Board of Governors of the Federal Reserve System

Division of Monetary Affairs

FOMC Secretariat

Date: June 5, 2018

To: Research Directors

From: Matthew M. Luecke

Subject: Supporting Documents for DSGE Models Update

The attached documents support the update on the projections of the DSGE models. Note that the Chicago Fed's DSGE documentation has not changed since the last quarter, so their guide from December is being redistributed.

The Current Outlook in EDO:

June 2018 FOMC Meeting

Class II FOMC – Restricted (FR)

Diego Vilán*

June 1, 2018

1 The EDO Forecast from 2018 to 2020

The EDO model's forecast is conditional on data through the first quarter of 2018 and on a preliminary Tealbook forecast for the second quarter of 2018.

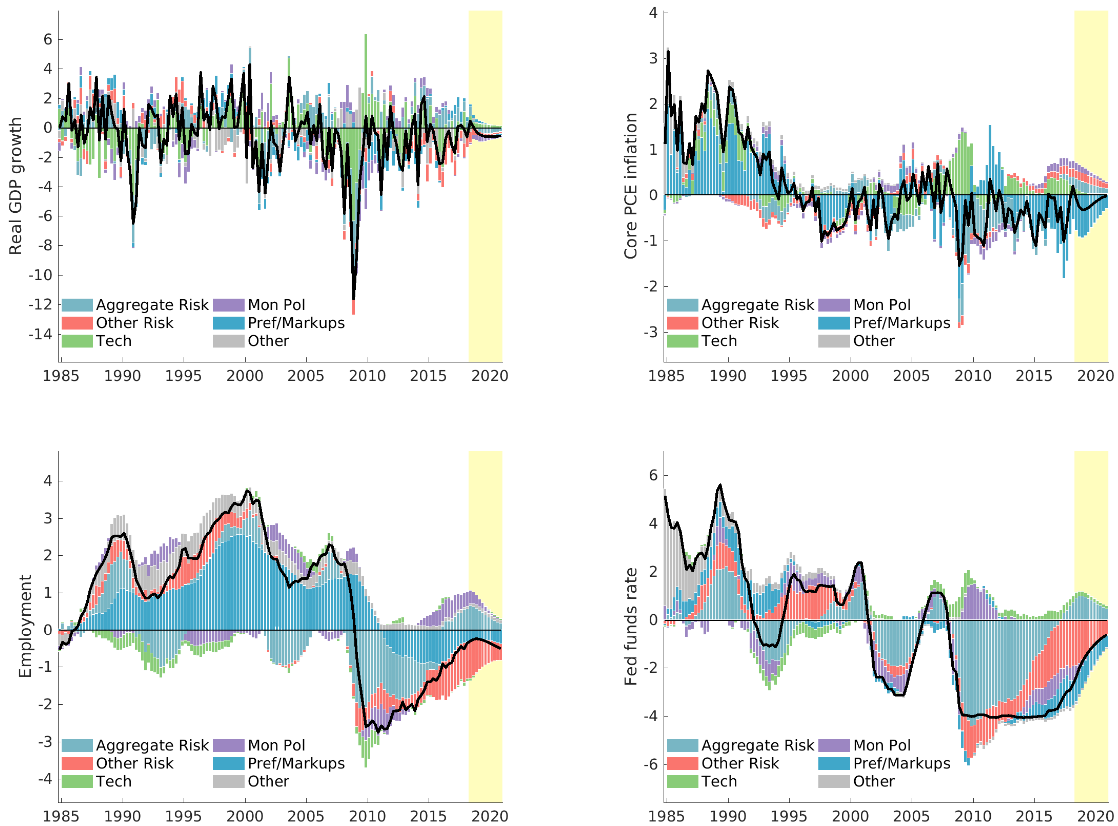
Real GDP growth is about 2.6 percent, on average, over the projection horizon, somewhat below its long-run value of 3 percent. Inflation reaches the Committee's 2 percent objective in the third quarter of 2020. Below-trend real GDP growth is driven by the slow fading of risk premium shocks and the waning effects of the currently accommodative stance of monetary policy. On the nominal side, for a number of years wages have been below the level consistent with the models wage Phillips curve, holding down marginal cost and depressing inflation over that time period. The persistence of these wage shocks accounts for most of the weakness in inflation over the forecast. By contrast, the rise in inflation at the beginning of 2018 is attributed by the model to transient shocks to the price Phillips curve, which largely dissipate by the first half of 2019.

The output gap is currently estimated to be -0.7 percent, and the economy slowly reaches its full potential by the end of 2020. The real natural rate of interest is estimated to be around 0.9 percent in the third quarter of 2018 and hovers around 1.7 percent thereafter until the end of 2020, 0.4 percentage point below its steady-state value of 2.1 percent. The trajectories of the natural rate of interest and the output gap are heavily driven by the model's view that capital stocks are currently well below those that would have prevailed in the absence of nominal rigidities, and that the investment-related shocks responsible for this condition are likely to dissipate slowly.

Consistent with the gradual return of inflation and the output gap to their long-run values, the federal funds rate is projected to increase gradually over the forecast horizon, reaching $3\frac{3}{4}$ percent by the end of 2020. At the end of the projection horizon, the federal funds rate is still below its

*The author is affiliated with the Division of Research and Statistics of the Federal Reserve Board. Sections 2 and 3 contain background material on the EDO model, as in previous rounds. These sections were co-written with Hess Chung and Jean-Philippe Laforte.

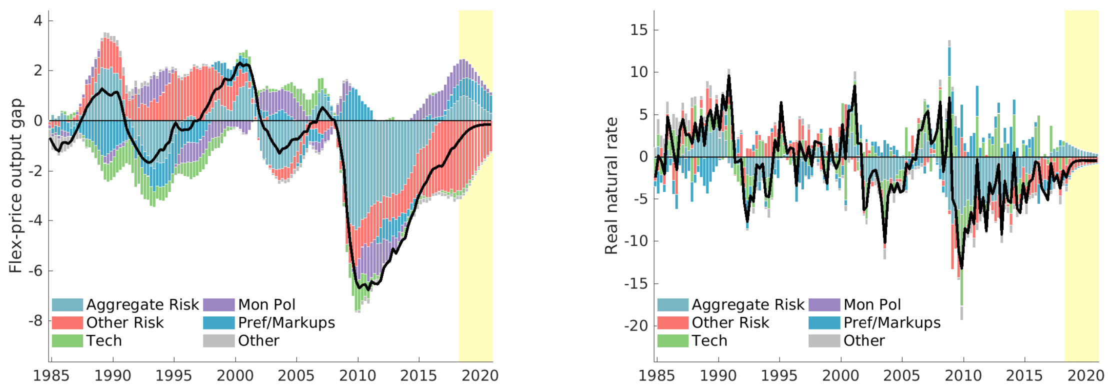
Figure 1: Recent History and Forecasts



long-run value of 4.1 percent, reflecting the inertia in the policy rule and the persistently negative output gap.

Compared with the March 2018 projection, the EDO model's forecast of real GDP growth in this round is stronger until the first half of 2019 because of various transitory factors; thereafter, real GDP growth is only a touch weaker relative to March. Core PCE inflation is, on average, unchanged over the forecast horizon. The output gap has been revised up, on average, 15 basis points over the projection period since March, while the forecast of the real natural rate of interest has been revised up 17 basis points, on average, since March, with a noticeable upward revision in 2017 due to positive revisions to the contribution of TFP growth shocks. The path of the federal funds is higher this round than in March, following the upward revision of the output gap.

Figure 2: Recent History and Forecasts: Latent Variables



2 An Overview of Key Model Features

Figure 3 provides a graphical overview of the model. While similar to most related models, EDO has a more detailed description of production and expenditure than most other models.[1]

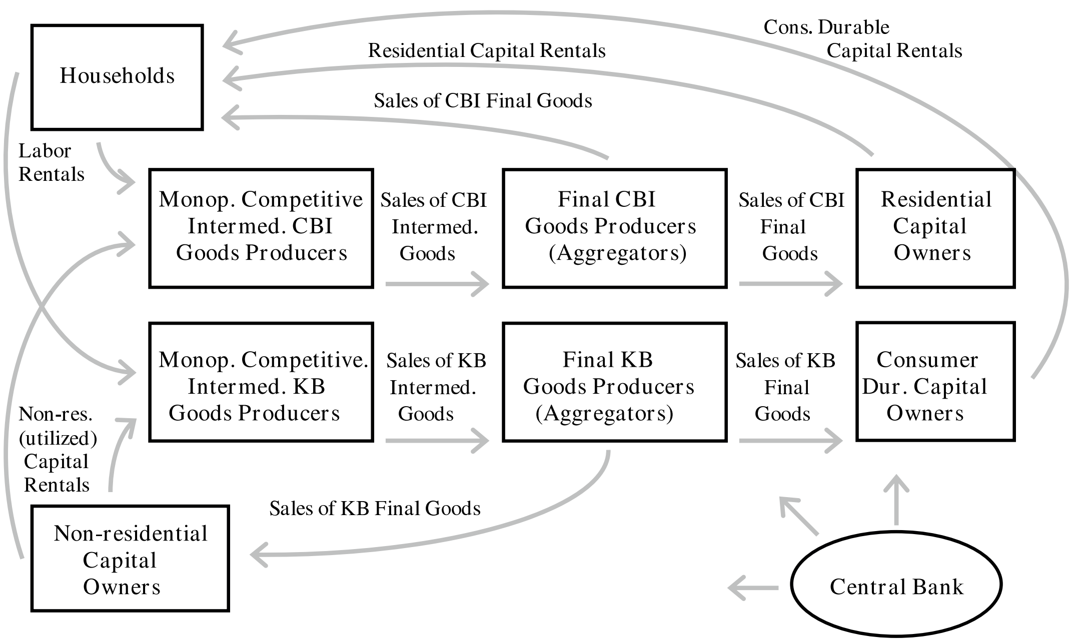
Specifically, the model possesses two final good sectors in order to capture key long-run growth facts and to differentiate between the cyclical properties of different categories of durable expenditure (for example, housing, consumer durables, and nonresidential investment). For example, technological progress has been faster in the production of business capital and consumer durables (such as computers and electronics).

The disaggregation of production (aggregate supply) leads naturally to some disaggregation of expenditures (aggregate demand). We move beyond the typical model with just two categories of (private domestic) demand (consumption and investment) and distinguish between four categories of private demand: consumer nondurable goods and nonhousing services, consumer durable goods, residential investment, and nonresidential investment. The boxes surrounding the producers in the figure illustrate how we structure the sources of each demand category. Consumer nondurable goods and services are sold directly to households; consumer durable goods, residential capital goods, and nonresidential capital goods are intermediated through capital-goods intermediaries (owned by the households), who then rent these capital stocks to households. Consumer nondurable goods and services and residential capital goods are purchased (by households and residential capital goods owners, respectively) from the first of economy's two final goods-producing sectors, while consumer durable goods and nonresidential capital goods are purchased (by consumer durable and residential capital goods owners, respectively) from the second sector. In addition to consuming the nondurable goods and services that they purchase, households supply labor to the intermediate goods-producing firms in both sectors of the economy.

The remainder of this section provides an overview of the main properties of the model. In

[1] Chung, Kiley, and Laforte (2010) provide much more detail regarding the model specification, estimated parameters, and model properties.

Figure 3: Model Overview



particular, the model has five key features:

- A New-Keynesian structure for price and wage dynamics. Unemployment measures the difference between the amount workers are willing to be employed and firms' employment demand. As a result, unemployment is an indicator of wage and, hence, price pressures as in Gali (2011).
- Production of goods and services occurs in two sectors, with differential rates of technological progress across sectors. In particular, productivity growth in the investment and consumer durable goods sector exceeds that in the production of other goods and services, helping the model match facts regarding long-run growth and relative price movements.
- A disaggregated specification of household preferences and firm production processes that leads to separate modeling of nondurables and services consumption, durables consumption, residential investment, and business investment.
- Risk premiums associated with different investment decisions play a central role in the model. These include, first, an aggregate risk premium, or natural rate of interest, shock driving a wedge between the short-term policy rate and the interest rate faced by private decisionmakers

(as in Smets and Wouters (2007)) and, second, fluctuations in the discount factor/risk premiums faced by the intermediaries financing household (residential and consumer durable) and business investment.

2.1 Two-sector production structure

It is well known (for example, Edge, Kiley, and Laforte (2008)) that real outlays for business investment and consumer durables have substantially outpaced those on other goods and services, while the prices of these goods (relative to others) has fallen. For example, real outlays on consumer durables have far outpaced those on other consumption while prices for consumer durables have been flat and those for other consumption have risen substantially; as a result, the ratio of nominal outlays in the two categories has been much more stable, although consumer durable outlays plummeted in the Great Recession. Many models fail to account for this fact.

EDO accounts for this development by assuming that business investment and consumer durables are produced in one sector and other goods and services in another sector. Specifically, production by firm j in each sector s (where s equals kb for the sector producing business investment and consumer durables and cbi for the sector producing other goods and services) is governed by a Cobb-Douglas production function with sector-specific technologies:

$$X_t^s(j) \;= \left(Z_t^m Z_t^s L_t^s(j)\right)^{1-\alpha} \left(K_t^{u,nr,s}(j)\right)^{\alpha}, \text{ for } s = cbi, kb. \tag{1}$$

In equation (1), Z^m represents (labor-augmenting) aggregate technology, while Z^s represents (labor-augmenting) sector-specific technology; we assume that sector-specific technological change affects the business investment and consumer durables sector only. L^s is labor input and $K^{u,nr,s}$ is capital input (that is, utilized *nonresidential business* capital (and hence the nr and u terms in the superscript). Growth in this sector-specific technology accounts for the long-run trends, while high-frequency fluctuations allow for the possibility that investment-specific technological change is a source of business cycle fluctuations, as in Fisher (2006).

2.2 The structure of demand

EDO differentiates between several categories of expenditure. Specifically, business investment spending determines nonresidential capital used in production, and households value consumer non-durables goods and services, consumer durable goods, and residential capital (for example, housing). Differentiation across these categories is important, as fluctuations in these categories of expenditure can differ notably, with the cycles in housing and business investment, for example, occurring at different points over the last three decades.

Valuations of these goods and services, in terms of household utility, is given by the following utility function:

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \varsigma^{cnn} \ln(E_t^{cnn}(i) - hE_{t-1}^{cnn}(i)) + \varsigma^{cd} \ln(K_t^{cd}(i)) + \varsigma^{r} \ln(K_t^{r}(i)) - \Lambda_t^{Lpref} \Theta_t^H \sum_{s=cbi,kb} \int_0^1 \varsigma^{l,s} L_t^s(i)^{\frac{1+\sigma_N}{1+\frac{\sigma_N}{1+\sigma_h}}} di \Big\}, \quad (2)$$

where E^{cnn} represents expenditures on consumption of nondurable goods and services, K^{cd} and K^r represent the stocks of consumer durables and residential capital (housing), Λ_t^{Lpref} represents a labor supply shock, Θ_t is an endogenous preference shifter whose role is to reconcile the existence of a long-run balance growth path with a small short-term wealth effect[2], L^{cbi} and L^{kb} represent the labor supplied to each productive sector (with hours worked causing disutility), and the remaining terms represent parameters (such as the discount factor, relative value in utility of each service flow, and the elasticity of labor supply). Gali, Smets, and Wouters (2011) state that the introduction of the endogenous preference shifter is key in order to match the joint behavior of the labor force, consumption, and wages over the business cycle.

By modeling preferences over these disaggregated categories of expenditure, EDO attempts to account for the disparate forces driving consumption of nondurables and durables, residential investment, and business investment —thereby speaking to issues such as the surge in business investment in the second half of the 1990s or the housing cycle in the early 2000s recession and the most recent downturn. Many other models do not distinguish between developments across these categories of spending.

2.3 Risk premiums, financial shocks, and economic fluctuations

The structure of the EDO model implies that households value durable stocks according to their expected returns, including any expected service flows, and according to their risk characteristics, with a premium on assets that have high expected returns in adverse states of the world. However, the behavior of models such as EDO is conventionally characterized under the assumption that this second component is negligible. In the absence of risk adjustment, the model would then imply that households adjust their portfolios until expected returns on all assets are equal.

Empirically, however, this risk adjustment may not be negligible and, moreover, there may be a variety of factors, not explicitly modeled in EDO, that limit the ability of households to arbitrage away expected return differentials across different assets. To account for this possibility, EDO features several exogenous shocks to the rates of return required by the household to hold the assets in question. Following such a shock —an increase in the premium on a given asset, for example —households will wish to alter their portfolio composition to favor the affected asset, leading to changes in the prices of all assets and, ultimately, to changes in the expected path of production underlying these claims.

[2] The endogenous preference shifter is defined as $\Theta_t^H = Z_t \Lambda_t^{cnn}$, where $Z_t = \frac{Z_{t-1}^{1-\nu}}{\Lambda_t^{cnn}}$ and Λ_t^{cnn} is the shadow price of nondurable consumption. The importance of the short-term wealth effect is determined by the parameter $\nu \in (0, 1]$.

The "sector specific" risk shocks affect the composition of spending more than the path of GDP itself. This occurs because a shock to these premiums leads to sizable substitution across residential, consumer durable, and business investment; for example, an increase in the risk premiums on residential investment leads households to shift away from residential investment and toward other types of productive investment. Consequently, it is intuitive that a large fraction of the non-cyclical, or idiosyncratic, component of investment flows to physical stocks will be accounted for by movements in the associated premiums.

Shocks to the required rate of return on the nominal risk-free asset play an especially large role in EDO. Following an increase in the premium, in the absence of nominal rigidities, the households' desire for higher real holdings of the risk-free asset would be satisfied entirely by a fall in prices, that is, the premium is a shock to the natural rate of interest. Given nominal rigidities, however, the desire for higher risk-free savings must be offset, in part, through a fall in real income, a decline which is distributed across all spending components. Because this response is capable of generating co-movement across spending categories, the model naturally exploits such shocks to explain the business cycle. Reflecting this role, we denote this shock as the "aggregate risk-premium."

Movements in financial markets and economic activity in recent years have made clear the role that frictions in financial markets play in economic fluctuations. This role was apparent much earlier, motivating a large body of research (for example, Bernanke, Gertler, and Gilchrist (1999)). While the range of frameworks used to incorporate such frictions has varied across researchers studying different questions, a common theme is that imperfections in financial markets —for example, related to imperfect information on the outlook for investment projects or earnings of borrowers —drives a wedge between the cost of riskless funds and the cost of funds facing households and firms. Much of the literature on financial frictions has worked to develop frameworks in which risk premiums fluctuate for endogenous reasons (for example, because of movements in the net worth of borrowers). Because the risk-premium shocks induces a wedge between the short-term nominal risk-free rate and the rate of return on the affected risky rates, these shocks may thus also be interpreted as a reflection of financial frictions not explicitly modeled in EDO. The sector-specific risk premiums in EDO enter the model in much the same way as does the exogenous component of risk premiums in models with some endogenous mechanism (such as the financial accelerator framework used Boivin, Kiley, and Mishkin (2010)), and the exogenous component is quantitatively the most significant one in that research.[3]

2.4 Labor market dynamics in the EDO model

This version of the EDO model assumes that labor input consists of both employment and hours per worker. Workers differ in the disutility they associate with employment. Moreover, the labor market is characterized by monopolistic competition. As a result, unemployment arises in equilibrium – some workers are willing to be employed at the prevailing wage rate, but cannot find employment because firms are unwilling to hire additional workers at the prevailing wage.

[3]Specifically, the risk premiums enter EDO to a first-order (log)linear approximation in the same way as in the cited research if the parameter on net worth in the equation determining the borrowers cost of funds is set to zero; in practice, this parameter is often fairly small in financial accelerator models.

As emphasized by Gali (2011), this framework for unemployment is simple and implies that the unemployment rate reflects wage pressures: When the unemployment rate is unusually high, the prevailing wage rate exceeds the marginal rate of substitution between leisure and consumption, implying that workers would prefer to work more.

The new preference specification and the incorporation of labor force participation in the information set impose discipline in the overall labor market dynamics of the EDO model. The estimated short-run wealth effect on labor supply is relatively attenuated with respect to previous versions of the EDO model. Therefore, the dynamics of both labor force participation and employment are more aligned with the empirical evidence.

In addition, in our environment, nominal wage adjustment is sticky, and this slow adjustment of wages implies that the economy can experience sizable swings in unemployment with only slow wage adjustment. Our specific implementation of the wage adjustment process yields a relatively standard New Keynesian wage Phillips curve. The presence of both price and wage rigidities implies that stabilization of inflation is not, in general, the best possible policy objective (although a primary role for price stability in policy objectives remains).

While the specific model on the labor market is suitable for discussion of the links between employment and wage/price inflation, it leaves out many features of labor market dynamics. Most notably, it does not consider separations, hires, and vacancies, and is hence not amenable to analysis of issues related to the Beveridge curve.

The decline in employment during the Great Recession primarily reflected, according to the EDO model, the weak demand that arose from elevated risk premiums that depressed spending, as illustrated by the light blue and red bars in figure 1. The role played by these demand factors in explaining the cyclical movements in employment is only determinant during the 1980s and during the Great Recession. As apparent in figure 1, the most relevant drivers of employment in the remaining of the sample are labor supply (preference) and markup shocks as shown by the blue bars. Specifically, favorable supply developments in the labor market are estimated to have placed upward pressure on employment until 2010; these developments have reversed, and some of the currently low level for employment growth is, according to EDO, attributable to adverse labor market supply developments. As discussed previously, these developments are simply exogenous within EDO and are not informed by data on a range of labor market developments (such as gross worker flows and vacancies).

2.5 New Keynesian price and wage Phillips curves

As in most of the related literature, nominal prices and wages are both “sticky” in EDO. This friction implies that nominal disturbances —that is, changes in monetary policy —have effects on real economic activity. In addition, the presence of both price and wage rigidities implies that stabilization of inflation is not, in general, the best possible policy objective (although a primary role for price stability in policy objectives remains).

Given the widespread use of the New Keynesian Phillips curve, it is perhaps easiest to consider the form of the price and wage Phillips curves in EDO at the estimated parameters. The price

Phillips curve (governing price adjustment in both productive sectors) has the form

$$\pi_t^{p,s} = 0.22\pi_{t-1}^{p,s} + 0.76E_t\pi_{t+1}^{p,s} + .017mc_t^s + \theta_t^s \tag{3}$$

where mc is marginal cost and θ is a markup shock. As the parameters indicate, inflation is primarily forward looking in EDO.

The wage (w) Phillips curve for each sector has the form

$$\triangle w_t^s = 0.01\triangle w_{t-1}^s + 0.95E_t\triangle w_{t+1}^s + .012\left(mrs_t^{c,l} - w_t^s\right) + \theta_t^w + adj.\,costs. \tag{4}$$

where mrs represents the marginal rate of substitution between consumption and leisure. Wages are primarily forward looking and relatively insensitive to the gap between households' valuation of time spent working and the wage.

The top right panel of figure 1 presents the decomposition of inflation fluctuations into the exogenous disturbances that enter the EDO model. As can be seen, aggregate demand fluctuations, including aggregate risk premiums and monetary policy surprises, contribute little to the fluctuations in inflation according to the model. This is not surprising: In modern DSGE models, transitory demand disturbances do not lead to an unmooring of inflation (so long as monetary policy responds systematically to inflation and remains committed to price stability). In the short run, inflation fluctuations primarily reflect transitory price and wage shocks, or markup shocks in the language of EDO. Technological developments can also exert persistent pressure on costs, most notably during and following the strong productivity performance of the second half of the 1990s, which is estimated to have lowered marginal costs and inflation through the early 2000s. More recently, disappointing labor productivity readings over the course of 2011 have led the model to infer sizable negative technology shocks in both sectors, contributing noticeably to inflationary pressure over that period (as illustrated by the blue bars in figure 1).

2.6 Monetary authority and a long-term interest rate

We now turn to the last agent in our model, the monetary authority. It sets monetary policy in accordance with an Taylor-type interest rate feedback rule. Policymakers smoothly adjust the actual interest rate R_t to its target level $\bar{R}_t$

$$R_t = (R_{t-1})^{\rho^r}\left(\bar{R}_t\right)^{1-\rho^r}\exp\left[\epsilon_t^r\right], \tag{5}$$

where the parameter ρ^r reflects the degree of interest rate smoothing, while ϵ_t^r represents a monetary policy shock. The central bank's target nominal interest rate, $\bar{R}_t$ depends on the deviation of output from the level consistent with current technologies and "normal" (steady-state) utilization of capital and labor ($\tilde{X}^{pf}$, the "production function" output gap). Also, the change in the output gap and consumer price inflation enter the target. The target equation is

$$\bar{R}_t = \left(\tilde{X}_t^{pf}\right)^{r^y} \left(d\tilde{X}_t^{pf}\right)^{r^{dy}} \left(\frac{\Pi_t^c}{\Pi_*^c}\right)^{r^\pi} R_*. \tag{6}$$

In equation (6), R_* denotes the economy's steady-state nominal interest rate, $d\tilde{X}_t^{pf}$ denotes the change in the output gap and r^y, r^{dy} and r^π denote the weights in the feedback rule. Consumer price inflation, Π_t^c, is the weighted average of inflation in the nominal prices of the goods produced in each sector, $\Pi_t^{p,cbi}$ and $\Pi_t^{p,kb}$:

$$\Pi_t^c = (\Pi_t^{p,cbi})^{1-w_{cd}} (\Pi_t^{p,kb})^{w_{cd}}. \tag{7}$$

The parameter w^{cd} is the share of the durable goods in nominal consumption expenditures.

The model also includes a long-term interest rate (RL_t), which is governed by the expectations hypothesis subject to an exogenous term premiums shock:

$$RL_t = \mathcal{E}_t \left[\Pi_{\tau=0}^N R_\tau\right] \cdot \Upsilon_t. \tag{8}$$

where Υ is the exogenous term premium, governed by

$$Ln\left(\Upsilon_t\right) = \left(1 - \rho^\Upsilon\right) Ln\left(\Upsilon_*\right) + \rho^\Upsilon Ln\left(\Upsilon_{t-1}\right) + \epsilon_t^\Upsilon. \tag{9}$$

In this version of EDO, the long-term interest rate plays no allocative role; nonetheless, the term structure contains information on economic developments useful for forecasting (for example, Edge, Kiley, and Laforte (2010)), and hence RL is included in the model and its estimation.

2.7 Summary of model specification

Our brief presentation of the model highlights several points. First, although our model considers production and expenditure decisions in a bit more detail, it shares many similar features with other DSGE models in the literature, such as imperfect competition, nominal price and wage rigidities, and real frictions like adjustment costs and habit-persistence. The rich specification of structural shocks (to aggregate and investment-specific productivity, aggregate and sector-specific risk premiums, and markups) and adjustment costs allows our model to be brought to the data with some chance of finding empirical validation.

Within EDO, fluctuations in all economic variables are driven by 13 structural shocks. It is most convenient to summarize these shocks into five broad categories:

- Permanent technology shocks: This category consists of shocks to aggregate and investment-specific (or fast-growing sector) technology.
- A labor supply shock: This shock affects the willingness to supply labor. As was apparent in our earlier description of labor market dynamics and in the presentation of the structural drivers below, this shock captures the dynamics of the labor force participation rate in the sample and those of employment. While EDO labels such movements labor supply shocks, an alternative

interpretation would describe these as movements in the labor force and employment that reflect structural features not otherwise captured by the model.

- Financial, or intertemporal, shocks: This category consists of shocks to risk premiums. In EDO, variation in risk premiums —both the premium households receive relative to the federal funds rate on nominal bond holdings and the additional variation in discount rates applied to the investment decisions of capital intermediaries —are purely exogenous. Nonetheless, the specification captures aspects of related models with more explicit financial sectors (for example, Bernanke, Gertler, and Gilchrist (1999)), as we discuss in our presentation of the model's properties below.
- Markup shocks: This category includes the price and wage markup shocks.
- Other demand shocks: This category includes the shock to autonomous demand and a monetary policy shock.

3 Estimation: Data and Properties

3.1 Data

The empirical implementation of the model takes a log-linear approximation to the first-order conditions and constraints that describe the economy's equilibrium, casts this resulting system in its state-space representation for the set of (in our case, 13) observable variables, uses the Kalman filter to evaluate the likelihood of the observed variables, and forms the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods.

The model is estimated using 13 data series over the sample period from 1984:Q4 to 2015:Q3. The series are the following:

1. The growth rate of real gross domestic product (ΔGDP);
2. The growth rate of real consumption expenditure on nondurables and services (ΔC);
3. The growth rate of real consumption expenditure on durables (ΔCD);
4. The growth rate of real residential investment expenditure (ΔRes);
5. The growth rate of real business investment expenditure (ΔI);
6. Consumer price inflation, as measured by the growth rate of the Personal Consumption Expenditure (PCE) price index ($\Delta P_{C,total}$);
7. Consumer price inflation, as measured by the growth rate of the PCE price index excluding food and energy prices ($\Delta P_{C,core}$);
8. Inflation for consumer durable goods, as measured by the growth rate of the PCE price index for durable goods (ΔP_{cd});
9. Hours, which equals hours of all persons in the nonfarm business sector from the Bureau of Labor Statistics (H);

10. Civilian employment-population ratio, defined as civilian employment from the Current Population Survey (household survey) divided by the noninstitutional population, age 16 and over (N);
11. Labor force participation rate;
12. The growth rate of real wages, as given by compensation per hour in the non-farm business sector from the Bureau of Labor Statistics divided by the GDP price index (ΔRW); and
13. The federal funds rate (R).

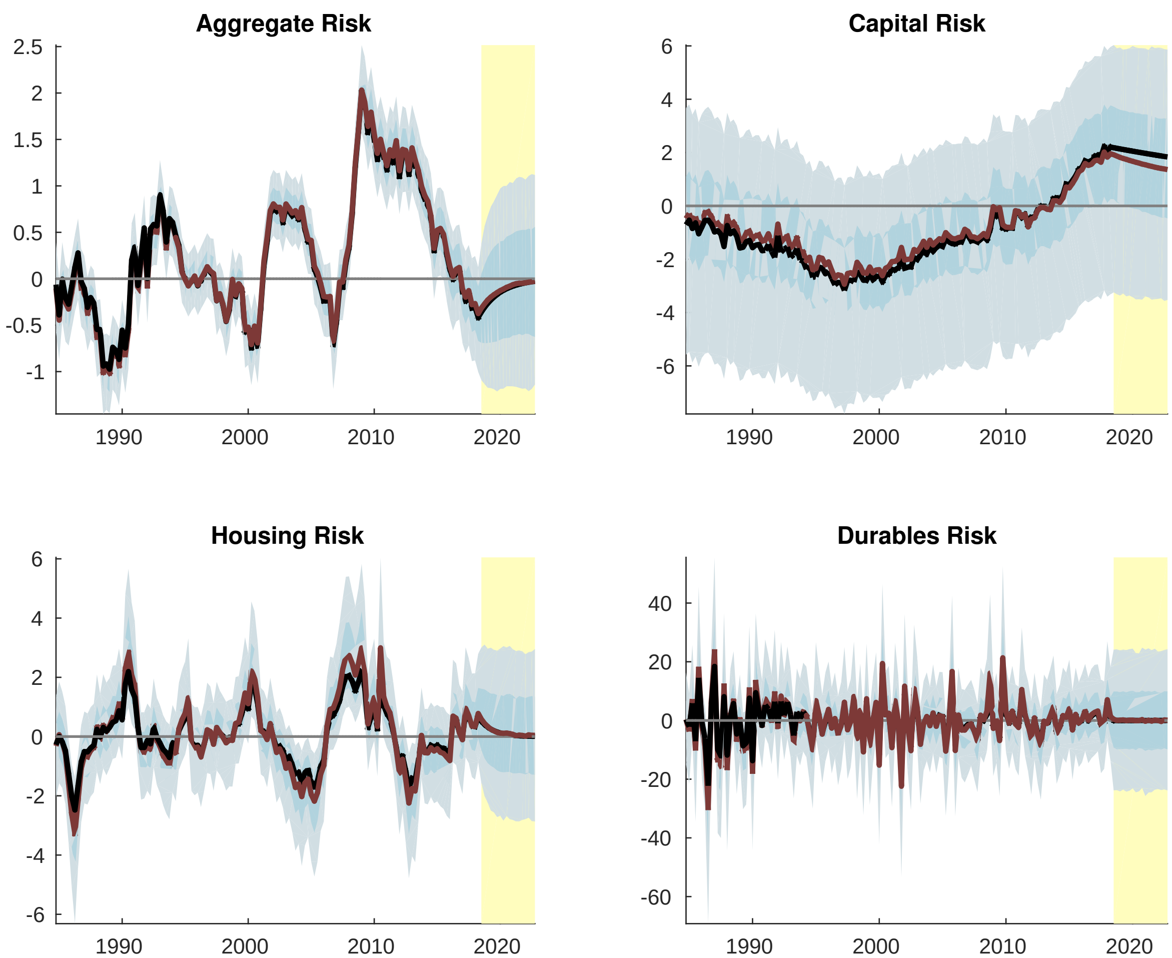
Our implementation adds measurement error processes to the likelihood implied by the model for all of the observed series used in estimation except the short-term nominal interest rate series.

3.2 Estimates of latent variable paths

Figures 4, 5, and 6 report estimates of the model's persistent exogenous fundamentals (for example, risk premiums and autonomous demand). These series have recognizable patterns for those familiar with U.S. economic fluctuations. For example, the risk premiums jump at the end of 2008, reflecting the financial crisis and the model's identification of risk premiums, both economy-wide and for housing, as key drivers.

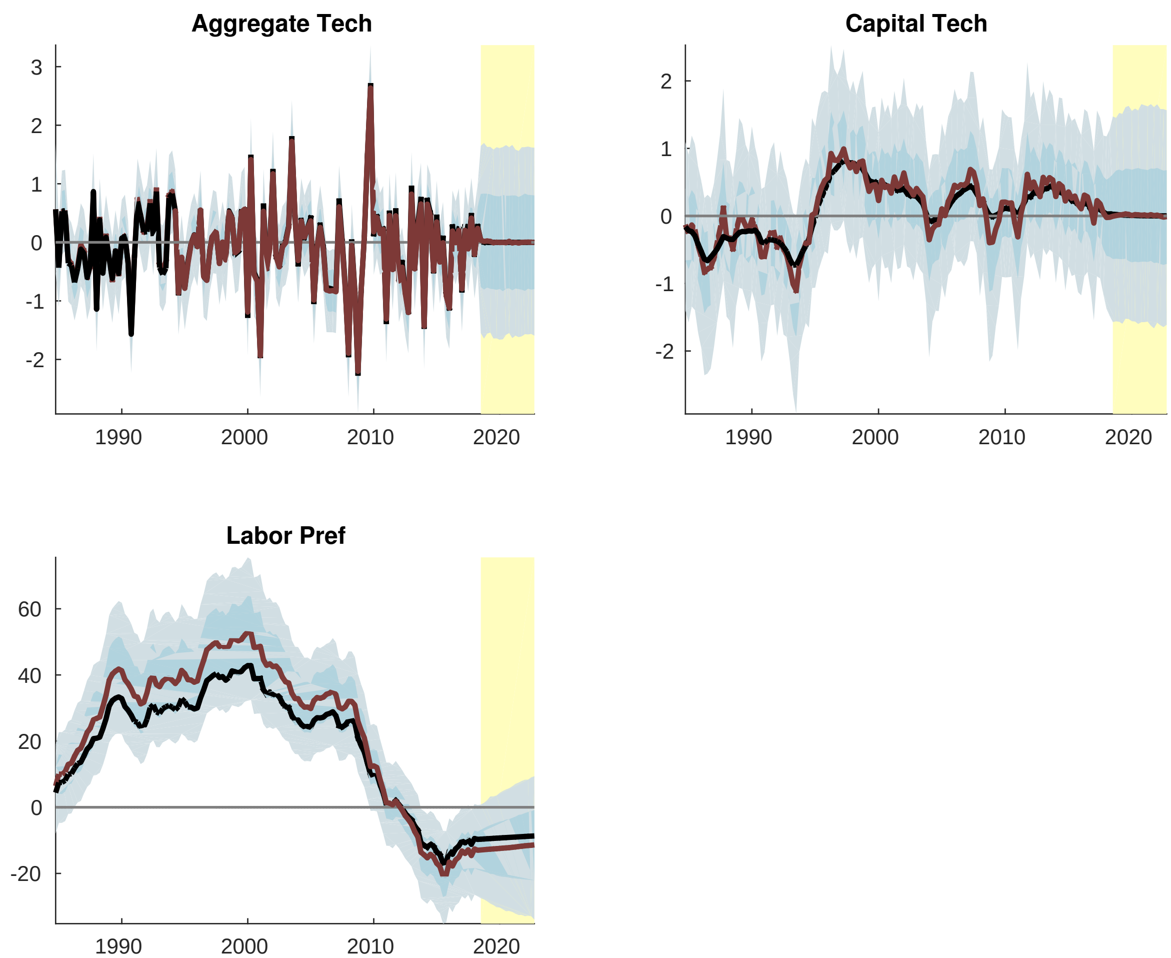
Of course, these stories from a glance at the exogenous drivers, yield applications for alternative versions of the EDO model and future model enhancements. For example, the exogenous risk premiums can easily be made to have an endogenous component, following the approach of Bernanke, Gertler, and Gilchrist (1999) (and, indeed, we have considered models of that type). At this point, we view incorporation of such mechanisms in our baseline approach as premature, pending ongoing research on financial frictions, banking, and intermediation in dynamic general equilibrium models. Nonetheless, the EDO model captured the key financial disturbances during the last several years in its current specification, and examining the endogenous factors that explain these developments will be a topic of further study.

Figure 4: Model Estimates of Risk Premiums



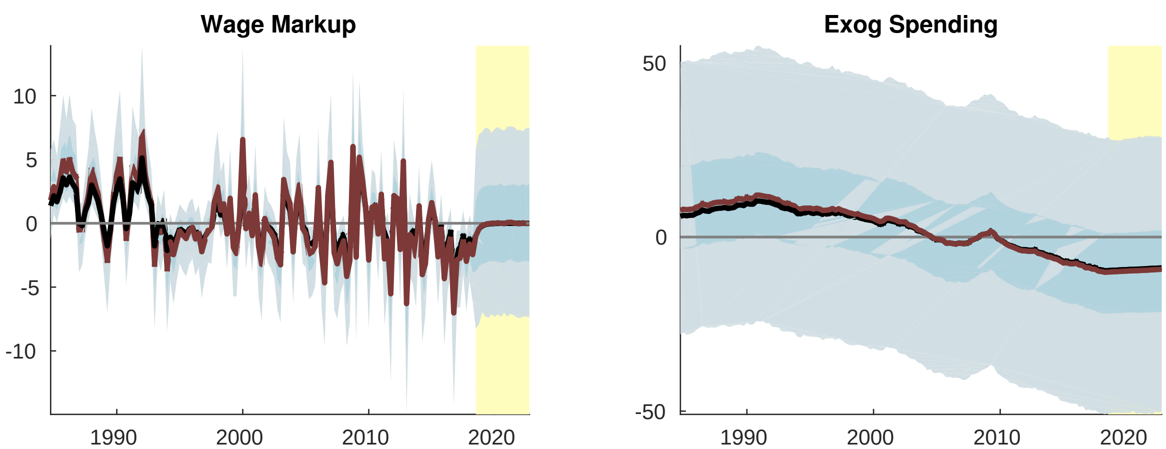
Black line: modal parameters. Red line: posterior median. Dark blue intervals: 68 percent credible set. Light blue intervals: 95 percent credible set.

Figure 5: Model Estimates of Key Supply-side Variables



Black line: modal parameters. Red line: posterior median. Dark blue intervals: 68 percent credible set. Light blue intervals: 95 percent credible set.

Figure 6: Model Estimates of Selected Other Exogenous Drivers



Black line: modal parameters. Red line: posterior median. Dark blue intervals: 68 percent credible set. Light blue intervals: 95 percent credible set.

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New York Fed DSGE Model: Research Directors Draft

May 31, 2018

Introduction

This document describes the New York Fed DSGE model, which we use both for internal forecasting and for creating our contributions to the System DSGE memo distributed quarterly to the FOMC. The document is structured as follows. First, we provide a description and interpretation of the forecast for the current forecast horizon. Next, we describe the structure of the DSGE model followed by the impulse response functions to various shocks.

Model Forecast

The New York Fed model forecasts are obtained using data released through 2018Q1, augmented for 2018Q2 with the New York Fed staff forecasts (as of May 31) for real GDP growth and core PCE inflation, and with values of the federal funds rate, the 10-year Treasury yield and the spread between Baa corporate bonds and 10-year Treasury yields based on 2018Q2 averages up to May 31.

Table 1 shows both the conditional and unconditional forecasts of real GDP growth, core PCE inflation, federal funds rate, real natural rate of interest and the output gap. *Unconditional* forecasts are obtained using data up to the quarter for which we have the most recent GDP release, as well as the federal funds rate, 10-year Treasury yield, and spreads data for the following ("current") quarter. *Conditional* forecasts further include the current-quarter New York Fed staff projections for GDP growth and core PCE inflation as additional data points.

Figure 1 plots the conditional and unconditional forecasts of real GDP growth, core PCE inflation and the federal funds rate. Figure 2 provides a comparison of current and previous quarterly forecasts while Figure 3 depicts the shock decomposition of the conditional forecasts, where different colored bars indicate the contribution of different shocks to the conditional forecast of GDP growth, inflation and federal funds rate. Finally, Figure 4 plots the historical estimates and forecast of the output gap in the top panel, and the real natural rate in the bottom panel.

The *output gap* is defined as the difference between actual output and potential output. Potential output is defined as the level that output would take in a world where capital and labor are fully utilized, i.e., where there are no nominal rigidities or shocks to markups.[1] A positive (negative) output gap indicates that output is above (below) its potential. The *natural rate of interest* is a concept analogous to potential output: it represents the rate of interest that would prevail in the economy absent nominal rigidities and markup shocks.

Current Forecast

We project real GDP growth of 2.3 percent in 2018 on a Q4/Q4 basis, in line with the March forecast. GDP growth is anticipated to decline to 1.9 percent in 2019 and 2020, slightly below the March forecast of 2.0 percent and 2.1 percent in 2019 and 2020 respectively. Inflation is forecast to be slightly higher in the short term, at 1.9 percent relative to 1.8 percent in the March projection. While inflation in 2018 is close to the FOMC's longer run goal of 2 percent, the model projects that inflation will decline to 1.5 percent in 2019 and 2020 (unchanged from the March projection).

The output gap is projected to be -0.4 percent in 2018, -0.3 percent in 2019 and -0.3 percent in 2020. This represents an improvement relative to the March forecast of the output gap (-0.7 percent, -0.6 percent and -0.4 percent in 2018, 2019 and 2020 respectively), especially in the near term. The forecast for the natural rate of interest is unchanged from March, rising from 1.0 percent in 2018 to 1.4 percent by the end of 2020.

The federal funds rate is forecast to be 0.1 percentage points lower than the March projections throughout the forecast horizon: 2.2, 2.6 and 2.8 percent in 2018, 2019 and 2020 respectively. This shallower path translates into approximately two more hikes in 2018, one more in 2019 and another in 2020.

The projections for all the variables are surrounded by significant uncertainty. For instance, the 68 percent posterior probability interval for GDP growth includes negative readings for the years between 2019 and 2020. In comparison, the posterior probability intervals for inflation are tighter, with their upper bound never exceeding 3 percent throughout the forecast horizon.

As in March, the model explains the above long-run average real GDP growth rate in 2018 with continued improvement in financial conditions, as captured by positive contributions of

[1] Markup shocks represent exogenous fluctuations in price and wage inflation arising from various sources, such as variations in the degree of market power, or in the price of commodities.

both the financial and marginal efficiency of investment shocks. Over the medium term however, these factors are offset by lower TFP growth and the gradual withdrawal of monetary accommodation (see the top panel of Figure 3). Sluggish TFP growth also explains, at least in part, the negative output gap over the forecast horizon. The model projects near-target inflation for 2018 (see middle panel of Figure 3), driven by a temporary increase in price markups. However, as in March, the model projects below-target inflation in 2019 and 2020, driven by a confluence of several factors. These factors include the lingering effects of the financial headwinds that have hampered the recovery, as well as negative shocks to wage and price markups. Finally, the federal funds rate is projected to remain below its long-run level of 4 percent throughout the forecast horizon owing to persistence in the interest rate rule, weak projected inflation and a persistently negative output gap (see lower panel of Figure 3).

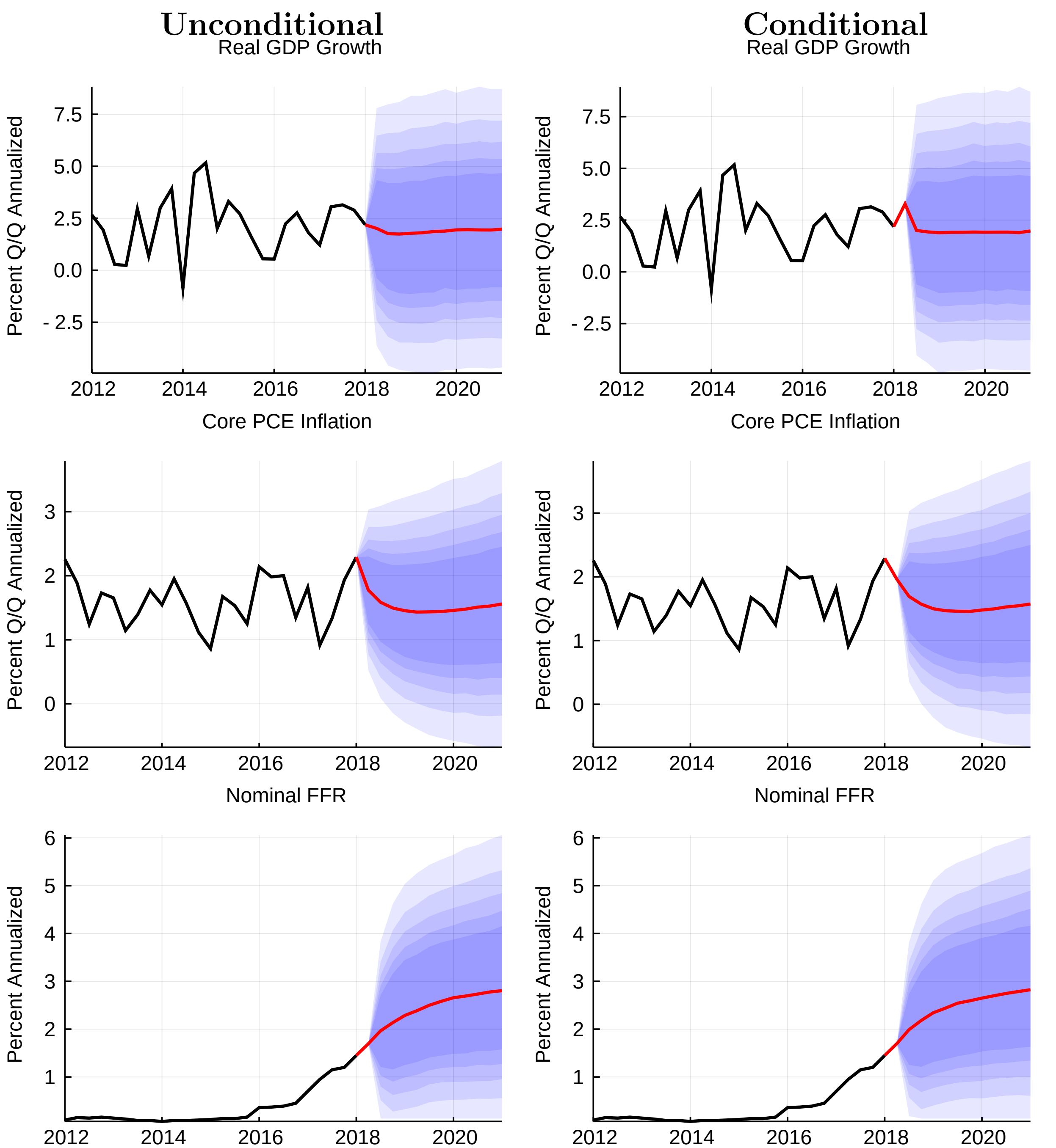
Table 1: Forecasts

	Unconditional Forecast					
	2018		2019		2020	
	Jun.	Mar.	Jun.	Mar.	Jun.	Mar.
Real GDP Growth (Q4/Q4)	1.9 (−0.2,3.8)	2.6 (0.1,5.0)	1.8 (−1.1,4.4)	2.1 (−0.7,4.7)	1.9 (−0.9,4.6)	2.1 (−0.7,4.7)
Core PCE Inflation (Q4/Q4)	1.8 (1.2,2.3)	1.4 (0.7,2.1)	1.4 (0.5,2.4)	1.4 (0.4,2.3)	1.5 (0.4,2.6)	1.5 (0.3,2.6)
Federal Funds Rate (Q4)	2.1 (0.7,3.6)	2.2 (0.8,3.6)	2.6 (1.0,4.4)	2.6 (1.0,4.4)	2.8 (1.0,4.7)	2.8 (1.0,4.8)
Real Natural Rate (Q4)	1.0 (−0.6,2.6)	1.0 (−0.6,2.7)	1.3 (−0.5,3.0)	1.3 (−0.5,3.1)	1.3 (−0.5,3.2)	1.4 (−0.5,3.3)
Output Gap (Q4)	−0.6 (−2.5,1.1)	−0.5 (−2.5,1.3)	−0.6 (−3.5,1.9)	−0.4 (−3.2,2.3)	−0.6 (−4.0,2.5)	−0.2 (−3.7,2.9)

	Conditional Forecast					
	2018		2019		2020	
	Jun.	Mar.	Jun.	Mar.	Jun.	Mar.
Real GDP Growth (Q4/Q4)	2.3 (0.8,3.8)	2.3 (0.2,4.2)	1.9 (−1.0,4.5)	2.0 (−0.8,4.7)	1.9 (−0.9,4.7)	2.1 (−0.8,4.8)
Core PCE Inflation (Q4/Q4)	1.9 (1.5,2.3)	1.8 (1.3,2.4)	1.5 (0.5,2.4)	1.5 (0.5,2.5)	1.5 (0.4,2.6)	1.5 (0.4,2.7)
Federal Funds Rate (Q4)	2.2 (0.7,3.7)	2.3 (0.9,3.7)	2.6 (1.0,4.4)	2.7 (1.0,4.4)	2.8 (1.1,4.7)	2.9 (1.1,4.8)
Real Natural Rate (Q4)	1.0 (−0.6,2.6)	1.0 (−0.7,2.6)	1.3 (−0.5,3.1)	1.3 (−0.6,3.1)	1.4 (−0.5,3.2)	1.4 (−0.6,3.3)
Output Gap (Q4)	−0.4 (−2.1,1.2)	−0.7 (−2.6,1.0)	−0.3 (−3.1,2.1)	−0.6 (−3.3,1.9)	−0.3 (−3.6,2.6)	−0.4 (−3.8,2.6)

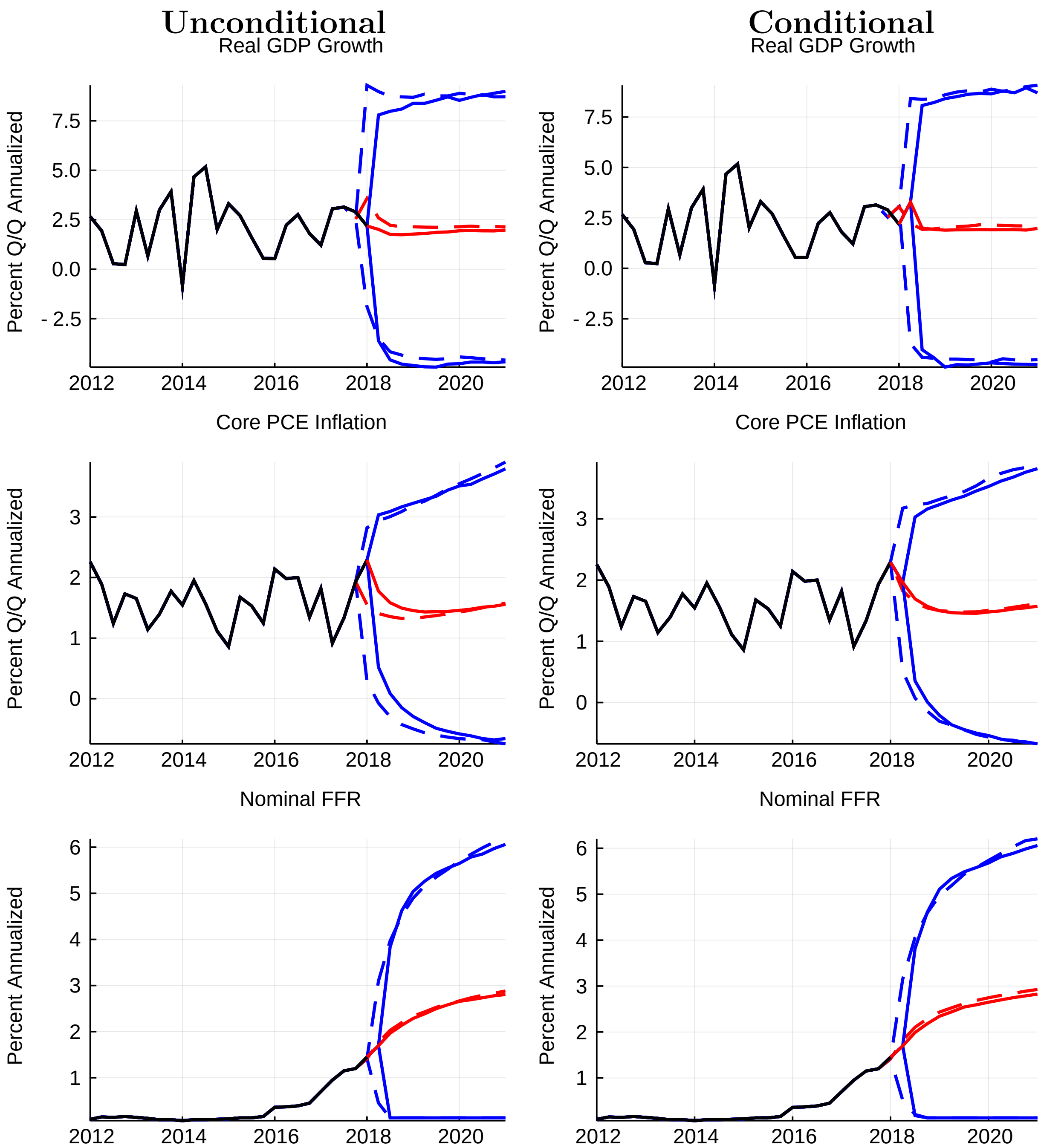
The unconditional forecasts use data up to the quarter for which we have the most recent GDP release, as well as the federal funds rate, 10-year Treasury yield, and spreads data for the following ("current") quarter. In the conditional forecasts, we further include the current-quarter New York Fed staff projections for GDP growth and core PCE inflation as additional data points. Numbers in parentheses indicate 68 percent probability intervals.

Figure 1: Forecasts



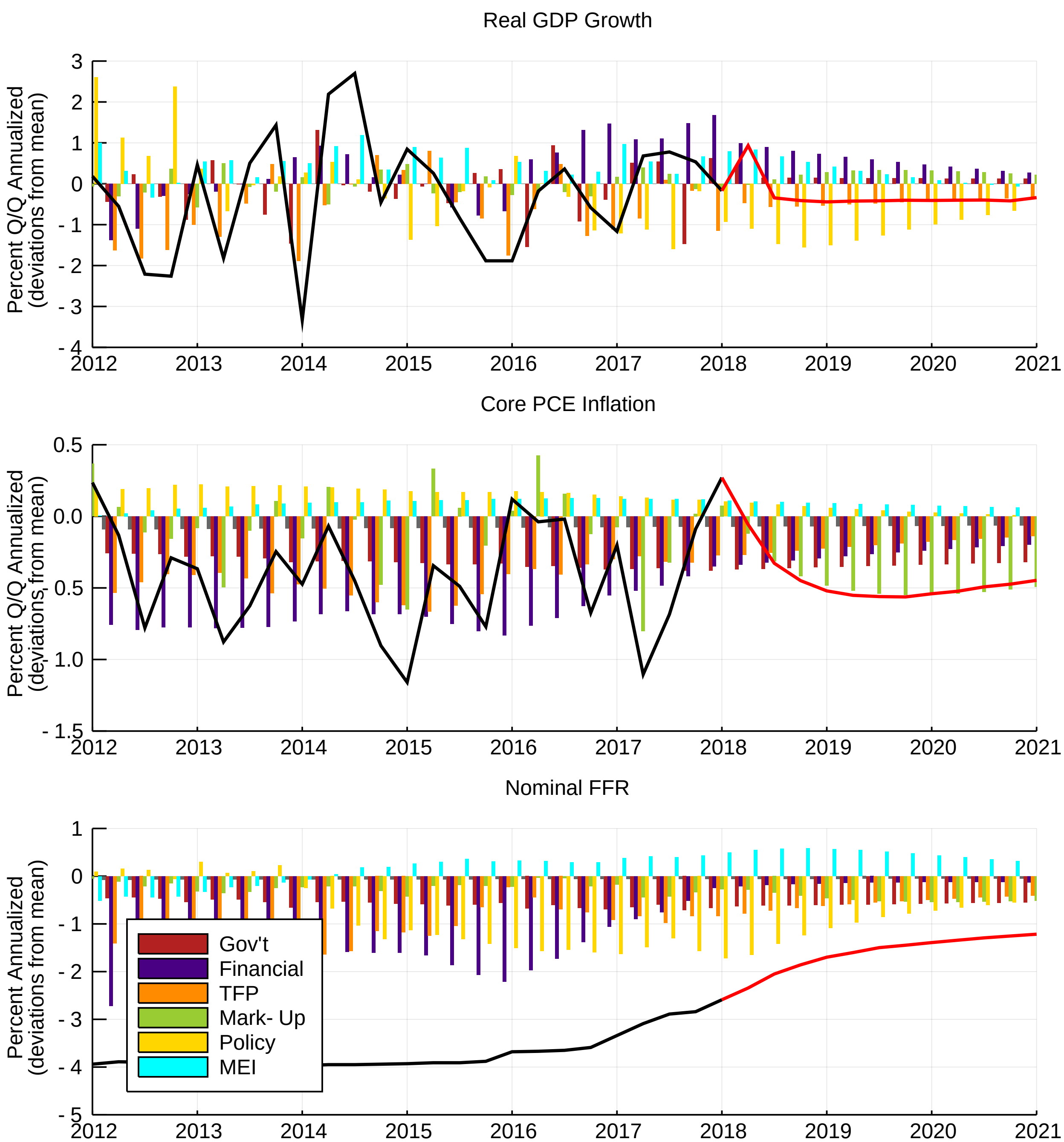
Quarterly forecasts, both unconditional (left panels) and conditional (right panels). The black line represents data, the red line indicates the mean forecast, and the shaded areas mark the 50, 60, 70, 80 and 90 percent probability intervals for the forecasts, reflecting both parameter and shock uncertainty.

Figure 2: Change in Forecasts



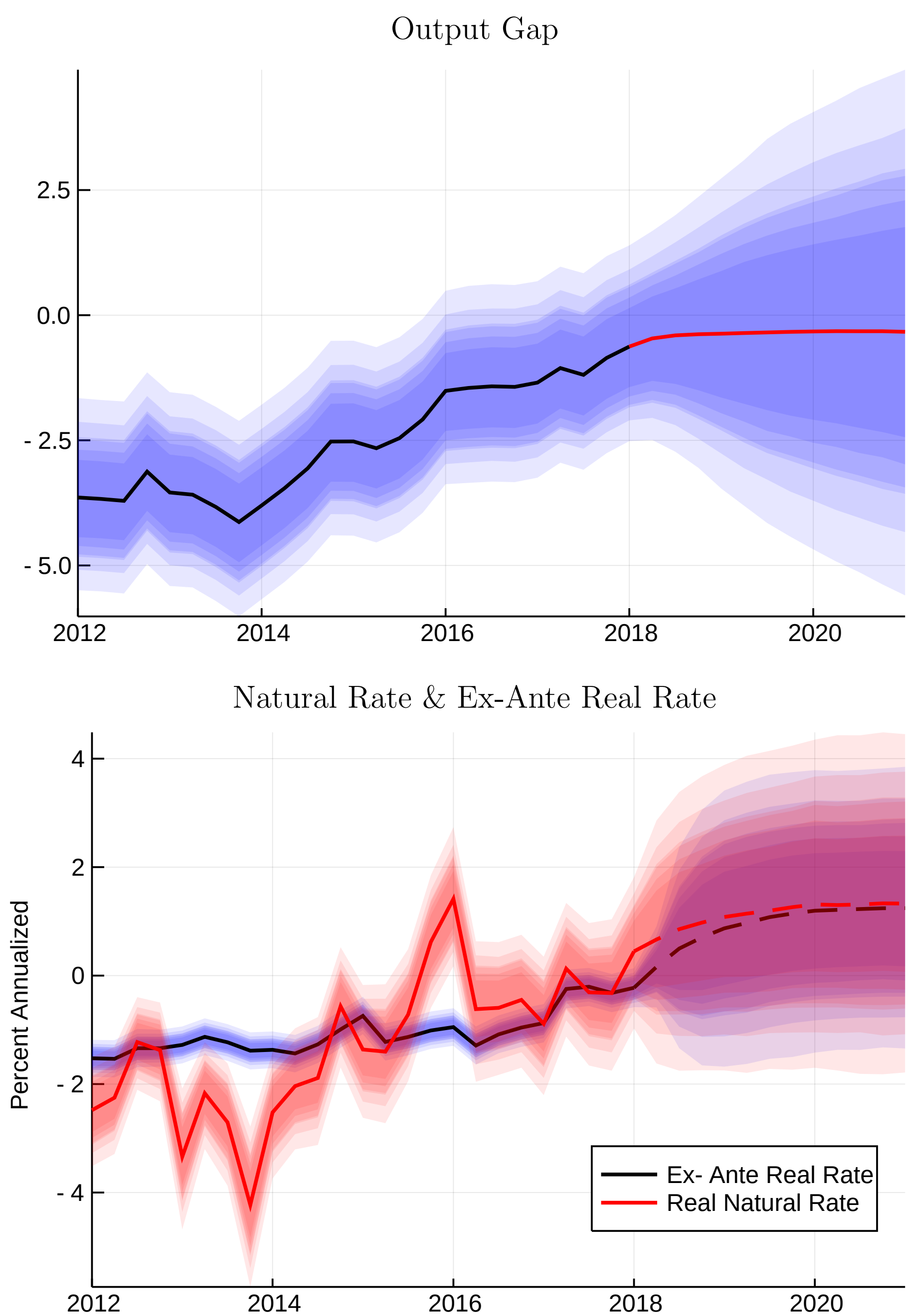
Comparison of current and previous quarterly forecasts. Solid (dashed) red and blue lines represent the mean and the 90 percent probability intervals, respectively, of the current (previous) forecast.

Figure 3: Shock Decomposition



Shock decomposition of the conditional forecast. The solid lines (black for realized data, red for mean forecast) show each variable in deviation from its steady state. The bars represent the shock contributions; specifically, the bars for each shock represent the counterfactual values for the observables (in deviations from the mean) obtained by setting all other shocks to zero.

Figure 4: Output Gap and Natural Interest Rate



Historical estimates and forecasts of the output gap (upper panel) and the real natural rate of interest and the ex-ante real interest rate (lower panel). In the upper panel, the black line represents the mean historical estimate, the red line the mean forecast. In the lower panel, the solid lines represent historical estimates and the dashed lines represent forecasts of the natural rate (red) and ex-ante rate (black). In both panels, the shaded areas mark the 50, 60, 70, 80, and 90 percent probability intervals for the historical estimates and forecasts, reflecting both parameter and shock uncertainty.

The Model

The following section contains a description of the New York Fed DSGE model and plots of impulse response functions.

General structure

The New York Fed DSGE model is a medium scale, one-sector dynamic stochastic general equilibrium model which is based on the New Keynesian model with financial frictions used in Del Negro et al. (2015). The core of the model is based on the work of Smets and Wouters (2007) (henceforth SW) and Christiano et al. (2005): It builds on the neo-classical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, habit formation in consumption. The model also includes credit frictions as in the *financial accelerator* model developed by Bernanke et al. (1999b) where the actual implementation of credit frictions follows closely Christiano et al. (2014), and accounts for forward guidance in monetary policy by including anticipated policy shocks as in Laseen and Svensson (2011).

The current version of the model has several features that improve upon the version presented in the New York Fed Staff Report no. 647. It features both a deterministic and a stochastic trend in productivity and allows for exogenous movements in risk premia; the inflation target is time-varying, following Del Negro and Schorfheide (2012); households preferences are non-separable in consumption and leisure; the Dixit-Stiglitz aggregator of intermediate goods has been replaced by the more flexible Kimball aggregator; we include indexation in the price and wage adjustment processes.

Here is a brief overview. The model economy is populated by eight classes of agents: 1) a continuum of households, who consume and supply differentiated labor; 2) competitive labor aggregators that combine labor supplied by individual households; 3) competitive final good-producing firms that aggregate the intermediate goods into a final product; 4) a continuum of monopolistically competitive intermediate good producing firms; 5) competitive capital producers that convert final goods into capital; 6) a continuum of entrepreneurs who purchase capital using both internal and borrowed funds and rent it to intermediate good producing firms; 7) a representative bank collecting deposits from the households and lending funds to the entrepreneurs; and finally 8) a government, composed of a monetary authority that sets short-term interest rates and a fiscal authority that sets public spending and collects taxes.

Growth in the economy is driven by technological progress. We specify a process for technology Z_t^* which includes both a deterministic and a stochastic trend, and a stationary component:

$$Z_t^* = e^{\frac{1}{1-\alpha}\tilde{z}_t} Z_t^p e^{\gamma t}, \tag{1}$$

where γ is the steady state growth rate of the economy, Z_t^p is a stochastic trend and $\tilde{z}_t$ is the stationary component.

The *production function* is

$$Y_t(i) = \max\{e^{\tilde{z}_t} K_t(i)^\alpha \left(L_t(i)e^{\gamma t} Z_t^p\right)^{1-\alpha} - \Phi Z_t^*, 0\}, \tag{2}$$

where ΦZ_t^* is a fixed cost.

Trending variables are divided by Z_t^* to express the model's equilibrium conditions in terms of the stationary variables. In what follows we present a summary of the log-linearized equilibrium conditions, where all variables are expressed in log deviations from their non-stochastic steady state.

Log-linear equilibrium conditions

The stationary component of productivity $\tilde{z}_t$ evolves as:

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \varepsilon_{z,t}. \tag{3}$$

Since Z_t^p is a non stationary process, we define its growth rate as $z_t^p = log(Z_t^p/Z_{t-1}^p)$ and assume that it follows an AR(1) process:

$$z_t^p = \rho_{z^p} z_{t-1}^p + \sigma_{z^p}\epsilon_{z^p,t},\ \epsilon_{z^p,t} \sim N(0,1). \tag{4}$$

It follows that

$$z_t \equiv log(Z_t^*/Z_{t-1}^*) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z \epsilon_{z,t} + z_t^p, \tag{5}$$

where γ is the steady-state growth rate of the economy. Steady-state values are denoted by $*$-subscripts, and steady-state formulas are provided in the technical appendix of Del Negro and Schorfheide (2012), which is available online.

The *optimal allocation of consumption* satisfies the following consumption Euler equation:

$$c_t = -\frac{(1-he^{-\gamma})}{\sigma_c(1+he^{-\gamma})}\left(R_t - \mathbb{E}_t[\pi_{t+1}] + b_t\right) + \frac{he^{-\gamma}}{(1+he^{-\gamma})}\left(c_{t-1} - z_t\right) + \frac{1}{(1+he^{-\gamma})}\mathbb{E}_t\left[c_{t+1} + z_{t+1}\right] + \frac{(\sigma_c - 1)}{\sigma_c(1+he^{-\gamma})}\frac{w_* L_*}{c_*}\left(L_t - \mathbb{E}_t[L_{t+1}]\right), \quad (6)$$

where c_t is consumption, L_t is labor supply, R_t is the nominal interest rate, and π_t is inflation. The exogenous process b_t drives a wedge between the intertemporal marginal utility of consumption and the riskless real return $R_t - \mathbb{E}_t[\pi_{t+1}]$, and is meant to capture risk-premium shocks.[2] This shock follows an AR(1) process with parameters ρ_b and σ_b. The parameters σ_c and h capture the degree of relative risk aversion and the degree of habit persistence in the utility function, respectively.

The *optimal investment decision* satisfies the following relationship between the level of investment i_t, measured in terms of consumption goods, and the value of capital in terms of consumption q_t^k:

$$i_t = \frac{q_t^k}{S'' e^{2\gamma}(1+\bar{\beta})} + \frac{1}{1+\bar{\beta}}\left(i_{t-1} - z_t\right) + \frac{\bar{\beta}}{1+\bar{\beta}}\mathbb{E}_t\left[i_{t+1} + z_{t+1}\right] + \mu_t. \quad (7)$$

This relationship shows that investment is affected by investment adjustment costs (S'' is the second derivative of the adjustment cost function) and by an exogenous process μ_t, which we call “marginal efficiency of investment”, that alters the rate of transformation between consumption and installed capital (see Greenwood et al. (1998)). The shock μ_t follows an AR(1) process with parameters ρ_μ and σ_μ. The parameter $\bar{\beta}$ depends on the intertemporal discount rate in the household utility function, β, on the degree of relative risk aversion σ_c, and on the steady-state growth rate γ: $\bar{\beta} = \beta e^{(1-\sigma_c)\gamma}$.

The *capital stock*, $\bar{k}_t$, which we refer to as “installed capital”, evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right)\left(\bar{k}_{t-1} - z_t\right) + \frac{i_*}{\bar{k}_*} i_t + \frac{i_*}{\bar{k}_*} S'' e^{2\gamma}(1+\bar{\beta})\mu_t, \quad (8)$$

where $i_*/\bar{k}_*$ is the steady state investment to capital ratio.

Capital is subject to variable capacity utilization u_t; *effective capital* rented out to firms,

[2] In the code, the b_t shock is normalized to be in the same units as consumption, i.e., we estimate the shock $\tilde{b}_t = -\frac{(1-he^{-\gamma})}{\sigma_c(1+he^{-\gamma})} b_t$.

k_t, is related to $\bar{k}_t$ by:

$$k_t \;=\; u_t - z_t + \bar{k}_{t-1}. \tag{9}$$

The optimality condition determining the *rate of capital utilization* is given by

$$\frac{1-\psi}{\psi} r_t^k = u_t, \tag{10}$$

where r_t^k is the rental rate of capital and ψ captures the utilization costs in terms of foregone consumption.

Real marginal costs for firms are given by

$$mc_t \;=\; w_t + \alpha L_t - \alpha k_t, \tag{11}$$

where w_t is the real wage and α is the income share of capital (after paying mark-ups and fixed costs) in the production function.

From the optimality conditions of goods producers it follows that all firms have the same *capital-labor ratio*:

$$k_t \;=\; w_t - r_t^k + L_t. \tag{12}$$

We include financial frictions in the model, building on the work of Bernanke et al. (1999a), Christiano et al. (2003), De Graeve (2008), and Christiano et al. (2014). We assume that banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus turn out to be too low to pay back the loans received by the banks. The banks therefore protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of entrepreneurs' leverage and riskiness.

The *realized return on capital* is given by:

$$\tilde{R}_t^k - \pi_t = \frac{r_*^k}{r_*^k + (1-\delta)} r_t^k + \frac{(1-\delta)}{r_*^k + (1-\delta)} q_t^k - q_{t-1}^k, \tag{13}$$

where $\tilde{R}_t^k$ is the gross nominal return on capital for entrepreneurs, r_*^k is the steady state value of the rental rate of capital r_t^k, and δ is the depreciation rate.

The *excess return on capital* (the spread between the expected return on capital and the riskless rate) can be expressed as a function of the entrepreneurs' leverage (i.e. the ratio of the value of capital to nominal net worth) and exogenous fluctuations in the volatility of entrepreneurs' idiosyncratic productivity:

$$E_t\left[\tilde{R}^k_{t+1} - R_t\right] = b_t + \zeta_{sp,b}\left(q^k_t + \bar{k}_t - n_t\right) + \tilde{\sigma}_{\omega,t}, \tag{14}$$

where n_t is entrepreneurs' net worth, $\zeta_{sp,b}$ is the elasticity of the credit spread to the entrepreneurs' leverage $(q^k_t + \bar{k}_t - n_t)$, and $\tilde{\sigma}_{\omega,t}$ captures mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano et al. (2014)). $\tilde{\sigma}_{\omega,t}$ follows an AR(1) process with parameters ρ_{σ_ω} and σ_{σ_ω}.

Entrepreneurs' net worth n_t evolves according to:

$$\begin{aligned} n_t \;=\; & \zeta_{n,\tilde{R}^k}\left(\tilde{R}^k_t - \pi_t\right) - \zeta_{n,R}\left(R_{t-1} - \pi_t + b_{t-1}\right) + \zeta_{n,qK}\left(q^k_{t-1} + \bar{k}_{t-1}\right) + \zeta_{n,n} n_{t-1} \\ & -\gamma_* \tfrac{v_*}{n_*} z_t - \tfrac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}} \tilde{\sigma}_{\omega,t-1}, \end{aligned} \tag{15}$$

where the ζ's denote elasticities, that depend among others on the entrepreneurs' steady-state default probability $F(\bar{\omega})$, where γ_* is the fraction of entrepreneurs that survive and continue operating for another period, and where v_* is the entrepreneurs' real equity divided by Z^*_t, in steady state.

The *production function* is

$$y_t = \Phi_p\left(\alpha k_t + (1-\alpha) L_t\right), \tag{16}$$

where $\Phi_p = \frac{y_* + \Phi}{y_*}$, and the *resource constraint* is:

$$y_t = g_* g_t + \frac{c_*}{y_*} c_t + \frac{i_*}{y_*} i_t + \frac{r^k_* k_*}{y_*} u_t. \tag{17}$$

where $g_t = log(\frac{G_t}{Z^*_t y_* g_*})$ and $g_* = 1 - \frac{c_* + i_*}{y_*}$.

Government spending g_t is assumed to follow the exogenous process:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz} \sigma_z \varepsilon_{z,t}.$$

The *price and wage Phillips curves* are, respectively:

$$\pi_t = \kappa\, mc_t + \frac{\iota_p}{1+\iota_p\bar{\beta}}\pi_{t-1} + \frac{\bar{\beta}}{1+\iota_p\bar{\beta}}\mathbb{E}_t[\pi_{t+1}] + \lambda_{f,t}, \tag{18}$$

and

$$\begin{aligned} w_t = {} & \frac{(1-\zeta_w\bar{\beta})(1-\zeta_w)}{(1+\bar{\beta})\zeta_w((\lambda_w-1)\epsilon_w+1)}\left(w_t^h - w_t\right) - \frac{1+\iota_w\bar{\beta}}{1+\bar{\beta}}\pi_t + \frac{1}{1+\bar{\beta}}\left(w_{t-1} - z_t + \iota_w\pi_{t-1}\right) \\ & + \frac{\bar{\beta}}{1+\bar{\beta}}\mathbb{E}_t\left[w_{t+1} + z_{t+1} + \pi_{t+1}\right] + \lambda_{w,t}, \end{aligned} \tag{19}$$

where $\kappa = \frac{(1-\zeta_p\bar{\beta})(1-\zeta_p)}{(1+\iota_p\bar{\beta})\zeta_p((\Phi_p-1)\epsilon_p+1)}$, the parameters ζ_p, ι_p, and ϵ_p are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices, and ζ_w, ι_w, and ϵ_w are the corresponding parameters for wages. w_t^h measures the household's marginal rate of substitution between consumption and labor, and is given by:

$$w_t^h = \frac{1}{1-he^{-\gamma}}\left(c_t - he^{-\gamma}c_{t-1} + he^{-\gamma}z_t\right) + \nu_l L_t, \tag{20}$$

where ν_l characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in the absence of wage rigidities). The mark-ups $\lambda_{f,t}$ and $\lambda_{w,t}$ follow exogenous ARMA(1,1) processes:

$$\lambda_{f,t} = \rho_{\lambda_f}\lambda_{f,t-1} + \sigma_{\lambda_f}\varepsilon_{\lambda_f,t} - \eta_{\lambda_f}\sigma_{\lambda_f}\varepsilon_{\lambda_f,t-1},$$

and

$$\lambda_{w,t} = \rho_{\lambda_w}\lambda_{w,t-1} + \sigma_{\lambda_w}\varepsilon_{\lambda_w,t} - \eta_{\lambda_w}\sigma_{\lambda_w}\varepsilon_{\lambda_w,t-1},$$

respectively.

Finally, the monetary authority follows a generalized *policy feedback rule*:

$$\begin{aligned} R_t &= \rho_R R_{t-1} + (1-\rho_R)\Big(\psi_1(\pi_t - \pi_t^*) + \psi_2(y_t - y_t^f)\Big) \\ & \quad +\psi_3\Big((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\Big) + r_t^m. \end{aligned} \tag{21}$$

where y_t^f is the flexible price/wage output, obtained from solving the version of the model without nominal rigidities and markup shocks (that is, Equations (6) through (20) with

$\zeta_p = \zeta_w = 0$, and $\lambda_{f,t} = \lambda_{w,t} = 0$), and the residual r_t^m follows an AR(1) process with parameters ρ_{r^m} and σ_{r^m}.

In this version of the model we have replaced a constant inflation target with a time-varying inflation target π_t^*, to capture the rise and fall of inflation and interest rates in the estimation sample. Although time-varying target rates have been frequently used for the specification of monetary policy rules in DSGE model (e.g., Erceg and Levin (2003) and Smets and Wouters (2003), among others), we follow the approach of Aruoba and Schorfheide (2008) and Del Negro and Eusepi (2011) and include data on long-run inflation expectations as an observable for the estimation of the model. At each point in time, long-run inflation expectations essentially determine the level of the target inflation rate. To the extent that long-run inflation expectations at the forecast origin contain information about the central bank's objective function, e.g. the desire to stabilize inflation at 2%, this information is automatically included in the forecast.

The time-varying *inflation target* evolves according to:

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*}\epsilon_{\pi^*,t}, \tag{22}$$

where $0 < \rho_{\pi^*} < 1$ and $\epsilon_{\pi^*,t}$ is an iid shock. We model π_t^* as a stationary process, although our prior for ρ_{π^*} will force this process to be highly persistent. The assumption that the changes in the target inflation rate are exogenous is, to some extent, a short-cut. For instance, the learning models of Sargent (1999) or Primiceri (2006) imply that the rise in the target inflation rate in the 1970's and the subsequent drop is due to policy makers learning about the output-inflation trade-off and trying to set inflation optimally. We are abstracting from such a mechanism in our specification.

Anticipated policy shocks

This section describes the introduction of anticipated policy shocks in the model, which follows Laseen and Svensson (2011). We modify the exogenous component of the policy rule (21) as follows:

$$r_t^m = \rho_{r^m} r_{t-1}^m + \epsilon_t^R + \sum_{k=1}^{K} \epsilon_{k,t-k}^R, \tag{23}$$

where ϵ_t^R is the usual contemporaneous policy shock, and $\epsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t.

We assume that $\epsilon^R_{k,t-k} \sim N(0, \sigma^2_{k,r}),\ i.i.d.$

In order to solve the model we need to express the anticipated shocks in recursive form. For this purpose, we augment the state vector s_t (described below) with K additional states ν^R_t,...,ν^R_{t-K} whose law of motion is as follows:

$$\begin{aligned}
\nu^R_{1,t} &= \nu^R_{2,t-1} + \epsilon^R_{1,t} \\
\nu^R_{2,t} &= \nu^R_{3,t-1} + \epsilon^R_{2,t} \\
&\vdots \\
\nu^R_{K,t} &= \epsilon^R_{K,t}
\end{aligned}$$

and rewrite the exogenous component of the policy rule (23) as[3]

$$r^m_t = \rho_{r^m} r^m_{t-1} + \epsilon^R_t + \nu^R_{1,t-1}.$$

Parameters

The following tables describe the parameters used in the New York Fed DSGE model. Table 2 gives the prior distributions for each parameter. Table 3 gives the posterior mean, 5th percentile, and 95th percentile for each parameter.

[3]It is easy to verify that $\nu^R_{1,t-1} = \sum_{k=1}^{K} \epsilon^R_{k,t-k}$, that is, $\nu^R_{1,t-1}$ is a "bin" that collects all anticipated shocks that affect the policy rule in period t.

Table 2: Priors

	Dist	Mean	Std Dev		Dist	Mean	Std Dev
Policy Parameters							
ψ_1	Normal	1.50	0.25	ρ_{r^m}	Beta	0.50	0.20
ψ_2	Normal	0.12	0.05	σ_{r^m}	InvG	0.10	2.00
ψ_3	Normal	0.12	0.05	σ_{ant1}	InvG	0.20	4.00
ρ_R	Beta	0.75	0.10				
Nominal Rigidities Parameters							
ζ_p	Beta	0.50	0.10	ζ_w	Beta	0.50	0.10
ι_p	Beta	0.50	0.15	ι_w	Beta	0.50	0.15
ϵ_p	-	10.00	fixed	ϵ_w	-	10.00	fixed
Other Endogenous Propagation and Steady State Parameters							
100γ	Normal	0.40	0.10	S''	Normal	4.00	1.50
α	Normal	0.30	0.05	ψ	Beta	0.50	0.15
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	π_*	-	0.50	fixed
σ_c	Normal	1.50	0.37	γ_{gdpdef}	Normal	1.00	2.00
h	Beta	0.70	0.10	δ_{gdpdef}	Normal	0.00	2.00
ν_l	Normal	2.00	0.75	$\bar{L}$	Normal	-45.00	5.00
δ	-	0.03	fixed	λ_w	-	1.50	fixed
Φ_p	Normal	1.25	0.12	g_*	-	0.18	fixed
Financial Frictions Parameters							
$F(\bar{\omega})$	-	0.03	fixed	$\zeta_{sp,b}$	Beta	0.05	0.00
SP_*	Gamma	2.00	0.10	γ_*	-	0.99	fixed
Exogenous Process Parameters							
ρ_g	Beta	0.50	0.20	σ_g	InvG	0.10	2.00
ρ_b	Beta	0.50	0.20	σ_b	InvG	0.10	2.00
ρ_μ	Beta	0.50	0.20	σ_μ	InvG	0.10	2.00
ρ_z	Beta	0.50	0.20	σ_z	InvG	0.10	2.00
ρ_{σ_ω}	Beta	0.75	0.15	σ_{σ_ω}	InvG	0.05	4.00
ρ_{π_*}	-	0.99	fixed	σ_{π_*}	InvG	0.03	6.00
ρ_{z^p}	Beta	0.50	0.20	σ_{z^p}	InvG	0.10	2.00
ρ_{λ_f}	Beta	0.50	0.20	σ_{λ_f}	InvG	0.10	2.00
ρ_{λ_w}	Beta	0.50	0.20	σ_{λ_w}	InvG	0.10	2.00
η_{λ_f}	Beta	0.50	0.20	η_{gz}	Beta	0.50	0.20
η_{λ_w}	Beta	0.50	0.20				
Measurement Error Parameters							

Note: For Inverse Gamma prior mean and SD, τ and ν reported.

σ_{ant1} through σ_{ant12} all have the same distribution.

	Dist	Mean	Std Dev		Dist	Mean	Std Dev
$\mathcal{C}_{me}$	-	1.00	fixed	ϱ_{gdp}	Normal	0.00	0.40
ρ_{gdp}	Normal	0.00	0.20	σ_{gdp}	InvG	0.10	2.00
ρ_{gdi}	Normal	0.00	0.20	σ_{gdi}	InvG	0.10	2.00
ρ_{10y}	Beta	0.50	0.20	σ_{10y}	InvG	0.75	2.00
ρ_{tfp}	Beta	0.50	0.20	σ_{tfp}	InvG	0.10	2.00
ρ_{gdpdef}	Beta	0.50	0.20	σ_{gdpdef}	InvG	0.10	2.00
ρ_{pce}	Beta	0.50	0.20	σ_{pce}	InvG	0.10	2.00

Note: For Inverse Gamma prior mean and SD, τ and ν reported.

Table 3: Posteriors

	Mean	(p5, p95)		Mean	(p5, p95)
Policy Parameters					
ψ_1	1.45	(1.21, 1.68)	σ_{ant1}	0.09	(0.07, 0.11)
ψ_2	0.05	(0.03, 0.08)	σ_{ant2}	0.09	(0.07, 0.10)
ψ_3	0.25	(0.20, 0.31)	σ_{ant3}	0.09	(0.07, 0.10)
ρ_R	0.66	(0.59, 0.73)	σ_{ant4}	0.08	(0.06, 0.10)
ρ_{r^m}	0.31	(0.20, 0.42)	σ_{ant5}	0.09	(0.07, 0.11)
σ_{r^m}	0.23	(0.21, 0.25)	σ_{ant6}	0.11	(0.08, 0.13)
Nominal Rigidities Parameters					
ζ_p	0.93	(0.91, 0.95)	ζ_w	0.92	(0.90, 0.94)
ι_p	0.22	(0.10, 0.35)	ι_w	0.55	(0.34, 0.76)
ϵ_p	10.00	fixed	ϵ_w	10.00	fixed
Other Endogenous Propagation and Steady State Parameters					
100γ	0.35	(0.28, 0.43)	S''	3.15	(2.11, 4.12)
α	0.18	(0.15, 0.20)	ψ	0.56	(0.42, 0.69)
$100(\beta^{-1} - 1)$	0.15	(0.07, 0.23)	π_*	0.50	fixed
σ_c	1.00	(0.77, 1.25)	γ_{gdpdef}	1.03	(0.95, 1.10)
h	0.48	(0.38, 0.58)	δ_{gdpdef}	0.00	(-0.05, 0.05)
ν_l	2.30	(1.50, 3.08)	$\bar{L}$	-48.03	(-50.06, -46.03)
δ	0.03	fixed	λ_w	1.50	fixed
Φ_p	1.10	(1.04, 1.16)	g_*	0.18	fixed
Financial Frictions Parameters					
$F(\bar{\omega})$	0.03	fixed	$\zeta_{sp,b}$	0.05	(0.05, 0.06)
SP_*	1.81	(1.67, 1.94)	γ_*	0.99	fixed
Exogenous Process Parameters					
ρ_g	0.99	(0.98, 1.00)	σ_g	2.26	(2.07, 2.45)
ρ_b	0.95	(0.94, 0.96)	σ_b	0.03	(0.02, 0.03)
ρ_μ	0.81	(0.75, 0.87)	σ_μ	0.44	(0.38, 0.50)
ρ_z	0.96	(0.94, 0.98)	σ_z	0.57	(0.52, 0.63)
ρ_{σ_ω}	0.98	(0.97, 1.00)	σ_{σ_ω}	0.04	(0.03, 0.05)
ρ_{π_*}	0.99	fixed	σ_{π_*}	0.03	(0.02, 0.04)
ρ_{z^p}	0.89	(0.84, 0.95)	σ_{z^p}	0.15	(0.10, 0.21)
ρ_{λ_f}	0.80	(0.69, 0.91)	σ_{λ_f}	0.08	(0.06, 0.10)
ρ_{λ_w}	0.36	(0.09, 0.61)	σ_{λ_w}	0.40	(0.35, 0.44)
η_{λ_f}	0.65	(0.46, 0.85)	η_{gz}	0.38	(0.10, 0.64)
η_{λ_w}	0.40	(0.18, 0.62)			
Measurement Error Parameters					
$\mathcal{C}_{me}$	1.00	fixed	ϱ_{gdp}	-0.15	(-0.79, 0.43)

	Mean	(p5, p95)		Mean	(p5, p95)
ρ_{gdp}	0.04	(-0.17, 0.25)	σ_{gdp}	0.24	(0.20, 0.28)
ρ_{gdi}	0.93	(0.89, 0.97)	σ_{gdi}	0.31	(0.28, 0.35)
ρ_{10y}	0.96	(0.94, 0.99)	σ_{10y}	0.12	(0.11, 0.13)
ρ_{tfp}	0.21	(0.10, 0.32)	σ_{tfp}	0.77	(0.70, 0.84)
ρ_{gdpdef}	0.51	(0.38, 0.64)	σ_{gdpdef}	0.16	(0.15, 0.18)
ρ_{pce}	0.30	(0.08, 0.51)	σ_{pce}	0.10	(0.08, 0.12)

Impulse Responses

The following figures depict impulse response functions to various shocks. Figure 5 depicts the response of the economy to a discount factor shock, Figure 6 to a spread shock, Figure 7 to a shock to the marginal efficiency of investment (MEI), Figure 8 to a TFP shock, Figure 9 to a price markup shock, and Figure 10 to a monetary policy shock.

Figure 5: Responses to a Discount Factor Shock b_t

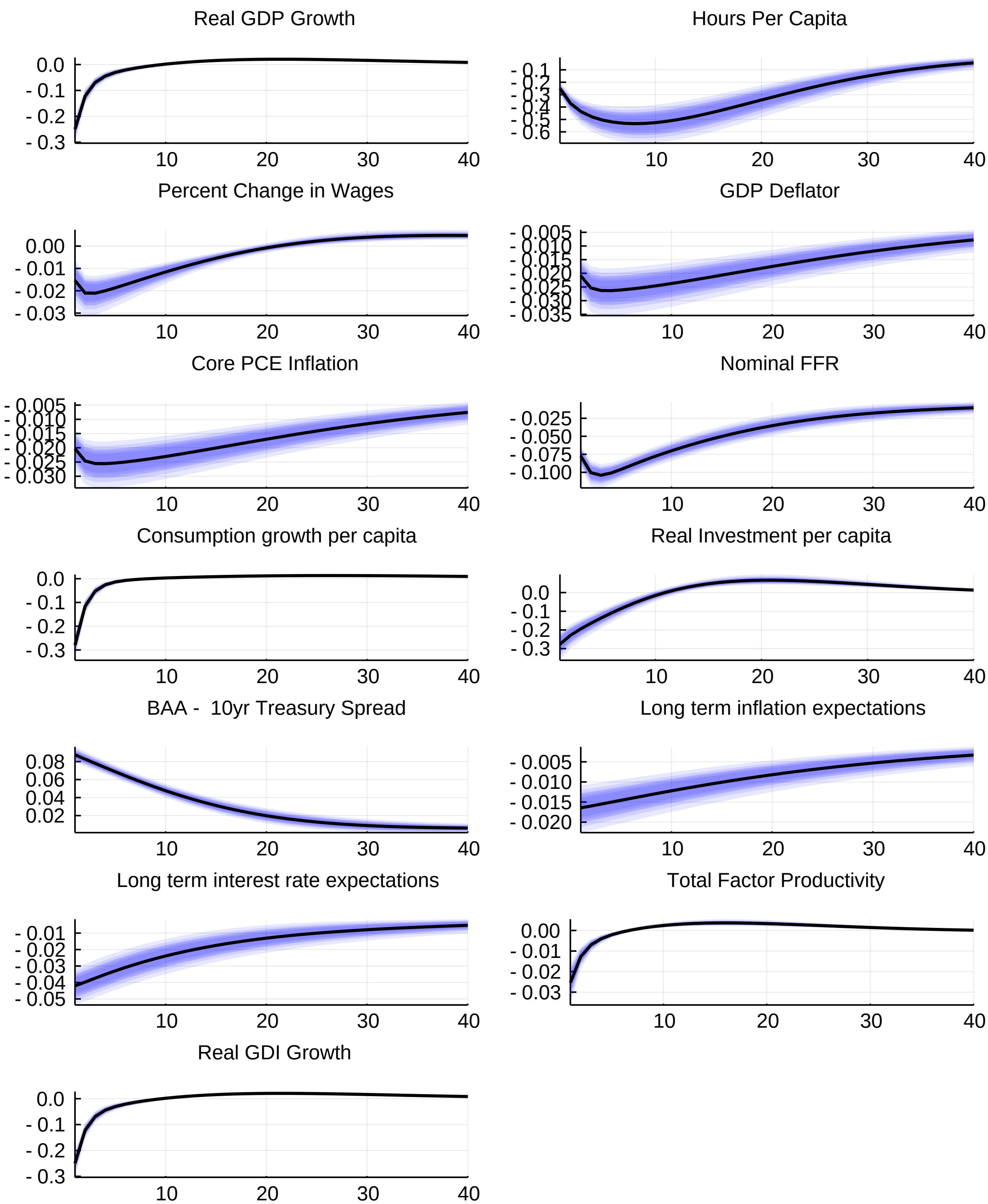


Figure 6: Responses to a Spread Shock $\tilde{\sigma}_{\omega,t}$

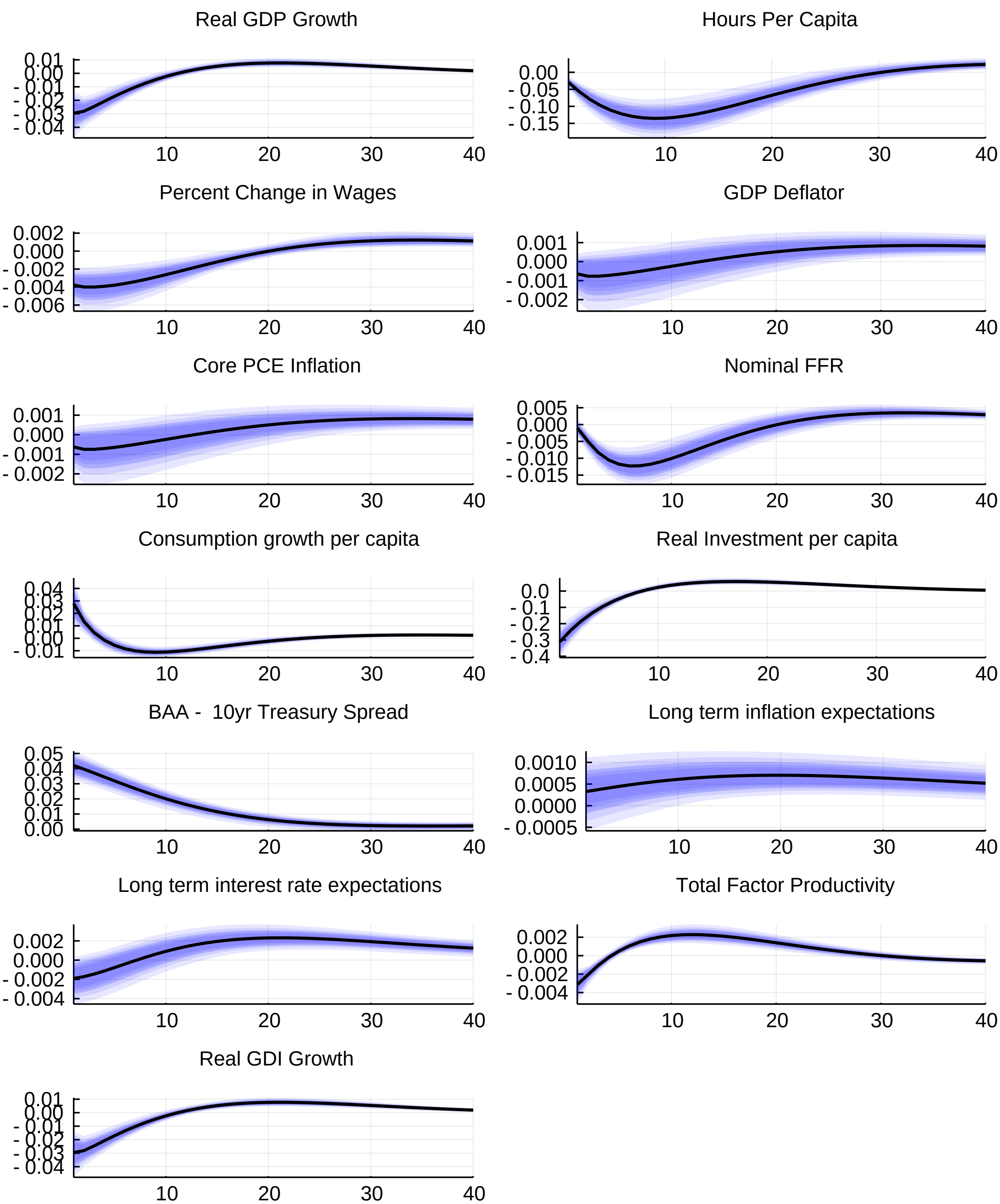


Figure 7: Responses to an MEI Shock μ_t

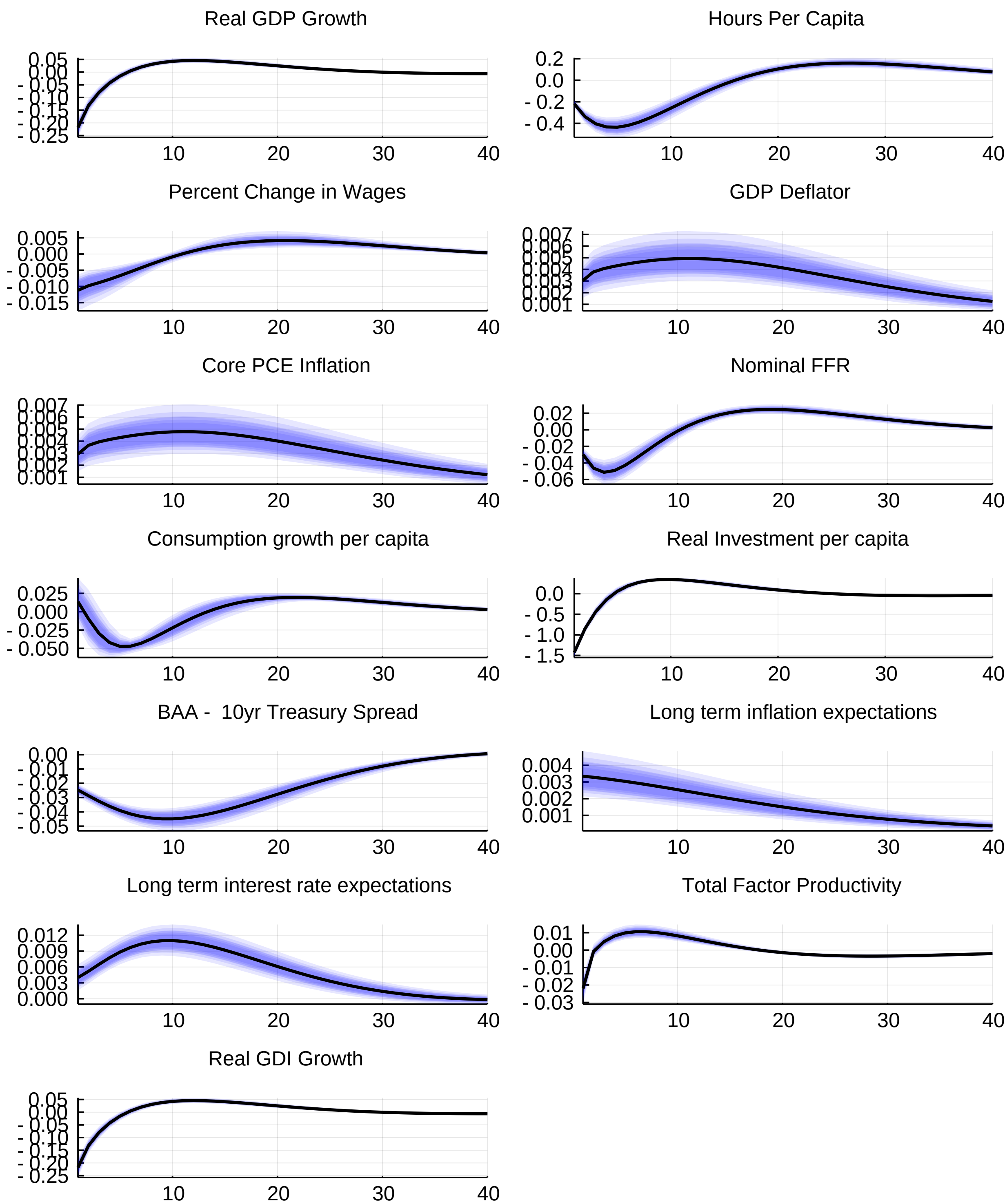


Figure 8: Responses to a TFP Shock $\tilde{z}_t$

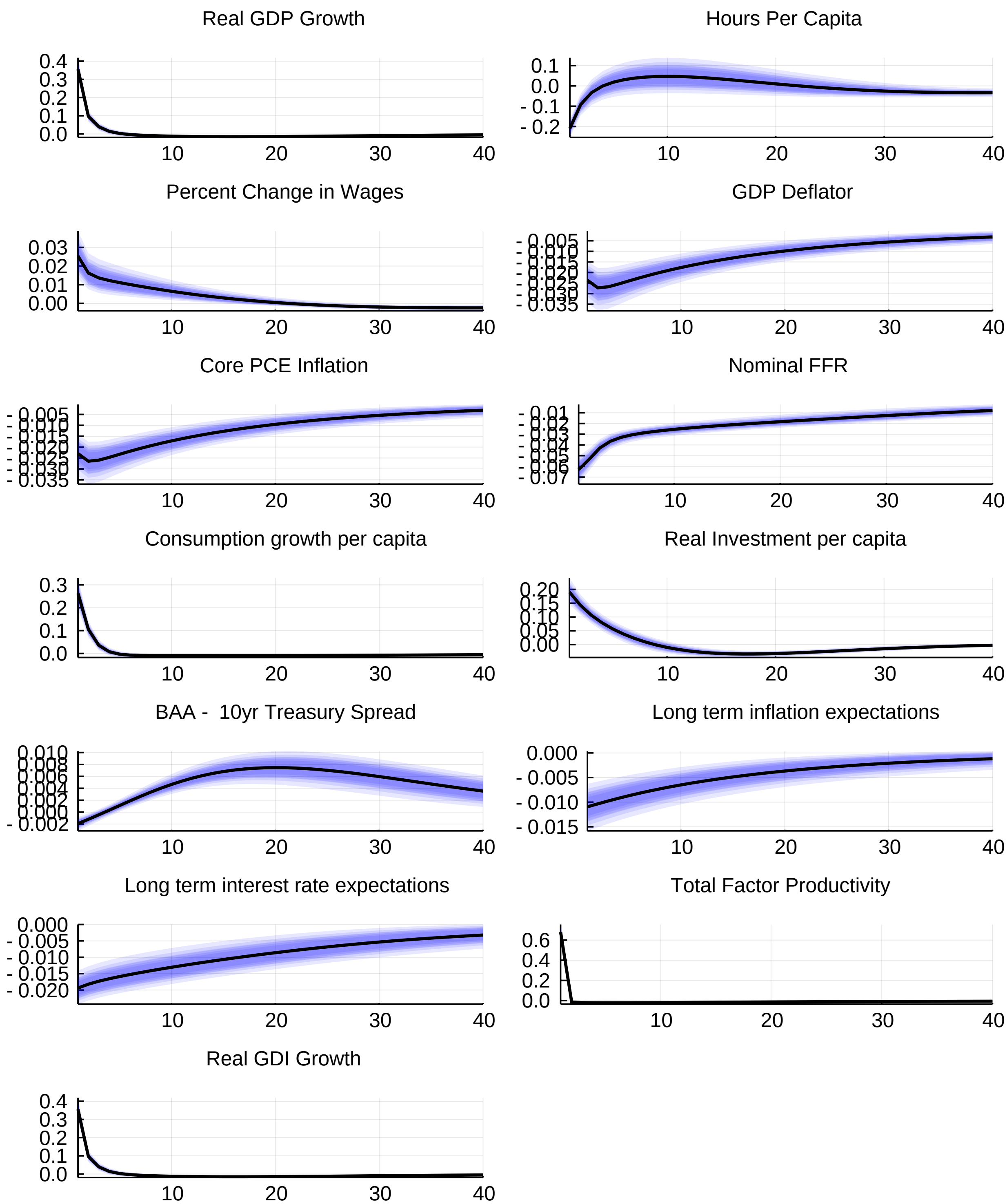


Figure 9: Responses to a Price Markup Shock $\lambda_{f,t}$

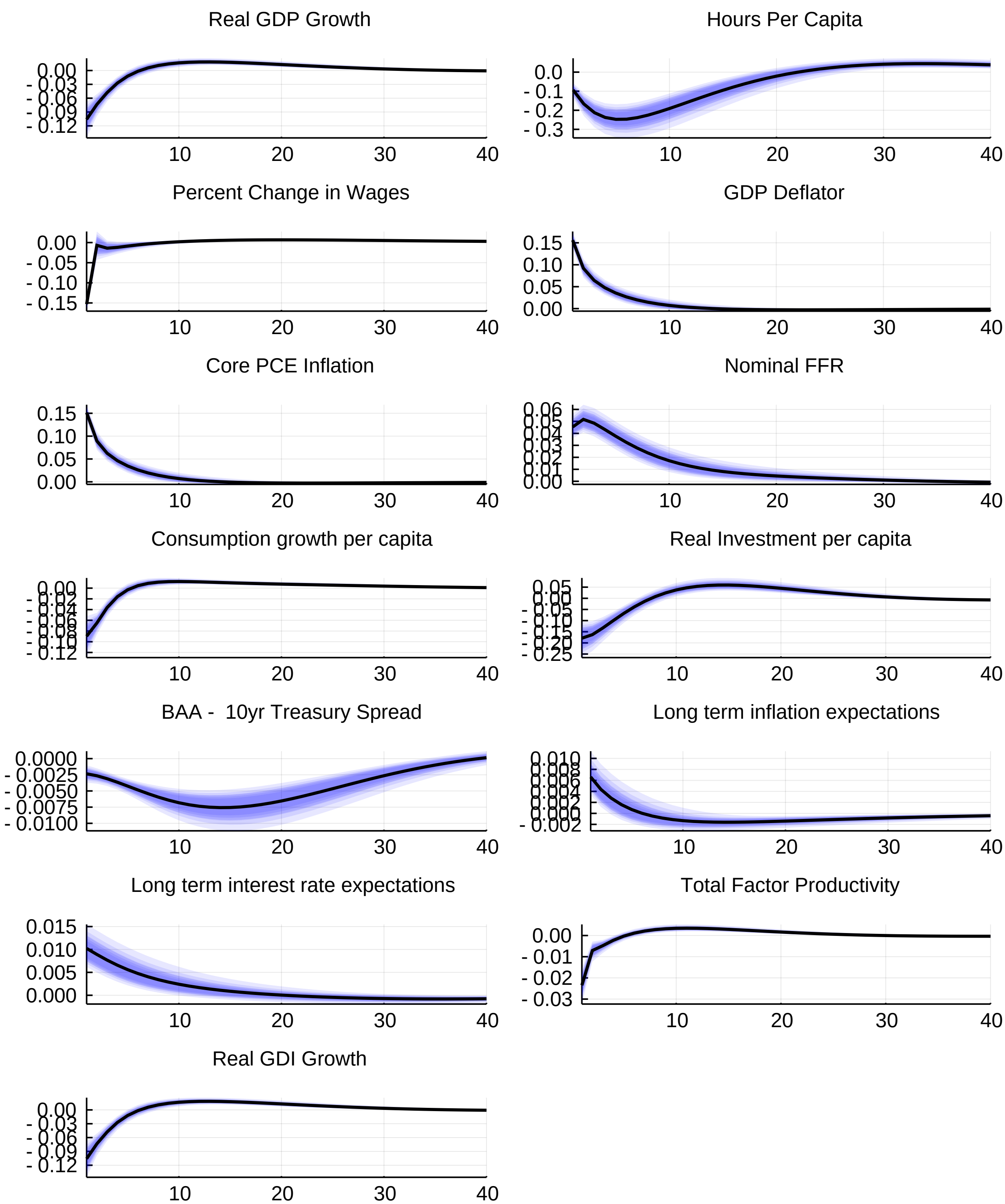
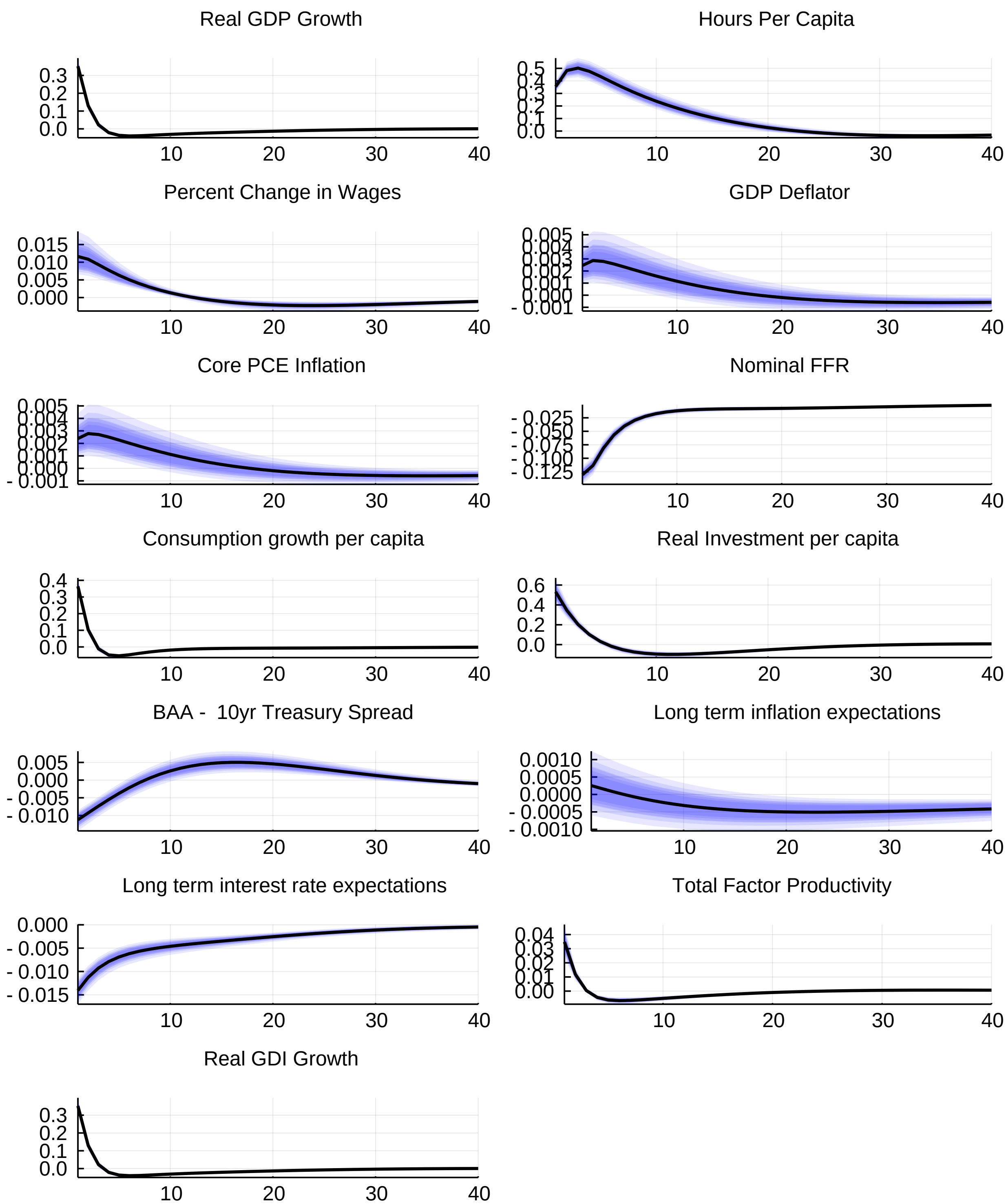


Figure 10: Responses to a Monetary Policy Shock r_t^m



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Detailed Philadelphia (PRISM) Forecast Overview

June 2018

Keith Sill

Forecast Summary

The FRB Philadelphia DSGE model denoted PRISM, projects that real GDP growth will be slightly above its trend pace over the next three years with real output growth running at about a 3 percent pace. Core PCE inflation is only marginally below the FOMC target of 2 percent over most of the next three years. The funds rate rises to 2.7 percent in 2018Q4 and increases steadily to reach 3.7 percent at the end of 2020. The current gap between the level of output and its trend level remains significant in the estimated model and, absent any shocks, the model continues to predict a recovery to the trend level. The relatively slow pace of growth and low inflation that have tended to characterize U.S. economic performance over the past few years require the presence of shocks to offset the strength of the model's internal propagation channels. The PRISM model does not take into account the recent tax reform except to the extent that it is represented in current data observations.

The Current Forecast and Shock Identification

The PRISM model is an estimated New Keynesian DSGE model with sticky wages, sticky prices, investment adjustment costs, and habit persistence. The model is similar to the Smets & Wouters 2007 model and is described more fully in Schorfheide, Sill, and Kryshko 2010. Unlike in that paper though, we estimate PRISM directly on core PCE inflation rather than projecting core inflation as a non-modeled variable. Details on the model and its estimation are available in a Technical Appendix that was distributed for the June 2011 FOMC meeting or is available on request.

The current forecasts for real GDP growth, core PCE inflation, and the federal funds rate are shown in Figures 1a-1c along with 68 percent probability coverage intervals. The forecast uses data through 2018Q1 supplemented by a 2018Q2 nowcast. The model takes the 2018Q2 nowcast for output growth of 3.2 percent as given and the projection begins with 2018Q3. PRISM anticipates that output growth edges down to 3 percent in 2018Q3, with growth then running at about that pace through the end of 2019. Growth tapers off slightly in 2020 to run at 2.8 percent by the end of the year. Overall, the growth forecast for this round is very similar to that from the March projection. While output growth is fairly robust going forward, core PCE inflation stays contained and runs at a pace slightly below the 2 percent target over the forecast horizon. Based on the 68 percent coverage interval, the model sees a minimal chance of deflation or recession (measured as negative quarters of real GDP growth) over the next 3 years. The federal funds rate is determined by an estimated policy rule and the funds rate rises from 1.7

percent in 2018Q2, to 2.7 percent in 2018Q4, 3.4 percent in 2019Q4, and 3.7 percent in 2020Q4. This path for the funds rate is similar to that in the March projection.

Figures 1d and 1e plot the model's estimates of the output gap and the natural real rate of interest. The output gap is defined as the log deviation of the level of output from the level that obtains under flexible wages and prices. The real natural rate of interest is the short term real interest rate that obtains in the model when prices and wages are flexible. The model currently estimates the output gap at -1.6 percent, a more or less continued improvement from the -3 percent reached during the recession. The output gap is expected to improve slowly, reaching about -0.9 percent at the end of 2020. The estimated real natural rate of interest is low at 0.13 percent in 2018Q2. The natural rate then rises over the next three years to reach 1.4 percent at the end of 2020. Note though that the natural rate estimate is extremely volatile and the 68 percent confidence bands are quite large.

The key factors driving the projection are shown in the forecast shock decompositions (Figures 2a-2e) and the smoothed estimates of the model's primary shocks (shown in Figure 3, where they are normalized by standard deviation). GDP growth is at an above trend pace in 2018Q2, boosted by shocks to labor supply, investment, and government spending (which includes net exports). Thereafter, growth is at a close to a trend pace over the forecast horizon. TFP, financial, and monetary policy shocks act as a drag on growth, but are largely offset by rebounds in government spending, investment, and labor supply. Over the course of the recession and recovery PRISM estimated a series of large positive shocks to leisure (negative shocks to labor supply) that have a persistent effect on hours worked and so pushed hours well below steady state. As these shocks unwind hours worked continue to rebound over the forecast horizon and so support higher output growth. Similarly, the unwinding of investment shocks contribute to output growth over the forecast horizon.

Consumption is expected to grow at a slightly below-trend pace over the forecast horizon (Figure 2d). Consumption growth is pulled down by shocks to technology, investment, monetary policy, and government spending. Consumption gets a boost from the unwinding of financial and labor shocks, but not enough to offset the downward pull from the other shocks in the model. Financial shocks exert a considerable drag on investment growth in early 2018 – these same shocks make a positive contribution to consumption growth (Figure 2d-e). However, strong and offsetting investment shocks are enough to keep investment growth near its trend over the forecast horizon. All told, the model now forecasts trend growth in investment (gross private domestic + durable goods consumption) in 2018 as the gradual unwinding of MEI shocks (see Figures 2e and 3) are offset by the effects of financial shocks.

The forecast for core PCE inflation continues to be a story of upward pressure from the unwinding of negative labor supply shocks and MEI shocks being offset by downward pressure from the waning of discount factor shocks. Negative discount factor shocks have a strong and persistent negative effect on marginal cost and inflation in the estimated model. But labor supply shocks that push down aggregate hours also serve to put upward pressure on the real wage and hence marginal cost. The effect is persistent -- as the labor supply shocks unwind over the forecast horizon they exert a waning upward push to inflation. On balance the effect of these opposing forces keep inflation slightly below target over the next 3 years.

The federal funds rate is projected to rise fairly quickly over the forecast horizon. The model attributes the current level of the funds rate primarily to a combination of monetary policy, discount factor and labor supply dynamics. Looking ahead, the positive contribution from labor supply shocks is more than offset by discount factor shock dynamics over the medium term, but as these shocks wane the funds rate gradually rises to 3.7 percent by the end of 2020.

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Figure 1a

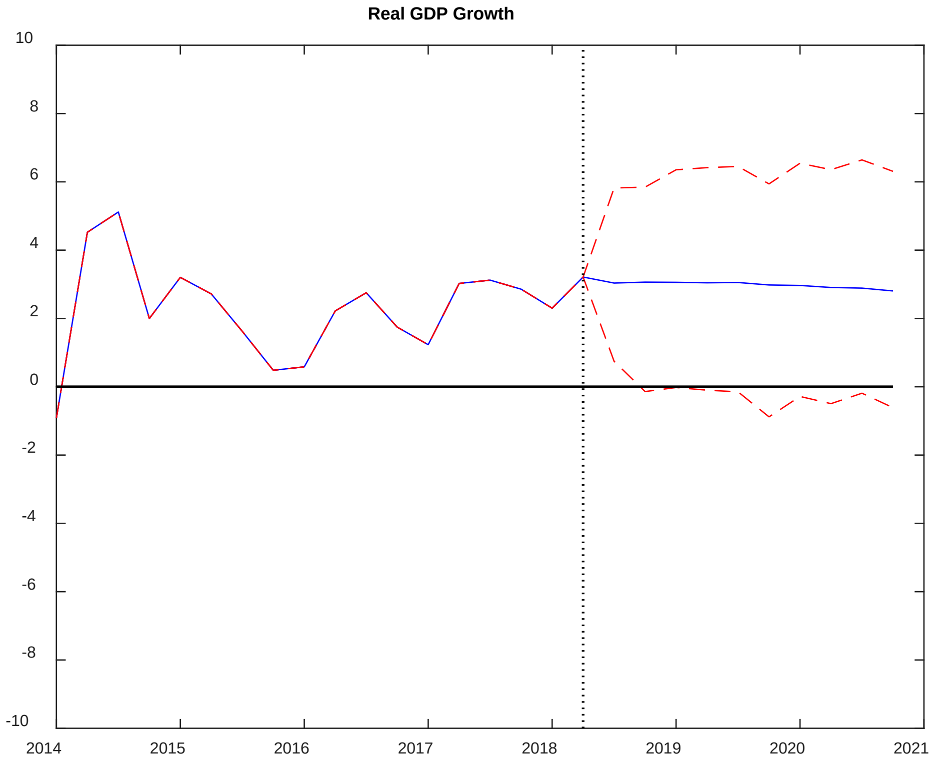

Real GDP Growth
10
8
6
4
2
0
-2
-4
-6
-8
-10
2014
2015
2016
2017
2018
2019
2020
2021

Figure 1b

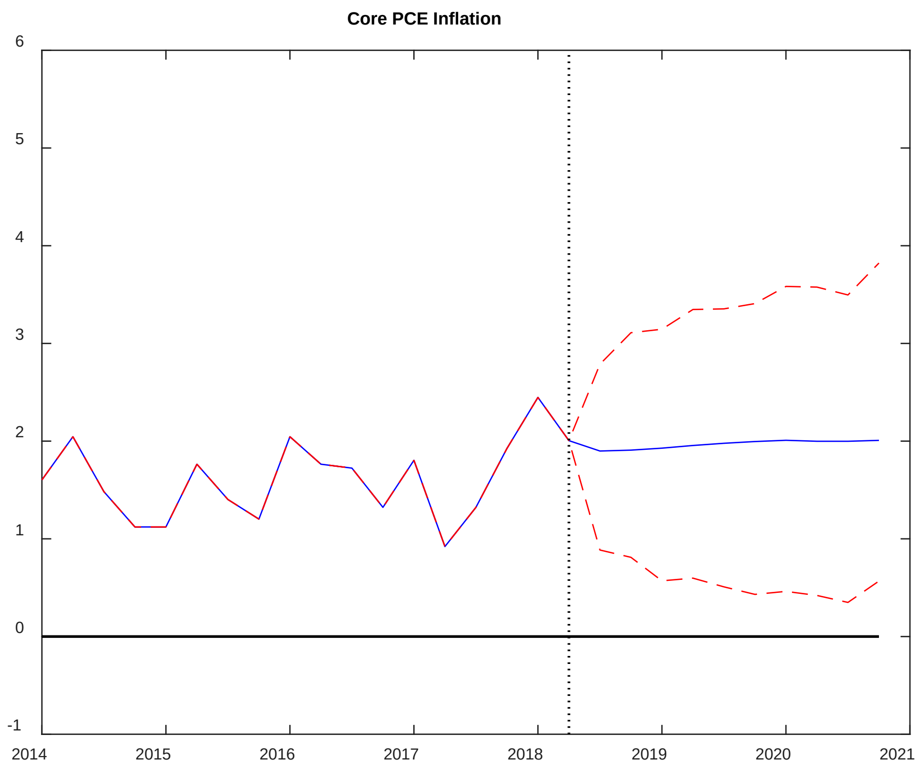

Core PCE Inflation
6
5
4
3
2
1
0
-1
2014
2015
2016
2017
2018
2019
2020
2021

Figure 1c

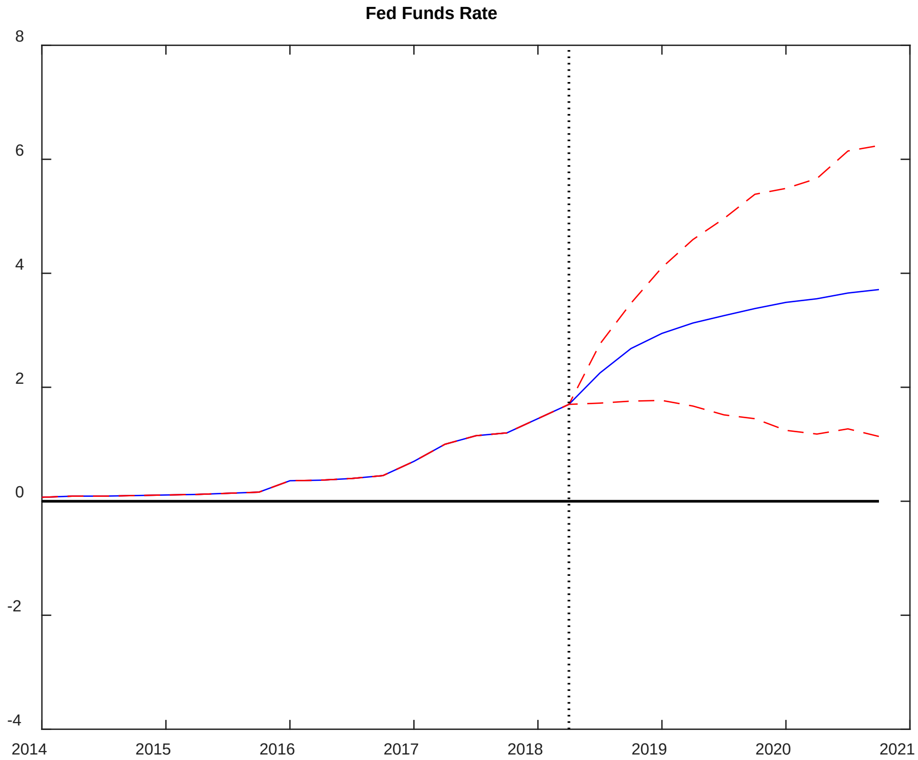

Fed Funds Rate
8
6
4
2
0
-2
-4
2014
2015
2016
2017
2018
2019
2020
2021

Figure 1d

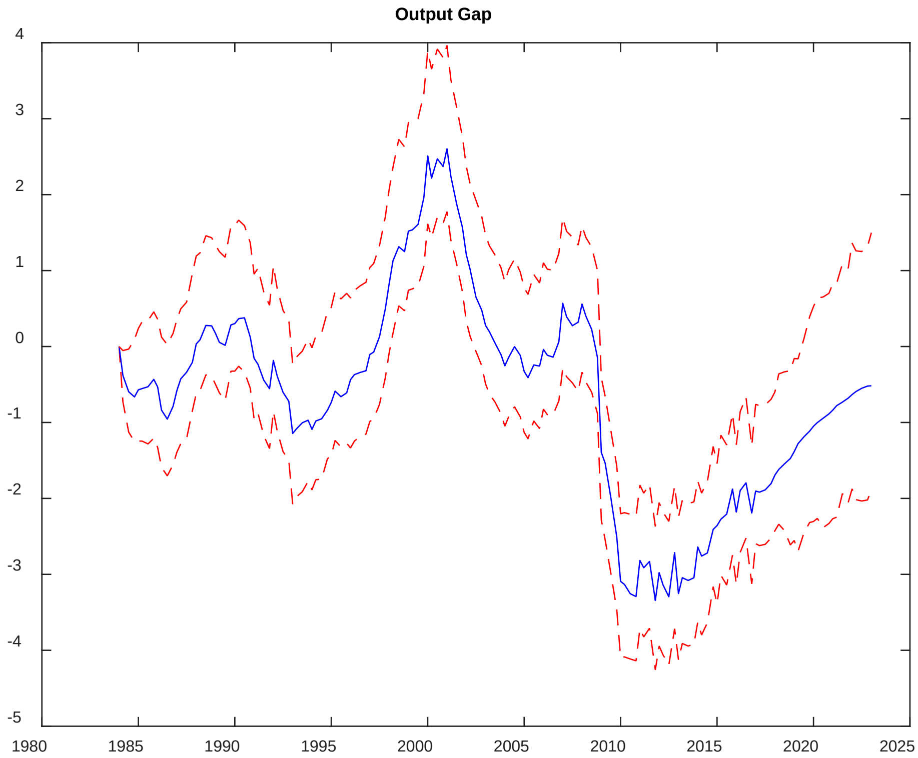

Output Gap
4
3
2
1
0
-1
-2
-3
-4
-5
1980
1985
1990
1995
2000
2005
2010
2015
2020
2025

Figure 1e

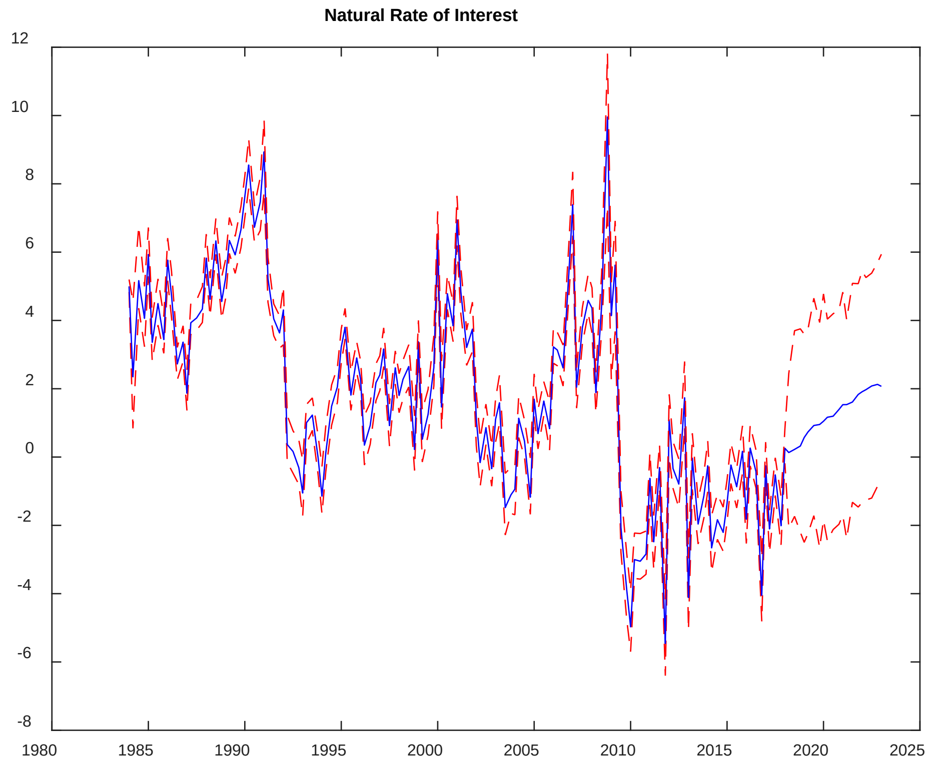
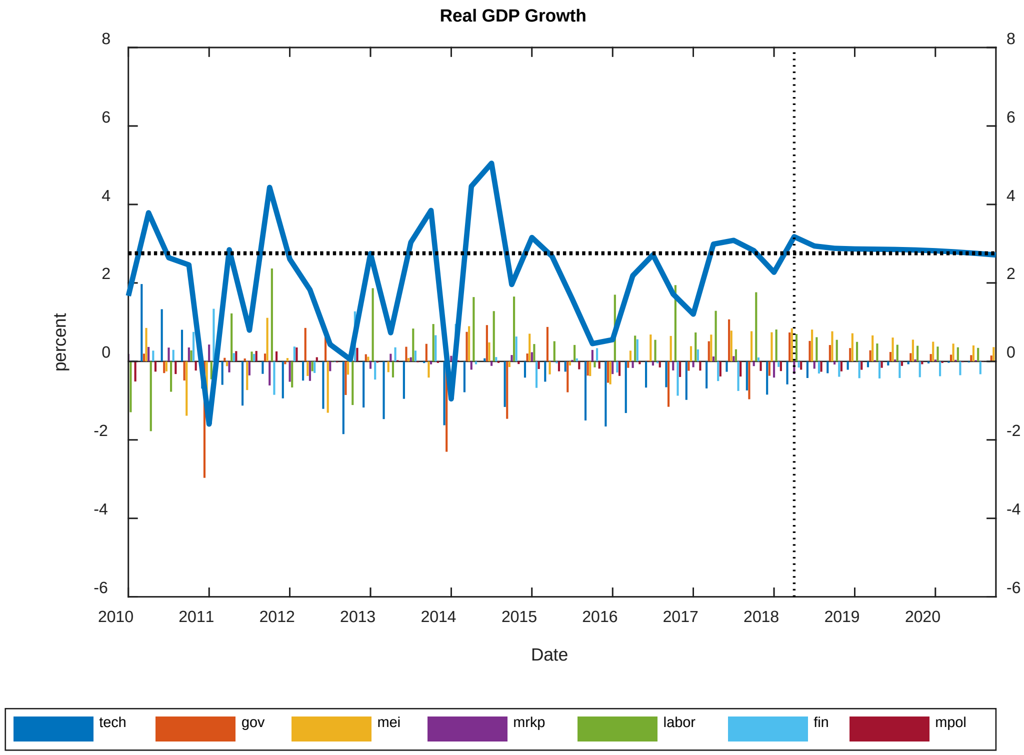

Natural Rate of Interest
12
10
8
6
4
2
0
-2
-4
-6
-8
1980
1985
1990
1995
2000
2005
2010
2015
2020
2025

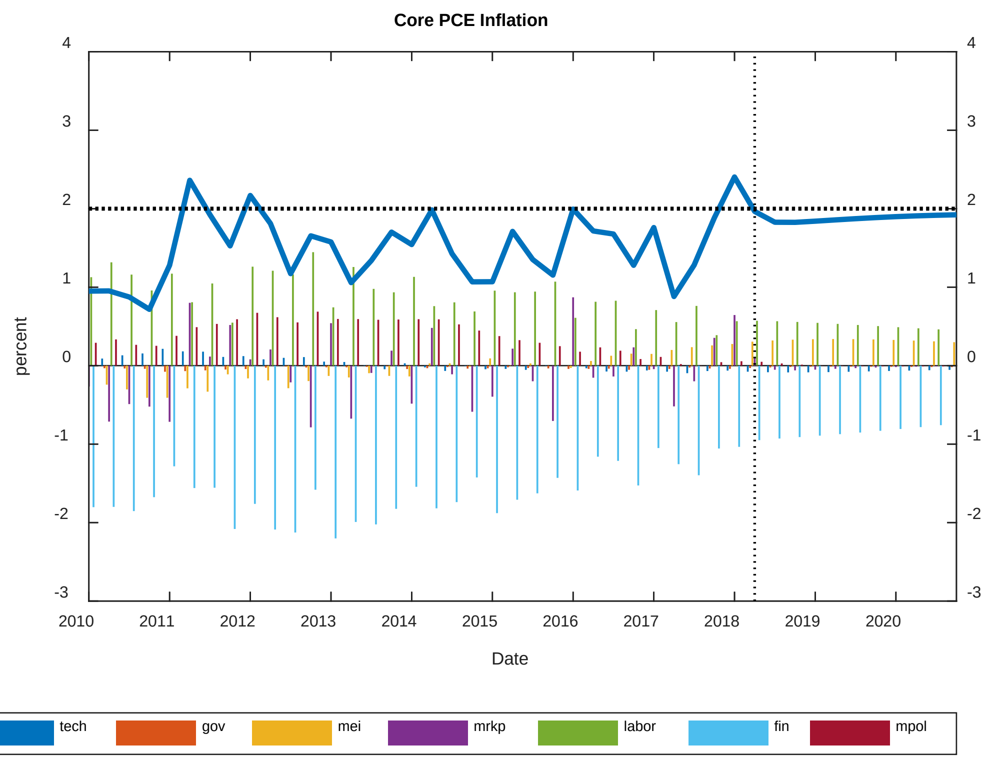
Figure 2a
Shock Decompositions



shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

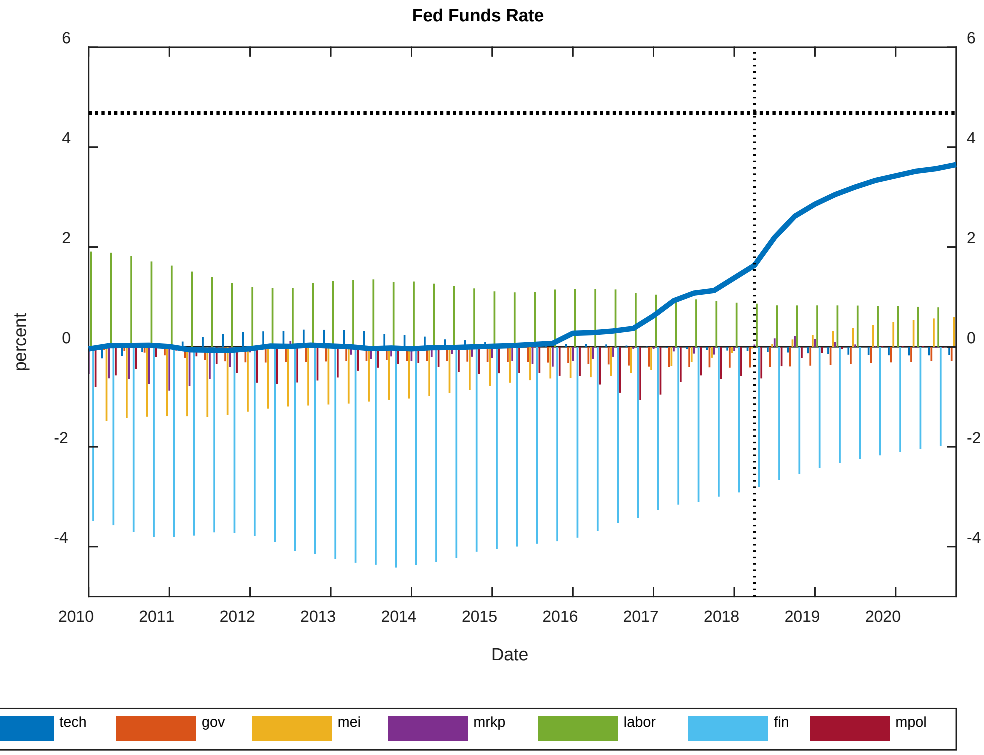
Figure 2b
Shock Decompositions



shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

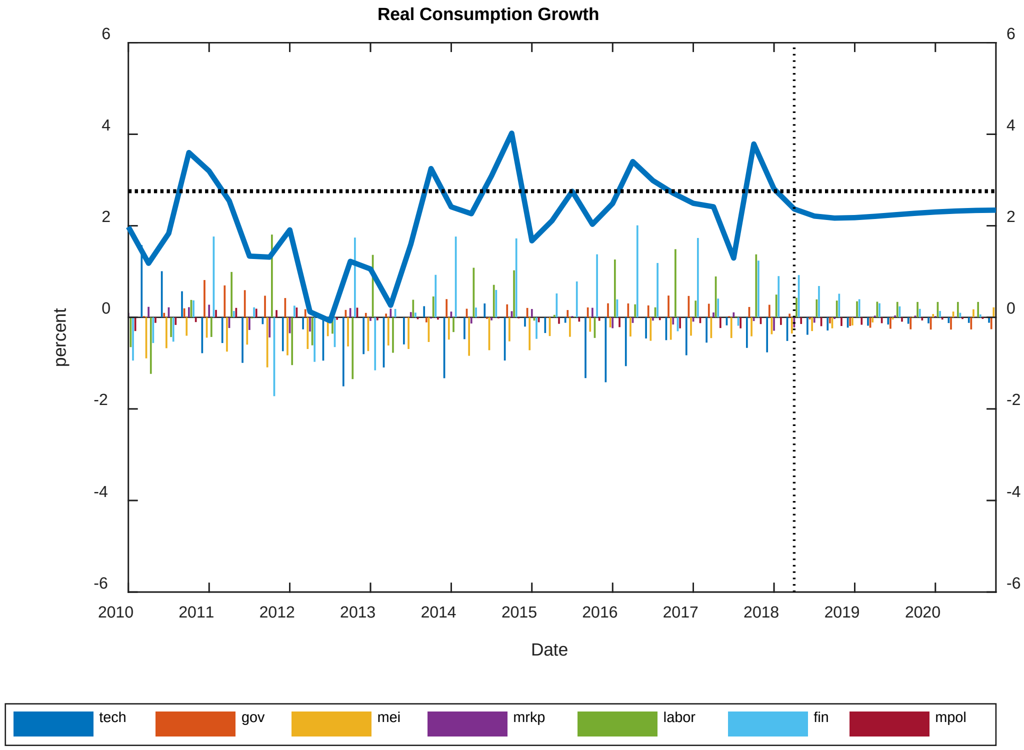
Figure 2c
Shock Decompositions



shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

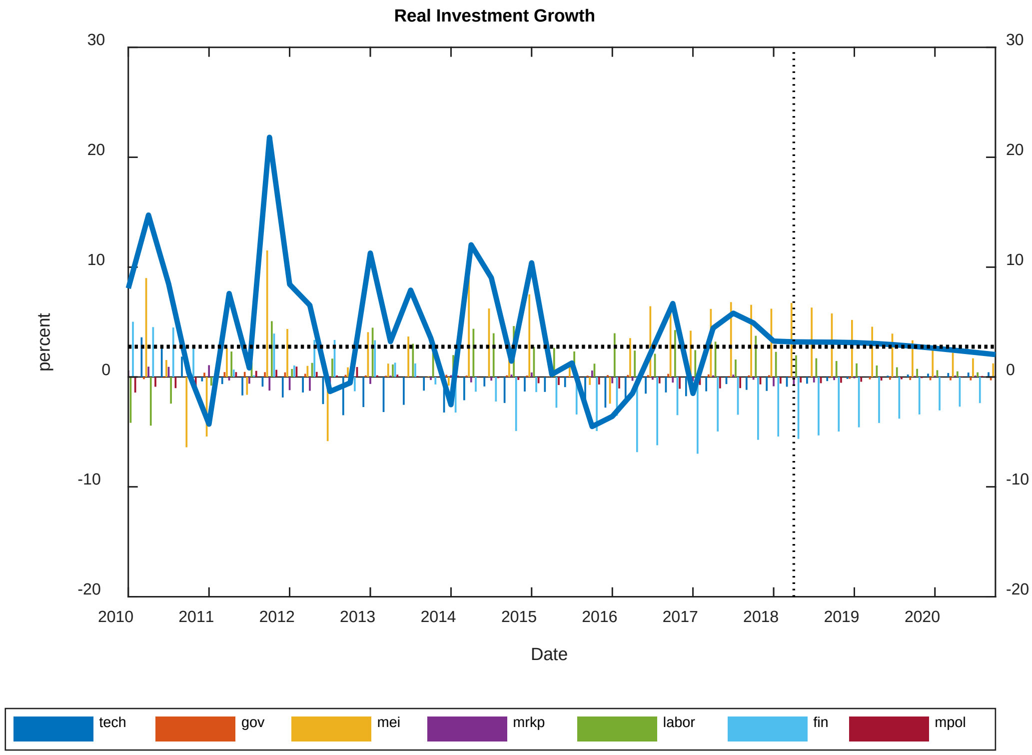
Figure 2d
Shock Decompositions



shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

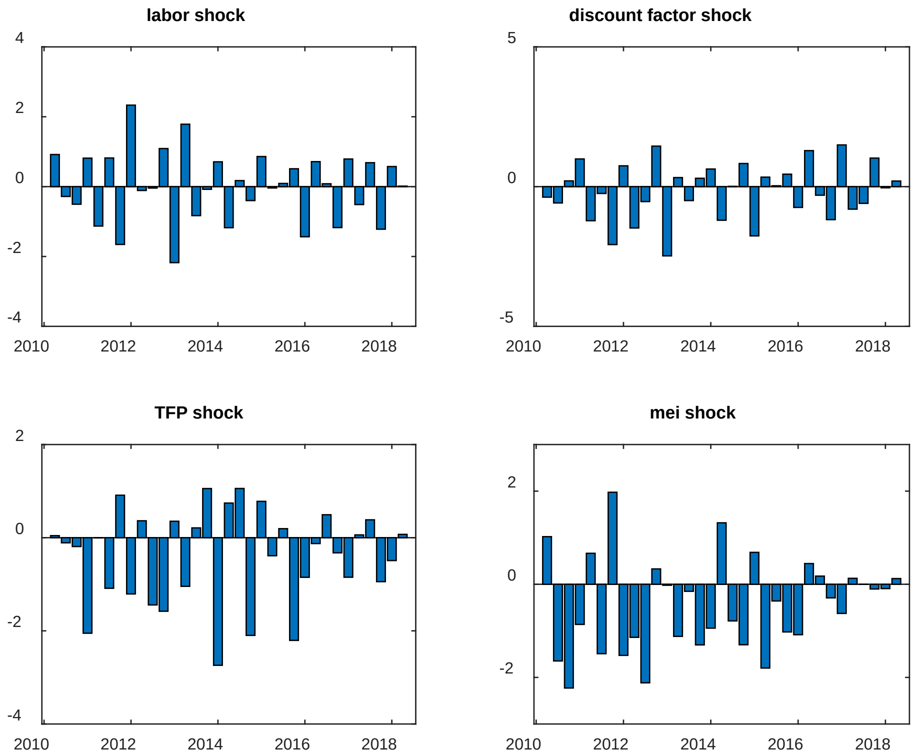
Figure 2e
Shock Decompositions



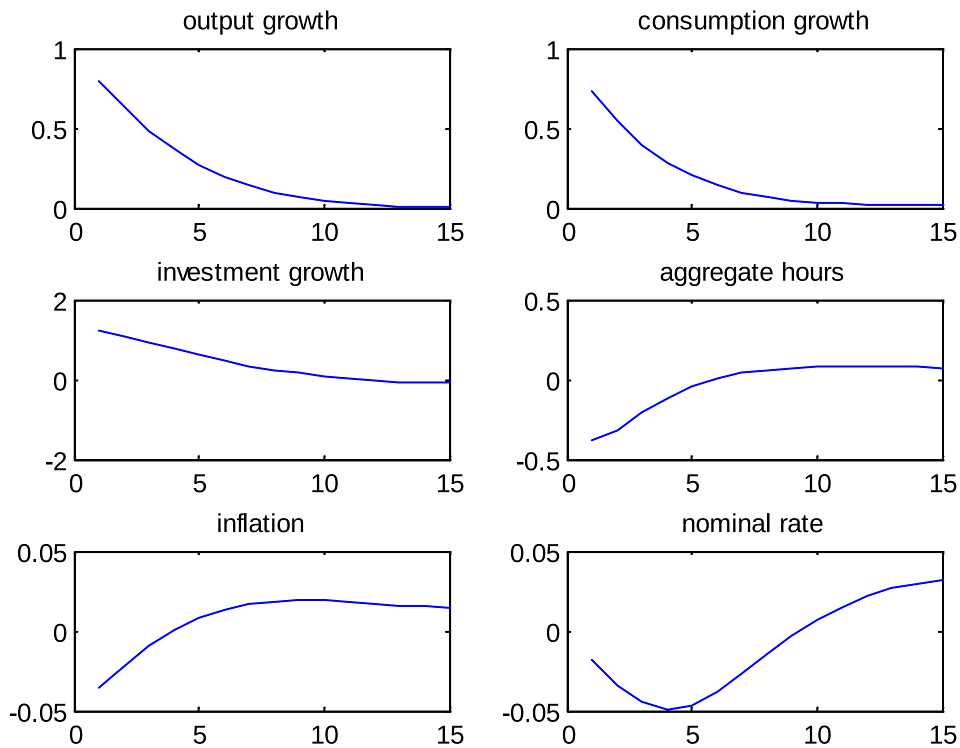
shocks:

TFP:	Total factor productivity growth shock
Gov:	Government spending shock
MEI:	Marginal efficiency of investment shock
MrkUp:	Price markup shock
Labor:	Labor supply shock
Fin:	Discount factor shock
Mpol:	Monetary policy shock

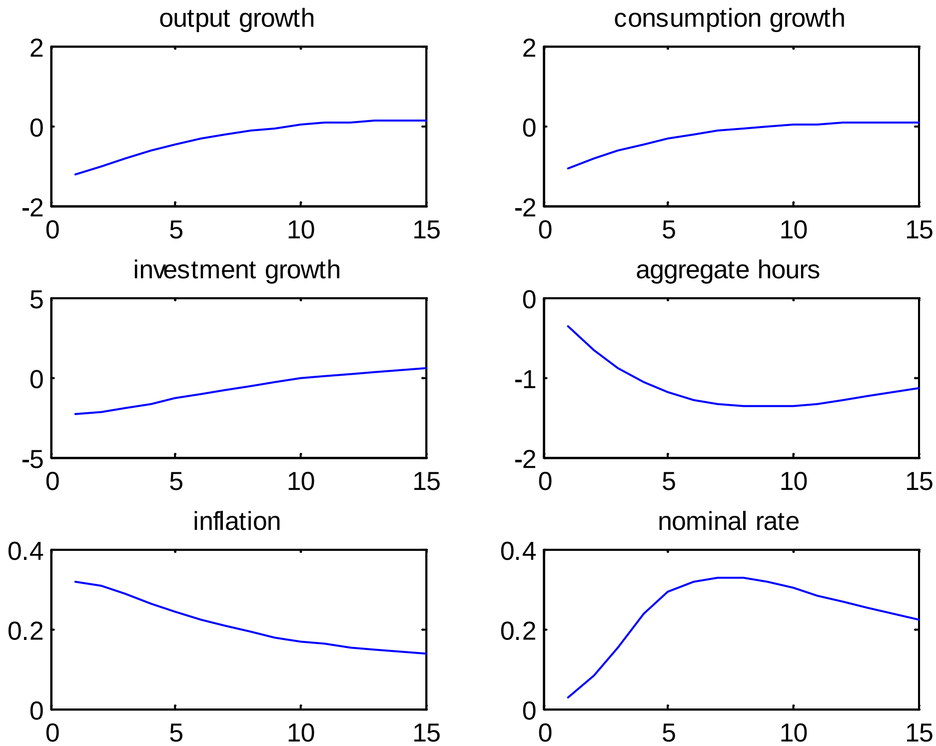
Figure 3
Smoothed Shock Estimates for Conditional Forecast Model
(normalized by standard deviation)



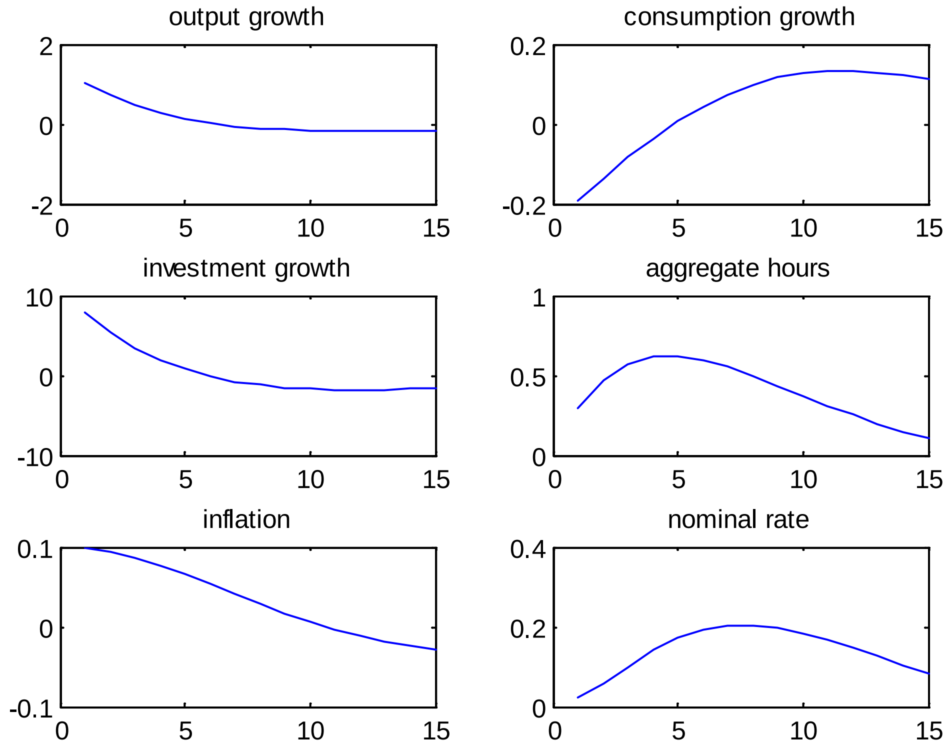
Impulse Responses to TFP shock



Impulse Response to Leisure Shock



Impulse Responses to MEI Shock



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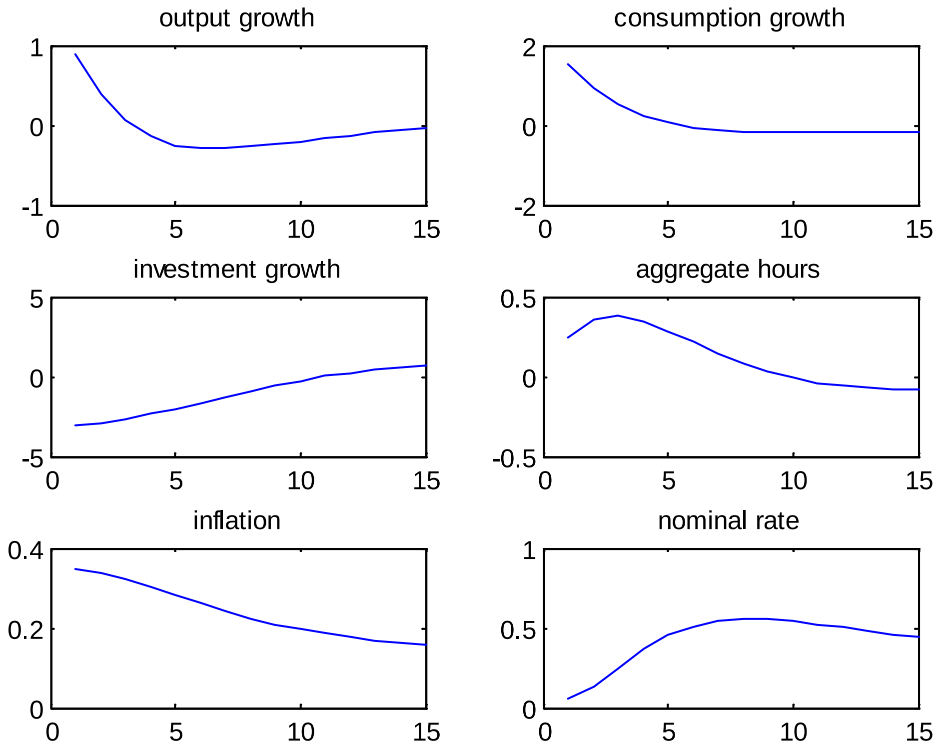

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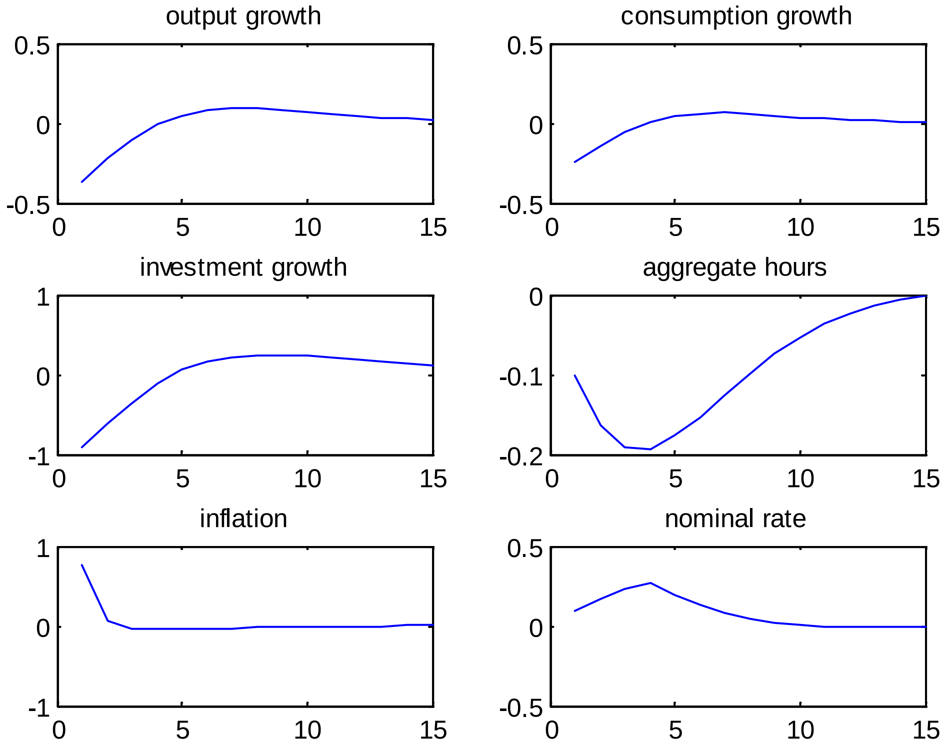

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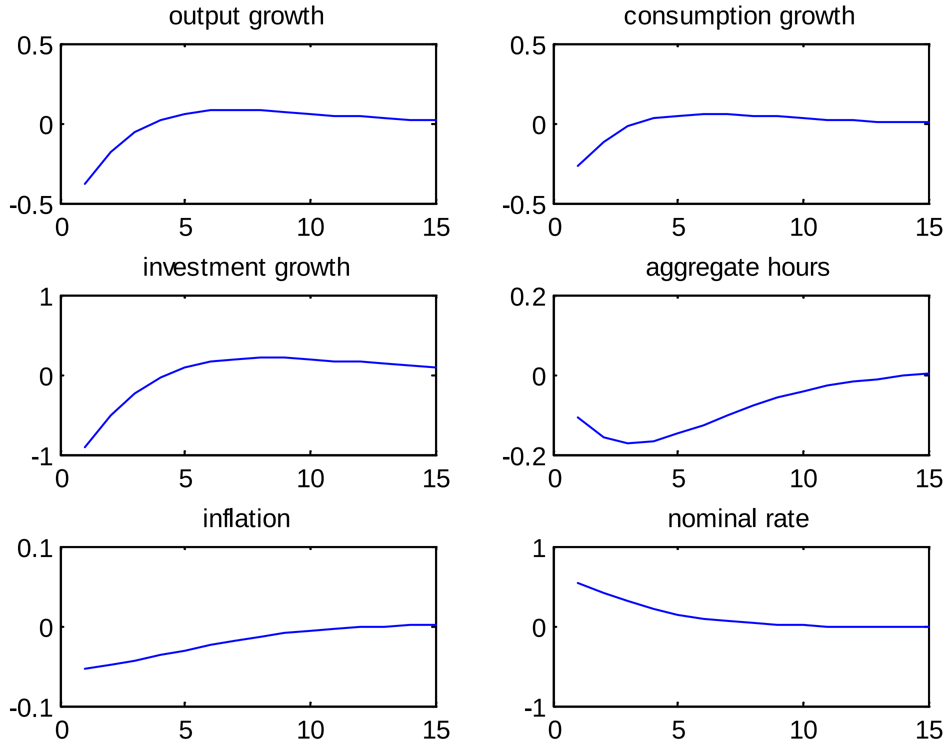
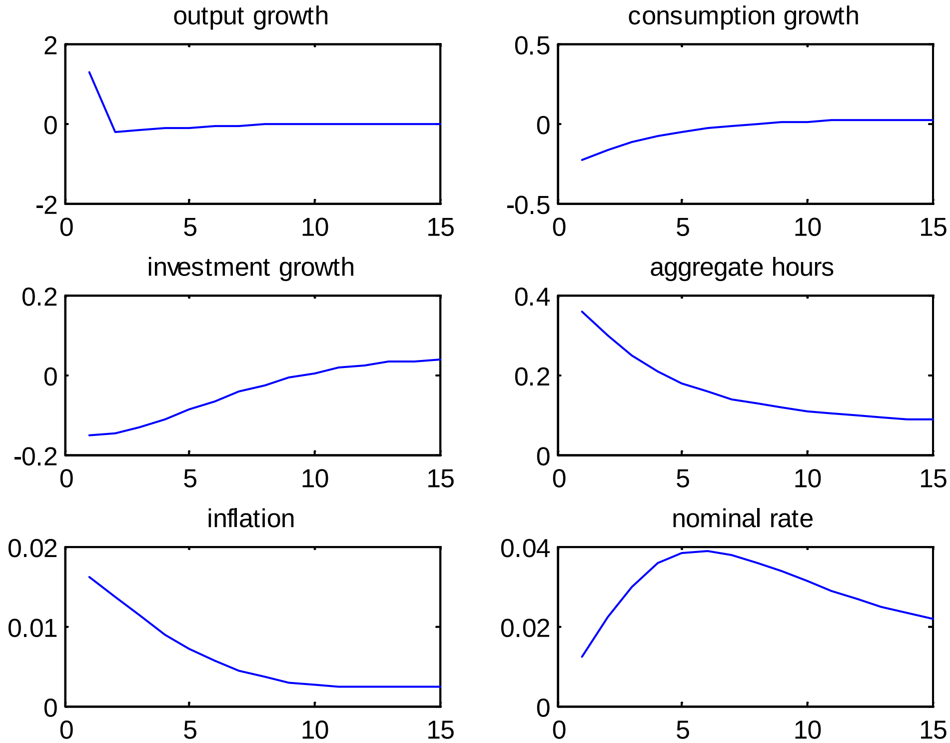


Impulse Responses to Govt Spending Shock



Research Directors' Guide to the Chicago Fed DSGE Model*

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This guide describes the construction and estimation of the Chicago Fed's DSGE model, which we use both for internal forecasting and for creating our contributions to the System DSGE memo distributed quarterly to the FOMC. The model has been in use and under ongoing development since 2010. Originally, it was largely based on Justiniano, Primiceri, and Tambalotti (2010). We published results based on simulations from the estimated model in Campbell, Evans, Fisher, and Justiniano (2012) and in Campbell, Fisher, Justiniano, and Melosi (2016).

The model contains many features familiar from other DSGE analyses of monetary policy and bussiness cycles. External habit in preferences, i-dot costs of adjusting investment, price and wage stickiness based on Calvo's (1983) adjustment probabilities, and partial indexation of unadjusted prices and wages using recently observed price and wage inflation. The features which distinguish our analysis from many otherwise similar undertakings are

- **Forward Guidance Shocks:** An interest-rate rule which depends on recent (and expected future) inflation and output and is subject to stochastic disturbances governs our model economy's monetary policy rate. Standard analysis prior to the great recession restricted the stochastic disturbances to be unforecastable. Our model deviates from this historical standard by including forward guidance shocks, as in Laséen and Svensson (2011). A j-quarter ahead forward guidance shock revealed to the public at time t influences the interest-rate rule's stochastic intercept only at time $t + j$. Each period, the model's monetary authority reveals a vector of these shocks with one element for each quarter from the present until the end of the forward guidance horizon. The

*The views expressed herein are the authors'. They do not necessarily represent those of the Federal Reserve Bank of Chicago, the Federal Reserve System, or its Board of Governors.

vector's elements may be correlated with each other, so the monetary authority could routinely reveal persistent shifts in the interest-rate rule's stochastic intercept. However, the forward guidance shocks are serially uncorrelated *over time*, as is required for them to match the definition of "news."

- **Investment-Specific Technological Change:** As in the Real Business Cycle models from which modern DSGE models decend (King, Plosser, and Rebelo, 1988a), stochastic trend productivity growth both short-run and long-run fluctuations. Our model features two such stochastic trends, one to Hicks-neutral productivity (King, Plosser, and Rebelo, 1988b) and one to the technology for converting consumption goods into investment goods (as in Fisher (2006)). This investment-specific technological change allows our model to reproduce the dynamics of the relative price of investment goods to consumption goods, which is a necessary input into the formula we use to create Fisher-ideal chain-weighted index of real GDP.

- **A Mixed Calibration-Bayesian Estimation Empirical Strategy:** Bayesian estimation of structural business cycle models attempts to match all features of the data's probability distribution using the model's parameters. Since no structural model embodies Platonic "truth," this exercise inevitably requires trading off between the model's ability to replicate first moments with its fidelty to the business cycles in second moments. Since the criteria for this tradeoff are not always clear, we adopt an alternative "first-moments-first" strategy. This selects the values of model parameters which govern the model's steady-state growth path, such as the growth rates of Hicks-neutral and investment-specific technology, to match estimates of selected first moments. These parameter choices are then fixed for Bayesian estimation, which chooses values for model parameters which only influence second moments, such as technology innovation variances. (Since we employ a log linear solution of our model and all shocks to its primitives have Gaussian distributions, our analysis has no non-trivial implications for third and higher moments of the data.)

The guide proceeds as follows. The next section presents the model economy's primitives, while Section 2 presents the agents' first-order conditions. Section 3 gives the formulas used to remove nominal and technological trends from model variables and thereby induce model stationarity, and Sections 4 and 5 discuss the stationary economy's steady state and the log linearization of its equilibrium necessary conditions around it. Section 6 discusses measurement issues which arise when comparing model-generated data with data measured by the BEA and BLS. Section 7 describes our mixed Calibration-Bayesian Estimation empirical strategy and presents the resulting parameter values we use for model simulations and forecasting.

1 The Model's Primitives

Eight kinds of agents populate the model economy:

- Households,
- Investment producers,
- Competitive final goods producers,
- Monopolistically-competitive differentiated goods producers,
- Labor Packers,
- Monopolistically-competitive guilds,
- a Fiscal Authority and
- a Monetary Authority.

These agents interact with each other in markets for

- final goods used for consumption
- investment goods used to augment the stock of productive capital
- differentiated intermediate goods
- capital services
- raw labor
- differentiated labor
- composite labor

- government bonds
- privately-issued bonds, and
- state-contingent claims.

The households have preferences over streams of an aggregate consumption good, leisure, and the real value of the fiscal authority's bonds in their portfolios. Our specification for preferences displays balanced growth. They also feature *external* habit in consumption; which creates a channel for the endogenous propagation of shocks. Our bonds-in-the-utility-function preferences follow those of Fisher (2015), and they allow us to incorporate a persistent spread between the monetary policy rate and the return on productive capital. The aggregate consumption good has a single alternative use, as the only input into the linear production function operated by investment producers. These firms sell their output to the households. In turn, households produce capital services from their capital stocks, which they then sell to differentiated goods producers. Producers of final goods operate a constant-returns-to-scale technology with a constant elasticity of substitution between its inputs, which are differentiated goods produced by the monopolistically-competitive firms. These firms operate technologies with affine cost curves (a constant fixed cost and linear marginal cost), which employs capital services and composite labor as inputs. The labor packers produce composite labor using a constant-returns-to-scale technology with a constant elasticity of substitution between its inputs, the differentiated labor sold by guilds. Each of these produces differentiated labor from the raw labor provided by the households with a linear technology, and they sell their outputs to the labor packers. There is a nominal unit of account, called the "dollar." The fiscal authority issues one-period nominally risk-free bonds, provides public goods through government spending, and assesses lump-sum taxes on households. The monetary authority sets the interest rate on the fiscal authority's one-period bond according to an interest-rate rule.

All non-financial trade is denominated in dollars, and all private agents take prices as given with two exceptions: the monopolistically-competitive differentiated-goods producers and guilds. These choose output prices to maximize the current value of expected future profits taking as given their demand curves and all relevant input prices. Financial markets are complete, but all securities excepting equities in differentiated-goods producers are in zero net supply. These producers' profits and losses are rebated to the households (who own the firms' equities) lump-sum period-by-period, as are the profits and losses of the guilds. Given both a process for government spending and taxes and a rule for the monetary authority's interest rate choice, a competitive equilibrium consists of allocations and prices that are consistent with households' utility maximization, firms' profit maximization, guilds' profit maximization, and market clearing.

The economy is subject to stochastic disturbances in technology, preferences, and government policy. Without nominal rigidities, the economy's real allocations in competitive equilibrium can be separated from inflation and other dollar-denominated variables. Specifically, monetary policy only influences inflation. To connect real and nominal variables in the model and thereby consider the impact of monetary policy on the business cycle, we introduce Calvo-style wage and price setting. That is, nature endows both differentiated goods producers and guilds with *stochastic* opportunities to incorporate all available information into their nominal price choices. Those producers and guilds without such a opportunity must set their prices according to simple indexing formulas. These two pricing frictions create two forward-looking Phillips curves, one for prices and another for wages, which form the core of the new Keynesian approach to monetary policy analysis.

The model economy is stochastic and features complete markets in state-contingent claims. To place these features on a sound footing, we base all shocks on a general Markovian stochastic process s_t. Denote the history of this vector from an initial period 0 through τ with $s^\tau \equiv (s_0, s_1, \ldots, s_\tau)$. The support of s^τ is Σ^τ, and the probability density of s^τ given s_t for some $t < \tau$ is $\samekh(s^\tau \mid s_t)$. (The Hebrew letter ס, pronounced "samekh," corresponds to the Greek letter σ.) All model shocks are implicit functions of s_t, and all endogenous variables are implicit functions of s^t. We refer to all such implicit functions as "state-contingent sequences." We use braces to denote such a sequence. For example, $\{X_t\}$ represents the state-contingent sequence for a generic variable X_t.

1.1 Households

Our model's households are the ultimate owners of all assets in positive net supply (the capital stock, differentiated goods producers, and guilds). They provide labor and divide their current after-tax income (from wages and assets) between current consumption, investment in productive capital, and purchases of financial assets, both those issued by the government and those issued by other households. The individual household divides its current resources between consumption and the available vehicles for intertemporal substitution (capital and financial assets) to maximize a discounted sum of current and expected future felicity.

$$\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau}\varepsilon^b_{t+\tau}\left(U_{t+\tau}+\varepsilon^s_{t+\tau}L\left(\frac{B_{t+\tau}}{P_{t+\tau}R_{t+\tau}}\right)\right)\right]$$

with

$$U_t=\frac{1}{1-\gamma_c}\left((C_t-\varrho\bar{C}_{t-1})(1-H_t^{1+\gamma_h})\right)^{(1-\gamma_c)} \tag{1}$$

The function $L(\cdot)$ is strictly increasing, concave, and differentiable everywhere on $[0,\infty)$. In particular, $L'(0)$ exists and is finite. Without loss of generality, we set $L'(0)$ to one. The argument of $L(\cdot)$ equals the real value of government bonds in the household's portfolio: their period $t+1$ redemption value B_t divided by their nominal yield R_t expressed in units of the consumption good with the nominal price index P_t. The time-varying coefficient multiplying this felicity from bond holdings, ε^s_t, is the liquidity preference shock introduced by Fisher (2015). A separate shock influences the household's discounting of future utility to the present, ε^b_t. Specifically, the household discounts a certain utility in $t+\tau$ back to t with $\beta^{\tau}\mathbb{E}_t\left[\varepsilon^b_{t+\tau}/\varepsilon^b_t\right]$. In logarithms, these two preference shocks follow independent autoregressive processes.

$$\ln\varepsilon^b_t = (1-\rho_b)\ln\varepsilon^b_* + \rho_b\ln\varepsilon^b_{t-1}+\eta^b_t, \eta^b_t\sim\mathbb{N}(0,\sigma_b) \tag{2}$$

$$\ln\varepsilon^s_t = (1-\rho_s)\ln\varepsilon^s_* + \rho_s\ln\varepsilon^s_{t-1}+\eta^s_t, \eta^s_t\sim\mathbb{N}(0,\sigma_s). \tag{3}$$

A household's wealth at the beginning of period t consists of its nominal government bond holdings, B_t, its net holdings of privately-issued financial assets, and its capital stock K_{t-1}. The household chooses a rate of capital utilization u_t, and the capital services resulting from this choice equal u_tK_{t-1}. The cost of increasing utilization is higher depreciation. An increasing, convex and differentiable function $\delta(U)$ gives the capital depreciation rate. We specify this as

$$\delta(u)=\delta_0+\delta_1(u-u_\star)+\frac{\delta_2}{2}(u-u_\star)^2.$$

A household can augment its capital stock with investment, I_t. Investment requires paying adjustment costs of the "i-dot" form introduced by Christiano, Eichenbaum, and Evans (2005). Also, an *investment demand shock* alters the efficiency of investment in augmenting the capital stock. Altogether, if the household's investment in the previous period was I_{t-1}, and it purchases I_t units of the investment good today, then the stock of capital available in the *next* period is

$$K_t = (1 - \delta(u_t))\, K_{t-1} + \varepsilon_t^i \left(1 - S\left(\frac{A_{t-1}^K I_t}{A_t^K I_{t-1}}\right)\right) I_t. \tag{4}$$

In (4), A_t^K equals the productivity level of capital goods production, described in more detail below, and ε_t^i is the investment demand shock. In logarithms, this follows a first-order autoregression with a normally-distributed innovation.

$$\ln \varepsilon_t^i = (1 - \rho_i) \ln \varepsilon_*^i + \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim \mathbb{N}(0, \sigma_i) \tag{5}$$

1.2 Production

The producers of investment goods use a linear technology to transform the final good into investment goods. The technological rate of exchange from the final good to the investment good in period t is A_t^I. We denote $\Delta \ln A_t^I$ with ω_t, which we call the *investment-specific technology shock* and which follows first-order autogregression with normally distributed innovations.

$$\omega_t = (1 - \rho_\omega)\omega_\star + \rho_\omega \omega_{t-1} + \eta_t^\omega, \eta_t^\omega \sim \mathbb{N}(0, \sigma_\omega^2) \tag{6}$$

Investment goods producers are perfectly competitive.

Final good producers also operate a constant-returns-to-scale technology; which takes as inputs the products of the differentiated goods producers. To specify this, let Y_{it} denote the quantity of good i purchased by the representative final good producer in period t, for $i \in [0, 1]$. The representative final good producer's output then equals

$$Y_t \equiv \left(\int_0^1 Y_{it}^{\frac{1}{1+\lambda_t^p}} di\right)^{1+\lambda_t^p}.$$

With this technology, the elasticity of substitution between any two differentiated products equals $1 + 1/\lambda_t^p$ in period t. Although this is constant across products within a time period, it varies stochastically over time according to an ARMA$(1, 1)$ in logarithms.

$$\ln \lambda_t^p = (1 - \rho_p) \ln \lambda_\star^p + \rho_p \ln \lambda_{t-1}^p - \theta_p \eta_{t-1}^p + \eta_t^p, \eta_t^p \sim \mathbb{N}(0, \sigma_p) \tag{7}$$

Given nominal prices for the intermediate goods P_{it}, it is a standard exercise to show that the final goods producers' marginal cost equals

$$P_t = \left(\int_0^1 P_{it}^{-\frac{1}{\lambda_t^p}} di \right)^{-\lambda_t^p} \tag{8}$$

Just like investment goods firms, the final goods' producers are perfectly competitive. Therefore, profit maximization and positive final goods output together require the competitive output price to equal P_t. Therefore, we can define inflation of the nominal final good price as $\pi_t \equiv \ln(P_t/P_{t-1})$.

The intermediate goods producers each use the technology

$$Y_{it} = (K_{it}^e)^\alpha \left(A_t^Y H_{it}^d \right)^{1-\alpha} - A_t \Phi \tag{9}$$

Here, K_{it}^e and H_{it}^d are the capital services and labor services used by firm i, and A_t^Y is the level of neutral technology. Its growth rate, $\nu_t \equiv \ln(A_t^Y / A_{t-1}^Y)$, follows a first-order autogregression.

$$\nu_t = (1 - \rho_\nu) \nu_* + \rho_v \nu_{t-1} + \eta_t^\nu, \eta_t^\nu \sim \mathbb{N}(0, \sigma_\nu), \tag{10}$$

The final term in (9) represents the fixed costs of production. These grow with

$$A_t \equiv A_t^Y \left(A_t^I \right)^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

We demonstrate below that A_t is the stochastic trend in equilibrium output and consumption, measured in units of the final good. We denote its growth rate with

$$z_t = \nu_t + \frac{\alpha}{1 - \alpha} \omega_t \tag{12}$$

Similarly, define

$$A_t^K \equiv A_t A_t^I \tag{13}$$

In the specification of the capital accumulation technology, we labelled A_t^K the "productivity level of capital goods production." We demonstrate below that this is indeed the case with the definition in (13).

Each intermediate goods producer chooses prices subject to a Calvo (1983) pricing scheme. With probability $\zeta_p \in [0,1]$, producer i has the opportunity to set P_{it} without constraints. With the complementary probability, P_{it} is set with the indexing rule

$$P_{it} = P_{it-1} \pi_{t-1}^{\iota_p} \pi_\star^{1-\iota_p}. \tag{14}$$

In (14), $\pi_\star$ is the gross rate of price growth along the steady-state growth path, and $\iota_p \in [0,1]$.[1]

1.3 Labor Markets

Households' hours worked pass through two intermediaries, guilds and labor packers, in their transformation into labor services used by the intermediate goods producers. The guilds take the households' homogeneous hours as their only input and produce differentiated labor services. These are then sold to the labor packers, who assemble the guilds' services into composite labor services.

The labor packers operate a constant-returns-to-scale technology with a constant elasticity of substitution between the guilds' differentiated labor services. For its specification, let H_{it} denote the hours of differenziated labor purchased from guild i at time t by the representative labor packer. Then that packer's production of composite labor services, H_t^s are given by

$$H_t^s = \left(\int_0^1 (H_{it})^{\frac{1}{1+\lambda_t^w}} di \right)^{1+\lambda_t^w}.$$

As with the final good producer's technology, an ARMA$(1,1)$ in logarithms governs

[1] To model firms' price-setting opportunities as functions of s_t, define a random variable u_t^p which is independent over time and uniformly distributed on $[0,1]$. Then, firm i gets a price-setting opportunity if either $u_t^p \geq \zeta_p$ and $i \in [u_t^p - \zeta_p, u_t^p]$ or if $u_t^p < \zeta_p$ and $i \in [0, u_t^p] \cup [1 + u_t^p - \zeta_p, 1]$.

the constant elasticity of substitution between any two guilds' labor services.

$$\ln \lambda_t^w = (1-\rho_w)\ln \lambda_\star^w + \rho_w \ln \lambda_{t-1}^w - \theta_w \eta_{t-1}^w + \eta_t^w, \eta_t^w \sim \mathbb{N}(0, \sigma_w^2) \tag{15}$$

Just as with the final goods producers, we can easily show that the labor packers' marginal cost equals

$$W_t = \left(\int_0^1 (W_{it})^{-\frac{1}{\lambda_t^w}} di \right)^{-\lambda_t^w}. \tag{16}$$

Here, W_{it} is the nominal price charged by guild i per hour of differentiated labor. Since labor packers are perfectly competitive, their profit maximization and positive output together require that the price of composite labor services equals their marginal cost.

Each guild produces it's differentiated labor service using a linear technology with the household's hours worked as its only input. A Calvo (1983) pricing scheme similar to that of the differentiated goods producers constrains their nominal prices. Guild i has an unconstrained opportunity to choose its nominal price with probability $\zeta_w \in [0,1]$. With the complementary probability, W_{it} is set with an indexing rule based on π_{t-1} and last period's trend growth rate, z_{t-1}.

$$W_{it} = W_{it-1} \left(\pi_{t-1} e^{z_{t-1}}\right)^{\iota_w} \left(\pi_\star e^{z_\star}\right)^{1-\iota_w}. \tag{17}$$

In (17), $z_\star \equiv \nu_\star + \frac{\alpha}{1-\alpha}\omega_\star$ is the unconditional mean of z_t and $\iota_w \in [0,1]$.

1.4 Fiscal and Monetary Policy

The model economy hosts two policy authorities, each of which follows exogenously-specified rules that receive stochastic disturbances. The fiscal authority issues bonds, B_t, collects lump-sum taxes T_t, and buys "wasteful" public goods G_t. Its period-by-period budget constraint is

$$G_t + B_{t-1} = T_t + \frac{B_t}{R_t}. \tag{18}$$

The left-hand side gives the government's uses of funds, public goods spending and the retirement of existing debt. The left-hand side gives the sources of funds, taxes and the proceeds of new debt issuance at the interest rate R_t. We assume that the fiscal authority keeps its budget balanced period-by-period, so $B_t = 0$. Furthermore, the fiscal authority sets public goods expenditure equal to a stochastic share of output, expressed in consumption units.

$$G_t = (1 - 1/g_t)Y_t, \tag{19}$$

with

$$\ln g_t = (1 - \rho_g)\ln s_\star^g + \rho_g \ln g_{t-1} + \eta_t^g, \eta_t^g \sim \mathbb{N}(0, \sigma_g^2). \tag{20}$$

The monetary authority sets the nominal interest rate on government bonds, R_t. For this, it employs a Taylor rule with interest-rate smoothing and forward guidance shocks.

$$\ln R_t = \rho_R \ln R_{t-1} + (1 - \rho_R)\ln R_t^n + \sum_{j=0}^{M} \xi_{t-j}^j. \tag{21}$$

The monetary policy disturbances in (21) are $\xi_t^0, \xi_{t-1}^1, \ldots, \xi_{t-M}^M$. The public learns the value of ξ_{t-j}^j in period $t - j$. The conventional unforecastable shock to current monetary policy is ξ_t^0, while for $j \geq 1$, these disturbances are *forward guidance shocks*. We gather all monetary shocks revealed at time t into the vector ε_t^1. This is normally distributed and *i.i.d.* across time. However, *its elements may be correlated with each other.* That is,

$$\varepsilon_t^1 \equiv \left(\xi_t^0, \xi_t^1, \ldots, \xi_t^M\right) \sim \mathbb{N}(0, \Sigma^1). \tag{22}$$

The off-diagonal elements of Σ^1 are not necessarily zero, so forward-guidance shocks need not randomly impact expected future monetary policy at two adjacent dates independently. Current economic circumstances influence R_t through the notional interest rate, R_t^n.

$$\ln R_t^n = \ln r_\star + \ln \pi_t^\star + \frac{\phi_1}{4}\mathbb{E}_t \sum_{j=-2}^{1} \left(\ln \pi_{t+j} - \ln \pi_t^\star\right) + \frac{\phi_2}{4}\mathbb{E}_t \sum_{j=-2}^{1} \left(\ln Y_{t+j} - \ln y^\star - \ln A_{t+j}\right). \tag{23}$$

The constant $r_\star$ equals the real interest rate along a steady-state growth path, and $\pi_t^\star$ is the central bank's intermediate target for inflation. We call this the *inflation-drift shock*. it follows a first-order autoregression with a normally-distributed innovation. Its unconditional mean equals $\pi_\star$, the inflation rate on a steady-state growth path.

$$\ln \pi_t^\star = (1-\rho_\pi)\pi_\star + \rho_\pi \ln \pi_{t-1}^\star + \eta_t^\pi, \eta_t^\pi \sim \mathbb{N}(0, \sigma_\pi^2) \tag{24}$$

Allowing $\pi_t^\star$ to change over time enables our model to capture the persistent decline in inflation from the early 1990s through the early 2000s engineered by the Greenspan FOMC.

1.5 Other Financial Markets and Equilibrium Definition

All households participate in the market for nominal risk-free government debt. Additionally, they can buy and sell two classes of privately issued assets without restriction. The first is one-period nominal risk-free *private* debt. We denote the value of household's net holdings of such debt at the beginning of period t with B_{t-1}^P and the interest rate on such debt issued in period t maturing in $t+1$ with R_{t+1}^P. The second asset class consists of a complete set of *real* state-contingent claims. As of the end of period t, the household's ownership of securities that pay off one unit of the aggregate consumption good in period τ if history s^τ occurs is $Q_t(s^\tau)$, and the nominal price of such a security in the same period is $J_t(s^\tau)$.

We define an equilibrium for our economy in the usual way: Households maximize their utility given all prices, taxes, and dividends from both producers and guilds; final goods producers and labor packers maximize profits taking their input and output prices as given; differentiated goods producers and guilds maximize the market value of their dividend streams taking as given all input and financial-market prices; differentiated goods producers and guilds produce to satisfy demand at their posted prices; and otherwise all product, labor, and financial markets clear.

2 First Order Conditions

In this section we present the first-order conditions associated with the optimization problems that the agents in our model solve.

2.1 Households

Given initial financial asset holdings holdings, a stock of productive capital, investment in the previous period (which influences investment adjustment costs), and the external habit stock; households' choices of consumption, capital investment, capital utilization, hours worked, and financial investments maximize utility subject to the constraints of the capital accumulation and utilization technology and a sequence of one-period budget constraints. To specify these budget constraints, denote the nominal wage-per-hour paid by labor guilds to households with W_t^h, the nominal rental rate for capital services with R_t^k, the nominal price of investment goods with P_t^I, and the dividends paid by labor guilds added to those paid by differentiated good producers with D_t. With this notation, writing the period t budget constraint with uses of funds on the left and sources of funds on the right yields

$$C_t + \frac{P_t^I I_t}{P_t} + \frac{B_t}{R_t P_t} + \frac{B_t^P}{R_t^P P_t} + \frac{T_t}{P_t} \leq \frac{B_{t-1}}{P_t} + \frac{B_{t-1}^P}{P_t} + \frac{W_t^h H_t}{P_t} + \frac{R_t^k u_t K_{t-1}}{P_t} + \frac{D_t}{P_t} \tag{25}$$

Denote the Lagrange multiplier on (25) with $\beta^t \Lambda_t^1$, and that on the capital accumulation constraint in (4) with $\beta^t \Lambda_t^2$. With these definitions, the first-order conditions for a household's utility maximization problem are

$$
\begin{aligned}
\Lambda_t^1 &= \varepsilon_t^b \left((C_t - \varrho \bar{C}_{t-1})(1 - \varepsilon_t^h H_t^{1+\gamma_h}) \right)^{-\gamma_c} (1 - \varepsilon_t^h H_t^{1+\gamma_h}) \\
\Lambda_t^1 \frac{W_t^h}{P_t} &= (1+\gamma_h) \varepsilon_t^b \left((C_t - \varrho \bar{C}_{t-1})(1 - \varepsilon_t^h H_t^{1+\gamma_h}) \right)^{-\gamma_c} (C_t - \varrho \bar{C}_{t-1}) \varepsilon_t^h H_t^{\gamma_h} \\
\frac{\Lambda_t^1}{R_t P_t} - \varepsilon_{t+q}^b L'(\frac{B_t}{R_t P_t}) \frac{\varepsilon_t^s}{R_t P_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{P_{t+1}} \right] \\
\frac{\Lambda_t^1}{R_t^P P_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{P_{t+1}} \right] \\
\Lambda_t^2 &= \beta \mathbb{E} \left[\Lambda_{t+1}^1 \frac{R_{t+1}^k u_{t+1}}{P_{t+1}} + \Lambda_{t+1}^2 (1 - \delta(u_{t+1})) \right] \\
\frac{\Lambda_t^1 R_t^k}{P_t} &= \Lambda_t^2 \delta'(u_t) \\
\Lambda_t^1 &= \varepsilon_t^i \Lambda_t^2 \left((1 - S_t(\cdot)) - S_t'(\cdot) \frac{i_t}{i_{t-1}} \right) \\
&\quad + \beta \mathbb{E}_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C) z_{t+1}} \lambda_{t+1}^2 S_{t+1}'(\cdot) \frac{i_{t+1}^2}{i_t^2} \right]
\end{aligned}
$$

In equilibrium, $\bar{C}_t = C_t$ always.

2.2 Goods Sector

2.2.1 Final Goods Producers

The nominal marginal cost of final goods producers equals the right-hand side of (8). A producer of final goods maximizes profit by shutting down if P_t is less than this marginal cost and can make an arbitrarily large profit if P_t exceeds it. When (8) holds, an individual final goods producer's output is indeterminate.

Final goods producers' demand for intermediate goods takes the familiar constant-elasticity form. If we use Y_t to denote total final goods output, then the amount of differentiated good i demanded by final goods producers is

$$
Y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}} .
$$

Given the choice of a reset price, we wish to calculate the overall price level. All intermediate goods producers with a price-setting opportunity choose $\tilde{P}_t$. The remaining producers use the price-indexing rule in (14). The aggregate price level is given by

$$P_t = \left[(1-\zeta_p)\tilde{P}_t^{\frac{1}{\lambda_{p,t-1}}} + \zeta_p \left((\pi_{t-1})^{\iota_p} (\pi_*)^{1-\iota_p} P_{t-1} \right)^{\frac{1}{\lambda_{p,t-1}}} \right]^{\lambda_{p,t-1}}$$

where $\tilde{P}_t$ is the optimal reset price

2.2.2 Intermediate Goods Producers

Intermediate goods producers' cost minimization reads as follows:

$$\max_{H_{t,i}, K^e_{i,t}} W_t H^d_{t,i} + R^k_t K^e_{i,t}$$
$$\text{s.t. } Y_{t,i} = \varepsilon^a_t (K^e_{t,i})^{\alpha} \left(A^y_t H^d_{t,i} \right)^{1-\alpha} - A_t \Phi$$

We get the following optimal capital-labor ratio.

$$\frac{\alpha}{1-\alpha} \frac{W_t}{R^k_t} = \frac{(K^e_{it})^s}{H^d_{t,i}}$$

Notice how for each firm, the idiosyncratic capital to labor ratio is not a function of any firm-specific component. Hence, each firm has the same capital to labor ratio. In equilibrium,

$$K^e_t = u_t K_{t-1}$$

To find the marginal cost, we differentiate the variable part of production with respect to output, and substitute in the capital-labor ratio.

$$MC_{t,i} = (\varepsilon^a_t)^{-1} (A^y_t)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{k\alpha} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$$

Again, notice that each firm as the same marginal cost.

Given cost minimization, a differentiated goods producer with an opportunity to adjust its nominal price does so to maximize the present-discounted value of profits earned until the next opportunity to adjust prices arrives. Formally,

$$\max_{\tilde{P}_{t,i}} E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Lambda^1_{t+s} P_t}{\Lambda^1_t P_{t+s}} \left[\tilde{P}_{t,i} X^y_{t,s} - MC_{t+s}\right] Y_{t+s,i}$$

$$\text{s.t. } Y_t(i) = \left(X^y_{t,s} \frac{\tilde{P}_{t,i}}{P_t}\right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} Y_t$$

$$\text{where } X^y_{t,s} = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p} & : s = 1, \ldots, \infty \end{array} \right\}$$

This problem leads to the following price-setting equation for firms that are allowed to reoptimize their price:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Lambda^1_{t+s} P_t}{\Lambda^1_t P_{t+s}} Y_{it+s} \left[\lambda_{p,t+s} MC_{t+s} - X_{t,s} \tilde{P}_{it} \right]$$

It can be shown that the producers that are allowed to reoptimize choose the same price. So henceforth, $\tilde{P}_{it} = \tilde{P}_t$.

2.2.3 Investment Goods Producers

Characterizing the profit-maximizing choices of investment goods and final goods producers is straightforward. If $P^I_t > P_t / A^I_t$, then each investment goods producer can make infinite profit by choosing an arbitrarily large output. On the other hand, if $P^I_t < P_t / A^I_t$, then investment goods producers maximize profits with zero production. Finally, their profit-maximizing production is indeterminate when

$$P^I_t = P_t / A^I_t. \tag{26}$$

The relative price of investment to consumption is equal to $(A^I_t)^{-1}$. Making this substitution into the household f.o.c and noting that $P_t Y^I_t$ is an intermediate input that will not be reflected in the aggregate resource constraint, it suffices to substitute the relative price $(A^I_t)^{-1}$ in the constraint for the household.

2.3 Labor Sector

2.3.1 Labor Packers

The labor packers choose the the labor inputs supplied by guilds, pack them into a composite labor service to be sold to the intermediate goods producers. Formally, labor packers' problem reads as follows:

$$\max_{H_t^s, H_{it}} W_t H_t^s - \int_0^1 W_{it} H_{it} di$$
$$s.t. \left[\int_0^1 H_{it}^{\frac{1}{1+\lambda_{w,t}}} di \right]^{1+\lambda_{w,t}} = H_t^s$$

We obtain the following labor demand equation for guild i:

$$H_{it} = \left(\frac{W_{it}}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} H_t \tag{27}$$

As for the goods sector, we can show that aggregate wage is given by the following equation:

$$W_t = \left[(1-\zeta_w) \tilde{W}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w \left(\left(e^{z_{t-1}} \pi_{t-1} \right)^{\iota_w} \left(\pi_* e^{z_*} \right)^{1-\iota_w} W_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

where $\tilde{W}$ is the optimal reset wage for guilds.

2.3.2 Guilds

Each guild with an opportunity to set its nominal price does so to maximize the current value of the stream of dividends returned to the household. Formally, their problem reads

$$\max_{\tilde{W}_{it}} E_t \sum_{s=0}^{\infty} \zeta_w^s \left(\frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} \right) \left[X_{t+s}^l \tilde{W}_{it} - W_{t+s}^h \right] H_{it+s}$$
$$\text{s.t. } H_{it+s} = \left(\frac{X_{t,s}^l \tilde{W}_{it}}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} H_{t+s}$$
$$\text{where } X_{t,s}^l = \begin{Bmatrix} 1 & : s = 0 \\ \prod_{j=1}^s \left(\pi_{t+j-1} \frac{A_{t+j-1}}{A_{t+j-2}} \right)^{1-\iota_w} \left(\pi e^{\gamma} \right)^{\iota_w} & : s = 1, \ldots, \infty \end{Bmatrix}$$

$\tilde{W}_t$ is the optimal reset wage. This optimal wage is chosen by the guilds who are allowed, with probability ζ_w, to change their prices in a given period. Also, we index the nominal wage inflation rate with ι_w.

This maximization problem gives a wage-setting equation that reads as follows:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} H_{it+s} \frac{1}{\lambda_{w,t+s}} \left((1 + \lambda_{w,t+s}) W_{t+s}^h - X_{t,s}^l \tilde{W}_{it} \right)$$

It can be shown that the guilds that are allowed to reoptimize choose the same wage. So henceforth, $\tilde{W}_{it} = \tilde{W}_t$.

3 Detrending

To remove nominal and real trends, we deflate nominal variables by their matching price deflators, and we detrend any resulting real variables influenced permanently by technological change. All scaled versions of variables are the lower-case counterparts.

$$c_t = \frac{C_t}{A_t} \qquad i_t = \frac{I_t}{A_t A_t^I}$$

$$k_t = \frac{K_t}{A_t A_t^I} \qquad k_t^e = \frac{K_t^e}{A_t A_t^I}$$

$$w_t = \frac{W_t}{A_t P_t} \qquad \tilde{w}_t = \frac{\tilde{W}_t}{A_t P_t}$$

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t} \qquad \pi_t = \frac{P_t}{P_{t-1}}$$

$$y_t = \frac{Y_t}{A_t} \qquad mc_t = \frac{MC_t}{P_t}$$

$$r_t^k = \frac{R_t^k A_t^I}{P_t} \qquad w_t^h = \frac{W_t^h}{A_t P_t}$$

$$\lambda_t^1 = \Lambda_t^1 A_t^{\gamma_C} \qquad \lambda_t^2 = \Lambda_t^2 A_t^{\gamma_C} A_t^I$$

$$\varepsilon_t^s = A_t^{\gamma_C} \varepsilon_t^s$$

3.1 Detrended Equations

The detrended equations describing our model are listed in the following sections.

Households' FOC

$$
\begin{aligned}
\lambda_t^1 &= \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \right]^{-\gamma_c} \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \\
\lambda_t^1 w_t^h &= (1+\gamma_h) \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{(1+\sigma_h)} \right) \right]^{-\gamma_c} \left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \varepsilon_t^h h_t^{\gamma_h} \\
\frac{\lambda_t^1}{R_t^P} &= \beta E_t \left[\frac{\lambda_{t+1}^1 e^{-\gamma_C z_{t+1}}}{\pi_{t+1}} \right] \\
\frac{\lambda_t^1}{R_t} - L'(0) \frac{\varepsilon_t^b \varepsilon_t^s}{R_t} &= \beta E_t \frac{\lambda_{t+1}^1}{\pi_{t+1}} e^{-z_{t+1} \gamma_C} \\
\lambda_t^1 &= \varepsilon_t^i \lambda_t^2 \left((1 - S_t(\cdot)) - S_t'(\cdot) \frac{i_t}{i_{t-1}} \right) + \beta E_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C) z_{t+1}} \lambda_{t+1}^2 S_{t+1}'(\cdot) \frac{i_{t+1}^2}{i_t^2} \right] \\
\lambda_t^2 &= \beta E_t \left[e^{-\gamma_C z_{t+1} - \omega_{t+1}} \left(\lambda_{t+1}^1 r_{t+1}^k u_{t+1} + \lambda_{t+1}^2 (1 - \delta(u_{t+1})) \right) \right] \\
\lambda_t^1 r_t^k &= \lambda_t^2 \delta'(u_t) \\
k_t &= (1 - \delta(u_t)) k_{t-1} e^{-z_t - \omega_t} + \varepsilon_t^i (1 - S(\cdot)) i_t \\
k_t^e &= u_t k_{t-1} e^{-z_t - \omega_t}
\end{aligned}
$$

Final Goods Price Index

$$
1 = \left[(1 - \zeta_p) \tilde{p}_t^{\frac{1}{1-\lambda_{p,t}}} + \zeta_p \left(\pi_{t-1}^{\iota_p} \pi^{*(1-\iota_p)} \pi_t^{-1} \right)^{\frac{1}{1-\lambda_{p,t}}} \right]^{1-\lambda_{p,t}}
$$

Intermediate Goods Firms: Capital-Labor Ratio

$$
\frac{k_t^e}{h_t^d} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k}
$$

Intermediate Goods Firms: Real Marginal Costs

$$
mc_t = \frac{w_t^{1-\alpha} \left(r_t^k \right)^{\alpha}}{\varepsilon_t^a \alpha^{\alpha} (1-\alpha)^{1-\alpha}}
$$

Intermediate Goods Firms: Price-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \lambda_{t+s}^1 \frac{\tilde{y}_{t,t+s}}{\lambda_{p,t+s} - 1} \left(\frac{A_{t+s}}{A_t} \right)^{1-\gamma_C} \left[\lambda_{p,t+s} mc_{t+s} - \tilde{X}_{t,s}^p \tilde{p}_t \right]$$

where

$$\tilde{X}_{t,s}^p = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \frac{\prod_{j=1}^s \pi_{t+j-1}^{1-\iota_p} \pi_*^{\iota_p}}{\prod_{j=1}^s \pi_{t+j}} & : s = 1, \ldots, \infty \end{array} \right\}$$

$\tilde{y}_{t,t+s}$ denotes the time $t+j$ output sold by the producers that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing producers all choose the same price, then we do not have to carry the i-subscript.

Labor Packers: Aggregate Wage Index

$$w_t = \left[(1 - \zeta_w) \tilde{w}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w \left(e^{\iota_w z_{t-1} - z_t} e^{(1-\iota_w) z_*} \pi_{t-1}^{\iota} \pi_t^{-1} \pi_*^{1-\iota_w} w_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

Guilds: Wage-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \beta \lambda_{t+s}^1 \left(\frac{A_{t+s}}{A_t} \right)^{1-\gamma_C} \frac{\tilde{h}_{t,t+s}}{\lambda_{w,t+s}} \left((1 + \lambda_{w,t+s}) w_{t+s}^h - \tilde{X}_{t,s}^l \tilde{w}_t \right)$$

where

$$\tilde{X}_{t,s}^l = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \frac{\prod_{j=1}^s \left(\pi_{t+j-1} e^{z_{t+j-1}} \right)^{1-\iota_w} (\pi\gamma)^{\iota_w}}{\prod_{j=1}^s \pi_{t+j} e^{z_{t+j}}} & : s = 1, \ldots, \infty \end{array} \right\}$$

$\tilde{h}_{t,t+s}$ denotes the time $t+j$ labor supplied by the guild that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing guilds all choose the same wage, then we do not have to carry the i-subscript.

Monetary Authority

$$R_t = R_{t-1}^{\rho_R}\left[r_*\pi_t^*\left(\prod_{j=-2}^{1}\frac{\pi_{t+j}}{\pi_t^*}\right)^{\frac{\psi_1}{4}}\left(\prod_{j=-2}^{1}\frac{y_{t+j}}{y^*}\right)^{\frac{\psi_2}{4}}\right]^{1-\rho_R}\prod_{j=0}^{M}\xi_{t-j,j}$$

The Aggregate Resource Constraint

$$\frac{y_t}{g_t} = c_t + i_t$$

Production Function

$$y_t = \varepsilon_t^a\left(k_t^e\right)^{\alpha}\left(h_t^d\right)^{1-\alpha} - \Phi$$

Labor Market Clearing Condition

$$h_t = h_t^d$$

4 Steady State

We normalize most shocks and the utilization rate:

$$u_\star = 1 \qquad \varepsilon^i = 1$$
$$\varepsilon^a = 1 \qquad \varepsilon^b = 1$$

Next, we set the following restriction on adjustment costs:

$$S(\cdot_*) \equiv 0$$
$$S'(\cdot_*) \equiv 0$$

4.1 Prices and Interest Rates

Given β, z_*, γ_C, and π_*, we can solve for the steady-state nominal interest rate on private bonds R_*^P by using the FOC on private bonds:

$$R_*^P = \frac{\pi_*}{(\beta e^{-\gamma_C z_*})} \tag{28}$$

From the definition of $\delta(u)$, we have

$$\begin{aligned}\delta(1) =& \delta_0 \\ \delta'(1) =& \delta_1.\end{aligned}$$

Next, given ω_*, δ_0, and the above, we can solve for the real return on capital r_*^k using the FOC on capital:

$$r_*^k = \frac{e^{\gamma_C z_* + \omega_*}}{\beta} - (1 - \delta_0) \tag{29}$$

4.2 Ratios

Moving to the production side, we can use the aggregate price equation to solve for $\tilde{p}_*$:

$$\tilde{p}_* = 1$$

Using this result and given $\lambda_{p,*}$, we can use the price Phillips curve to solve for mc_*:

$$mc_* = \frac{1}{1 + \lambda_{p,*}} \tag{30}$$

Given values for α and ε_*^a, we can use the marginal cost equation to solve for w_*:

$$w_* = \left(mc_* \alpha^\alpha (1 - \alpha)^{1-\alpha} (r_*^k)^{-\alpha}\right)^{\frac{1}{1-\alpha}} \tag{31}$$

The definition of effective capital gives us a value for k^e_* in terms of k_*:

$$k^e_* = k_* e^{-z_* - \omega_*}$$

Calculating y_* using the labor share of output $1 - \alpha$:

$$y_* = \frac{w_* h_*}{1 - \alpha}$$

Using capital shares based off our value of α, we can calculate the output to capital ratio as follows:

$$\frac{y_*}{k^e_*} = \frac{r^k_*}{\alpha}$$

$$\frac{y_*}{k_*} = e^{-z_* - \omega_*} \frac{r^k_*}{\alpha}$$

Using the capital accumulation equation, we can get a value for $\frac{i_*}{k_*}$

$$\frac{i_*}{k_*} = 1 - (1 - \delta_0) e^{-z_* - \omega_*}$$

Using the resource constraint, we can get $\frac{c_*}{k_*}$:

$$\frac{c_*}{k_*} = \frac{y_*}{k_* s^g_*} - \frac{i_*}{k_*}$$

These ratios will give us the remaining steady-state levels and ratios:

$$k_* = y_* \left(\frac{y_*}{k_*} \right)^{-1} \qquad i_* = \frac{i_*}{k_*} k_*$$

$$c_* = \frac{c_*}{k_*} k_* \qquad g_* = g_y y_*$$

4.3 Liquidity Premium

Using the aggregate wage equation, we can get the following for $\tilde{w}_*$:

$$\tilde{w}_* = w_*$$

Combining this result with the wage Phillips curve, we get the following:

$$w_*^h = \frac{w_*}{1 + \lambda_{w,*}}$$

We can use the FOC for consumption and the labor supply to pin down ε^h and λ_*^1

$$\varepsilon^b \left[c_* \left(1 - \frac{\varrho}{e^z} \right) \right]^{-\gamma_c} \left(1 - \varepsilon^h h_*^{(1+\gamma_h)} \right) - \lambda_*^1 = 0$$

$$-(1 + \gamma_h) \varepsilon^b c_*^{(1-\gamma_c)} \left(1 - \frac{\varrho}{\varepsilon^z} \right)^{(1-\gamma_c)} \left(1 - \varepsilon^h h_*^{(1+\gamma_h)} \right)^{-\gamma_c} \varepsilon^h h_*^{\gamma_h} + \lambda_*^1 w_*^h = 0$$

Finally, the government bond rate is calculated from

$$\begin{aligned} \lambda_*^1 - \varepsilon_*^b \varepsilon_*^s &= \beta R_* \frac{\lambda_*^1}{\pi_*} e^{-\gamma_C z} \\ \underbrace{\frac{\pi_*}{\beta e^{-\gamma_C z}}}_{R_*^P} - \varepsilon_*^b \varepsilon_*^s \frac{\pi_*}{\beta e^{-\gamma_C z} \lambda_*^1} &= R_* \end{aligned}$$

Noting that $R_*^P = \frac{\pi_*}{\beta e^{-\gamma_C z}}$ we can write

$$\frac{R_*^P - R_*}{R_*^P} = \frac{\varepsilon_*^b \varepsilon_*^s}{\lambda_*^1}.$$

This is the liquidity premium in steady state.

5 Log Linearization

Hatted variables refer to log deviations from steady-state ($\hat{x} = \ln\left(\frac{x_t}{x_*}\right)$):

$$\ln \varepsilon_t^j = \rho_j \ln \varepsilon_{t-1}^j + \eta_t^j$$

In the cases of z_t, ω_t, and ν_t, we have that $\hat{x} = x_t - x_*$ as these variables are already in logs.

Households' First Order Conditions

$$\hat{\varepsilon}^b_t - \hat{\lambda}^1_t - \gamma_c \frac{1}{1-\frac{\varrho}{e^z}} \hat{c}_t + \gamma_c \frac{\frac{\varrho}{e^z}}{1-\frac{\varrho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \tag{32}$$

$$\hat{\lambda}^1_t + \hat{w}^h_t - \hat{\varepsilon}^b_t - \hat{\varepsilon}^h_t - \frac{1-\gamma_c}{1-\frac{\varrho}{e^z}} \hat{c}_t + (1-\gamma_c) \frac{\frac{\varrho}{e^z}}{1-\frac{\varrho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \tag{33}$$

$$- \left(\gamma_h + \gamma_c (1+\gamma_h) \frac{\varepsilon^h h_*^{1+\gamma_h}}{(1-\varepsilon^h h_*^{1+\gamma_h})^2} \right) \hat{h}_t = 0$$

$$\hat{\lambda}^1_t = \frac{R^P_* - R_*}{R^P_*} (\hat{\varepsilon}^s_t + \hat{\varepsilon}^b_t) + \frac{R_*}{R^P_*} (\hat{R}_t + E_t[(\hat{\lambda}^1_{t+1} - \hat{\pi}_{t+1} - \gamma_C \hat{z}_{t+1}]) \tag{34}$$

$$\hat{\lambda}^1_t = E_t \left[\hat{\lambda}^1_{t+1} - \gamma_C \hat{z}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1} \right] \tag{35}$$

$$\hat{\lambda}^1_t = \left(\ln \varepsilon^i_t + \hat{\lambda}^2_t \right) - S'' (\hat{\imath}_t - \hat{\imath}_{t-1}) + \beta e^{(1-\gamma_C)\gamma} S'' E_t (\hat{\imath}_{t+1} - \hat{\imath}_t) \tag{36}$$

$$\lambda^2_* \hat{\lambda}^2_t = \beta e^{-\gamma_C z_* - \omega_*} \left[\lambda^1_* u_* r^k_* E_t \left(-\gamma_C \hat{z}_{t+1} - \hat{\omega}_{t+1} + \hat{\lambda}^1_{t+1} + \hat{r}^k_{t+1} + \hat{u}_{t+1} \right) \right] + \tag{37}$$

$$+ \beta e^{-\gamma_C z_* - \omega_*} \left[(1-\delta_0) \lambda^2_* E_t \left(-\gamma_C \hat{z}_{t+1} - \hat{\omega}_{t+1} + \hat{\lambda}^2_{t+1} \right) - \lambda^2_* \delta_1 u_* E_t \hat{u}_{t+1} \right]$$

$$\hat{\lambda}^1_t = \hat{\lambda}^2_t + \frac{\delta_2}{\delta_1} u_* \hat{u}_t - \hat{r}^k_t \tag{38}$$

$$\hat{k}_t = \left(1 - \frac{\varepsilon^i_* i_*}{k_*} \right) \left(\hat{k}_{t-1} - \hat{z}_t - \hat{\omega}_t \right) + \frac{\varepsilon^i_* i_*}{k_*} \left(\hat{\varepsilon}^i_t + \hat{\imath}_t \right) - \delta_1 u_* e^{-z_* - \omega_*} \hat{u}_t \tag{39}$$

$$\hat{k}^e_t = \hat{u}_t + \hat{k}_{t-1} - \hat{z}_t - \hat{\omega}_t \tag{40}$$

Capital-Labor Ratio

$$\hat{k}^e_t = \hat{w}_t - \hat{r}^k_t + \hat{h}^d_t \tag{41}$$

Real Marginal Costs

$$\widehat{mc}_t = (1-\alpha) \hat{w}_t + \alpha \hat{r}^k_t - \hat{\varepsilon}^a_t \tag{42}$$

The New Keynesian Phillips Curve for Inflation

$$\hat{\pi}_t = \frac{(1-\beta\zeta_p e^{(1-\gamma_C)z_*})(1-\zeta_p)}{(1+\beta\iota_p e^{(1-\gamma_C)z_*})\zeta_p} \left[\frac{\lambda_{p,*}}{1+\lambda_{p,*}} \hat{\lambda}_{p,t} + \widehat{mc}_t \right] + \tag{43}$$

$$+ \frac{\iota_p}{1+\beta\iota_p e^{(1-\gamma_C)z_*}} \hat{\pi}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta\iota_p e^{(1-\gamma_C)z_*}} E_t \hat{\pi}_{t+1}$$

Wage Mark-Up

$$\hat{\mu}_t^w = \hat{w}_t - \hat{w}_t^h \tag{44}$$

The New Keynesian Phillips Curve for Wages

$$\hat{w}_t = \frac{1}{1+\beta e^{(1-\gamma_C)z_*}}\hat{w}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\hat{w}_{t+1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\left(E_t\hat{\pi}_{t+1} + E_t\hat{z}_{t+1}\right) + \tag{45}$$

$$\frac{\iota_w}{1+\beta e^{(1-\gamma_C)z_*}}\left(\hat{\pi}_{t-1} + \hat{z}_{t-1}\right) - \frac{1+\iota_w\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\left(\hat{\pi}_t + \hat{z}_t\right) +$$
$$\frac{1-\beta\zeta_w e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\frac{1-\zeta_w}{\zeta_w}\left[\frac{\lambda_{w,*}}{1+\lambda_{w,*}}\hat{\lambda}_{w,t} - \hat{\mu}_t^w\right]$$

The Aggregate Resource Constraint

$$\frac{y_*}{g_*}\left(\hat{y}_t - \hat{g}_t\right) = \frac{c_*}{c_* + i_*}\hat{c}_t + \frac{i_*}{c_* + i_*}\hat{\imath}_t \tag{46}$$

The Production Function

$$\hat{y}_t = \frac{1}{mc_*}\left(\ln\varepsilon_t^a + \alpha\hat{k}_t^e + (1-\alpha)\,\hat{h}_t^d\right) \tag{47}$$

Labor Market Clearing Condition

$$\hat{h}_t = \hat{h}_t^d \tag{48}$$

Monetary Authority's Reaction Function

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[(1-\psi_1)\,\hat{\pi}_t^* + \frac{\psi_1}{4}\left(\sum_{j=-2}^{1}\hat{\pi}_{t+j}\right) + \frac{\psi_2}{4}\left(\sum_{j=-2}^{1}\hat{y}_{t+j}\right)\right] + \sum_{j=0}^{M}\hat{\xi}_{t-j,j} \tag{49}$$

6 Measurement

6.1 National Income Accounts

The model economy's basic structure, with the representative household consuming a single good and accumulating capital using a different good, differs in some important ways from the accounting conventions of the Bureau of Economic Analysis (BEA) underlying the National Income and Product Accounts (NIPA). In particular

1. The BEA treats household purchases of long-lived goods inconsistently. If classifies purchases of residential structures as investment and treats the service flow from their stock as part of Personal Consumption Expenditures (PCE) on services. The BEA classifies households purchases of all other durable goods as consumption expenditures. No service flow from the stock of household durables enters measures of current consumption. In the model, all long-lived investments add to the productive capital stock.

2. The BEA treats all government purchases as government consumption. However, government at all levels makes purchases of investment goods on behalf of the populace. In the model, these should be treated as additions to the single stock of productive capital.

3. The BEA sums PCE and private expenditures on productive capital (Business Fixed Investment and Residential Investment), with government spending, inventory investment, and net exports to create Gross Domestic Product. The model features only the first three of these.

To bridge these differences, we create four *model consistent* NIPA measures from the BEA NIPA data.

1. Model-consistent GDP. Since the model's capital stock includes both the stock of household durable goods and the stock of government-purchased capital, a model-consistent GDP series should include the value of both stocks' service flows. To construct these, we followed a five-step procedure.

   (a) We begin by estimating a constant (by assumption) service-flow rate by dividing the nominal value of housing services from NIPA Table 2.4.5 by the beginning-of-year value of the residential housing stock from the

BEA's Fixed Asset Table 1.1. We use annual data and average from 1947 through 2014. The resulting estimate is 0.096. That is, the annual value of housing services equals approximately 10 percent of the housing stock's value each year.

(b) In the second step, we estimate estimate constant (by assumption) depreciation rates for residential structures, durable goods, and government capital. We constructed these by first dividing observations of value lost to depreciation over a calendar year by the end-of-year stocks. Both variables were taken from the BEA's Fixed Asset Tables. (Table 1.1 for the stocks and Table 1.3 for the deprecation values.) We then averaged these ratios from 1947 through 2014. The resulting estimates are 0.021, 0.194, and 0.044 for the three durable stocks.

(c) In the third step, we calculated the average rates of real price depreciation for the three stocks. For this, we began with the nominal values and implicit deflators for PCE Nondurable Goods and PCE Services from NIPA Table 1.2. We used these series and the Fisher-ideal formula to produce a chain-weighted implicit deflator for PCE Nondurable Goods and Services. Then, we calculated the price for each of the three durable good's stocks in consumption units as the ratio of the implicit deflator taken from Fixed Asset Table 1.2 to this deflator. Finally, we calculated average growth rates for these series from 1947 through 2014. The resulting estimates equal 0.0029, −0.0223, and 0.0146 for residential housing, household durable goods, and government-purchased capital.

(d) The fourth combines the previous steps' calculations to estimate constant (by assumption) service-flow rates for household durable goods and government-purchased capital. To implement this, we assumed that all *three* stocks yield the same financial return along a steady-state growth path. These returns sum the per-unit service flow with the appropriately depreciated value of the initial investment. This delivers two equations in two unknowns, the two unknown service-flow rates. The resulting estimates are 0.29 and 0.12 for household durable goods and government-purchased capital.

(e) The fifth and final step uses the annual service-flow rates to calculate real and nominal service flows from the real and nominal stocks of durable goods and government-purchased capital reported in Fixed Asset Table 1.1. This delivers an annual series. Since the stocks are measured as of the end of the calendar year, we interpret these as the service flow values in the *next* year's first quarter. We create quarterly data by linearly interpollating between these values.

With these real and nominal service flow series in hand, we create nominal model-consistent GDP by summing the BEA's definition of nominal GDP with the nominal values of the two service flows. We create the analogous series for model-consistent real GDP by applying the Fisher ideal formula to the nominal values and price indices for these three components.

2. Model-consistent Investment. The nominal version of this series sums nominal Business Fixed Investment, Residential Investment, PCE Durable Goods, and government investment expenditures. The first three of these come from NIPA Table 1.1.5, while government investment expenditures sums Federal Defense, Federal Nondefense, and State and Local expenditures from NIPA Table 1.5.5. We construct the analogous series for real Model-consistent Investment by combining these series with their real chain-weighted counterparts found in NIPA Tables 1.1.3 and 1.5.3 using the Fisher ideal formula. By construction, this produces an implicit deflator for Model-consistent investment as well.

3. Model-consistent Consumption. The nominal version of this series sums nominal PCE Nondurable Goods, PCE Services, and the series for nominal services from the durable goods stock. The first two of thse come from NIPA Table 1.1.5. We construct the analogous series for real Model-consistent consumption by combining these series with their real chain-weighted counterparts using the Fisher ideal formula. The two real PCE series come from NIPA Table 1.1.3. Again, this produces an implicit deflator for Model-consistent consumption as a by-product.

4. Model-consistent Government Purchases. Conceptually, the model's measure of Government Purchases includes all expenditures not otherwise classified as Investment or Consumption: Inventory Investment, Net Exports, and actual Government Purchases. We construct the nominal version of this series simply

by subtracting nominal Model-consistent Investment and Consumption from nominal Model-consistent GDP. We calculate the analogous real series using "chain subtraction." This applies the Fisher ideal formula to Model-consistent GDP and the *negatives* of Model-consistent Consumption and Investment.

Our empirical analysis requires us to compare model-consistent series measured from the NIPA data with their counterparts from the model's solution. To do this, we begin by solving the log-linearized system above, and then we feed the model specific paths for all exogenous shocks starting from a particular initial condition. for a given such simulation, the growth rates of Model-consistent Consumption and Investment equal

$$\begin{aligned} \Delta \ln C_t &= z_* + \Delta \hat{c}_t + z_t \text{ and} \\ \Delta \ln I_t &= z_* + \omega_* + \Delta \hat{i}_t + z_t + \omega_t \end{aligned}$$

The measurement of GDP growth in the model is substantially more complicated, because the variables Y_t and y_t denote model output *in consumption units.* In contrast, we mimic the BEA by using a chain-weighted Fisher ideal index to measure model-consistent GDP. Therefore, we construct an analogus chain-weighted GDP index from model data. Since such an ideal index is invariant to the units with which nominal prices are measured, we can normalize the price of consumption to equal one and employ the prices of investment goods and government purchases relative to current consumption. Our model identifies the first of these relative prices as with investment-specific technology. However, the model characterizes only government purchases *in consumption units*, because private agents do not care about their division into "real" purchases and their relative price. For this reason, we use a simple autoregression to characterize the evolution of the price of government services in consumption units. Denote this price in quarter t with P_t^g. We construct this for the US economy by dividing the Fisher-ideal price index for model-consistent government purchases by that for model-consistent consumption. Then, our model for its evolution is

$$\ln(P_t^g/P_{t-1}^t) = \mu_g + \theta_{gg1} \ln(P_{t-1}^g/P_{t-2}^g) + \theta_{gg2} \ln(P_{t-2}^g/P_{t-3}^g) + \varepsilon_t^{gg}. \tag{50}$$

Here, $\varepsilon_t^{gg} \sim \mathbb{N}(0, \sigma_{gg}^2)$. Given an arbitrary normalization of P_t^g to one for some time period, simulations from (**??**) can be used to construct simulated values of P_t^g for all other time periods. With these and a simulation from the model of all other variables in hand, we can calculate the simulation's values for Fisher ideal GDP growth using

$$\frac{Q_t}{Q_{t-1}} \equiv \sqrt{\dot{Q}_t^P \dot{Q}_t^L}, \tag{51}$$

where the Paasche and Laspeyres indices of quantity growth are

$$\dot{Q}_t^P \equiv \frac{C_t + P_t^I I_t + P_t^G (G_t / P_t^G)}{C_{t-1} + P_t^I I_{t-1} + P_t^G (G_{t-1} / P_{t-1}^g)} \text{ and} \tag{52}$$

$$\dot{Q}_t^L \equiv \frac{C_t + P_{t-1}^I I_t + P_{t-1}^G (G_t / P_t^G)}{C_{t-1} + P_{t-1}^I I_{t-1} + P_{t-1}^G (G_{t-1} / P_{t-1}^G)}. \tag{53}$$

In both (52) and (53), P_t^I is the relative price of investment to consumption. In equilibrium, this always equals A_t^I.

The above gives a complete recipe for *simulating* the growth of model-consistent real GDP growth. However, we also embody its insights into our estimation with a log-linear approximation. For this, we start by removing stochastic trends from all variables in (52) and (53), and we proceed by taking a log-linear approximation of the resulting expression. Details are available from the authors upon request.

6.2 Hours Worked Measurement

Empirical work using DSGE models like our own typically measure labor input with hours worked per capita, constructed directly from BLS measures of hours worked and the civilian non-institutional population over age 16. However, this measure corresponds poorly with business cycle models because it contains underlying low frequency variation. This fact led us to construct a new measure of hours for the model using labor market trends produced for the FRB/US model and for the Chicago Fed's in-house labor market analysis.

We begin with a multiplicative decomposition of hours worked per capita into hours per worker, the employment rate of those in the labor force, and the labor-force participation rate. The BLS provides CPS-based measures of the last two rates for the US as a whole. However, its measure of hours per worker comes from the Establishment Survey and covers only the private business sector. If we use hours per worker in the business sector to approximate hours per worker in the economy as a whole, then we can measure hours per capita as

$$\frac{H_t}{P_t} = \frac{H_t^E}{E_t^E}\frac{E_t^C}{L_t^C}\frac{L_t^C}{P_t^C}.$$

Here, H_t and P_t equal total hours worked and the total population, H_t^E/E_t^E equals hours per worker measured with the Establishment survey, E_t^C/L_t^C equals one minus the CPS based unemployment rate, and L_t^C/P_t^C equals the CPS based labor-force participation rate. Our measure of *model-relevant* hours worked deflates each component on the right-hand side by an exogenously measured trend. The trend for the unemployment rate comes from the Chicago Fed's Microeconomics team, while those for hours per worker and labor-force participation come from the FRB/US model files.

6.3 Inflation

Our empirical analysis compares model predictions of price inflation, wage inflation, inflation in the price of investment goods relative to consumption goods, and inflation expectations with their observed values from the U.S. economy. We describe our implementations of these comparisons sequentially below.

6.3.1 Price Inflation

Our model directly characterizes the inflation rate for Model-consistent Consumption. In principle, this is close to the FOMC's preferred inflation rate, that for the implicit deflator of PCE. However, in practice the match between the two inflation rates is poor. In the data, short-run movements in food and energy prices substantially influences the short-run evolution of PCE inflation. Our model lacks such a volatile sector, so if we ask it to match observed short-run inflation dynamics, it will attribute those to transitory shocks to intermediate goods' producers' desired markups driven by λ_t^p.

To avoid this outcome, we adopt a different strategy for matching model and data inflation rates, which follows that of Justiniano, Primiceri, and Tambalotti (2013). This relates three observable inflation rates – core CPI inflation, core PCE inflation, and market-based PCE inflation – to Model-consistent consumption inflation using auxiliary observation equations. For core PCE inflation, this equation is

$$\pi_t^{p1} = \pi_* + \pi_*^{p1} + \beta_1^{p1}\hat{\pi}_t + \beta_2^{p1}\pi_t^D + \varepsilon_t^{p1} \tag{54}$$

In (54) as elsewhere, π_* equals the long-run inflation rate. The constant π_*^1 is an adjustment to this long-run inflation rate which accounts for possible long-run differences between realized inflation and the FOMC's goal of $\pi_\star$. The right-hand side's inflation rates, $\hat{\pi}_t$ and π_t^D equal Model-consistent consumption inflation and PCE Durables inflation. We refer to the coefficients multiplying them, β_1^{p1} and β_2^{p1}, as the *inflation loadings*. We include PCE Durables inflation on the right-hand side of (54) because the principle adjustment required to transform Model-consistent inflation into core PCE inflation is the replacement of the price index for durable goods services with that for durable goods purchases. The disturbance term ε_t^{p1} follows a first-order autoregression with autocorrelation φ_{p1} and normally distributed innovations with mean zero and standard deviation σ_{p1}.

The other two observed inflation measures, market-based PCE inflation and core CPI inflation, have identically specified observation equations. We use $p2$ and $p3$ in superscripts to denote these equations parameters and error terms, and we use the same expressions as subscripts to denote the parameters governing the evolution of their error terms. We assume that the error terms ε_t^{p1}, ε_t^{p2}, and ε_t^{p3} are independent of each other at all leads and lags.

To produce forecasts of inflation with these these three observation equations, we must forecast their right-hand side variables. The model itself gives forecasts of $\hat{\pi}_t$. The forecasts of durable goods inflation come from a second-order autoregression.

$$\pi_t^D = \theta_0^D + \theta_1^D\pi_{t-1}^D + \varepsilon_t^D \tag{55}$$

Its innovation is normally distributed and serially uncorrelated with standard deviation σ_D.

6.3.2 Wage Inflation

Although observed wage inflation does not feature the same short-run variability as does price inflation, it does include the influences of persistent demographic labor-market trends which we removed ex ante from our measure of hours worked.

Therefore, we follow the same general strategy of relating observed measures of wage inflation to the model's predicted wage inflation with a error-augmented observation equation. For this, we employ two measures of compensation per hour, Earnings per Hour and Total Compensation per Hour. In parallel with our notation for inflation measures, we use $w1$ and $w2$ to denote these two wage measures of wage inflation. The observation equation for Earnings per Hour is

$$\pi_t^{w1} = z_* + \pi_*^{w1} + \beta^{w1}\hat{\pi}_t^{w} + \varepsilon_t^{w1} \tag{56}$$

Just as with the price inflation measurement errors, ε_t^{w1} follows a first-order autoregression with autocorrelation φ_{w1} and innovation standard deviation σ_{w1}. The observation equation for Total Compensation per Hour is analogous to (56).

6.3.3 Relative Price Inflation

To empirically ground investment-specific technological change in the model, we use an error-augmented observation equation to relate the relative price of investment to consumption, both model-consistent measures constructed from NIPA and Fixed Asset tables as described above, with the model's growth rate of the rate of technological transformation between these two goods, ω_t.

$$\pi_t^{C/I} = \omega_t + \varepsilon_t^{C/I}$$

Here, we use the superscript C/I to indicate that the variables characterize the price of Consumption relative to Investment. The measurement error $\varepsilon_t^{C/I}$ follows a first-order autoregression with autocorrelation $\varphi_{C/I}$ and normally-distributed innovations with standard deviation $\sigma_{C/I}$.

6.3.4 Inflation Expectations

We also discipline our model's inferences about the state of the economy by comparing expectations of one-yea and 10-year inflation from the Survey of Professional Forecasters with the analogous expectations from our model. Just as with all of the other inflation measures, we allow these two sets of expectations to differ from each other by including serially correlated measurement errors. The observation equations are

$$\begin{aligned}
\pi_t^{e4} &= \pi_* + \pi_*^{e4} + \frac{1}{4}\sum_{i=1}^{4}\mathbb{E}_t\left[\hat{\pi}_{t+i}\right] + \varepsilon_t^{e4} \\
\pi_t^{e40} &= \pi_* + \pi_*^{e40} + \frac{1}{40}\sum_{i=1}^{40}\mathbb{E}_t\left[\hat{\pi}_{t+i}\right] + \varepsilon_t^{e40}
\end{aligned}$$

The two measurement errors follow mutually-independent first-order autoregressions with autocorrelations φ_{e4} and φ_{e40} and innovation standard deviations σ_{e4} and σ_{e40}.

6.4 Interest Rates and Monetary Policy Shocks

Since our model features forward guidance shocks, it has non-trivial implications for the current policy rate as well as for expected future policy rates. We use two distinct but complementary approaches to disciplining the parameters governing their realizations, the elements of Σ_1, using data. The first method compares the model's monetary policy shocks to high-frequency interest-rate innovations informed by event studies, such as that of Gürkaynak, Sack, and Swanson (2005). Those authors applied a factor structure to innovations in implied expected interest rates from futures prices around FOMC policy announcement dates. Specifically, they show that the vector of M implied interest rate changes following an FOMC policy announcement, Δr, can be written as

$$\Delta r = \Lambda f + \eta$$

Where f is a 2×1 vector of factors, Λ is a $M \times 2$ matrix of factor loadings, and η is an $M \times 1$ vector of mutually independent shocks. Denoting the 2×2 diagonal variance covariance matrix of f with Σ_f and the $M \times M$ diagonal variance-covariance matrix of η with Ψ, we can express the observed variance-covariance matrix of Δr as $\Lambda \Sigma_f \Lambda' + \Psi$.

Our model has implications for this same variance covariance matrix. For this, use the model's solution to express the changes in current and future expected interest rates following monetary policy shocks as $\Delta r = \Gamma_1 \varepsilon^1$. Here, ε^1 is the vector which collects the current monetary policy shock with $M - 1$ forward guidance shocks, and Γ_1 is an $M \times M$ matrix. In general, Γ_1 does *not* simply equal the identity matrix, because current and future inflation and output gaps respond to the monetary policy shocks and thereby influence future monetary policy "indirectly" through the interest rate rule. Given this solution for Δr, we can calculate its variance-covariance matrix as $\Gamma_1 \Sigma_1 \Gamma_1'$. Equating these two expressions and solving for Σ_1 yields

$$\Sigma_1 = \Gamma_1^{-1} \left(\Lambda \Sigma_f \Lambda' + \Psi \right) \Gamma_1'^{-1}.$$

The second approach to disciplining Σ_1 is more traditional: directly compare quarterly observations of the current policy rate and expected future interest rates – from market prices, surveys of market participants, or both – with their implied values from the model given a particular realization of the vector of monetary policy shocks. We use both methods in the estimation procedure described below.

7 Calibration and Bayesian Estimation

As we noted in the introduction, we follow a two-stage approach to the estimation of our model's parameters. In a calibration stage, we set the values of selected parameters so that the model has empirically-sensible implications for long-run averages from the U.S. economy. In this stage, we also enforce several normalizations and a judgemental restriction on one of the measurement error variances. In the second stage, we estimate the model's remaining parameters using standard Bayesian methods.

We employ standard prior distributions, but those governing monetary policy shocks deserve further elaboration. Our estimation requires the variance-covariance matrix of monetary policy shocks to be consistent with the factor-structure of interest rate innovations used by Gürkaynak, Sack, and Swanson (2005), as described above. Therefore, we parameterize Σ_1 in terms of Λ, Σ_f, Ψ, and the model parameters which influence Γ_1. We then center our priors for Λ, Σ_f, and Ψ at their estimates from event-studies. However, we do not require our estimates to equal their prior values. Our Bayesian estimation procedure employs quarterly data on expected future interest rates, the posterior likelihood function includes Λ, Σ_f, and Ψ as free parameters. It is well known that Λ and Σ_f are not separately identified, so we impose two scale normalizations and one rotation normalization on Λ. The rotation normalization requires that the first factor, which we label "Factor A", is the only factor influence the current policy rate. That is, the second factor, "Factor B" influences only future policy rates. Gürkaynak, Sack, and Swanson (2005) call Factors A and B the "target" and "path" factors.

Our estimation's sample period begins in the first quarter of 1993 and ends in the fourth quarter of 2016. Of course, the FOMC substantially changed its operating procedures in the aftermath of its persistent stay at the Zero Lower Bound, so it would be unwise to imagine the data from this entire period being generated from our model with time-invariant parameters. For this reason, we estimate the model twice. For the *first sample*, which runs from 1993Q1 through 2008Q3, we estimate all model parameters while allowing for four quarters of forward guidance. For the *second sample*, we estimate the parameters governing monetary policy shocks allowing for ten quarters of forward guidance, adjust the average rate of Hicks-neutral productivity growth to bring potential GDP growth rate from its first-sample value of 3 percent down to 2 percent, and hold all other model parameters fixed at their first-sample posterior-mode values.

We report the results of our two-stage two-sample estimation in a series of tables. Table 1 reports our most notable calibration targets. The long-run policy rate equals 1.1 percent on a quarterly basis. We target a two percent growth rate of per capita GDP. Given an average population growth rate of one percent per year, this implies that our potential GDP growth rate equals three percent. The other empirical moments we target are a nominal investment to output ratio of 26 percent and nominal government purchases to output ratio of 15 percent. Finally, we target a capital to output ratio of approximately 10 on a quarterly basis.

Table 2 lists the parameters which we calibrate along with their given values. The table includes many more parameters than there are targets in Table 1. This is because Table 1 omitted calibration targets which map one-to-one with particular parameter values. For example, we calibrate the steady-state capital depreciation rate (δ_0) using standard methods applied to data from the Fixed Asset tables. It is also because Table 2 lists several parameters which are normalized prior to estimation. Most notable among these are the three factor loadings listed at the table's bottom.

Tables 3 and 4 report prior distributions and posterior modes for the model's remaining paramters, for the first and second samples respectively.

Table 1: Calibration Targets

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R	1.011
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.005
Investment to Output Ratio	I_t/Y_t	0.260
Capital to Output Ratio	K_t/Y_t	10.763
Fraction of final good output spent on public goods	G_t/Y_t	0.153

Table 2: First Sample Calibrated Parameters

Parameter	Symbol	Value
Discount Factor	β	0.986
Steady-State Measured TFP Growth (quarterly)	z_*	0.489
Investment-Specific Technology Growth Rate	ω_*	0.371
Elasticity of Output w.r.t Capital Services	α	0.401
Steady-State Wage Markup	λ_*^w	1.500
Steady-State Price Markup	λ_*^p	1.500
Steady-State Scale of the Economy	H_*	1.000
Steady-State Inflation Rate (quarterly)	π_*	0.500
Steady-State Depreciation Rate	δ_0	0.016
Steady-State Marginal Depreciation Cost	δ_1	0.039
Nominal Output over Nominal Private Purchases	g_*	0.847
Std. Dev Long-Run Inflation Expectations Measurement Error	σ_{e40}	0.010
Long-Run Inflation Expectations (Constant CPI Adjustment)	π_*^{e40}	0.122
Average Earnings Constant	π_*^{w1}	-0.237
Average Total Compensation Constant	π_*^{w2}	-0.202
Loading Compensation	β_1^{w2}	1.000
Loading Core PCE	β_1^{p1}	1.000
Constant for Relative Price Inflation	π_*^G	0.252
Loading 0 Factor A	$\lambda_{0,1}$	0.981
Loading 0 Factor B	$\lambda_{0,2}$	0.000
Loading 4 Factor B	$\lambda_{4,2}$	0.951

Table 3: First Sample Estimated Parameters

Parameter	Symbol	Density	Prior Mean	Std.Dev	Posterior Mode
Depreciation Curve	$\frac{\delta_2}{\delta_1}$	G	1.0000	0.150	0.499
Active Price Indexation Rate	ι_p	B	0.5000	0.150	0.280
Active Wage Indexation Rate	ι_w	B	0.5000	0.150	0.082
External Habit Weight	λ	B	0.7500	0.025	0.790
Labor Supply Elasticity	γ_H	N	0.6000	0.050	0.591
Price Stickiness Probability	ζ_p	B	0.8000	0.050	0.833
Wage Stickiness Probability	ζ_w	B	0.7500	0.050	0.904
Adjustment Cost of Investment	φ	G	3.0000	0.750	4.326
Elasticity of Intertemporal Substitution	γ_c	N	1.5000	0.375	1.915
Interest Rate Response to Inflation	ψ_1	G	1.7000	0.150	1.833
Interest Rate Response to Output	ψ_2	G	0.2500	0.100	0.488
Interest Rate Smoothing Coefficient	ρ_R	B	0.8000	0.100	0.791
Autoregressive Coefficients of Shocks					
Discount Factor	ρ_b	B	0.5000	0.250	0.850
Inflation Drift	ρ_π	B	0.9900	0.010	0.998
Exogenous Spending	ρ_g	B	0.6000	0.100	0.920
Investment	ρ_i	B	0.5000	0.100	0.759
Liquidity Preference	ρ_s	B	0.6000	0.200	0.841
Price Markup	ρ_{λ_p}	B	0.6000	0.200	0.687
Wage Markup	ρ_{λ_w}	B	0.5000	0.150	0.668
Neutral Technology	ρ_ν	B	0.3000	0.150	0.496
Investment Specific Technology	ρ_ω	B	0.3500	0.100	0.407
Moving Average Coefficients of Shocks					
Price Markup	θ_{λ_p}	B	0.4000	0.200	0.608
Wage Markup	θ_{λ_w}	B	0.4000	0.200	0.306
Standard Deviations of Innovations					
Discount Factor	σ_b	U	0.5000	2.000	1.187
Inflation Drift	σ_π	I	0.0150	0.0075	0.094
Exogenous Spending	σ_g	U	1.0000	2.000	2.500

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	Symbol	Density	Prior Mean	Std.Dev	Posterior Mode
Investment	σ_i	I	0.2000	0.200	0.618
Liquidity Preference	σ_s	U	0.5000	2.000	0.390
Price Markup	σ_{λ_p}	I	0.1000	1.000	0.069
Wage Markup	σ_{λ_w}	I	0.1000	1.000	0.031
Neutral Technology	σ_ν	U	0.5000	0.250	0.504
Investment Specific Technology	σ_ω	I	0.2000	0.100	0.183
Relative Price of Cons to Inv	$\sigma_{\frac{c}{i}}$	I	0.0500	2.000	0.215
Monetary Policy					
Unanticipated	σ_{μ_0}	N	0.0050	0.0025	0.012
1Q Ahead	σ_{μ_1}	N	0.0050	0.0025	0.012
2Q Ahead	σ_{μ_2}	N	0.0050	0.0025	0.007
3Q Ahead	σ_{μ_3}	N	0.0050	0.0025	0.009
4Q Ahead	σ_{μ_4}	N	0.0050	0.0025	0.010
Total Earnings					
Loading 1	β_1^{w1}	N	0.8000	0.100	0.824
Standard Deviation	σ_{w1}	I	0.0500	0.100	0.147
AR(1) Coefficient	φ_{w1}	B	0.4000	0.100	0.624
Total Compensation					
Standard Deviation	σ_{w2}	I	0.0500	0.100	0.169
AR(1) Coefficient	φ_{w2}	B	0.4000	0.100	0.343
Core PCE					
Constant	π_*^{p1}	N	-0.1000	0.100	-0.087
Loading 2	β_2^{p1}	N	0.0000	1.000	0.014
Standard Deviation	σ_{p1}	I	0.0500	0.100	0.048
AR(1) Coefficient	φ_{p1}	B	0.2000	0.100	0.091
Market-Based Core PCE					
Constant	π_*^{p2}	N	-0.1000	0.100	-0.123
Loading 1	β_1^{p2}	N	1.0000	0.100	1.102
Loading 2	β_2^{p2}	N	0.0000	1.000	0.028
Standard Deviation	σ_{p2}	I	0.0500	0.100	0.039

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	Symbol	Density	Prior Mean	Std.Dev	Posterior Mode
AR(1) Coefficient	φ_{p2}	B	0.2000	0.100	0.128
Core CPI					
Constant	π_*^{p3}	N	0.0500	0.100	0.047
Loading 1	β_1^{p3}	N	1.0000	0.100	0.804
Loading 2	β_2^{p3}	N	0.0000	1.000	0.119
Standard Deviation	σ_{p3}	I	0.1000	0.100	0.076
AR(1) Coefficient	φ_{p3}	B	0.4000	0.200	0.597
PCE Durable Goods Inflation					
Constant	θ_*^D	N	-0.3500	0.100	-0.356
1st Lag Coefficient	θ^{D1}	N	0.4500	0.200	0.430
2nd Lag Coefficient	θ^{D2}	N	0.4000	0.200	0.362
Standard Deviation	σ_D	I	0.2000	2.000	0.287
Relative Price Inflation					
1st Lag Coefficient	θ^{G1}	N	0.0000	0.500	0.279
2nd Lag Coefficient	θ^{G2}	N	-0.1000	0.500	0.006
Standard Deviation	σ_G	I	0.5000	2.000	0.811
Factor A					
Loading 1	$\lambda_{1,1}$	N	0.6839	0.200	1.256
Loading 2	$\lambda_{2,1}$	N	0.5224	0.200	0.857
Loading 3	$\lambda_{3,1}$	N	0.4314	0.200	0.361
Loading 4	$\lambda_{4,1}$	N	0.3243	0.200	0.032
Standard Deviation	σ_{F_1}	N	0.1000	0.0750	0.041
Factor B					
Loading 1	$\lambda_{1,2}$	N	0.3310	0.200	0.698
Loading 2	$\lambda_{2,2}$	N	0.6525	0.200	1.162
Loading 3	$\lambda_{3,2}$	N	0.8059	0.200	1.199
Standard Deviation	σ_{F_2}	N	0.1000	0.0750	0.072

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

Table 4: Second Sample Estimated Parameters

Parameter	**Symbol**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
Total Earnings				
Constant	β_*^{w1}	-0.2370	0.200	-0.096
Loading 1	β_1^{w1}	0.8242	0.200	0.252
Standard Deviation	σ_{w1}	0.1468	0.100	0.183
AR(1) Coefficient	φ_{w1}	0.6239	0.200	0.529
Total Compensation				
Constant	β_*^{w2}	-0.2023	0.200	-0.142
Standard Deviation	σ_{w2}	0.1687	0.100	0.233
AR(1) Coefficient	φ_{w2}	0.3430	0.200	0.351
Core PCE				
Loading 2	β_2^{p1}	0.0281	0.100	0.232
Standard Deviation	σ_{p1}	0.0481	0.100	0.143
AR(1) Coefficient	φ_{p1}	0.0913	0.150	0.256
Market PCE				
Constant	π_*^{p2}	-0.1230	0.100	-0.114
Loading 1	β_1^{p2}	1.1022	0.150	0.358
Loading 2	β_2^{p2}	0.0139	0.100	0.219
Standard Deviation	σ_{p2}	0.0755	0.100	0.121
AR(1) Coefficient	φ_{p2}	0.5972	0.150	0.527
CPI				
Constant	π_*^{p3}	0.0475	0.100	-0.022
Loading 1	β_1^{p3}	0.8039	0.150	0.305
Loading 2	β_2^{p3}	0.1192	0.100	0.206
Standard Deviation	σ_{p3}	0.0388	0.100	0.085
AR(1) Coefficient	φ_{p3}	0.1278	0.150	0.220
Durable Goods Inflation				
Constant	π_*^{D}	-0.4500	0.200	-0.463
Standard Deviation	σ_D	0.5000	0.150	0.291
Relative Price Inflation				

Second Sample Estimated Parameters (Continued)

Parameter	Symbol	Prior Mean	Std.Dev	Posterior Mode
Constant	π_*^G	0.8900	0.400	-0.146
Standard Deviation	σ_G	0.8143	0.080	1.023
Factor A				
Loading 0	$\lambda_{0,1}$	0.0180	0.250	0.158
Loading 1	$\lambda_{1,1}$	0.0574	0.250	0.356
Loading 2	$\lambda_{2,1}$	0.1941	0.250	0.496
Loading 3	$\lambda_{3,1}$	0.3996	0.250	0.623
Loading 4	$\lambda_{4,1}$	0.6520	0.250	0.820
Loading 6	$\lambda_{6,1}$	1.2266	0.250	0.990
Loading 7	$\lambda_{7,1}$	1.5237	0.250	1.122
Loading 8	$\lambda_{8,1}$	1.8139	0.250	1.102
Loading 9	$\lambda_{9,1}$	2.0914	0.250	1.087
Loading 10	$\lambda_{10,1}$	2.3523	0.250	2.515
Standard Deviation	σ_{F_1}	0.0442	0.100	0.072
Factor B				
Loading 0	$\lambda_{0,2}$	-0.0181	0.300	0.029
Loading 1	$\lambda_{1,2}$	0.2211	0.300	0.039
Loading 2	$\lambda_{2,2}$	0.3679	0.300	0.070
Loading 3	$\lambda_{3,2}$	0.4424	0.300	0.095
Loading 4	$\lambda_{4,2}$	0.4612	0.300	0.123
Loading 5	$\lambda_{5,2}$	0.4370	0.300	0.138
Loading 6	$\lambda_{6,2}$	0.3817	0.300	0.167
Loading 7	$\lambda_{7,2}$	0.3032	0.300	0.184
Loading 9	$\lambda_{9,2}$	0.1074	0.300	0.229
Standard Deviation	σ_{F_2}	0.0334	0.100	0.429
Standard Deviations of Monetary Policy Innovations				
Unanticipated	σ_{μ_0}	0.0061	0.005	0.011
1Q Ahead	σ_{μ_1}	0.0021	0.005	0.010
2Q Ahead	σ_{μ_2}	0.0004	0.005	0.010
3Q Ahead	σ_{μ_3}	0.0019	0.005	0.009

Second Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
4Q Ahead	σ_{μ_4}	0.0001	0.005	0.010
5Q Ahead	σ_{μ_5}	0.0025	0.005	0.010
6Q Ahead	σ_{μ_6}	0.0019	0.005	0.010
7Q Ahead	σ_{μ_7}	0.0011	0.005	0.010
8Q Ahead	σ_{μ_8}	0.0001	0.005	0.009
9Q Ahead	σ_{μ_9}	0.0014	0.005	0.010
10Q Ahead	$\sigma_{\mu_{10}}$	0.0028	0.005	0.010

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