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# Housing, House Prices, and the Equity Premium Puzzle

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**Abstract**

Many recent papers have claimed that when housing services are treated separately from other forms of consumption in utility, a wide range of economic puzzles such as the equity premium puzzle can be explained. Our paper challenges these claims. The key assumption embedded in this literature is that households are not very willing to substitute housing services for consumption. We show that housing services and consumption must be much more substitutable than has been assumed for a neoclassical consumption model to be consistent with U.S. house price data. Further, when forced to match both historical house prices and stock returns, the lowest risk-free rate the model can generate is 11 percent.

Keywords: House Prices, Housing, Equity Premium

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# 1 Introduction

An increasingly popular technique used to explain various econonomic and financial puzzles has been to treat housing services separately from all other consumption in utility. This modeling approach has been adopted by Lustig and Van Nieuwerburgh (LVN) (2004) and Piazzesi et. al. (2004) in attempts to resolve the equity premium puzzle. Other authors have used this framework to explain wealth and expenditure puzzles (Gruber and Martin (2004) and Fernandez-Villaverde and Krueger (2001)) and time-series data on consumption (Fratantoni (2001) and Martin (2002)). These papers take as given that housing services and consumption are quite complementary. That is, if per-period utility of consumption (excluding the consumption of housing services) $c_t$ and housing services $h_t$ is equal to

$$\frac{\left(\gamma c_t^{\alpha} + (1-\gamma) h_t^{\alpha}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma}, \tag{1}$$

then these authors assume $\alpha$ is 0 or less.[1]

To understand how this specification of utility may help resolve the equity premium puzzle, recall the standard consumption asset pricing formula: $1 = E\left[m_{t+1}^{*} R_{t+1}\right]$ with $m_{t+1}^{*} = \beta \left(\frac{c_{t+1}^{*}}{c_t^{*}}\right)^{-\sigma}$, where $R_{t+1}$ denotes the return on stocks and $c_t^{*}$ denotes consumption including the consumption of housing services. The equity premium puzzle arises from the fact that $c_t^{*}$ does not co-vary sufficiently with $R_{t+1}$ to support low risk free rates and price historical equity returns.

In comparison, the utility function in (1) yields the following asset pricing formula

$$1 = E\left[\beta \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \left\{ \left(\frac{\gamma + (1-\gamma)\left(h_{t+1}/c_{t+1}\right)^{\alpha}}{\gamma + (1-\gamma)\left(h_t/c_t\right)^{\alpha}}\right)^{\frac{1-\sigma-\alpha}{\alpha}} \right\} R_{t+1}\right]. \tag{2}$$

The term in braces allows the relative growth rates of consumption and housing services to affect the pricing kernel. Using this framework, Piazzesi et. al. show the equity premium puzzle is resolved if consumption and housing services are sufficiently complementary. The recent papers explaining wealth expenditure and

[1]For example, LVN assume that $\alpha = -5$ whereas Piazzesi et. al. assume it equals 0.05.

consumption puzzles rely on similar mechanisms and depend on nontrivial complementarity between consumption and housing.

In this paper, we estimate bounds for the value of the intra-temporal elasticity of substitution between housing services and consumption using moment conditions implied by a neoclassical consumption model, the same model used in Piazzesi et. al. To be consistent with U.S. housing stock and price data, we show $\alpha$ must be greater than 0.2, implying an intra-temporal elasticity of substitution no less than 1.25. The intuition of this result is simple; given estimated average growth rates of housing services and consumption, house prices have not grown fast enough over the past 35 years to support a smaller elasticity of substitution.

Further, when we force the model to be consistent with both historical housing prices and stock returns, we find $\alpha$ must be larger than 0.55, implying an elasticity of substitution at least 2.2. This estimate is larger than found in previous empirical studies.[2] These other studies use different models and different data sources and therefore their estimates are not directly comparable to ours.

Finally, we cannot find any parameters that allow the model to simultaneously match housing prices, stock returns, and real Treasury returns. Our parameter estimates imply a real risk-free rate of at least 11 percent per-year. We conclude that this framework, without further modification, is insufficient to resolve the equity premium puzzle and, we conjecture, the other macro puzzles mentioned above.

## 2 Model

We study an economy with a single representative agent that has the following per-period utility:

$$U(c_t, h_t) = \frac{\left(\gamma c_t^{\alpha} + (1-\gamma) h_t^{\alpha}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma}. \tag{3}$$

[2] Ogaki and Reinhart (1998) estimate an intra-temporal elasticity of substitution between consumption and durable goods of 1.17; Rupert et. al. (1995) estimate an elasticity of substitution between market consumption and home consumption of 1.0 (single males) and 1.8 (single females); and, McGratten et. al. (1997) estimate an elasticity of substitution between home and market consumption of about 1.75.

$h_t$ denotes housing services and $c_t$ denotes consumption exclusive of housing services, hereafter called simply consumption. The parameter $\gamma \in (0,1)$ weights housing services and consumption in utility, $\sigma \in (1,\infty)$ measures the degree of relative risk aversion, and $\alpha \in (-\infty,1)$ captures the intra-temporal elasticity of substitution between housing services and consumption. Preferences are Cobb-Douglas in the limit as $\alpha \to 0$, $lim_{\alpha\to 0} U(c_t,h_t) \approx \frac{\left(c_t^{\gamma} h_t^{1-\gamma}\right)^{1-\sigma}}{1-\sigma}$, and separable when $\alpha = 1-\sigma$, $U(c_t,h_t)_{\alpha=1-\sigma} = \frac{\gamma c_t^{1-\sigma}}{1-\sigma} + \frac{(1-\gamma)h_t^{1-\sigma}}{1-\sigma}$.

We assume that real housing services are always proportional to the real stock of housing, that is

$$h_t = \kappa H_t, \tag{4}$$

where $H_t$ is the real stock of housing and $\kappa$ maps the stock of housing into housing services. With this assumption, we can replace housing services with the housing stock in utility given, as we will show, we appropriately redefine $\gamma$. This assumption is not without loss of generality, and in our concluding section we discuss some alternatives.

The representative agent maximizes his discounted expected remaining lifetime utility,

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, H_t), \tag{5}$$

where $\beta$ is the discount factor, by choosing sequences of $c_t$, $H_t$, $\mathbf{A}_{t+1}$ (consumption of nondurable goods, housing, and a vector[3] of financial assets, one of which may be risk free) subject to the following per-period budget constraint:

$$c_t + p_t H_t + \mathbf{A}_{t+1} = \mathbf{R}_t \mathbf{A}_t + p_t H_{t-1}. \tag{6}$$

$p_t$ denotes the price of one unit of housing in units of consumption and $\mathbf{R_t}$ is a vector of total returns on the financial assets, including both capital gains and dividends. The expectation in (5) is taken over the future sequences of $\{c_t, H_t, \mathbf{R_t}\}_{t=0}^{\infty}$.

Denote $\lambda_t$ as the Lagrange multiplier on the budget constraint at period $t$. Assuming interior solutions,

[3] We denote vectors in bold.

the model implies the following set of first-order conditions,

$$c_t \quad : \quad U_c\left(c_t, H_t\right) - \lambda_t = 0 \tag{7}$$

$$H_t \quad : \quad U_H\left(c_t, H_t\right) - p_t\lambda_t + \beta E_t p_{t+1}\lambda_{t+1} = 0 \tag{8}$$

$$A_{t+1} \quad : \quad -\lambda_t + \beta E_t \mathbf{R}_{t+1}\lambda_{t+1} = 0 \tag{9}$$

Inserting analytic expressions for marginal utilities and indexing the financial assets in $\mathbf{R}_{t+1}$ by $i = 1, \ldots, I$ yields $I + 1$ Euler equations:

$$\begin{array}{rcl} 1 - \frac{1}{p_t}\frac{1-\gamma}{\gamma}\left(\frac{H_t}{c_t}\right)^{\alpha-1} - E_t\left[m_{t+1}\frac{p_{t+1}}{p_t}\right] & = & 0 \\ 1 - E_t\left[m_{t+1}R^i_{t+1}\right] & = & 0, \end{array} \tag{10}$$

where the pricing kernel, $m_{t+1}$, is defined as

$$m_{t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left[\frac{\gamma + (1-\gamma)\left(H_{t+1}/c_{t+1}\right)^{\alpha}}{\gamma + (1-\gamma)\left(H_t/c_t\right)^{\alpha}}\right]^{\frac{1-\sigma-\alpha}{\alpha}} \tag{11}$$

In the housing Euler equation, we assume the dividend, $\frac{U_H(c_t,H_t)}{U_c(c_t,H_t)} = \frac{1-\gamma}{\gamma}\left(H_t/c_t\right)^{\alpha-1}$, is paid in the current period and thus separate the dividend from its capital gain, unlike our treatment of financial assets.

Of course, the expectations in (10) are not observed. In order to estimate the parameters of this model, we specify that

$$E_t\left[m_{t+1}\frac{p_{t+1}}{p_t}\right] = m_{t+1}\frac{p_{t+1}}{p_t} + \epsilon^h_{t+1} \tag{12}$$

$$E_t\left[m_{t+1}R^i_{t+1}\right] = m_{t+1}R^i_{t+1} + \epsilon^i_{t+1} \tag{13}$$

and assume that the ex-post housing errors $\epsilon^h_{t+1}$ and ex-post vector of financial errors that are indexed by $i$, $\epsilon^i_{t+1}$, are not forecastable using any information dated at time $t$ or before. We rewrite (10) as

$$\epsilon^h_{t+1} = 1 - \frac{1}{p_t}\frac{1-\gamma}{\gamma}\left(\frac{H_t}{c_t}\right)^{\alpha-1} - m_{t+1}\frac{p_{t+1}}{p_t} \tag{14}$$

and

$$\epsilon^i_{t+1} = 1 - m_{t+1}R^i_{t+1}, \tag{15}$$

and use moment conditions involving $\epsilon_{t+1}^{h}$ and $\epsilon_{t+1}^{i}$ to estimate the parameters of the model.

Note that we are not able to identify $\kappa$. To understand why, suppose we rescale $c_t$ and $H_t$ by $k_c$ and $k_h$ respectively. This transforms the utility function as follows:

$$\frac{\left(\gamma\left(k_c c_t\right)^{\alpha}+(1-\gamma)\left(k_h H_t\right)^{\alpha}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma} = \tag{16}$$

$$\left(\gamma k_c^{\alpha}+(1-\gamma)\, k_h^{\alpha}\right)^{\frac{1-\sigma}{\alpha}} \frac{\left(\frac{\gamma k_c^{\alpha}}{\gamma k_c^{\alpha}+(1-\gamma) k_h^{\alpha}}\left(c_t\right)^{\alpha}+\frac{(1-\gamma) k_h^{\alpha}}{\gamma k_c^{\alpha}+(1-\gamma) k_h^{\alpha}}\left(H_t\right)^{\alpha}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma} = \tag{17}$$

$$A \frac{\left(\hat{\gamma}\left(c_t\right)^{\alpha}+(1-\hat{\gamma})\left(H_t\right)^{\alpha}\right)^{\frac{1-\sigma}{\alpha}}}{1-\sigma} . \tag{18}$$

Equations (14) and (15) are invariant to different values for $A$, and the rescaling of $c_t$ and $H_t$ has no effect on the solution except to affect our estimate of $\gamma$.

Finally, note that average value of $\epsilon_{t+1}^{h}$ can always mechanically be set to zero because we observe a price index for housing and not a price level. To see this, define the price of houses as $\delta p_t^h$, where $p_t^h$ is our observed price index for housing and $\delta$ is the parameter that links our price index to the true price level. Given the other parameters of the model ($\beta$, $\sigma$, $\alpha$, and $\gamma$), the value of $\delta$ that sets the average value of $\epsilon_{t+1}^{h}$ to zero satisfies

$$\frac{1}{\delta} = \frac{1-\frac{1}{T-1}\sum_{t=1}^{T-1}\left(m_{t+1}\frac{p_{t+1}^h}{p_t^h}\right)}{\frac{1}{T-1}\sum_{t=1}^{T-1}\frac{1-\gamma}{\gamma}\left(H_t/c_t\right)^{\alpha-1}\left(1/p_t^h\right)} \tag{19}$$

where $T$ is the last period of data and $m_{t+1}$ is defined as in equation (11). In the parameter estimates that we subsequently report, $\delta$ is always set such that (19) is satisfied.

# 3 Data

Our consumption and housing data are expressed as real per-capita quantities. Our estimates of the population for the entire United States are taken from the Regional Economic Accounts, produced by the Bureau of Economic Analysis (BEA).[4] From 1970 to 2004, the US population has grown at approximately 1 percent per year.

*Consumption.* To measure consumption, we subtract real consumption of housing services (as reported in line 14 of Table 2.3.6 of the National Income and Product Accounts) from line 1 of that same table, total real consumption inclusive of expenditures on consumer durable goods.[5] Our price index for consumption used to measure inflation in the calculation of real returns, denoted $p_t^{c*}$, is consistent with this definition.[6] Note that in the process of purging housing services from overall consumption, we may introduce a tiny amount of measurement error into our consumption series, increasing the volatility of consumption and inducing negative serial correlation in the growth rates. To understand the latter effect, denote observed consumption in period $t$ as $c_t^o$, true consumption as $c_t$, and an i.i.d. measurement error draw as $e_t$. If $\log(c_t^o) = \log(c) + e_t$ then $\Delta \log(c_t^o) = \Delta \log(c) + \Delta e_t$; obviously, $\Delta e_t$ is negatively serially correlated. We assume that the true process for $\log(c_t)$ is a random walk, that is $\Delta \log(c) = u_t$ with innovations $u_t$ that are uncorrelated with $e_s$ for any $s$ and $t$, and use the Kalman Filter to uncover an underlying series for $\log(c_t)$ that we use in our empirical analysis. The growth rates of the filtered series are less negatively autocorrelated than the unfiltered growth rates, but the level of the two series are basically identical (not shown).

*Stocks and Treasury Bills.* We use nominal beginning-of-quarter 3-month Treasury yields to proxy for nominal risk-free bond returns. For nominal quarterly equity returns, we study the six Fama-French portfolios that are available on Kenneth French's web site, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.[7] Quickly summarizing, the Fama-French portfolios classify stocks as either "small" or "big," where the split-

[4] The Regional Accounts publish annual population estimates, approximately for July of each year. We convert these to a quarterly frequency by assuming constant growth between years.

[5] We appropriately chain-weight the data.

[6] We set our price indexes and housing stock data to be beginning-of-quarter estimates; T-Bill returns and inflation for any period t are measured from the beginning-of-quarter t to the beginning-of-quarter t+1; and, consumption is measured as a flow throughout the quarter. Also, a * superscript denotes a nominal price index.

[7] We assume that these returns are consistent with our timing convention.

point is the median NYSE market equity. For both small and big stocks, the portfolios are split again into three groups based on the ratio of book-to-market equity; stocks with the highest book-to-market equity ratios are called "value" stocks, the lowest are the "growth" stocks, and those in between "neutral." The split points are the 30th and 70th NYSE percentiles. For our bond and stock portfolios, we convert nominal to real returns by appropriately accounting for realized consumer-price inflation, $p_{t+1}^{c^*}/p_t^{c^*}$.

*Price and Quantity of Housing Services.* We use quarterly data from Davis and Heathcote (2004) on house prices and the aggregate housing stock. The Davis and Heathcote data run from 1970:Q1 through 2004:Q2. For nominal house prices, Davis and Heathcote use the Freddie Mac USA Conventional Mortgage Home Price Index (CMHPI), a price index of house values for the entire United States. The CMHPI is a repeat-sales price index that approximately holds the quality of houses constant between any two consecutive periods but allows the quality of the stock to change over time as the quality of the aggregate stock changes. We calculate the relative price of housing $p_t^h$ as the nominal price index for housing $p_t^{h^*}$ divided by $p_t^{c^*}$.

Davis and Heathcote calculate the aggregate nominal stock of housing (owned and rented) using a perpetual accounting method that adjusts the aggregate nominal value of housing in period $t$, $p_t^{h*}H_t$, for capital gains as measured by the growth rate in the CMHPI, $\frac{p_{t+1}^{h*}}{p_t^{h*}}$, and then adds to that net-new aggregate investment in housing, $p_{t+1}^{h*}\Delta H_{t+1}$, that is derived from BEA data on investment in residential structures,

$$p_{t+1}^{h*}H_{t+1} = \left(\frac{p_{t+1}^{h*}}{p_t^{h*}}\right) p_t^{h*}H_t + p_{t+1}^{h*}\Delta H_{t+1}. \tag{20}$$

This system is benchmarked to an estimate of the aggregate nominal value of housing in 2000 that is derived from micro data reported in the 2000 Decennial Census of Housing. Davis and Heathcote find that this perpetual inventory system, benchmarked only to the 2000 Decennial Census of Housing, matches the nominal value of the housing stock within 5 percent to estimates derived from the 1980 Decennial Census of Housing.[8]

We define the real stock of housing in constant 2000 dollars as the nominal stock $p_t^{h*}H_t$ divided by the

[8]See Davis and Heathcote (2004) for more details.

price index $p_t^{h*}$ and set the real value such that it equals the nominal in 2000. In some ways our measure of the the real housing stock is similar to the real consumption of housing services series that is published by the BEA in the NIPA and has been used in previous studies. For example, the growth rates of the two series in per-capita terms are highly correlated (a correlation of 0.62) and both are weakly correlated (0.11) with our filtered per-capita consumption estimate. The two series are different in a very important way, however: Since 1970, our measure of the real per-capita stock of housing has grown at just 0.4 percent per year whereas the real per-capita consumption of housing services as published by the BEA in the NIPA has grown 1.7 percent per year.

We believe the Davis and Heathcote estimate is more accurate than the BEA estimate for three reasons. First, the Davis and Heathcote data are defined to be consistent with both Decennial Census of Housing data and repeat-sales measures of house prices. Second, the Personal Consumption Expenditure handbook that is published by the BEA (U.S. Department of Commerce, 1990) describes the growth in the estimate of owner-occupied nominal rent (the hypothetical amount owner-occupiers would pay to rent their housing units from a landlord) as being affected by year-by-year "judgmental" (p. 61) adjustments for changes to the quality of the housing stock. Finally, and related, the BEA deflates the nominal estimate of "space rent" in the NIPA by the CPI for either owners' equivalent rent or tenant rent, depending on the specific line item, to produce a real estimate. The CPI estimate of changes to rental prices likely has a large downward bias (Gordon and vanGoethem (2004)), which would induce an upward bias to growth of real housing services as published in the NIPA. We show later that the fact that our estimate of the housing stock is growing more slowly than the BEA estimate has important implications for our parameter estimates.[9]

[9]All of our data are available for download at http://morris.marginalq.com/.

## 4 Estimation and Analysis

*Housing Moment.* Any model that uses housing to help explain financial returns should be consistent with housing returns, and we examine the extent to which we can restrict the feasible set of parameters using only moment conditions involving $\epsilon_{t+1}^h$. Remember that the average value of $\epsilon_{t+1}^h$ is set to zero by construction. Therefore, to make headway on uncovering the parameters of this model we impose the next most basic moment restriction for housing: That the housing errors do not trend over time, $corr\left(\epsilon_t^h, t\right) = 0$. If the housing errors trend over time, then, holding parameters fixed, the average value of $\epsilon_t^h$ will move away from 0 as we extend the sample.

This one moment condition is not sufficient to identify $\beta$, $\sigma$, $\alpha$, and $\gamma$. Instead, given values for $\beta$ and $\sigma$, we find pairs of $(\gamma, \alpha)$ such that the sample $corr\left(\epsilon_t^h, t\right) = 0$.[10] Figure 1 graphs the absolute value of $corr\left(\epsilon_t^h, t\right)$ with $\sigma = 5$ and $\beta = 0.995$. This figure shows that for any $\alpha > 0.4$ there appears to be a corresponding value of $\gamma$ such that $corr\left(\epsilon_t^h, t\right)$ is exactly 0.

Any numerical minimization algorithm worth its salt should be able to find parameter estimates along the "valley" of this graph that runs from the pair $(\gamma = 0.46, \alpha = 0.999)$ to the pair $(\gamma = 0.999, \alpha = 0.38)$. There is an exact $(\gamma, \alpha)$ pair that minimizes the absolute value of $corr\left(\epsilon_t^h, t\right)$ and it occurs at about $\alpha = 0.70$. The difference in the value of the housing moment between $\alpha = 0.70$ and any other $(\gamma, \alpha)$ pair in the valley of this graph likely reflects numerical error.[11] In what follows we will treat all $(\gamma, \alpha)$ pairs in this valley (and others like it with different values for $\beta$ and $\sigma$) as equally viable candidates for our parameter estimates.

Figure 1 simply shows the feasible set of $(\gamma, \alpha)$ pairs at one particular combination of $\beta$ and $\sigma$. We now wish to characterize how this one housing moment restricts the entire parameter space. It turns out that for all reasonable values of $\beta$ and $\sigma$, the highest feasible value of $\alpha$ is always 0.999. Therefore, Figure 2 plots at several different values of $\beta$, the *lowest* feasible value of $\alpha$ that, when paired with the appropriate

[10] In all analysis that follows, our sample period is 1970:Q1 through 2004:Q2.

[11] The difference in the value of the housing moment between any two $(\gamma, \alpha)$ pairs in this valley is of the order of magnitude of $10^{-11}$.

$\gamma$, minimizes the absolute value of $corr\left(\epsilon_t^h, t\right)$ given $\sigma$. Notice that $\alpha < 0$ is only feasible when $\sigma > 20$. If we believe that $\sigma < 10$, $\alpha$ must be larger than 0.2.

Let's step back and consider the basic intuition for this result. It can be shown that house prices satisfy the following equation

$$p_t = E_t \sum_{s=0}^{\infty} \left( \prod_{j=1}^{s} m_{j+t} \right) \frac{U_H \left(c_{s+t}, H_{s+t}\right)}{U_c \left(c_{s+t}, H_{s+t}\right)}. \tag{21}$$

The current price is equal to the present discounted value of user costs, which is exactly analogous to the common result for pricing securities.

Suppose for a moment that we live in a deterministic world, implying we can ignore the expectations operator, and where $m$ is well approximated by a constant. In this world, the growth rate of prices should roughly match the growth rate of user costs. Given our framework, these assumptions imply

$$\Delta \log p_t \approx (1-\alpha)\left(\Delta \log c_t - \Delta \log H_t\right) \tag{22}$$

The average quarterly growth rates (expressed at an annual rate) of real house prices, real per-capita consumption, and real per-capita housing stock in our sample are 1.4%, 2.3%, and 0.4% respectively. Based on equation (22), the value of $\alpha$ that reconciles these average growth rates is 0.26. The last few years of atypically fast house price growth have reduced this estimate of $\alpha$ by quite a bit. For example, if the last eight years of data are omitted from the sample, an estimate of $\alpha \approx 0.5$ is obtained. Given the historical average growth rates of real consumption and the housing stock, if $\alpha$ were a negative number, say $\alpha = -0.5$, then the growth rate of house prices would have been double the average observed growth rate over the entire sample.

*Stock Moment.* Next, we try to further restrict the range of parameter estimates by explicitly using historical information on real stock returns. Specifically, we will restrict our parameter estimates further by including the additional moment condition that the average value of $\epsilon_{t+1}^i$ is zero for an equally-weighted bundle of the six Fama-French stock portfolios. Similar to the previous section, given values for $\beta$ and $\sigma$, we can find a range of pairs of $(\gamma, \alpha)$ such that the absolute value of the average value of $\epsilon_{t+1}^i$ is minimized.

Figure 3 plots the absolute value of this average error for $\sigma = 5$ and $\beta = 0.995$. As with the housing moment, there appears to be a valley along which pairs of $(\gamma, \alpha)$ satisfy this moment condition for stocks.

Our strategy to further refine our admissible parameter space is to find pairs, if they exist, of $(\gamma, \alpha)$ that set both the absolute values of $corr\left(\epsilon_t^h, t\right)$ and the average value of $\epsilon_{t+1}^i$ to zero given values for $\beta$ and $\sigma$. Restated, we are looking for $(\gamma, \alpha)$ pairs that lie in the valleys of both the housing and stock moments. Figure 4 shows, for several values of $\beta$, the values for $\sigma$ and $\alpha$ (with $\gamma$ suppressed) that satisfy the criteria $|corr\left(\epsilon_t^h, t\right)| < 0.001$ and $|avg\left(\epsilon_t^i\right)| < 0.001$. Notice, the line in the bottom-right corner in $(\sigma, \alpha)$ space is for $\beta = 0.999$. $\beta = 0.999$ gives the lowest feasible value of $\alpha$, $\alpha = 0.55$. For any $\beta$, larger values of $\sigma$ are associated with higher values of $\alpha$.[12] But, as $\beta$ is reduced, the set of admissible $(\sigma, \alpha)$ pairs shrinks toward the point $(\sigma = 1, \alpha = 1)$.

*Equity Premium Puzzle.* Finally, to understand if this model is capable of explaining the equity premium puzzle, we invent a risk-free asset that pays a constant real return of $x$ percent each quarter in our historical sample. We then search for the smallest value of $x$ such that the average value of the Euler equation error for this hypothetical risk free return is zero, and, our moment conditions for stock returns and house prices are also satisfied. The lowest value of $x$ is 11 percent per year. We conclude that the neoclassical consumption asset-pricing framework, with housing treated seperately from consumption in utility, cannot resolve the equity premium puzzle.

## 5 Concluding Remarks

The fact that the housing stock, and thus by assumption housing services, grow on average at a different rate than consumption implies that real interest rates are not stationary without additional assumptions on the utility function. This can be seen immediately from the pricing kernel, equation (11). Interest rates will

[12]This stands in contrast to the results of Figure 2, in which we graph the lowest value of $\alpha$ that satisfies our housing moment. In Figure 2, higher values of $\sigma$ are associated with decreasing lower bounds for $\alpha$.

trend over time unless: (A) $H$ and $c$ grow at the same average rate, or (B) $\alpha \in \{1-\sigma, 0\}$. $\alpha = 1-\sigma$ implies separable utility and $h$ is not an argument in the kernel. $\alpha = 0$ implies Cobb-Douglas preferences and the fact that housing services have grown much less quickly than consumption does not matter.

An alternative possibility, which we are currently exploring, is that the assumption that housing services are linearly proportional to the housing stock where the proportionality factor is constant is a poor choice. If instead housing services are produced as say $zf(H)$, where $H$ is the stock of housing, there exists a large class of processes for $z$ such that

$$1 = E\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\left(\frac{\gamma + (1-\gamma)\left(\frac{z_{t+1}f(H_{t+1})}{c_{t+1}}\right)^{\alpha}}{\gamma + (1-\gamma)\left(\frac{z_t f(H_t)}{c_t}\right)^{\alpha}}\right)^{\frac{1-\sigma-\alpha}{\alpha}} R_{t+1}\right], \tag{23}$$

interest rates are stationary, and the moment condition for housing is also satisfied. Given $z$ is unobservable, we will have to assume its stochastic properties in order to estimate the parameters of the model. If we think of housing services as being produced, and $z$ is viewed as a productivity process, some discipline may be found in the home production literature (see for example Greenwood, Rogerson, and Wright (1995)).

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Figure 1:  Housing Correlation Moment Sigma=5, Beta=.995

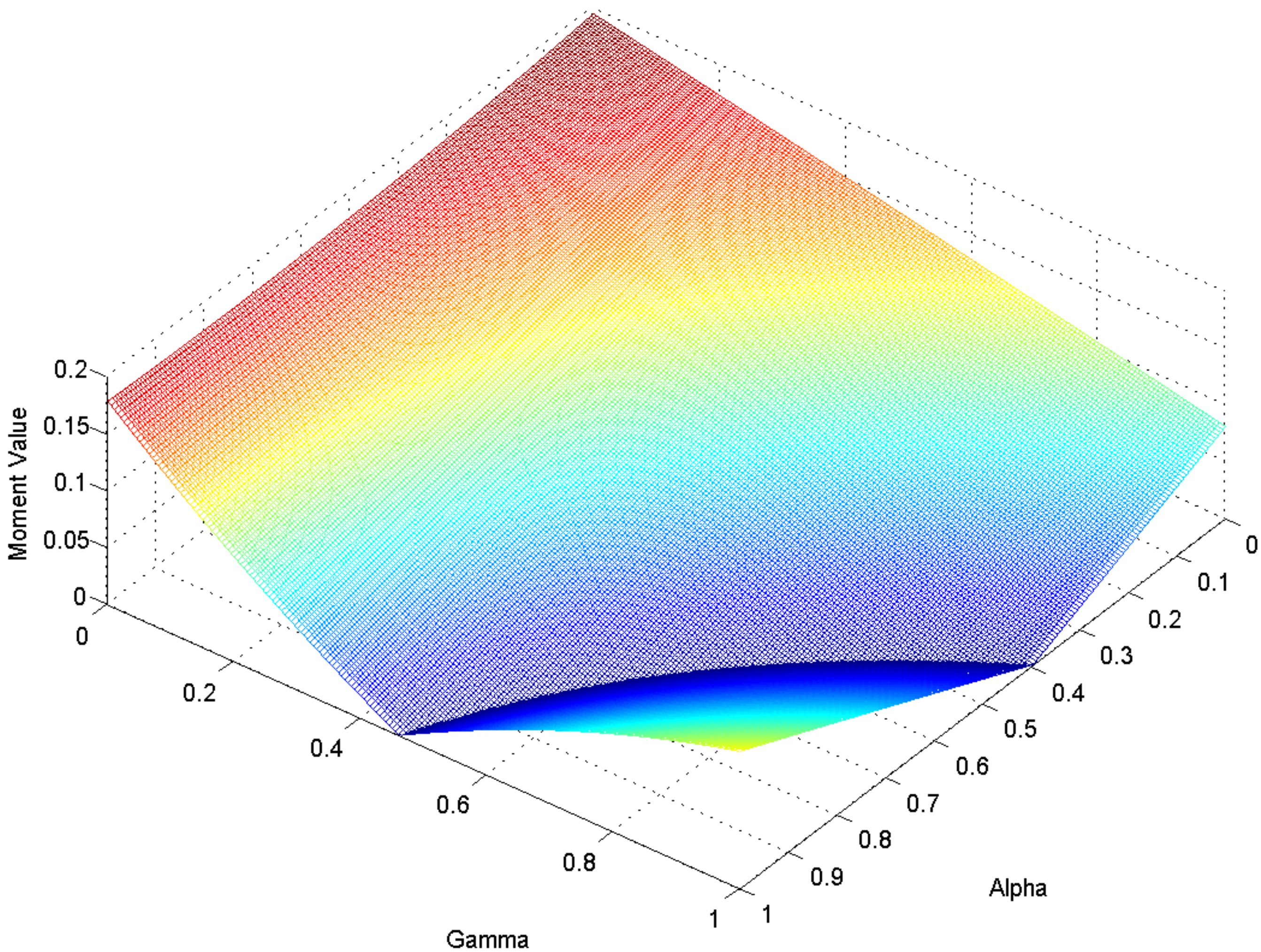

**Figure2: Minimum Alpha Consistent with Housing Moment**

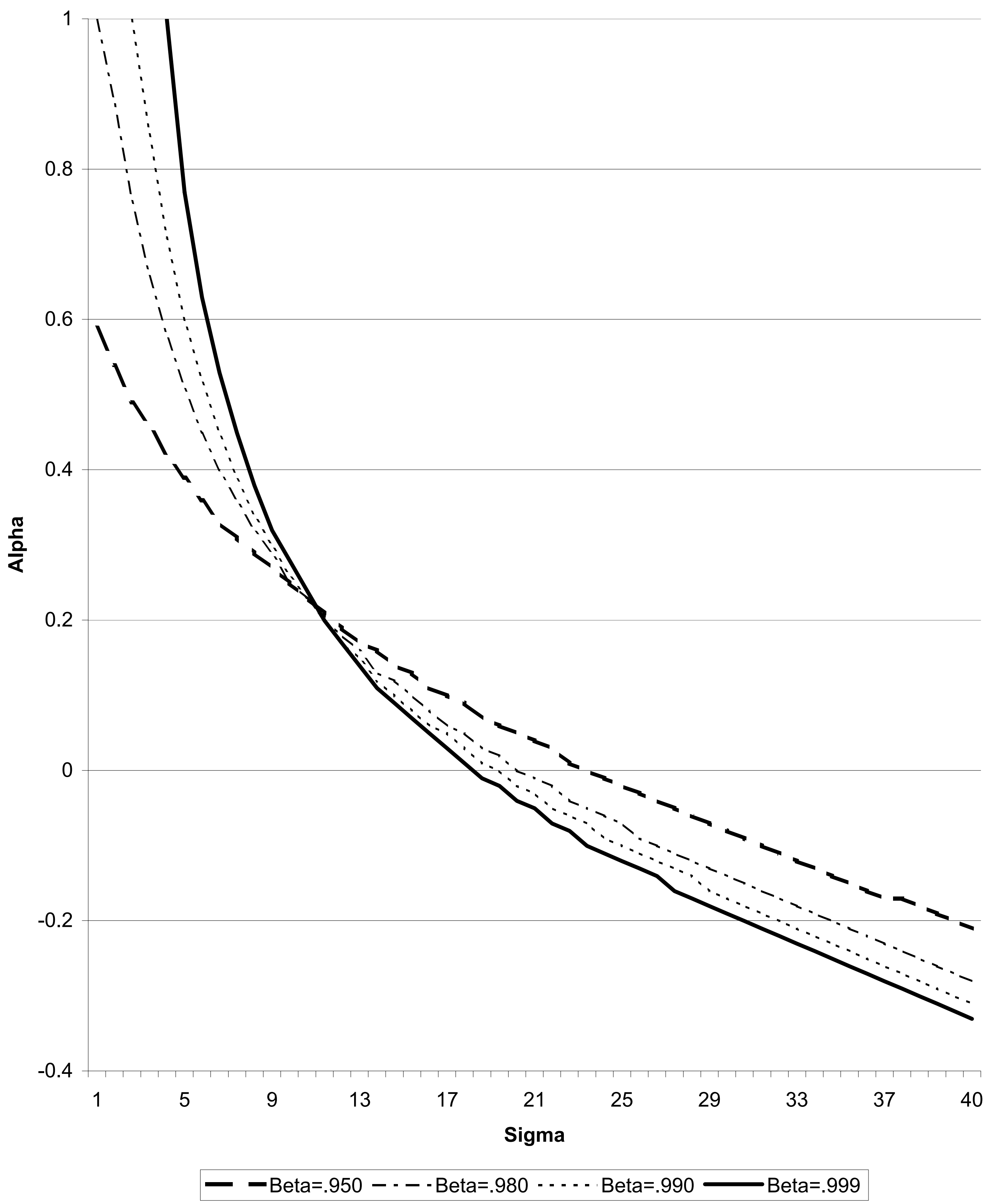

Figure 3: Equity Moment Sigma=5, Beta=.995

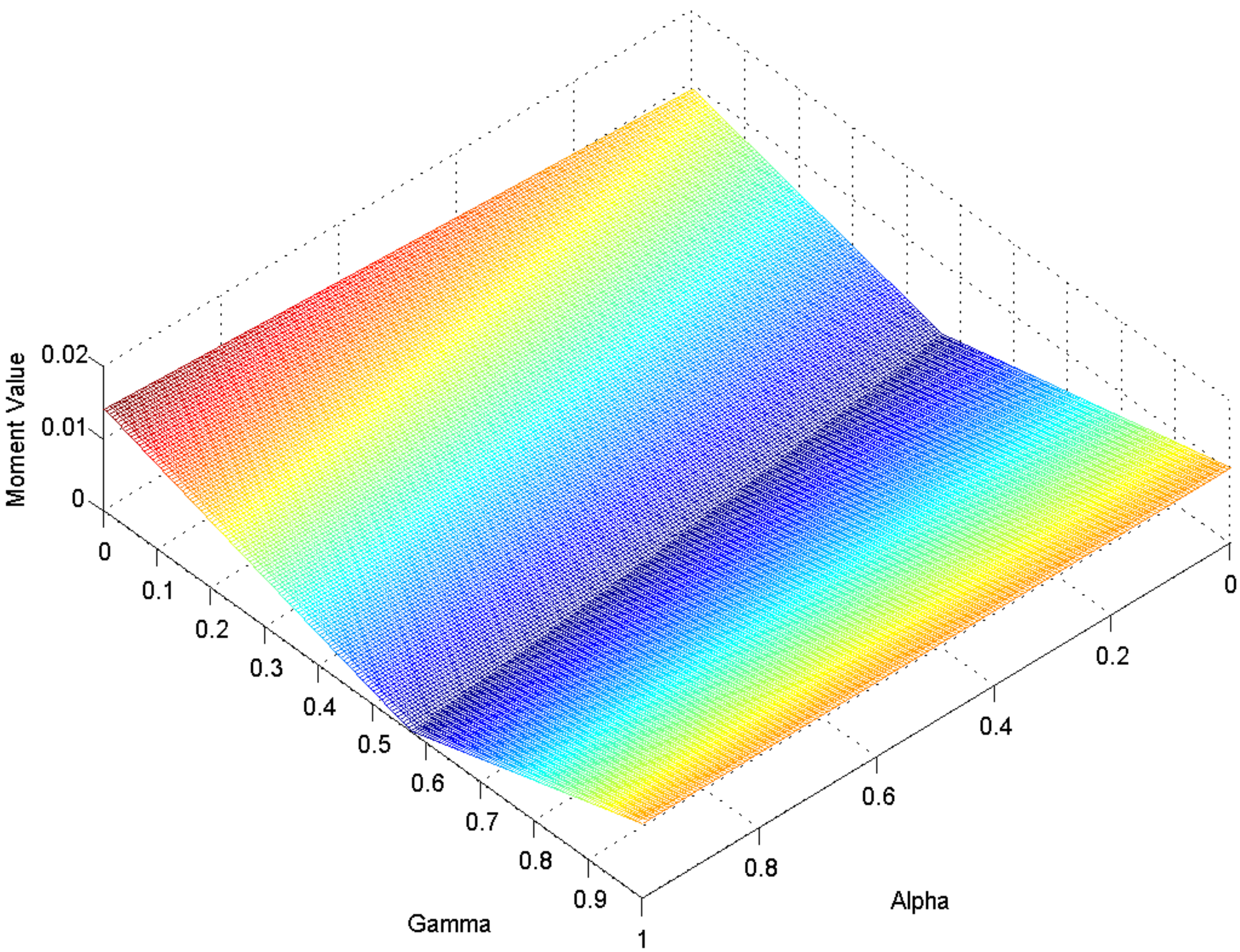

Figure 4: Feasible Parameter Set: Equities and Housing