00	1.124	[0.37 , 1.86]
		Firms			
α	$\mathcal{B}$	0.33	0.10	0.16	[0.13 , 0.19]
ζ_p	$\mathcal{B}$	0.60	0.20	0.90	[0.89 , 0.92]
S''	$\mathcal{G}$	4.00	1.50	5.30	[3.22 , 7.25]
λ_f	$\mathcal{G}$	0.15	0.10	0.16	[0.01 , 0.31]
		Monetary Policy			
$400\pi^*$	$\mathcal{N}$	3.00	1.50	3.31	[2.60 , 4.17]
ψ_1	$\mathcal{G}$	1.50	0.40	2.25	[1.90 , 2.64]
ψ_2	$\mathcal{G}$	0.20	0.10	0.06	[0.04 , 0.08]
ρ_R	$\mathcal{B}$	0.50	0.20	0.81	[0.77 , 0.86]

Table 1: PRIOR AND POSTERIOR OF DSGE MODEL PARAMETERS (PART 2)

Name	Density	Prior Para (1)	Para (2)	Posterior Mean	90% Intv.
		Shocks			
400γ	$\mathcal{G}$	2.00	1.00	1.66	[1.17 , 2.13]
g^*	$\mathcal{G}$	0.30	0.10	0.28	[0.13 , 0.41]
ρ_a	$\mathcal{B}$	0.20	0.10	0.25	[0.14 , 0.36]
ρ_μ	$\mathcal{B}$	0.80	0.05	0.85	[0.80 , 0.90]
ρ_{λ_f}	$\mathcal{B}$	0.60	0.20	0.16	[0.07 , 0.26]
ρ_g	$\mathcal{B}$	0.80	0.05	0.96	[0.95 , 0.98]
ρ_b	$\mathcal{B}$	0.60	0.20	0.91	[0.87 , 0.95]
ρ_ϕ	$\mathcal{B}$	0.60	0.20	0.71	[0.56 , 0.91]
σ_a	$\mathcal{IG}$	0.75	2.00	0.63	[0.56 , 0.71]
σ_μ	$\mathcal{IG}$	0.75	2.00	0.39	[0.32 , 0.45]
σ_{λ_f}	$\mathcal{IG}$	0.75	2.00	0.17	[0.15 , 0.20]
σ_g	$\mathcal{IG}$	0.75	2.00	0.35	[0.31 , 0.39]
σ_b	$\mathcal{IG}$	0.75	2.00	0.50	[0.36 , 0.62]
σ_ϕ	$\mathcal{IG}$	4.00	2.00	9.08	[3.44 , 14.16]
σ_R	$\mathcal{IG}$	0.20	2.00	0.14	[0.12 , 0.16]

Notes: Para (1) and Para (2) list the means and the standard deviations for the Beta ($\mathcal{B}$), Gamma ($\mathcal{G}$), and Normal ($\mathcal{N}$) distributions; the upper and lower bound of the support for the Uniform ($\mathcal{U}$) distribution; s and ν for the Inverse Gamma ($\mathcal{IG}$) distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-(\nu+1)}e^{-\nu s^2/2\sigma^2}$. The joint prior distribution is obtained as a product of the marginal distributions tabulated in the table and truncating this product at the boundary of the determinacy region. Posterior summary statistics are computed based on the output of the posterior sampler. The following parameters are fixed: $\delta = 0.025$, $\lambda_w = 0.3$. Estimation sample: 1984:I to 2010:I.

4.1 Impulse Responses

Impulse responses are to a 1-standard deviation shock.

Figure 1: Response to technology shock

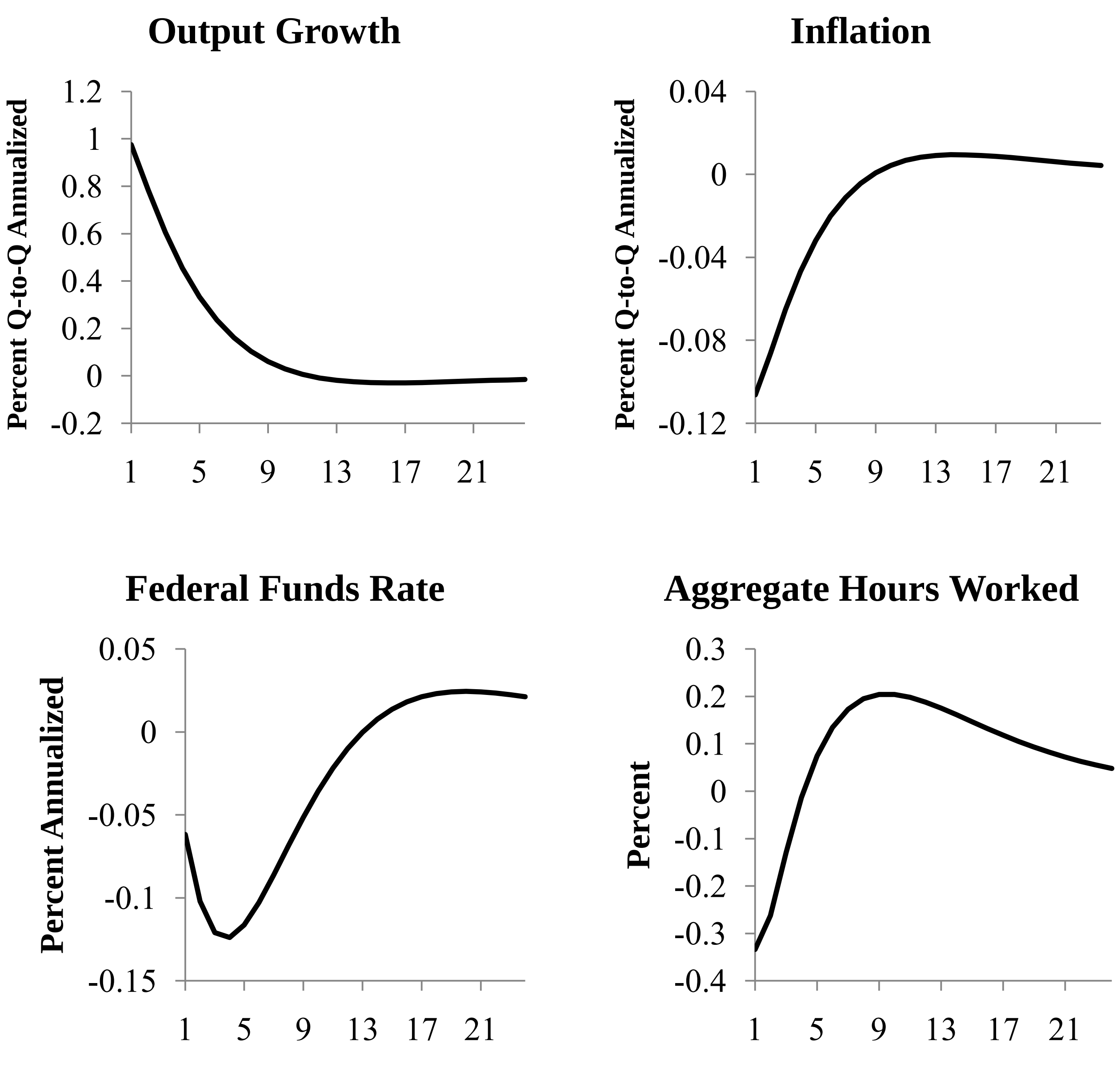

Figure 1: Response to technology shock

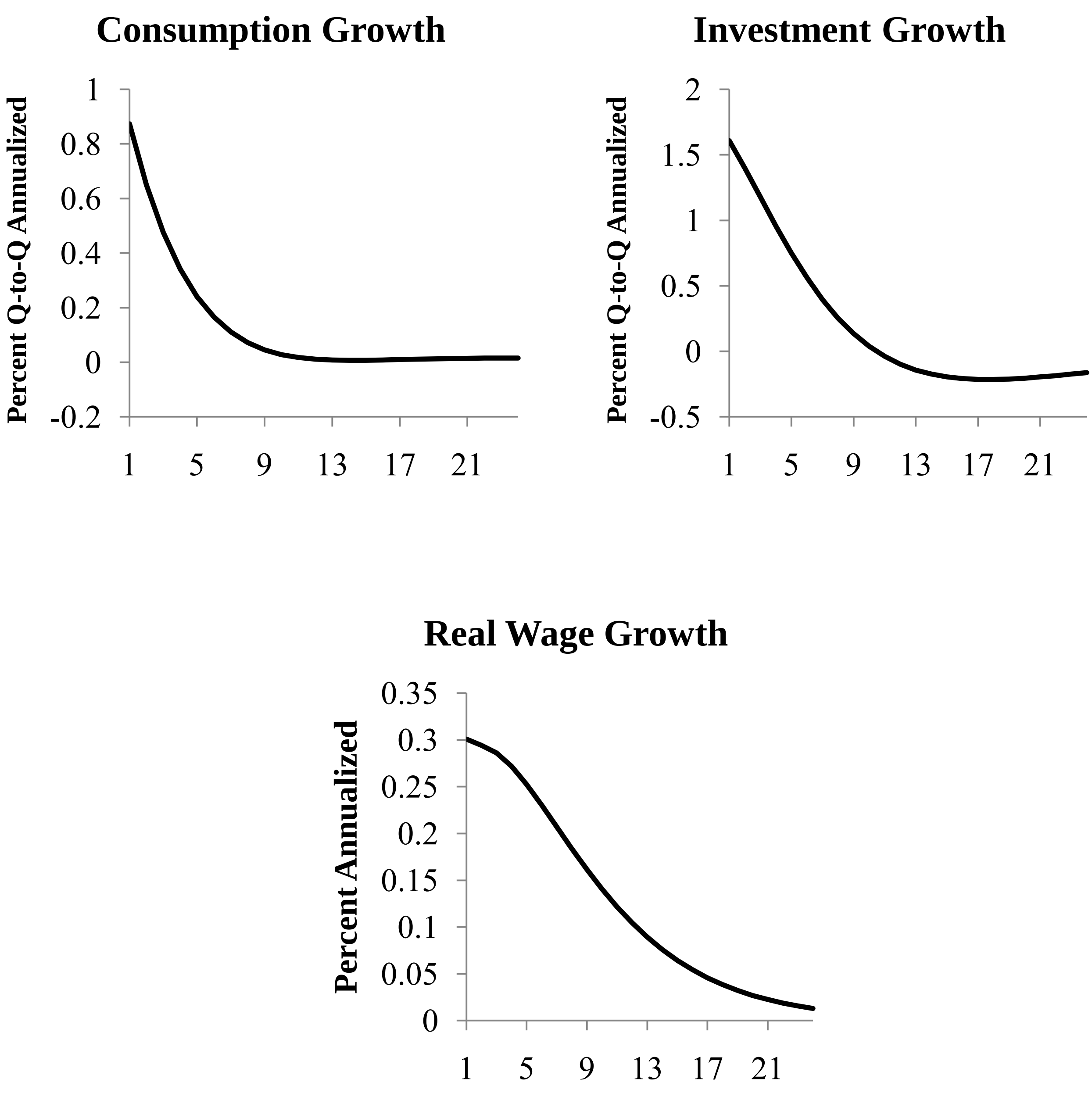

Figure 2: Response to mei shock

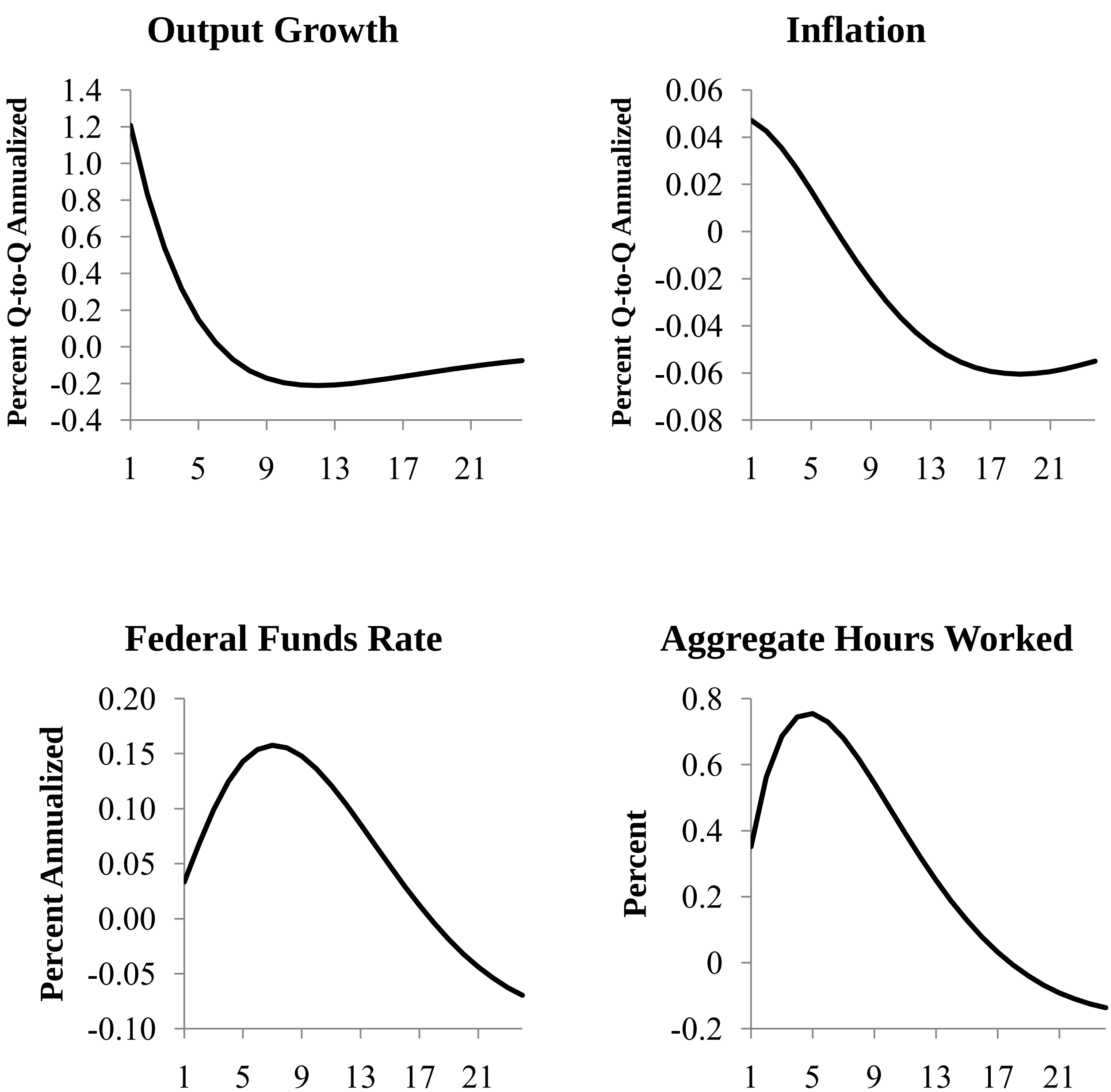

Figure 2: Response to mei shock

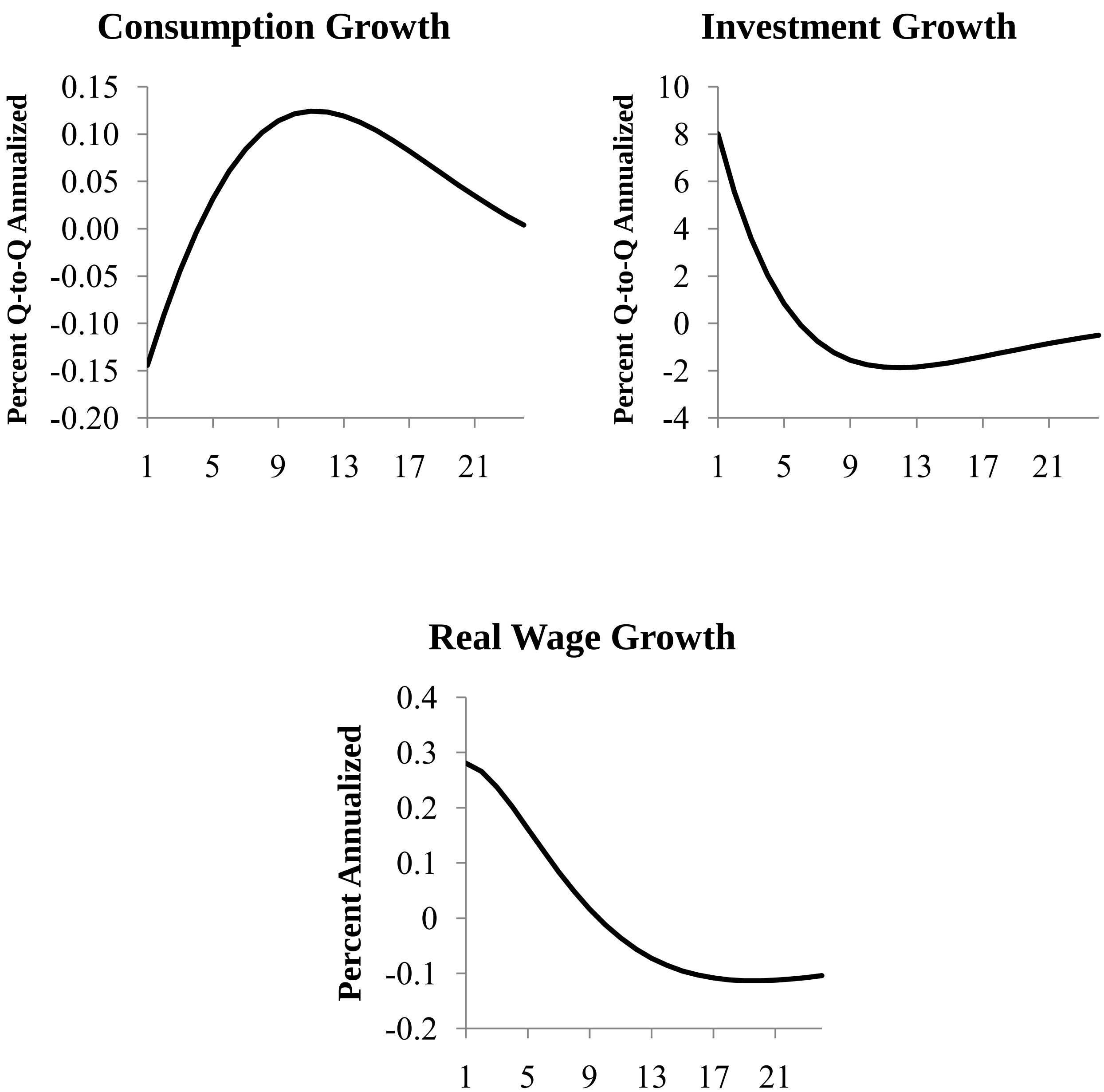

Figure 3: Response to leisure shock

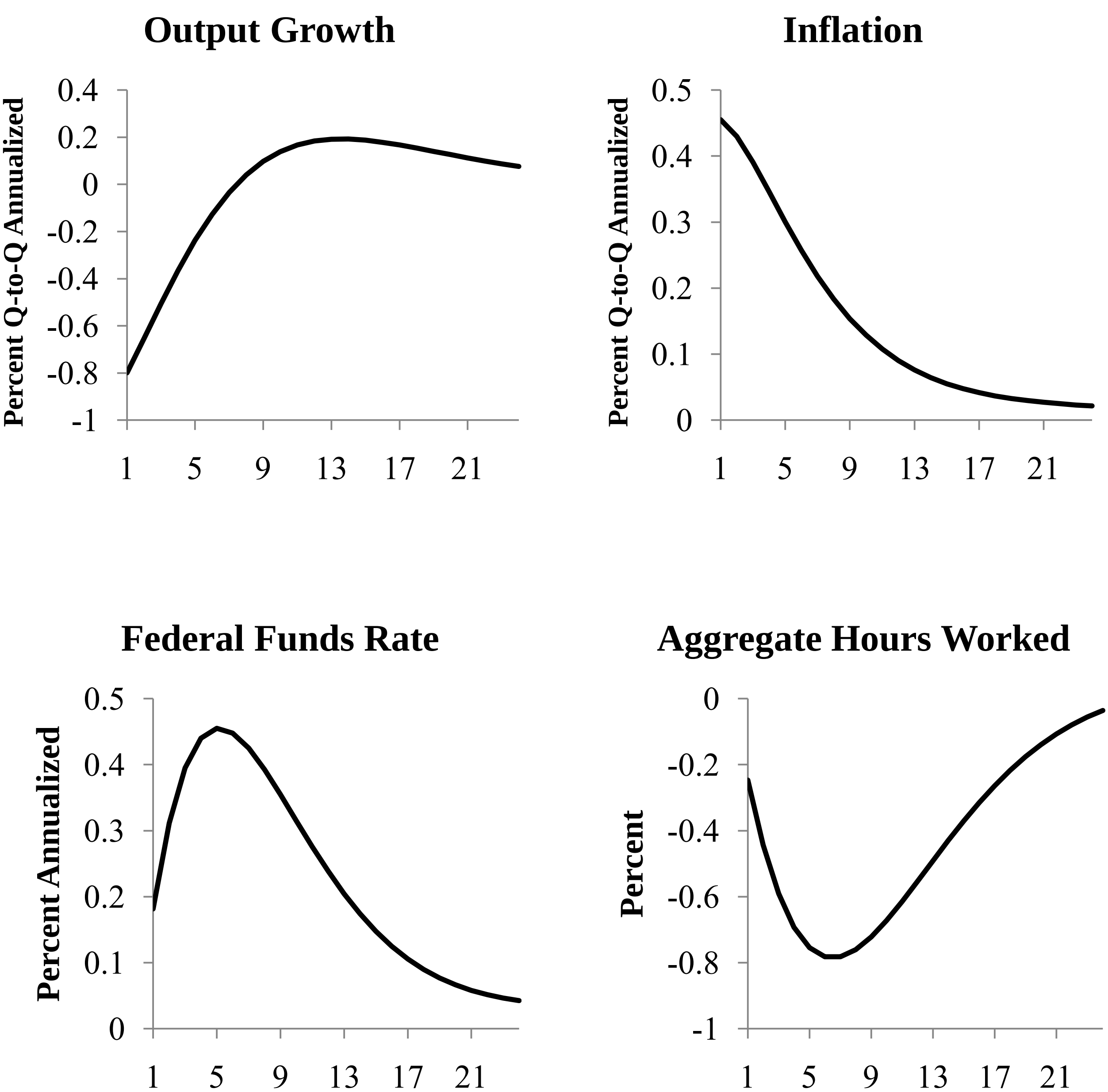

Figure 3: Response to leisure shock

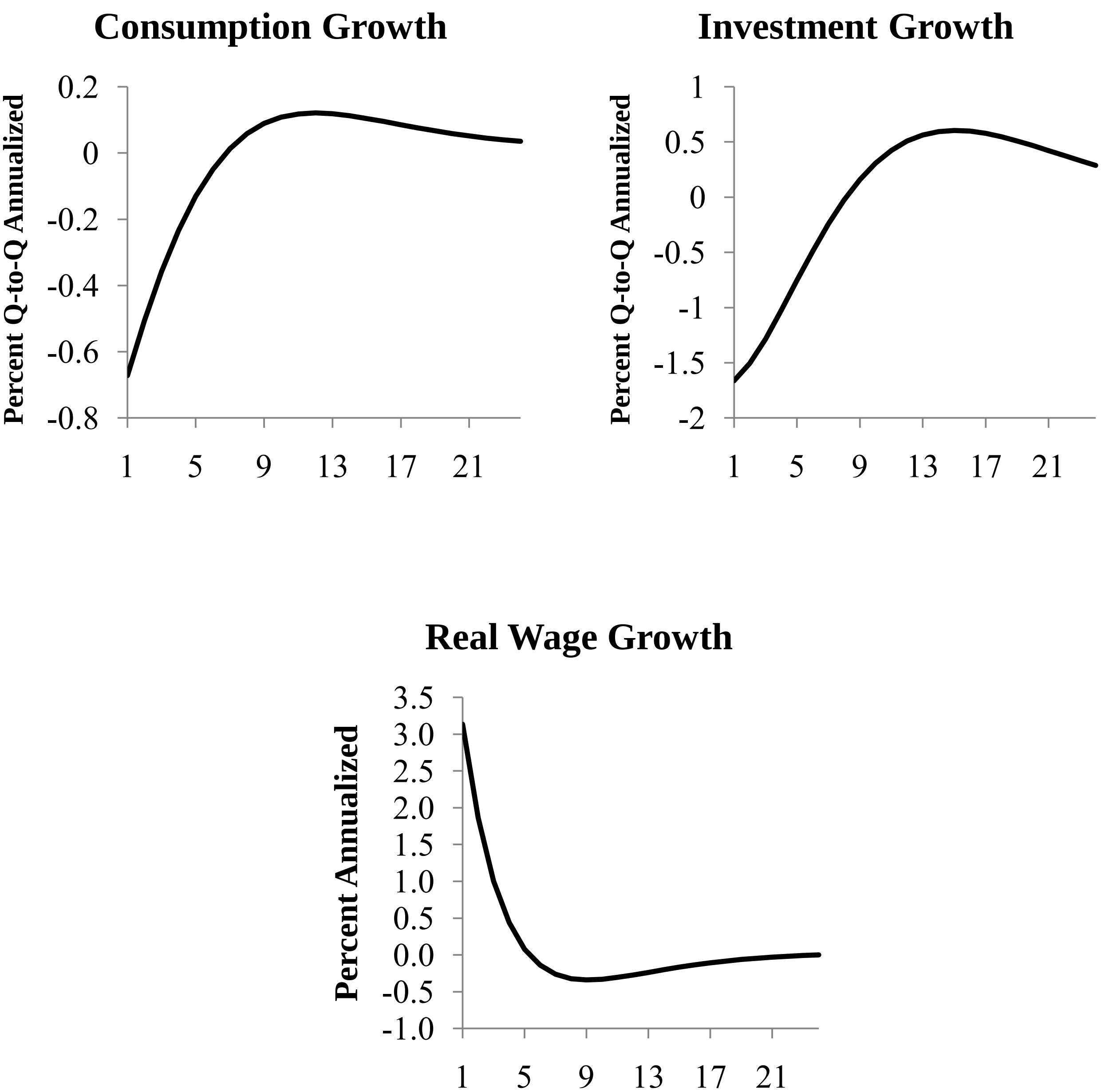

Figure 4: Response to financial shock

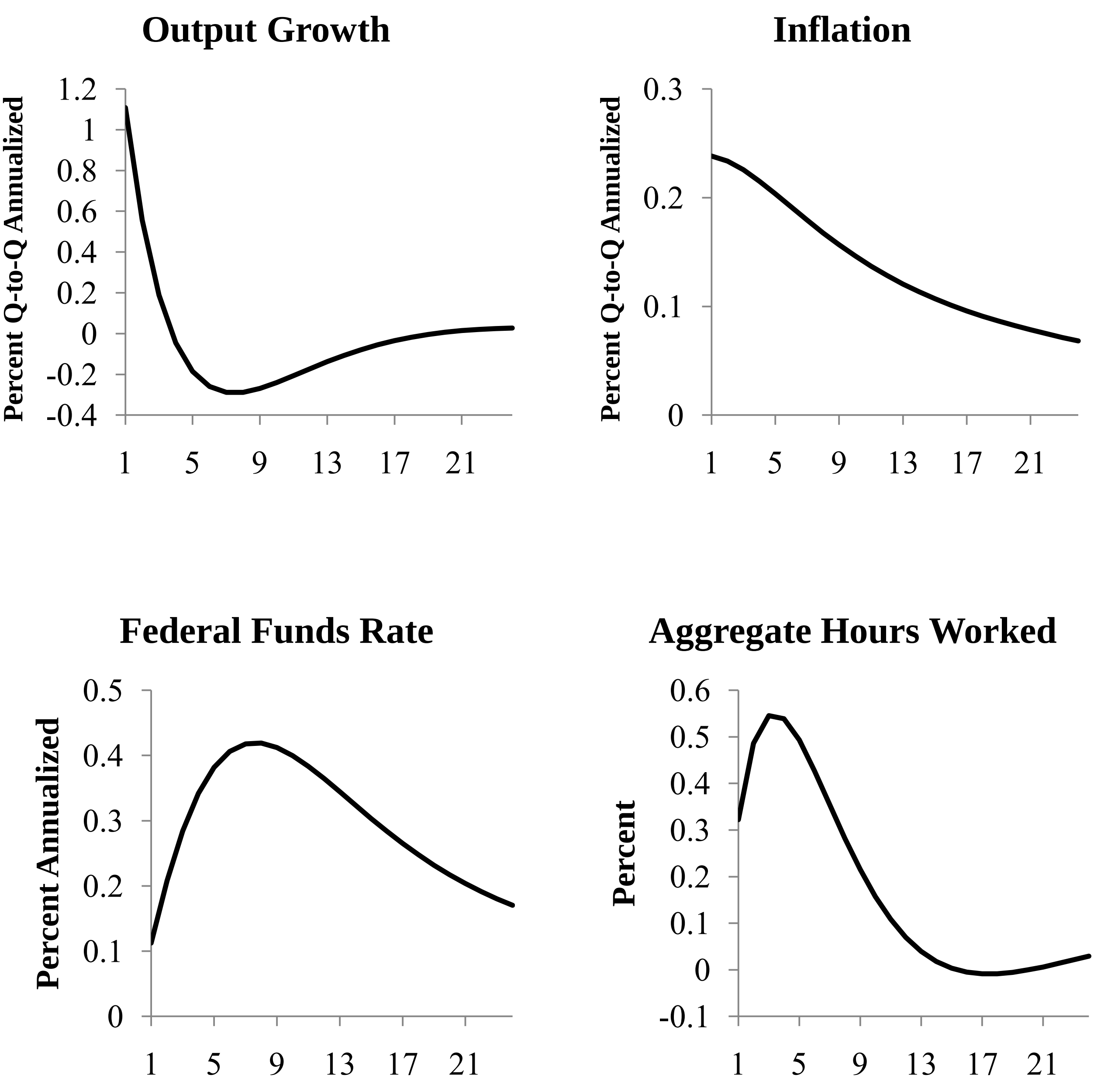

Figure 4: Response to financial shock

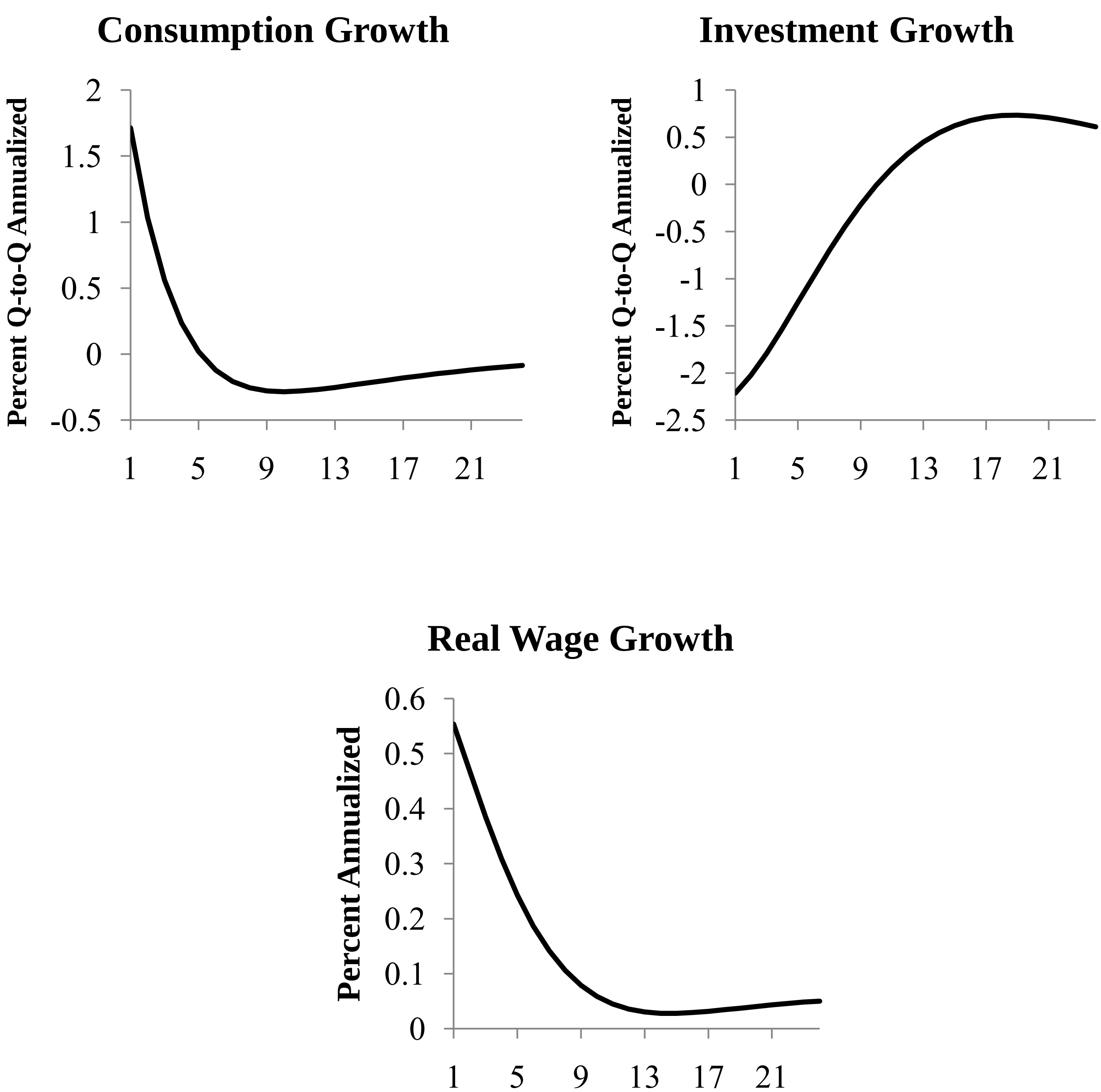

Figure 5: Response to markup shock

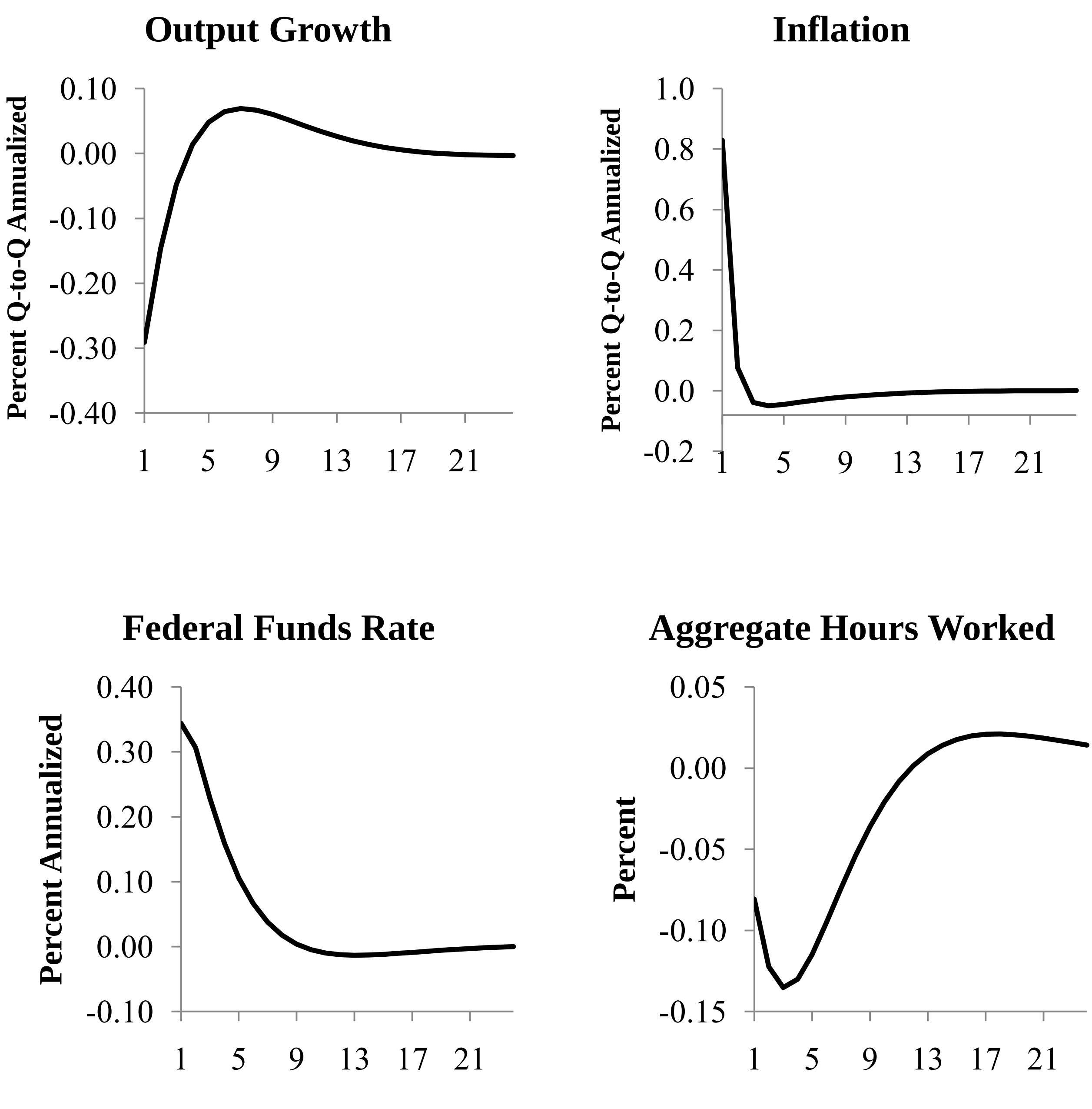

Figure 5: Response to markup shock

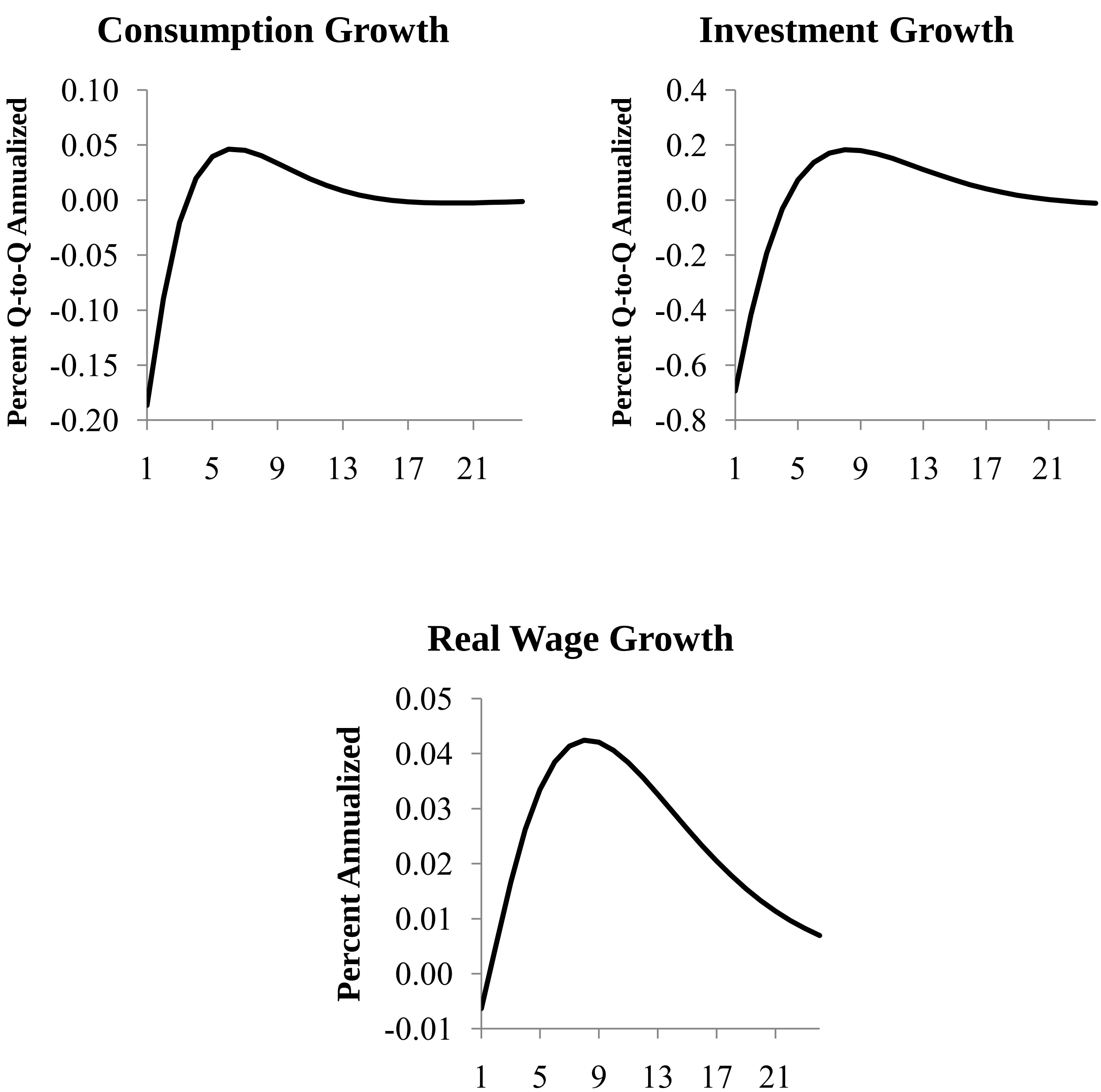

Figure 6: Response to monetary policy shock

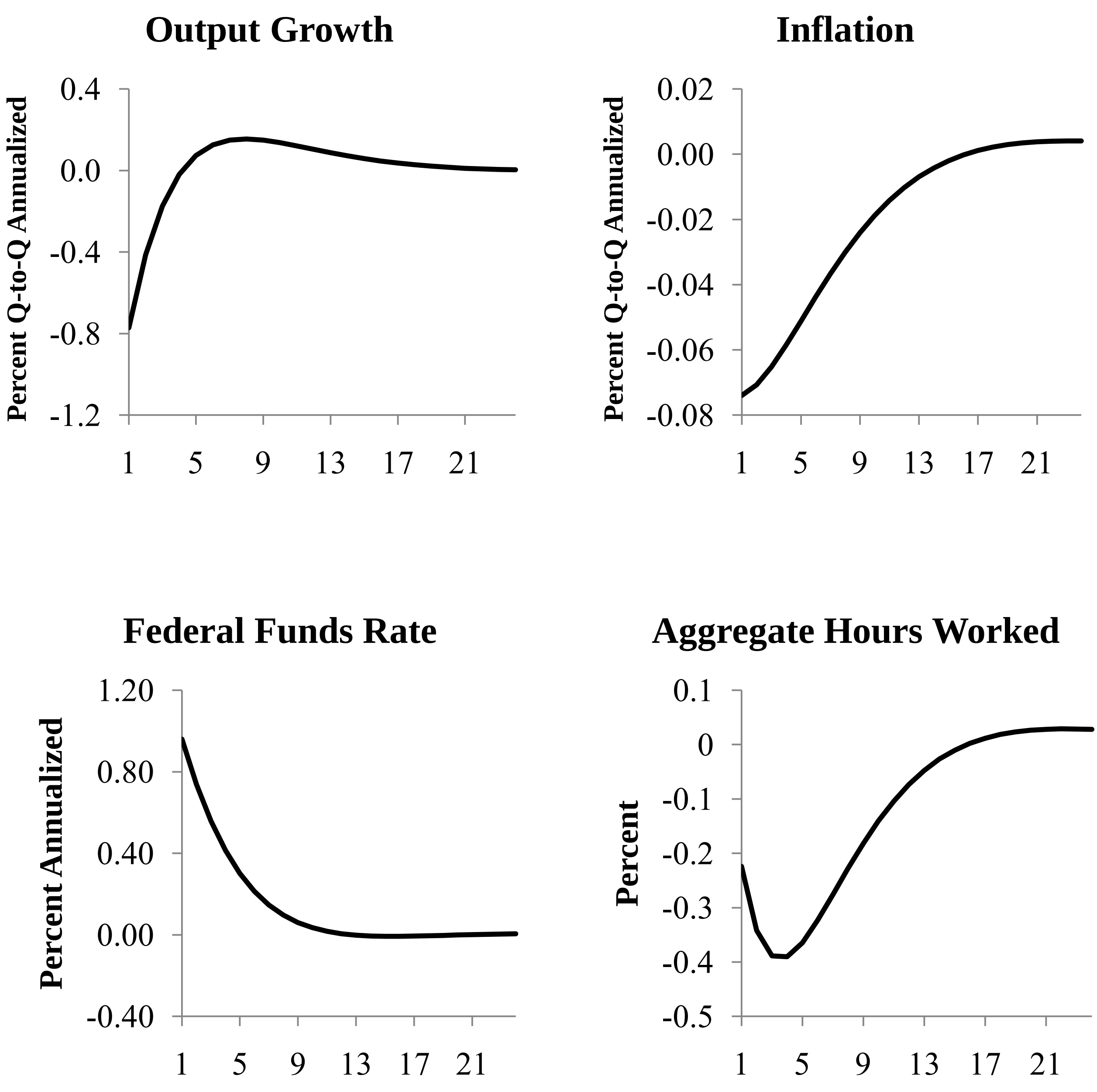

Figure 6: Response to monetary policy shock

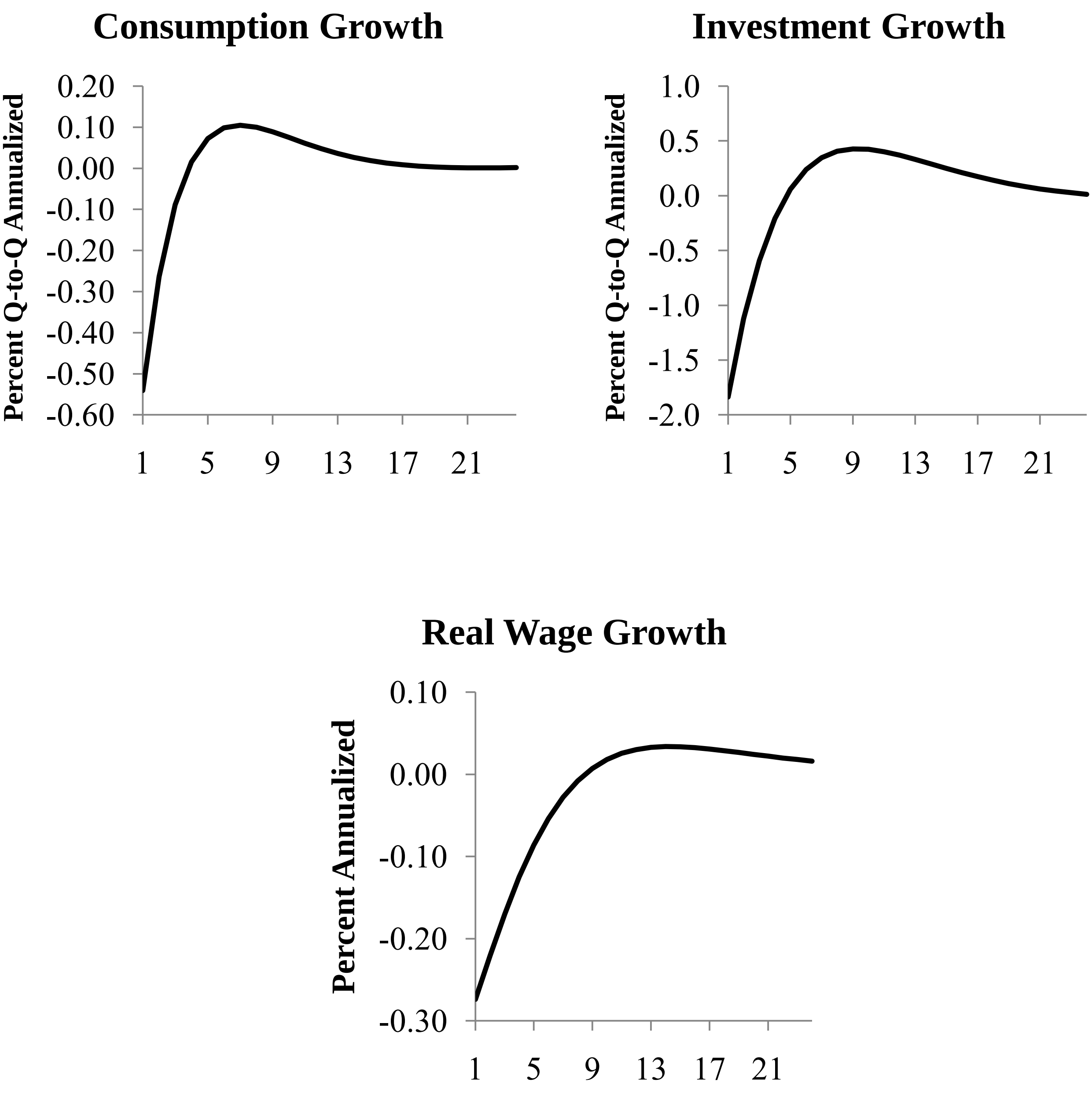

Figure 7: Response to government shock

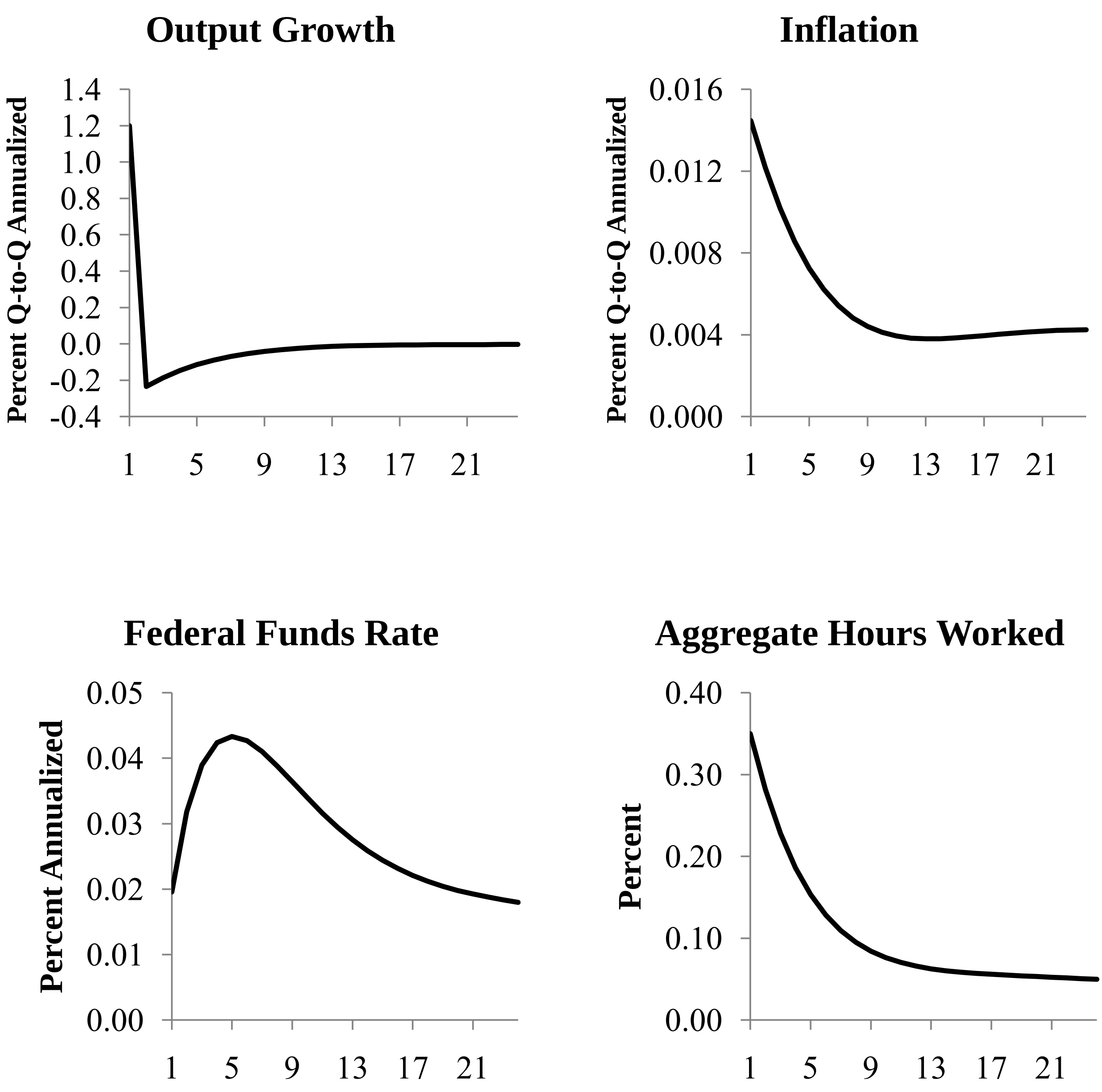

Figure 7: Response to government shock