October 13, 2015

Estimates of Short-Run r* from DSGE Models

Hess Chung, Marco Del Negro, Thiago Ferreira, Cristina Fuentes-Albero, Marc Giannoni, Manuel P. Gonzalez-Astudillo, Luca Guerrieri, Matteo Iacoviello, Evan F. Koenig, Jean-Philippe Laforte, Matthias Paustian, Damjan Pfajfar, Andrea Raffo, Andrea Tambalotti[1]

1. Introduction

The target range for the federal funds rate has been set at the effective lower bound on nominal interest rates for several years. Over this period, the headwinds depressing economic activity and inflation have been substantial. Indeed, Board staff estimates of the real interest rate needed to close the output gap over a 12-quarter time frame—the short-run r* regularly reported in the Tealbook—remained consistently below the actual real federal funds rate from late 2008 through the third quarter of 2013.

As noted in the first of the memos on r*, the Tealbook concept is only one of several available notions of r*.[2] In the academic literature, a related short-run r* concept has been developed within the New-Keynesian modeling framework. This so-called *natural rate of interest*, defined as the real interest rate that would prevail in the absence of sluggish adjustment in nominal prices and wages, is the subject of this memo.[3]

The main reason for focusing on this particular notion of r* is that, in simple New-Keynesian models, setting the real interest rate at the natural interest rate achieves both price stability and full employment, therefore providing a useful benchmark for the actual real policy rate. In larger, more realistic Dynamic Stochastic General Equilibrium (DSGE) models, this "divine coincidence" between the natural rate of interest, price stability, and full employment does not hold. Nonetheless, a monetary policy strategy in which the real policy rate tracks the natural rate generally promotes stable inflation and economic activity even in those models.

The DSGE approach to r* has several advantages. First, the connection between the natural rate of interest and the movement in observable variables established by the models provides a framework for the measurement of this otherwise unobservable object. Estimates based on this framework are shown in Section 3. Second, the models can identify the fundamental sources of changes in the natural rate and relate them to the factors underlying macroeconomic fluctuations. For instance, adverse financial disturbances, which depress both inflation and real activity, account for the overwhelming majority of the

[1] We would like to thank Michael Droste, Sara Shahanaghi, Micah Smith, and Eric Till for excellent research assistance and David Altig, Christopher Erceg, Michael Kiley, Thomas Laubach, David López-Salido, Jamie McAndrews, Jonathan McCarthy, Steve Meyer, Paolo Pesenti, Argia Sbordone, and Kei-Mu Yi for very valuable comments and suggestions.
[2] See "r*: Concepts, Measures, and Uses" by Christopher Gust, Benjamin K. Johannsen, David López-Salido, and Robert Tetlow. (October 2015).
[3] Many researchers have contributed to this literature, but the framework we employ is most closely associated with Woodford (2003). Neiss and Nelson (2003) were the first to evaluate the properties of the natural rate in a calibrated DSGE model. Edge, Kiley, and Laforte (2008), Justiniano and Primiceri (2010), and Barsky, Justiniano, and Melosi (2014) do so in estimated models.

decline in the natural rate of interest during the Great Recession, as illustrated in Section 4. Third, DSGE models connect the measurement of the natural rate to the study of its policy implications through the use of alternative policy simulations. For instance, we explore what would happen to inflation and output if the monetary authority in our models set the real policy rate exactly equal to the natural rate. This question is taken on in Section 5.

One drawback of the DSGE approach is that the measurement of the natural rate is tightly linked to the specific features of the model used to estimate it. As an attempt to guard against this drawback, this memo includes five estimates of r*, produced by five different models developed and maintained by economists from around the Federal Reserve System.

Our analysis reaches three main conclusions. First, despite important differences in their construction, the models examined provide a coherent assessment of the movements in the natural rate of interest over time and of the determinants of those movements. Second, all the models suggest that the natural rate plunged to its historical lows during the Great Recession and remained subdued throughout the recovery. Third, the natural rate provides a useful benchmark for the real policy rate, even though it is not optimal in any of our models.

2. Defining the natural rate of interest

One of the key features of New-Keynesian models is that nominal price and wage rigidities prevent rapid and efficient adjustment of the real wage and other relative prices, leading employment and economic activity to respond inefficiently to shocks. The simplest versions of such models feature only nominal price rigidity. In that environment, setting the real interest rate to the level that would prevail in the absence of price rigidity—the natural rate of interest—leads to price stability. Moreover, because nominal price rigidity is the only friction, setting the real rate to the natural rate also leads to full employment.

These simple models cannot, however, adequately describe the complex factors driving economic fluctuations. Therefore researchers have built upon this simple framework, producing empirical DSGE models that include a larger number of frictions to better account for those observed fluctuations. These empirical models typically include both nominal price and wage rigidity. Many of these models also feature frictions in credit markets, as well as shocks that can affect those frictions. In these more realistic models, the natural rate of interest is still commonly defined as the real interest rate that would prevail in the absence of nominal rigidities. This is the definition adopted here.[4]

Unlike in the simple models discussed above, setting the policy rate to track this natural rate does not deliver either price stability or full employment in empirical DSGE models because their richer structure implies a trade-off between multiple objectives.[5] Nonetheless, Section 5 demonstrates that this policy

[4] In models with more than just nominal frictions, other definitions are possible, depending on which frictions are eliminated when computing the natural rate. We discuss some of these alternatives in this memo's Appendix.
[5] In general, larger DSGE models imply a complex set of trade-offs because their multiple frictions create distortions on many fronts, which monetary policy cannot address with only one instrument. The details of these trade-offs, and hence of optimal policy, are very model-dependent. For this reason, the computation of an optimal interest rate is outside the scope of this memo. For examples of this type of optimal policy exercise in empirical DSGE models see Levin, Onatski, Williams, and Williams (2005) and Justiniano, Primiceri, and Tambalotti (2013).

strategy does deliver more stable inflation *and* output gaps than the estimated monetary policy rules in most of our models.

3. Estimates of the natural rate of interest from a battery of models

This section presents historical estimates of the natural rate of interest for the United States from five different models. These models are:

1. EDO, the DSGE model of the U.S. economy developed at the Board of Governors.
2. FRBNY-DSGE, the model of the Federal Reserve Bank of New York.
3. FR, a model based on the work of Christiano, Motto, and Rostagno (2014) and maintained by Thiago Ferreira and Andrea Raffo at the Board of Governors.
4. GIH, a model developed by Luca Guerrieri and Matteo Iacoviello, also at the Board of Governors.
5. DALLAS, an empirical model of r* developed by Evan Koenig and Alan Armen at the Federal Reserve Bank of Dallas.

EDO and FRBNY-DSGE may be relatively familiar, since they are both part of the System DSGE project, which has been presenting forecasts to the Committee on a regular basis since 2011.[6] FR is an extension of the framework for the estimation of the natural rate of interest in advanced foreign economies (AFEs) that was featured in a box of the June Tealbook ("Equilibrium Interest Rates in the Advanced Foreign Economies"). GIH was developed by Guerrieri and Iacoviello and was featured in the QS Summary.[7] Finally, DALLAS is an estimated time-series model used to analyze the implications of monetary policy for unemployment and inflation.[8]

These models include many important economic frictions, as summarized in Table 1. FRBNY-DSGE, FR, and GIH include various forms of financial frictions, which affect both households and firms. In GIH, which stresses the role of housing as collateral, these frictions take the form of occasionally binding borrowing constraints. As a result, their intensity varies over the business cycle, depending on whether the constraints bind, or do not bind, generating an asymmetry between expansions and recessions. In FRBNY-DSGE and FR, instead, the financial frictions create an explicit connection between firms' balance sheets and the spread on external financing that those firms pay.

[6] Details on these first two models can be found in the Research Director Drafts made available as part of the System DSGE project.
[7] Details on this model can be found in Guerrieri and Iacoviello (2013).
[8] DALLAS is not a DSGE model, but its estimated r* shares several characteristics with those obtained from the other models. In particular, this model's r* fluctuates over the business cycle, and it provides a useful benchmark for the setting of the policy rate, as explained in detail in the memo by Evan F. Koenig and Alan Armen ("Assessing Monetary Accommodation: A Simple Empirical Model of Monetary Policy and its Implications for Unemployment and Inflation"). Therefore, this approach provides an alternative read on short-run r*.

Table 1. Frictions included in the DSGE Models

	EDO	FRBNY	FR	GIH
Preferences and Technology				
Habit in consumption	X	X	X	X
Preference for housing				X
Variable capacity utilization	X	X	X	
Investment adjustment costs	X	X	X	X
New-Keynesian Frictions				
Monopolistic competition	X	X	X	X
Price stickiness	X	X	X	X
Wage stickiness	X	X	X	X
Financial Frictions				
Firm borrowing subject to interest rate spreads		X	X	
Collateral constraints on firms and households				X
Monetary Policy				
Interest rate feedback rule	X	X	X	X

Each model adopts somewhat different specifications for the preferences of the agents and the technological constraints that they face, as well as for the shocks that perturb the economy. For instance, EDO features a richer specification of the consumption and investment sectors compared to the other models, providing a complementary perspective on some of the headwinds that might have restrained activity during the recovery. Finally, the models adopt different estimation strategies. These differences include the use of different sets of macroeconomic and financial variables for the estimation, and of linear or non-linear techniques to back out values for the unobservable variables, including the natural rate of interest, spanning many of the practices that are common in the literature.[9]

This heterogeneity in the structure of the models and their estimation is a strength of this work, since it provides a range of estimates for the natural rate of interest that accounts for model uncertainty. Recognizing model uncertainty is especially important for policy purposes, given the overall uncertainty surrounding any estimate of r*, as stressed in the accompanying memo by Gust *et al.* (2015).

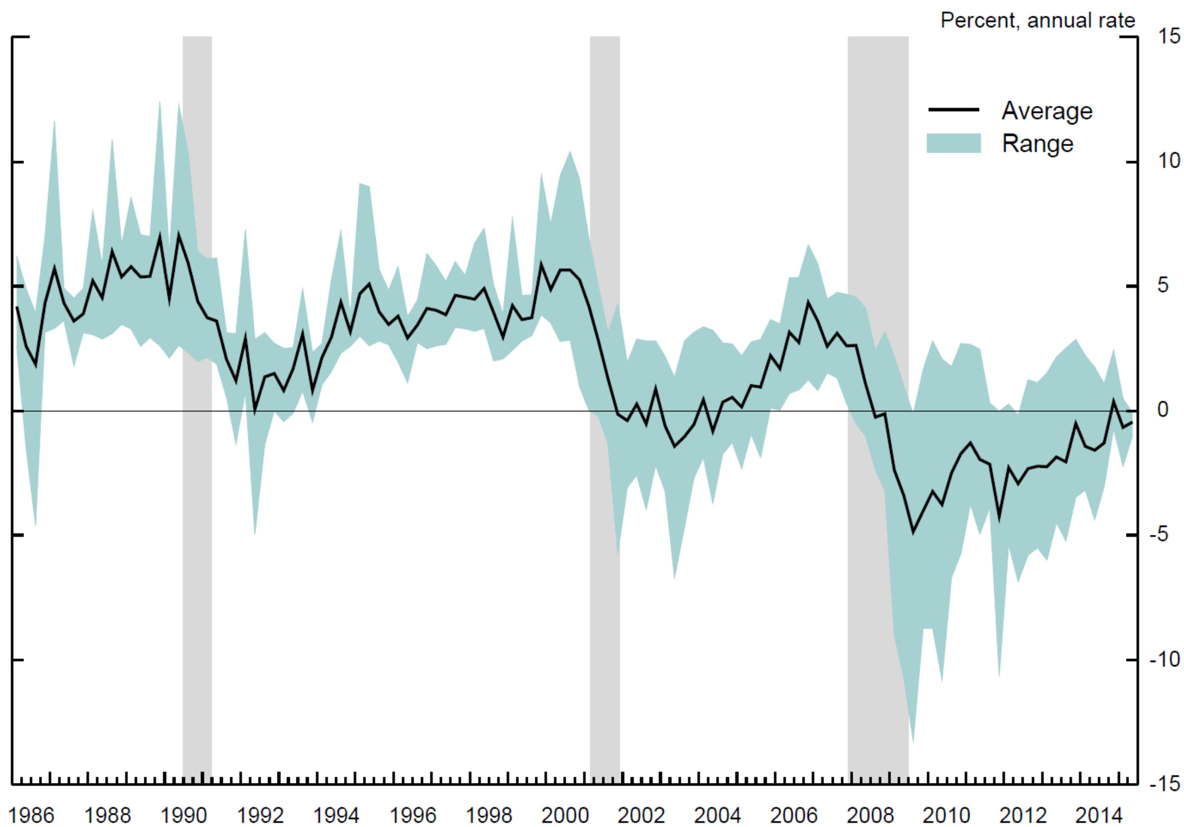
Figure 1 presents a summary of the time series estimates of the real natural rate of interest from the five models, starting in 1985 and running through 2015:Q2.[10] The black line is the average estimate across the

[9] One feature that all our DSGE models share is that they are built around neoclassical growth foundations. Therefore, they embed a long-run relationship between the natural rate of interest and output growth of the kind also discussed in the second of the memos on r* (see "Real Interest Rates over the Long-Run" by Kei-Mu Yi and Jing Zhang, October 2015). However, our models' version of this relationship is particularly simple, since it is based on the existence of a steady state, in which output growth and the natural rate are both constant. Therefore, our models abstract from the kind of long-run fluctuations in r* highlighted in Yi and Zhang (2015). Nevertheless, persistent shocks can result in very persistent deviations of the natural rate from its steady state even in our framework.

[10] Some of the models also produce estimates of r* going back further in history. These estimates share many features with the more recent ones, such as the fact that they tend to rise in booms and fall in recessions. They also

models. The blue band represents the range from the minimum to the maximum estimate at any point in time.

Figure 1: Estimates of the real natural rate of interest



Note. Estimates from EDO, FRBNY, FR, GIH, and Dallas.

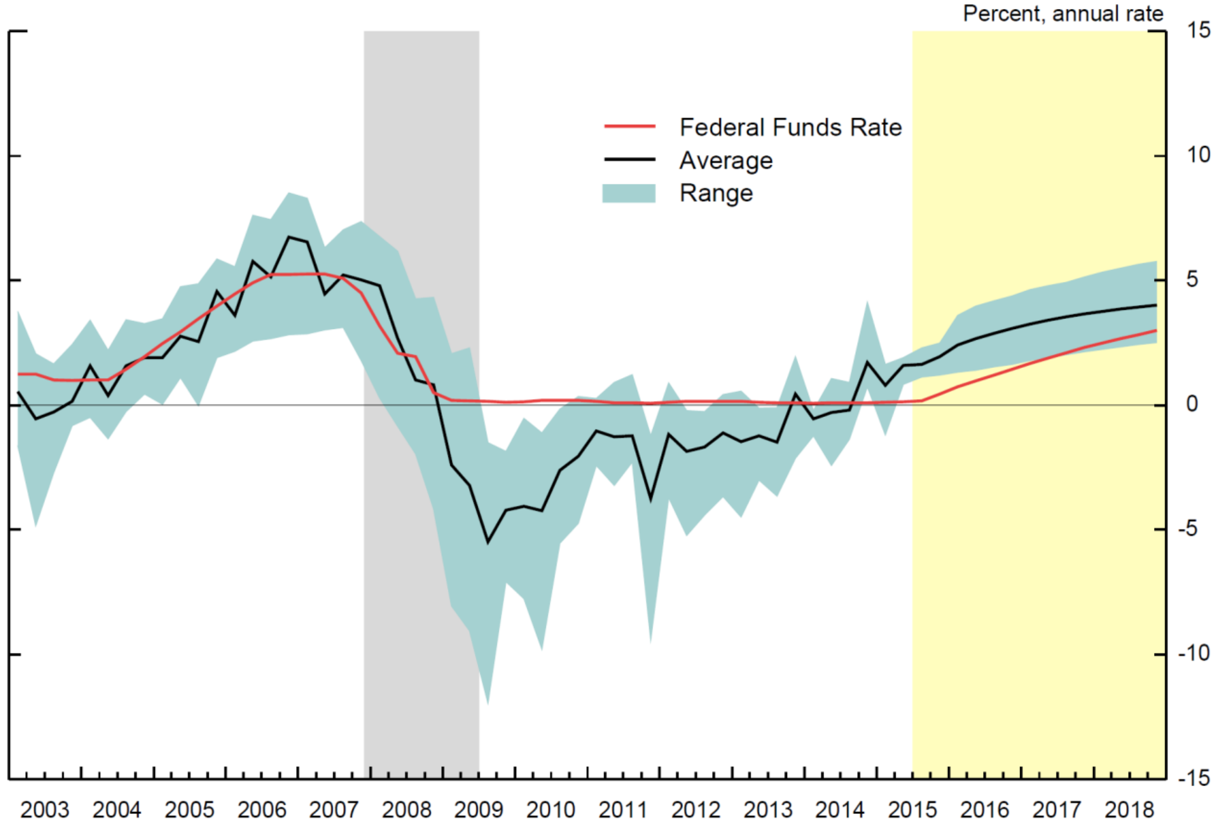
The first notable feature of Figure 1 is how much the average natural rate moves over time. These movements are highly correlated across models over the business cycle, resulting in a clear cyclical pattern. The estimated natural rate tends to rise during booms and declines abruptly in downturns. This decline is especially pronounced during the Great Recession, when all the models see the natural rate fall into negative territory. Over this period, the range of our estimates is in line with the deep drop in short-run r* estimated in the Tealbook. The recovery in the natural rate following the recession is extremely gradual, with the average edging above zero for the first time at the end of 2014. The sluggishness with which the natural rate has been returning towards more normal levels over the past several years mostly reflects the negative influence of financial headwinds, as discussed in Section 4.[11] Finally, some of the

tend to conform to the conventional wisdom on some notable episodes in monetary history. For instance, they imply that monetary policy was extremely restrictive in the early 1980s during the Volcker disinflation.

[11] The average natural rate across models also appears to reflect a slight downward trend, mostly associated with the years since the financial crisis. As discussed in footnote 9, over time the natural rate of interest in all our models returns to a constant steady state level by construction, even if deviations from this steady state can be very persistent. In estimation, the steady state level of the natural rate is pinned down by the mean of the actual real interest rate, which is around 2% in our sample.

estimates also display fairly pronounced high frequency variation: this is most evident in the edges of the blue range, but it is also apparent in the mean estimate. These gyrations reflect the fact that the natural rate fluctuates quarterly in reaction to the transitory shocks that buffet the economy.[12]

Figure 2: Federal funds rate vs estimates of the nominal natural rate of interest



Note. Estimates from EDO, FRBNY, FR, and GIH.

To focus more directly on the implications of the estimated natural rates for monetary policy over the recent past and over the next few years, Figure 2 reports estimates and projections of the natural rate of interest in *nominal* terms between 2003 and the end of 2018.[13] This conversion makes it possible to compare our estimates directly to the federal funds rate and to the baseline assumption from the September Tealbook for the forecast period. The average nominal natural rate across the models tracks the federal funds rate closely through 2008. Once the policy rate hits the effective lower bound, though, the natural rate continues to fall into negative territory, reaching a trough of about -5 percent on average, and as low as -12 percent according to EDO. This evidence suggests that the lower bound on nominal interest rates was a significant constraint on the ability of monetary policy to stimulate the economy over the past several years, at least through the conventional channel of a lower policy rate. However, this

[12] This high-frequency volatility in the estimated natural rate, although pretty common in DSGE models, is not shared by all our models. The estimates of both GIH and the FRBNY-DSGE, for instance, are fairly smooth.
[13] These are computed by adding expected inflation from each model to its estimate of the natural real rate. DALLAS is not included in this figure because it does not project the natural rate.

constraint no longer appears to be binding. In fact, all the models see the nominal natural rate crossing into positive territory some time in 2014.

Going forward, the average natural rate is projected to normalize at the same pace as the federal funds rate in the Tealbook assumption, even though it lies above it. However, the point estimates in the four models that underlie the range reported in the figure differ on the degree of accommodation implied by the September Tealbook assumption for the federal funds rate. FRBNY and FR project the natural rate to reach only 2.5 percent in 2018. By contrast, EDO and GIH estimate the nominal natural rate to be already close to 2 percent and project it to approach 6 percent at the end of the forecast horizon.

Although these estimates of the natural rate provide a useful benchmark to gauge the degree of accommodation or restraint associated with any given path of the policy rate, they are not optimal prescriptions for the setting of the policy rate in our models. This is especially the case after a long period in which the effective lower bound on the policy rate has been binding, and economic slack has persisted. According to the models, this slack is partly attributable to the difficulty of generating enough stimulus at the lower bound. This past shortfall, therefore, might need to be compensated by keeping the policy rate below its natural level for a period, so as to absorb the accumulated slack faster.

4. Sources of Fluctuations in the Natural Rate of Interest in the United States and in the Advanced Foreign Economies

One of the advantages of the DSGE approach is that it connects the estimated movements in the natural rate back to the same fundamental shocks that drive business cycles. In this respect, the natural rate represents a summary of the shocks that buffet the economy at any given point in time, filtered through the lens of their effect on the real rate of return in the frictionless economy. Here, we explore the extent to which our models provide a reasonable description of the main drivers of cyclical fluctuations, with a particular focus on the Great Recession and its aftermath.

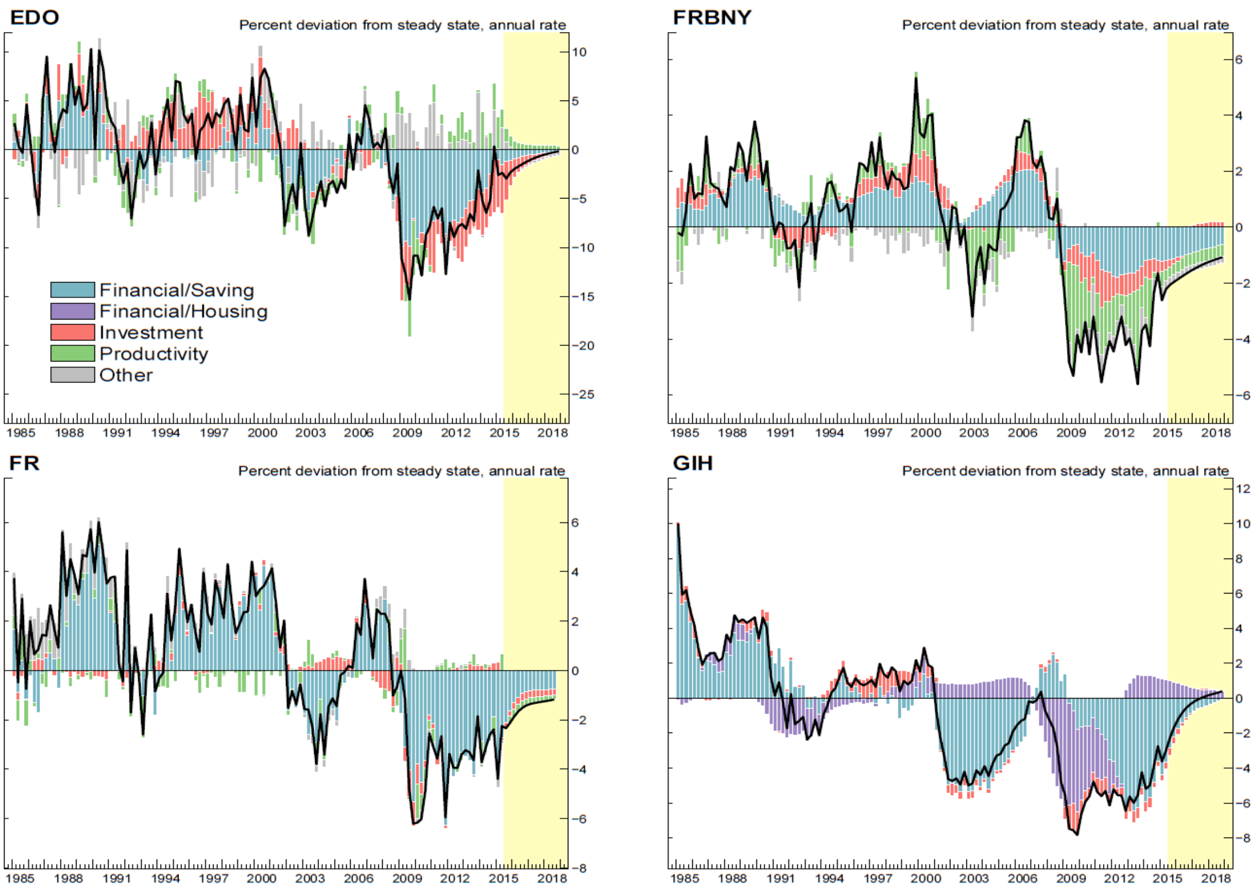
Figure 3 displays this historical decomposition of the natural rate in our four DSGE models. The panels in the figure group the shocks that affect the natural rate into four groups, based on the economic decisions that they influence most directly.[14] Negative financial/saving shocks (in blue) increase households' desire to save and to hold safe bonds. In some models, these shocks increase the external finance premium for firms, thus reducing investment. Investment shocks (in red) also curtail firms' willingness or ability to invest in physical capital. Technology shocks (the green bars) reduce total factor productivity, depressing the marginal product of capital and hence desired investment. Finally, in GIH, financial/housing shocks (the purple bars) tighten borrowing constraints of households and firms, weighing on aggregate demand. All these shocks, when negative, depress the natural rate because they boost desired saving and/or reduce desired investment.

All four models ascribe the unprecedented decline in the natural rate during the recent financial crisis to an unusual combination of severe negative shocks. This configuration is most striking in the FRBNY-DSGE model, in which financial, investment, and productivity shocks all turn from lifting the natural rate before 2007 to depressing it in 2008 and 2009. In EDO and FR, financial and investment shocks push the

[14] In this decomposition, the natural rate is reported in deviation from its mean, so as to make the contributions of the shocks to its fluctuations more evident.

natural rate to very low levels during the recession, with the former accounting for the majority of this decline. In GIH, the decline in house prices and housing wealth that started in 2006, and the associated tightening of borrowing constraints for both households and firms, explains most of the decline in aggregate demand and the natural rate through 2009.

Figure 3: Historical decomposition of the real natural rate of interest



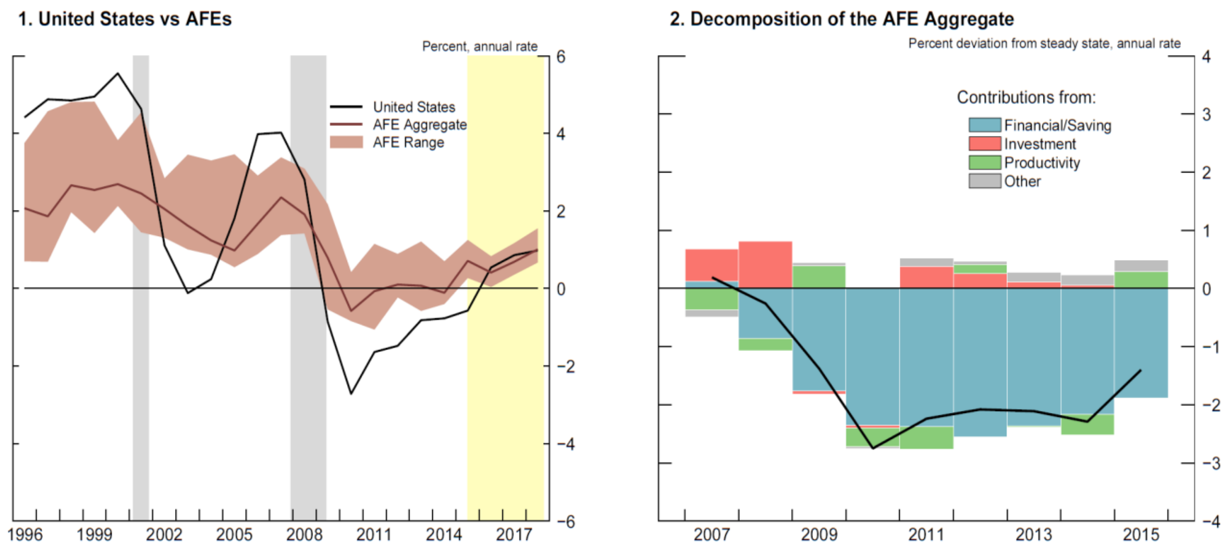
Financial and investment shocks also generate persistent headwinds well into the recovery, which keep the natural rate below its mean throughout this period. Currently, all estimates suggest that the natural real rate is about 2.5 percentage points below its long-run value, with financial headwinds accounting for a large portion of this shortfall. For FRBNY and FR, the slow unwinding of these factors keeps the natural rate below its long-run value over the forecast period, while in EDO and GIH the natural rate is back to its long-run mean by 2018.

Similar factors to those just highlighted for the United States also appear to have lowered the natural rate in the major AFEs. Panel 1 of Figure 4 compares the evolution of the natural rate of interest in the United States (black line) to that in the AFE aggregate (brown line), both estimated using the FR model.[15] The shaded area represents the range of estimates for the AFE countries. These estimates indicate

[15] The natural rate for the AFE aggregate is the trade-weighted aggregate of estimates for Canada, the euro area, and the United Kingdom.

synchronization across countries, with the natural rate falling markedly between 2008 and 2010. Moreover, panel 2 shows that adverse financial shocks (the blue bars) accounted for most of this fall and have kept the AFE average natural rate persistently depressed since the Great Recession, as in the United States.

Figure 4: The real natural rate of interest in the AFEs



Note. AFE aggregate refers to the trade-weighted average of Canada, euro area, and United Kingdom estimates.

Note. AFE aggregate refers to the trade-weighted average of Canada, euro area, and United Kingdom estimates.

All told, our DSGE models are consistent with the view that the Great Recession in the United States and abroad resulted from a combination of unprecedented adverse financial shocks. This consistency between the models' narrative and the consensus view on important historical events suggests that these models can provide valuable insights to policymakers. In particular, it supports the use of our DSGE models to perform counterfactual policy analysis that takes as given the path of the estimated shocks. We turn to an example of this type of analysis in the next Section.

5. Evaluating the Natural Rate of Interest as a Benchmark for Monetary Policy

As we discussed in Section 2, setting the policy rate so as to close the gap between the actual and the natural real rate of interest results in stable inflation and full employment in simple models. However, this policy strategy is not optimal in richer models. In these more complex environments, monetary policy faces numerous trade-offs, which make it impossible to stabilize all variables around their desired levels with only one policy instrument. To evaluate the extent to which the natural rate remains a useful guide to appropriate policy in these models, this section presents the results of a counterfactual experiment in which the central bank follows a natural rate targeting rule.[16]

---

[16] A natural rate targeting rule maintains a zero real interest rate gap through time by setting the nominal interest rate equal to the natural rate plus expected inflation, where the latter is computed from each model.

Table 2 below reports the standard deviations of inflation and the output gap in three of our models under this targeting rule, relative to those under each model's estimated historical monetary policy rule. In this exercise, the output gap is defined as the gap between actual output and its natural level, which is the level of output that would prevail in the absence of price and wage rigidities. For simplicity, these calculations abstract from the presence of the effective lower bound both under the historical and the counterfactual policy.[17]

Table 2: Volatility of inflation and the output gap under the natural rate targeting rule

Model	Inflation	Output Gap
FRBNY-DSGE	61%	14%
EDO	29%	33%
GIH	87%	125%

Note. The numbers in the table are the ratios of the standard deviation computed under the natural rate targeting rule over the standard deviation under the estimated policy rule in each model.

The table shows that targeting the natural rate significantly reduces the volatility of *both* inflation and the output gap in FRBNY-DSGE and EDO. Under the counterfactual policy, the standard deviation of these variables is between 14 and 61 percent of that obtained under the estimated policy rule. In GIH, targeting the natural rate also lowers the volatility of inflation, but at the cost of a higher standard deviation of the output gap. The primary reason for this tension is the interaction between financial frictions and shocks to desired markups, which play a more substantial role in GIH than in other models, generating an especially severe trade-off between the stability of inflation and the output gap.

Overall, these results indicate that maintaining the policy rate close to its natural counterpart over time can deliver desirable outcomes along both dimensions of the dual mandate, even in environments in which this policy is not optimal, providing a rationale for the use of the natural rate as an input in monetary policy deliberations.

6. Conclusions

This memo presented a particular approach to the measurement of r* based on estimated DSGE models. In these models, r* is commonly defined as the real interest rate that would prevail in the absence of sluggish adjustment of prices and wages, and labeled the natural rate of interest. This natural rate can be a useful input in monetary policy decisions, since maintaining the actual policy rate close to the nominal natural rate promotes more stable inflation and economic activity.

Our estimates suggest that the financial shocks that hit the U.S. economy during the Great Recession pushed the nominal natural rate well into negative territory, turning the effective lower bound into a tightly binding constraint on the Federal Reserve's ability to provide stimulus through conventional means. Moreover, these same shocks generated persistent headwinds well into the recovery, which now appear to be subsiding. Currently, all the models estimate a positive nominal natural rate and project it to be on a gradual normalization path going forward.

[17] The fourth memo addresses the interaction between r* and the effective lower bound on nominal interest rates (see "Monetary Policy at the Lower Bound with Imperfect Information about r*," by Christopher Gust, Benjamin K. Johannsen, David López-Salido, and Robert Tetlow, October 2015).

References

Edge, Rochelle M., Michael T. Kiley, and Jean-Philippe Laforte, 2008. "Natural rate measures in an estimated DSGE model of the U.S. economy," *Journal of Economic Dynamics and Control,* 32(8): 2512–2535.

Barsky, Robert, Alejandro Justiniano and Leonardo Melosi, 2014. "The natural rate of interest and its Usefulness for Monetary Policy," *American Economic Review,* 104(5): 37–43.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno, 2014. "Risk Shocks," *American Economic Review,* 104(1): 27-65.

Guerrieri, Luca and Matteo Iacoviello, 2013. "Collateral Constraints and Macroeconomic Asymmetries," International Finance Discussion Papers 1082.

Justiniano, Alejandro and Giorgio Primiceri, 2010. "Measuring the Equilibrium Real Interest Rate," *Federal Reserve Bank of Chicago Economic Perspectives,* 34(1): 14-27.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti, 2013. "Is There a Tradeoff Between Inflation and Output Stabilization?" *American Economic Journal: Macroeconomics*, 5(2): 1–31.

Levin, Andrew T., Alexei Onatski, John Williams, and Noah M. Williams, 2005. "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models," *NBER Macroeconomics Annual,* 20: 229-312.

Neiss, Katherine, Nelson, Edward, 2003. "The Real-Interest-Rate Gap as an Inflation Indicator," *Macroeconomic Dynamics,* 7: 239–262.

Woodford, Michael. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.