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r^*: Concepts, Measures, and Uses

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1. Introduction

Most central banks implement monetary policy by setting a target (or a range) for an overnight interbank interest rate. The level of the nominal short-term interest rate does not, by itself, reveal the level of monetary policy accommodation because spending decisions depend on real interest rates. In addition, policymakers may find it useful to monitor the evolution of the real policy rate relative to some benchmark; this benchmark real rate is often referred to as r^*.

This memo serves as a primer on the concept of r^*, its measurement, and its use in informing monetary policy. Researchers and policymakers have employed various concepts of r^*. In the next section, we offer a taxonomy for several of these concepts. In Section 3 we discuss the connection between modeling concepts and their empirical counterparts, along with associated measurement challenges. We close with some concluding remarks and with a roadmap of what the three memos that follow this one offer to readers.

2. A Taxonomy

Table 1 identifies different concepts that have been referred to as r^*. In some cases, these terms have a very specific interpretation, one that is tightly linked to a model or class of models. In other cases, the meaning of a term is looser and usually statistical in nature.

[1] In preparing this memo we have benefitted from the comments of David Altig, Bill English, Etienne Gagnon, Thomas Laubach, Steve Meyer, and Daniel Sullivan.

Table 1
Alternative Definitions of r^*

Neutral Real Rate: The short-term real interest rate corresponding to a stance of monetary policy that is neither expansionary nor contractionary.

Natural (Wicksellian) Real Rate: The short-term real interest rate that would prevail if there were no nominal rigidities (for example, price and wage stickiness), and hence is consistent with no deviation of output from its natural level.

Efficient Real Rate: The short-term real interest rate that would prevail if there were no nominal rigidities and also no real distortions (for example, imperfect competition and distortionary taxes) that might cause output to deviate from its efficient level.

Optimal Real Rate: The short-term real interest rate that would be prescribed by optimal monetary policy (defined as the interest rate plan that maximizes a welfare criterion).

Long-Run Real Rate: The average short-term real interest rate measured over a long period of time.

Steady-State Real Rate: The short-term real interest rate that would prevail in the long-run once all structural shocks die down.

FRB/US r^*: The short-term real interest rate which, if maintained for twelve quarters, would close the output gap in the FRB/US model in exactly twelve quarters.

The *neutral real rate*, defined as the short-term real interest rate corresponding to a stance of monetary policy that is neither expansionary nor contractionary, is the most general definition of r^* (as in Taylor, 1993). The remaining definitions in Table 1 offer a collection of concepts that might be best thought of as different benchmarks against which to assess the stance of monetary policy. That is, the other entries in Table 1 reflect refinements by the economics literature in conceptualizing the stance of monetary policy in the context of formal models (as in Woodford, 2003). However, the concept of the neutral rate may be useful even in a purely statistical setting, where tight and loose monetary policy could be well-defined—for example, by changes in inflation relative to target and/or changes in the output gap—without imposing the structure of a fully specified economic model (as in Laubach and Williams, 2003).

Estimates of the values of these alternative definitions of short-run r^* typically move over the business cycle in response to various shocks to the economy. The *natural (or "Wicksellian") real rate* is the real interest rate that would prevail in the absence of nominal rigidities such as sticky prices and wages. This means that it is also the rate that is consistent with full utilization of resources.[2] The concept of the natural real rate is distinct from the *efficient real rate*, which is the real rate that would prevail without such nominal rigidities and also without any real distortions (for instance, imperfect competition and distortionary taxes) that move allocations away from their efficient level.[3] The *optimal real rate* refers to the short-term interest rate that would be prescribed by optimal monetary policy, defined as the interest rate plan that maximizes an appropriate welfare criterion, taking into account all of the distortions in the economy.[4] In some relatively simple models, the natural, efficient, and optimal real rate are the same.

The *long-run average real rate* assumes little about how the economy works over the business cycle; it defines r^* as the average short-term real interest rate measured over a long period of time. This definition is statistical and does not rely on a specific economic model. By contrast, the concept of the *steady-state real rate* requires a fully specified economic model as it is the short-term real interest rate that would prevail in the long run once all structural shocks (within the economic model) die down.[5] These two concepts are often used interchangeably in the context of simple policy rules with constant intercept terms.

Finally, there is the FRB/US r^*, which is a related though different concept of r^*. As often discussed in the context of the Monetary Policy Strategies (MPS) section of Tealbook B, the FRB/US r^* is the level of the real federal funds rate which, if maintained for twelve quarters, will close the output gap in three years, according to the FRB/US model and taking as given the Tealbook baseline outlook. This concept of r^* can be thought of as a practical attempt to

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[2] As discussed in more detail in Chung *et al.*, "Estimates of Short-Run r^* from DSGE Models" memo to the Federal Open Market Committee (October 9, 2015), in the context of so-called New Keynesian models, the natural real rate reflects all the shocks borne by the economy and as a result it can move around substantially.

[3] When the number of distortions is greater than the number of policy instruments, it is the case that the efficient allocations (that is, the path for the real economic variables that would prevail in the hypothetical economy in which those economic frictions are absent) are not achievable. For example, if monopolistic competition and price stickiness are the main distortions in a model, two policy instruments (a path for taxes and a path for the nominal interest rate) can be optimally designed to achieve the efficient allocation.

[4] The real federal funds rate associated with the optimal control simulations shown in the Monetary Policy Strategies (MPS) section of Tealbook B is the optimal real rate for the FRB/US model, where optimality is defined with reference to the standard loss function used in these simulations.

[5] As discussed in more detail in Kei-Mu Yi and Jing Zhang, "Real Interest Rates over the Long-run" memo to the Federal Open Market Committee (October 9, 2015), in the context of the so-called Neoclassical model, the *long-run average real rate* can also exhibit some low-frequency variation.

provide, in the form of a single number, an answer to the question of what real rate would return output to full-employment levels over a reasonable horizon.

Given a definition of r^*, the "real rate gap," that is, the gap between the actual real rate and r^*, is one indicator of the monetary policy stance. If the actual real rate is low relative to that benchmark, policy is expansionary; conversely, if the actual real rate is high relative to that benchmark level, policy is contractionary.[6] For practical purposes, policymakers with different r^* benchmarks (that is, with different real rate gaps) could describe the same nominal interest rate as consistent with tight or loose monetary policy.

3. Conceptual Issues

The previous discussion hinted at a connection between r^* as a concept and the intercept term that usually appears in simple monetary policy rules like those that are employed in the MPS section of Tealbook B. In this section we relate some r^* concepts to the canonical policy rule of Taylor (1993), who proposed that the short-term nominal interest rate can be described as follows:

$$i_t = 2 + \pi_t + 0.5(\pi_t - 2) + 0.5y_t$$

This policy rule specifies setting the nominal interest rate (i_t) in response to observed values of inflation (π_t) relative to a target inflation rate of 2 percent, and of the output gap (y_t)—defined as the percentage difference between actual output and its potential value.[7] The constant intercept term of 2 percent implies that when inflation is equal to target (when $\pi_t = 2$) and actual output is equal to potential output (so $y_t = 0$), the real short-term interest rate is set to 2 percent.[8] If 2 percent happens to be the value of r^*, monetary policy can be described as restrictive (accommodative) if the gap between the actual real rate and this "benchmark" real rate is positive (negative), something that can be inferred by looking at the sign of the sum of the equally weighted deviations of output from potential and inflation from its target.

There are two justifications underlying the choice of a constant intercept in Taylor's original work. First, using a constant value reflects the view that, by conducting policy in a simple and

[6] In the case of the natural rate, the optimal rate and the efficient rate, the benchmark is the real rate that completely counters the original shock, so a real rate that differs from r^* in that instance is one that is too tight or too easy, relative to a notion of optimality. For the neutral real rate, the long-run real rate and the steady-state real rate, the benchmark is a rate that prevails only in the limit or over extended periods of time.

[7] In Taylor (1993), the potential level of output was defined as an estimated (linear) trend on the (log-) level of real GDP.

[8] As Taylor pointed out, the real rate is assumed to be constant and consistent with a "steady state" (and constant) output growth of about 2 percent.

systematic manner that is consistent with policymakers' objectives for inflation and economic activity, which allows policymakers to obtain superior outcomes by avoiding "too much" discretion. Second, the theoretical grounds for Taylor's assumption of a constant intercept are consistent with the policy rate taking its steady-state value along a balanced-growth path.[9] This steady-state real rate could be approximated by taking averages of the actual real rate over long periods of time on the assumption that over long periods of time resource slack probably averages to near zero, and thus the sample average of the real interest rate will be an estimate of the *long-run* real rate.[10]

However, this steady-state concept may not be the most relevant for policy for several reasons (Woodford (2001), Williams (2003)).[11] Over the medium to longer-run, the *long-run real rate* measure is unlikely to be constant; it will vary with slow-moving changes in underlying variables such as productivity growth, demographic trends and other factors affecting the growth of the labor force, and changes in capital and labor shares. These supply-side elements may display highly persistent changes such that, even over medium to long-horizons, policymakers should not treat the steady-state rate as fixed as is often assumed with simple monetary policy rules. There can be significant and persistent changes in the longer-run benchmark that policymakers would need to take into account.[12]

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[9] The notion of balanced growth can be traced back to Solow's growth model (Solow (1956)). Along a balanced growth path, there are no shocks, the economy is characterized by constant growth in productivity and the labor force, all markets are in equilibrium and expectations of the future are consistent with actual outcomes. Also along this balanced growth path, the private sector's savings ratio is constant and the capital stock receives a constant real return. Of course, there is active research on the determinants of long-run growth beyond Solow's neoclassical framework that introduces some additional low frequency variation in long-run growth. In particular, there are two main strands of the literature aiming at modeling endogenous technological change (Aghion and Howitt, 1998) and recognizing explicitly the role of institutions as key determinants of long-run growth (Acemoglu, 2009).
[10] It is not always the case that the "steady-state" real rate and the long-run estimated real rate are the same (see Yi and Zhang, "Real Interest Rates over the Long-run" memo to the Federal Open Market Committee (October 9, 2015)).
[11] The natural real rate is also called the Wicksellian real interest rate, which is the short-term risk-free real interest rate that balances planned saving and investment (or actual real GDP and potential real GDP) in the medium term at full employment and with inflation at its target level. Although originally associated with the economist Knut Wicksell (1898), the term has been re-coined and revitalized in academic circles by Woodford (2003) as a key element of the New Keynesian macroeconomic literature.
[12] Blanchard (1997) constitutes an attempt to characterize the role of these supply side elements in the transition from near-term or business cycle fluctuations to long-run economic growth, with notable effects on r^*. Recently, Comin and Gertler (2006) offer another attempt to present a unified approach to explaining the high- and medium frequency variation in the data. Solow (2000) sketches some key elements of a much needed macroeconomics of the medium run.

There is a recent literature that has extensively studied the implications of price and wage stickiness for monetary policy within the New Keynesian (NK) framework.[13] Such nominal frictions amplify and affect the transmission of economic shocks through the economy; they also introduce a role for monetary policy and the associated need for policymakers to distinguish the shocks to which they should not respond from those that need to be offset. The literature has used the NK framework to derive normative implications for monetary policy, including how monetary policy should adjust short-term interest rates to achieve stable prices and output growing at potential. In the basic NK model, it is optimal for policymakers to make the short-term real interest rate equal to the natural real interest rate on a period-by-period basis. This natural real rate represents the real rate of return required to keep the economy's output equal to potential output, which, in turn, is the level of output that would obtain if the economy exhibited flexible prices and wages and constant markups in goods and labor markets.[14] If the only nominal friction is the lack of adjustment in the price of goods, this simple policy prescription would completely stabilize both output and inflation. As a result, the natural rate is *efficient* and *optimal* in such a simple model. In the simplest New Keynesian models, the *natural*, *efficient*, and *optimal* real rates coincide.[15]

In practice, and in more complicated and realistic models, there may be a trade-off between the two objectives of output and inflation stabilization. For example, labor market frictions that keep nominal wages from adjusting fully period by period, as well as financial frictions associated with nominal credit market contracts, can create such a short-run policy tradeoff. In these contexts, the *natural* rate can be used as a benchmark for the conduct of monetary policy. Notably, the *natural* real rate will differ from the *optimal* and/or *efficient* real rate, because policymakers will face more than one nominal friction and can only act using a single instrument. Thus, the NK literature offers a framework that rigorously defines and tightly connects the *natural*, *efficient*, *optimal*, and *steady-state* real rate concepts. However,

[13] In these models, there is an important role for expected future short-term interest rates in aggregate demand determination. The supply block features two main ingredients. First, firms face a constant elasticity of demand for their products, which means that the profit maximizing markup of price over marginal cost is constant –invariant to shifts in demand or in the cost of production (Dixit and Stiglitz, 1977). Second, the presence of sticky prices is introduced through a variant of the staggered contract Taylor model which assumes instead that a fixed nominal price is reset at randomly varying intervals (see Calvo (1983), Yun (1996), Goodfriend and King (1997)). Woodford (2003, chapter 4), refers to this as a *Neo-Wicksellian* framework. Erceg, Henderson and Levin (2000), Christiano, Eichenbaum and Evans (2005), and Smets and Wouters (2007) incorporate wage stickiness and capital accumulation. Other forms of monetary non-neutralities have been introduced in the models—most prominently those arising from specific financial frictions.

[14] This formal definition resembles that of Knut Wicksell, so Woodford, among others, calls this natural rate the *Neo-Wicksellian* interest rate (or Wicksellian for short). In other words, the main policy implication of the previous observation is that policymakers concerned with maintaining output close to its potential level should set short-term nominal interest rates to minimize the gap between the real interest rate and the Wicksellian real rate.

[15] The so-called "divine coincidence" (see, e.g., Blanchard and Galí (2007)).

policymakers may judge that these models, although valuable, are too simple to be taken literally in the policymaking process.

From a practical policymaking perspective, the previous discussion leaves open some questions that the three companion memos will address. The next two memos will discuss two of the different measures of r^*. The memo by Kei-Mu Yi and Jing Zhang, titled "Real Interest Rates over the Long-Run" documents that the long-run marginal product of capital, total factor productivity growth, and working-age population trends are all consistent with low long-run real interest rates. However, there is no single, simple story to explain long-run real interest rate movements over the past 60 years. The memo by Hess Chung *et al.*, titled "Estimates of Short-Run r^* from DSGE Models" discusses shorter-run definitions and also addresses issues related to measuring r^* in the context of Dynamic Stochastic General Equilibrium (DSGE) models. In particular, even though the short-run concepts of r^* map into alternative DSGE models in different ways, all of the models considered suggest that the natural rate plunged to its historical lows during the Great Recession and has remained subdued throughout the recovery. Both memos document considerable uncertainty about the level of r^* and its expected evolution. The final memo by David López-Salido *et al.*, titled "Monetary Policy at the Lower Bound with Imperfect Information about r^*" discusses monetary policy design considering uncertainty about r^* and the effective lower bound (ELB). Near the ELB, policymakers optimally respond to imperfect incoming information about r^* by even less than they would during normal times in order to "take out insurance" against situations in which their misperceptions about r^* might cause policy to be "too tight" and in which the ELB makes the policymaker unable to offset those misperceptions in subsequent periods.

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