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To: William B. English

From: Canlin Li and Min Wei

Subject: Term Structure Modeling with Supply Factors and an Application to Maturity Extension Program Evaluation

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This memo summarizes recent staff analysis of term structure models with supply factors and then applies the methodology to evaluate the likely effects on Treasury yields of a maturity extension program of the form discussed in a recent staff memo.[1]

Motivation and Model Setup

The standard term structure literature gives little scope for the relative supplies of financial assets to influence the determination of Treasury yields. In contrast, in Vayanos and Vila (2009),[2] a role for supply factors arises from the existence of so-called "preferred-habitat" investors who concentrate their securities investments in a particular maturity range. Real-world examples include long-term investors, such as pension funds and insurance companies, that prefer to hold long-term bonds to match their long-duration liabilities, and short-term investors, such as money market mutual funds and foreign reserve managers, that prefer to hold Treasury bills and short-dated notes to maintain a high degree of liquidity in their portfolio. The existence of risk-averse arbitrageurs with limited capital, who have no maturity preference and actively trade to take advantage of existing arbitrage opportunities, ensures a smooth yield curve. However, this framework with preferred habitat investors and constraints on arbitrage implies that variations in the relative quantities of assets can affect the risk premiums that

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[1] See Bowman, Kiley, Levin, Meyer, Nelson, and Reifschneider (2011).
[2] See Vayanos and Vila (2009).

arbitrageurs demand for absorbing shocks to the size and composition of financial assets held by the private sector.

We are interested in applying the basic rationale underlying this model to assess the extent to which changes in the supply of privately-held Treasury securities would affect the risk premiums that arbitrageurs demand for bearing risk. Vayanos and Vila (2009) show that, under certain parameterizations, the yield impact of variations in relative supplies depends on the dollar duration of the purchases made by the arbitrageurs. Our first model (**Model 1A**) therefore regresses the 10-year term premium implied by the staff's three-factor nominal term structure model on the amount of Treasury debt in the hands of the private sector, measured in terms of 10-year equivalents[3] and normalized by nominal GDP, as well as other macroeconomic variables that can be expected to affect bond yields.[4,5] Our second model (**Model 1B**) replaces the ten-year equivalents variable with the average maturity and the par amount of privately-held Treasury debt (normalized by nominal GDP) and allows these variables to enter separately.[6] Our third model (**Model 1C**) retains the average maturity variable but replaces the par debt-to-GDP ratio in Model 1B by with ten-year equivalent debt-to-GDP ratio. The regression results, shown in Table 1, suggest that term premiums are significantly and positively related to the average maturity of the total stock of privately held Treasury debt and to both measures of the magnitude of Treasury debt supply—the par amount or ten-year equivalents—after controlling for other economic factors in most cases.

[3] The ten-year equivalents of a fixed-income portfolio are calculated as the par amount of on-the-run 10-year Treasury notes that would have the same par value times duration as the portfolio under consideration. In mathematical terms, ten-year equivalents = par value of portfolio * average portfolio duration / duration of ten-year on-the-run Treasury note.

[4] The other variables include capacity utilization, one-year-ahead CPI inflation forecast from Blue Chip Economic Indicators survey, implied volatilities from options on ten-year Treasury note futures, implied volatilities from options on the S&P 100 index, and foreign custody holdings of Treasury securities at FRBNY.

[5] This regression is also examined by D'Amico, King, Li, Stebunovs, and Wei (2010).

[6] We choose to use average maturity rather than average duration to be comparable to Hamilton and Wu (2011)'s analysis and to avoid the problem that average duration would respond to yield changes even if the maturity composition of Treasury debt outstanding remained the same.

Table 1: OLS regression results of the 10-year term premium (Term premium in percentage point; March 1994-July 2007)

	Model 1A	Model 1B	Model 1C
Constant	1.503 (1.531)	–1.419 (1.647)	–1.220 (1.690)
Ten year imp. Vol_{t-1}	0.144*** (0.027)	0.141*** (0.026)	0.156*** (0.026)
SP100 imp. Vol_t	–0.009*** (0.004)	–0.016*** (0.004)	–0.016*** (0.004)
Foreign holdings	–0.046*** (0.020)	0.037 (0.025)	0.011 (0.026)
Capacity utilization	–0.019 (0.021)	–0.056** (0.021)	–0.031 (0.021)
Blue chip CPI forecast	0.377*** (0.130)	0.450*** (0.121)	0.527*** (0.134)
10-year equiv.-to-GDP ratio	0.022 (0.031)		0.062* (0.032)
Par debt-to-GDP ratio		0.055*** (0.016)	
Average Maturity		0.558*** (0.124)	0.319*** (0.095)
Number of Obs	161	161	161
Adjusted R^2	67.79%	71.29%	69.80%

Newey-West standard error (12 lags) in parentheses.

Models 1A-1C are simple reduced form regressions. To incorporate supply variables into a formal term structure model, we estimate three no-arbitrage preferred-habitat term structure models with two yield factors—a level factor measured by the five-year Treasury yield and a slope factor measured by the spread between the five-year and the three-month Treasury yields—and one or two supply factors.[7,8,9] Following the structure for the simple regression models 1A-1C, the supply factors are specified as the ten-year equivalent debt-to-GDP ratio alone (**Model 2A**), the average maturity and the par debt-to-GDP ratio (**Model 2B**), and the average maturity and the ten-year equivalent debt-to-GDP ratio (**Model 2C**). We assume that the factors, f_t, follow a block-diagonal first-order VAR, in which yield factors and supply factors only load on their own lags, and that the nominal prices of risk are affine functions of all factors.

$$f_{t+1} = c + \rho f_t + \Sigma \varepsilon_{t+1}, \tag{1}$$

We also impose the restriction that the short-term rate loads only on the two yield factors, so that the two supply factors do not carry their *own* risk premiums but affect bond yields through their effect on interest rate risk premiums only. It is straightforward to derive mappings from the four observable factors to bond yields of various maturities:

$$y_t^n = a_n + b_n f_t. \tag{2}$$

We estimate model parameters by minimizing yield fitting errors at maturities of 6, 12, 24, 72 and 120 months over the sample of March 1994 to July 2007. The table below reports the model-implied loadings of yields on the two supply factors. Yields are reported in percentage terms.

These estimates suggest that the supply factors act like slope factors. For example, Model 2A implies that a one percentage point increase in the ten-year equivalent debt-to-GDP ratio would raise the 10-year yield by about 6 basis points but reduce the 6-month yield by about ½ of a basis point. Results from Model 2B suggest that average maturity has a strong steepening effect on the yield curve, with a one-year increase in the average maturity of privately-held Treasury debt raising the ten-year Treasury yield by about 30 basis points and lowering the six-month yield by about 6 basis points; a one percentage point increase in the par Treasury debt-to-GDP ratio, on the

[7] The term structure literature shows that the level and the slope factors are the most important for explaining the cross section of yields. In our sample, the two yield factors alone explain about 98 percent of yield variations.
[8] The yield factors are included to capture other economic forces that drive Treasury yields.
[9] A similar model is studied in Li and Wei (2011).

other hand, would raise the 10-year yield by about 3 basis points but have a small negative effect on the 6-month yield. The estimated effects of changes in average maturity are slightly smaller in Model 2C than in Model 2B, as the 10-year equivalent debt-to-GDP ratio used in Model 2C also captures some of the maturity effects.

Table 2. Estimated Term Premium Loadings on Supply Factors (basis points)

	Model 2A	**Model 2B**		**Model 2C**	
Maturity (Years)	10y Equiv. Debt-to-GDP (%)	Average Maturity (years)	Par Debt-to-GDP (%)	Average Maturity (years)	10y Equiv. Debt-to-GDP (%)
0.5	−0.52	−5.81	−0.26	−4.97	−0.28
1	−1.24	−14.33	−0.64	−12.29	−0.68
2	−1.66	−21.46	−0.90	−18.61	−0.93
5	1.00	−3.65	0.32	−5.10	0.82
7	3.35	13.92	1.50	8.78	2.58
10	5.81	30.33	2.84	21.63	4.85

Evaluating the A Hypothetical Maturity Extension Program[10]

The maturity extension program discussed in Bowman, Kiley, Levin, Meyer, Nelson, and Reifschneider (2011) is aimed at "extend[ing] the duration of the SOMA portfolio without changing its size by selling some or all of the debt securities now held in the SOMA that have a fairly short remaining time to maturity and buying assets with a long time to maturity, thus reducing the average duration of the private sector's holdings of Treasury securities."

Under the hypothetical maturity extension program discussed in the Bowman et. al. memo, the Desk would sell $400 billion of debt securities in the SOMA portfolio that have remaining maturities of 3 years or less, and buy the same par amount of Treasury securities with remaining maturities of 8 years or more (while continuing to reinvest principal from MBS and maturing debt securities in longer-term Treasuries). By

[10] This hypothetical maturity extension program differs somewhat from the program discussed in the September 12, 2011 staff memo to the FOMC by Carpenter, Castelo, Clouse, Ihrig, Klee, Li, Miller, Morse, Quinn, Tallarini, and Wei from the Board and Burke, Ezer, Frost, Gooriah, Hilton, Liu, McGowan, Moore, Stowe, and Remache from the Federal Reserve Bank of New York.

comparison, LSAP2 involved the outright purchase of $600 billion of Treasury securities, with the bulk of purchases spread across maturities between 2 and 10 years. The illustrative program would increase the average duration of the SOMA debt portfolio by 2.4 years (from 4.85 to 7.25 years) and reduce the average duration of privately held Treasury debt outstanding by about 0.6 year (from 3.8 to 3.2 years). In contrast, the LSAP2 purchases had a much smaller effect on the average duration of privately-held Treasury debt outstanding, reducing it by 0.1 year.

Table 3. Estimated Yield Effects of Maturity Extension Program

Panel A Results from Regression Models

Maturity (Years)	Model 1A	Model 1B	Model 1C
2	9.78	-32.13	-33.77
5	4.83	-39.07	-42.63
7	-1.67	-38.07	-43.42
10	-8.48	-35.30	-42.34

Par B. Results from Term Structure Models

Maturity (Years)	Model 2A	Model 2B	Model 2C
2	6.29	13.05	14.50
5	-3.79	2.02	-0.10
7	-12.70	-8.85	-14.96
10	-22.02	-19.11	-31.14

Panel C. Summary

Maturity (Years)	min	max	median	average
2	14.50	-33.77	8.04	-3.71
5	4.83	-42.63	-1.94	-13.12
7	-1.67	-43.42	-13.83	-19.94
10	-8.48	-42.34	-26.58	-26.40

Table 3 summarizes the estimated yield effects of this assumed maturity extension program from all six models (1A-1C) and (2A-2C), while Table 4 reports the results for the LSAP2 program for comparison. A comparison between the two tables shows that, as reported in Bowman et. al. (2011), the maturity-extension program would reduce the 10-year Treasury yield by 10 to 40 basis points, roughly the same amount as the estimated effects of the LSAP2 program. The results on the short end of the yield curve are much more uncertain, with the regression-based models suggesting that short-term yields will decline almost as much as the long-term yields, while the term structure models suggest that the maturity extension program would flatten the yield curve and might lead the two-year yield to rise up to 15 basis points.[11]

Table 4. Estimated Yield Effects of the LSAP2 Program

Panel A Results from Regression Models

Maturity (Years)	Model 1A	Model 1B	Model 1C
2	7.85	-20.29	-13.63
5	3.88	-29.66	-19.53
7	-1.34	-33.11	-21.93
10	-6.80	-35.26	-23.50

Par B. Results from Term Structure Models

Maturity (Years)	Model 2A	Model 2B	Model 2C
2	5.05	7.53	5.53
5	-3.04	-1.04	-1.75
7	-10.19	-9.38	-9.12
10	-17.67	-18.34	-17.89

Panel C. Summary

Maturity (Years)	min	max	median	average
2	7.85	-20.29	5.29	-1.32
5	3.88	-29.66	-2.40	-8.53
7	-1.34	-33.11	-9.78	-14.18
10	-6.80	-35.26	-18.12	-19.91

[11] However, it is arguable that, in an environment in which expectations about near-term funds rate policy are well anchored, there is little room for near-term Treasury yields to rise.

Extension to Incorporate Series of Shocks

The analysis so far considers one-period shocks to the supply variables with the implicit assumption that following the shocks, those supply variables will resume their evolution over time according to their historical dynamics. In contrast, the asset purchase programs completed or under consideration are usually implemented over a period of time, resulting in predictable changes to both the levels and the dynamics of the supply factors during and after the purchases.

This observation motivates us to model those programs as generating a series of shocks to the supply variables, which become known to the investors once the programs are announced:

$$\tilde{f}_t = \{f_t^y, \tilde{f}_t^s\}, \qquad \tilde{f}_t^s = f_t^s + u_t, \tag{3}$$

To measure these shocks, we start by forming projections for total marketable Treasury debt outstanding and SOMA Treasury holdings under a baseline scenario with no purchases and under the purchase program. Based on these projections, we can calculate the par or ten-year equivalent debt-to-GDP ratio and the average maturity of privately held Treasury securities under both scenarios. Finally, the supply shocks are measured as the differences between estimates of the supply variables under the alternative scenario and under the baseline scenario. As shown in Panels A and B of Chart 1, the maturity extension program is expected to reduce the average maturity and the par debt-to-GDP ratio of private Treasury holdings by up to 0.8 year and 2.6 percentage points, respectively, compared to the baseline scenario. In comparison, the LSAP2 program is estimated to have reduced the average maturity and the par debt-to-GDP ratio by up to 0.14 years and 4 percentage points, respectively,

We extend the no-arbitrage term structure model to incorporate these shocks as follows. By assumption, both options considered in the memo eventually cause Treasury holdings by the private sector to return to their baseline path, so that supply shocks disappear and bond yields follow the standard formula (2) at a sufficiently distant horizon T. Bond yields for period T–1 can then be derived from the basic pricing relation that the price of an n-period bond today should equal the discounted, risk-adjusted expected value of the price of an (n–1)-period bond next period. Applying this pricing equation recursively backward allows one to derive the bond pricing formulae in an economy with deterministic supply shocks.

$$\tilde{y}_t^n = \tilde{a}_{n,t} + b_n \tilde{f}_t, \tag{4}$$

Finally, we measure the effects of the two programs on long-term interest rates as the difference between the bond yields implied by Equation (4) and those implied by the standard formula (2):

$$\tilde{y}_t^n - y_t^n = b_n^s u_t + \sum_{i=1}^{T-t} \frac{n-i}{n} b_{n-i}^s \left(u_{t+i} - \rho^{ss} u_{t+i-1}\right), \tag{5}$$

where b_n^s denotes the loadings of yields on supply factors and ρ^{ss} denotes the autoregressive matrix of the supply variables. This measure captures the cumulative effect of supply shocks on yields over the life of the bond.

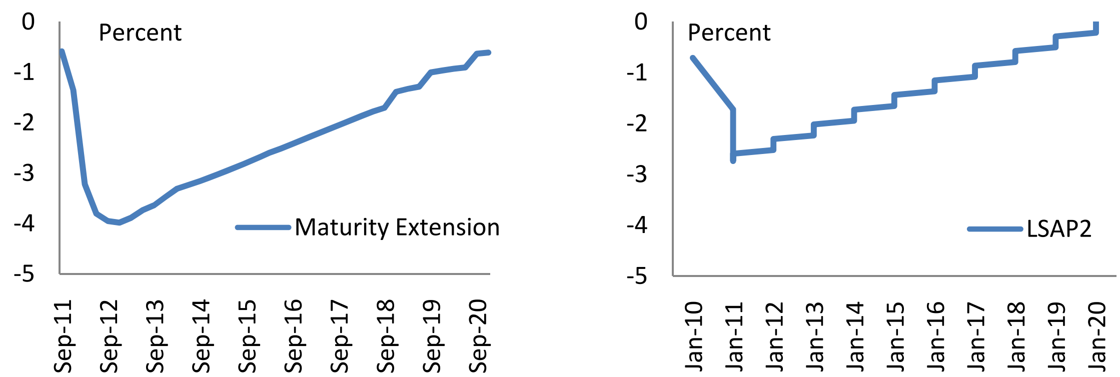
The results from extending Model 2A using the methodology outlined above are reported in Chart 1. Panel A shows the path of the shocks to the ten-year equivalent debt-to-GDP ratio under the maturity extension and the LSAP2 program. Panel B shows that the maturity extension programs is estimated to reduce ten-year Treasury yields by about 25 basis points upon their announcement; these effects are expected to dissipate over time to about 5 basis points by the end of the projection period.[12] The LSAP2 program, on the other hand, is estimated to have lowered the 10-year Treasury yield by about 12 basis points when it was first announced. This estimate is about half of that reported in the October 2010 Tealbook, which was based on a model in which yields are affected by the dollar amount but not the duration of private Treasury coupon holdings. This estimated effect is also about half of that reported for the maturity extension program. Both observations can be explained by the fact that the amount and duration of purchases jointly determines the interest rate effects in the models developed here. The LSAP2 purchases had an average maturity close to that of outstanding Treasuries, resulting in a

[12] By the end of the projection period in 2020, the paths have not yet converged. We assume that they would do so in a linear fashion in the years following the end of the projection period.

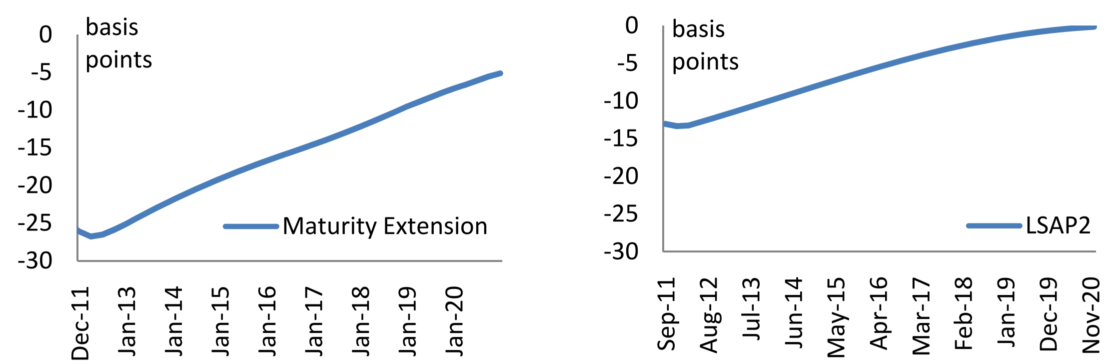
smaller reduction in the ten-year equivalents of private Treasury holdings than the proposed maturity extension program. Finally, the estimated maximum effects of both programs shown in Chart 1 are slightly smaller than those reported for Model 2A in Tables 3 and 4, once we taken into account the fact that the shocks dissipate over time at a faster pace than suggested by the historical dynamics of the supply variables.

Chart 1. Supply Shocks and Effects on Ten-Year Treasury Yields

Panel A. Shocks to ten-year equivalent debt/GDP



Panel B. Term Premium Effect



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Appendix: A Comparison with Hamilton and Wu (2011)

Hamilton and Wu (2011) also analyze the yield effects of a maturity swap exercise, under which the Federal Reserve would sell all of its holdings of Treasury securities of less than one-year maturity and use the proceeds to purchase Treasury securities with the longest available maturities. They estimated that in the pre-2007 environment and ignoring the zero lower bound, such an exercise would amount to a

$400 billion asset swap that would retire all privately-held Treasury debt outstanding of maturities of more than 10 years. They estimate that such a swap would reduce 10-year Treasury yield by 14 basis points and raise the six-month Treasury yield by 11 basis points. They also estimated that in the post-2007 environment and taking into account the zero lower bound, the asset swap would depress long-term yields by the same amount but without producing any rise in short-term yields.

Our methodology and results differ from those in the Hamilton and Wu paper in two important aspects. First, Hamilton and Wu (2011) do not impose restrictions from the underlying preferred-habitat term structure model in their estimation. As a result, their analysis is essentially based on a regression of the slope of the yield curve, defined as the difference between the 10-year and the 6-month Treasury yields, on the average maturity of privately-held Treasury debt outstanding. In contrast, we estimate Models 2A to 2C in manners that respect the restrictions of the model. Second, Hamilton and Wu (2011) ignore any effect on yields arising from changes in the overall supply of Treasury debt, whereas our analysis takes this effect into account. The maturity swap exercises will reduce the 10-year equivalent debt-to-GDP ratio as well as the average duration of privately held debt, the former of which would exert additional downward pressure on yields.