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Distributional dynamics under smoothly state-dependent pricing

James Costain and Anton Nakov

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James Costain[a], Anton Nakov[b,*†]

[a] Banco de España; [b] Federal Reserve Board

Abstract

Starting from the assumption that firms are more likely to adjust their prices when doing so is more valuable, this paper analyzes monetary policy shocks in a DSGE model with firm-level heterogeneity. The model is calibrated to retail price microdata, and inflation responses are decomposed into "intensive", "extensive", and "selection" margins. Money growth and Taylor rule shocks both have nontrivial real effects, because the low state dependence implied by the data rules out the strong selection effect associated with fixed menu costs. The response to sector-specific shocks is gradual, but inappropriate econometrics might make it appear immediate.

Keywords: Nominal rigidity, state-dependent pricing, menu costs, heterogeneity, Taylor rule

JEL classification: E31, E52, D81

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*Corresponding author: James Costain, Banco de Espana, Calle Alcalá 48, 28014 Madrid, Spain, +34-91-338-5732, james.costain@bde.es

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1. Introduction

Sticky prices are an important ingredient in modern dynamic general equilibrium models, including those used by central banks for policy analysis. But how best to model price stickiness, and to what extent stickiness of individual prices implies rigidity of the aggregate price level, remains controversial. Calvo's (1983) assumption of a constant adjustment probability is popular for its analytical tractability, and implies that monetary shocks have large and persistent real effects. However, Golosov and Lucas (2007, henceforth GL07) have argued that microfounding price rigidity on a fixed "menu cost"and calibrating to microdata implies that monetary shocks are almost neutral.

This paper calibrates and simulates a general model of state-dependent pricing that nests the Calvo (1983) and fixed menu cost (FMC) models as two opposite limiting cases, with a continuum of smooth intermediate cases lying in between. As in Dotsey, King, and Wolman (1999) and Caballero and Engel (2007), the setup rests on one fundamental property: firms are more likely to adjust their prices when doing so is more valuable. Implementing this assumption requires the selection of a parameterized family of functions to describe the adjustment hazard; the exercise is disciplined by fitting the model to the size distribution of price changes found in recent US retail microdata (Klenow and Kryvtsov 2008; Midrigan 2011; Nakamura and Steinsson 2008).[1] One of the calibrated parameters controls the degree of state dependence; matching the smooth distribution of price changes seen in microdata requires rather low state dependence. Therefore, impulse responses reveal substantial monetary nonneutrality, with real effects only slightly weaker than the Calvo model implies.

The impulse response analysis considers a number of issues unaddressed by previous work on state-dependent pricing. GL07 restricted attention to *iid* money growth shocks; this paper also considers the autocorrelated case, and shows that the shape and persistence of responses is primarily determined by the degree of state dependence, not by the autocorrelation of the driving process. Moreover, this paper also studies monetary policy governed by a Taylor rule, as opposed to an exogenous money growth process, which reinforces the conclusion that a calibrated model of state-dependent pricing implies nontrivial real effects. This paper also decomposes inflation into an "intensive margin" relating to

[1] A companion paper, Costain and Nakov (2011), discusses the calibration in greater detail, documenting the steady-state model's fit to cross-sectional microdata on price adjustments, both for low and high trend inflation rates.

the average desired price change, an "extensive margin" relating to the fraction of firms adjusting, and a "selection effect" relating to *which* firms adjust. This decomposition corroborates the claim of GL07, which was challenged by Caballero and Engel (2007), that the selection effect is crucial for the behavior of the FMC model. A fourth contribution of this paper is to argue that prices respond slowly to sector-specific as well as aggregate shocks, despite some recent empirical claims to the contrary. The paper also implements an algorithm for computing heterogeneous-agent economies which is well-suited to modeling state-dependent pricing but has not yet been applied in this context.

1.1. Relation to previous literature

Most previous work on state-dependent pricing has obtained solutions by limiting the analysis, either focusing on partial equilibrium (*e.g.* Caballero and Engel, 1993, 2007; Klenow and Kryvtsov, 2008), or assuming firms face aggregate shocks only (*e.g.* Dotsey *et al.*, 1999), or making strong assumptions about the distribution of idiosyncratic shocks (*e.g.* Caplin and Spulber, 1987; Gertler and Leahy, 2005). But Klenow and Kryvtsov (2008) argue convincingly that firms are often hit by large idiosyncratic shocks. And while heterogeneity may average out in many macroeconomic contexts, it is hard to ignore in the debate over nominal rigidities, because firm-level shocks could greatly alter firms' incentives to adjust prices. GL07 were the first to confront these issues directly, by studying a menu cost model in general equilibrium with idiosyncratic productivity shocks. They obtained a striking near-neutrality result. However, their model's fit to price data is questionable, as our Figure 1 shows. A histogram of retail microdata displays a wide range of price adjustments, whereas their FMC model generates two sharp spikes of price increases and decreases occurring near the (S,s) bounds.

Other micro facts have been addressed in more recent papers on state-dependent pricing. Eichenbaum, Jaimovich, and Rebelo (2011) and Kehoe and Midrigan (2010) modeled "temporary" price changes (sales), assuming that these adjustments are cheaper than other price changes. However, they ultimately conclude that the possibility of sales has little relevance for monetary transmission, which depends instead on the frequency of regular non-sale price changes. Guimaraes and Sheedy's (2011) model of sales as stochastic price discrimination has the same implication. Thus, since the model developed in this paper has no natural motive for sales, it will be compared to a dataset of "regular" price changes from which apparent sales have been removed. In another branch of the literature, Boivin, Giannoni, and Mihov (2009) and Mackowiak, Moench, and Wiederholt (2009) estimate that prices respond much more quickly to sectoral shocks than to aggregate shocks. However, the present paper performs a Monte Carlo exercise that shows that this finding should be treated with caution. Remarkably, even when the true response to a sector-specific shock is lagged and transitory, the estimation routine of Mackowiak *et al.* can erroneously conclude that sector-specific shocks have an immediate, permanent impact on prices.

Our need to allow for firm-specific shocks complicates computation, because it implies that the distribution of prices and productivities across firms is a relevant state variable. This paper shows how to compute a dynamic general equilibrium with state-dependent pricing via the two-step algorithm of Reiter (2009), which calculates steady-state equilibrium using backwards induction on a grid, and then linearizes the equations at every grid point to calculate the dynamics. This approach avoids some complications (and simplifying assumptions) required by other methods. In contrast to GL07, it is not necessary to assume that aggregate output stays constant after a money shock. In contrast to Dotsey, King, and Wolman (2008), it more fully exploits the recursive structure of the model, tracking the price distribution without needing to know who adjusted when. In contrast to the method of Krusell and Smith (1998), used by Midrigan (2011), there is no need to find an adequate summary statistic for the distribution. In contrast to Den Haan (1997), there is no need to impose a specific distributional form. The nonlinear, nonparametric treatment of firm-level heterogeneity in Reiter's algorithm makes it straightforward to calculate the time path of cross-sectional statistics, like our inflation decomposition; the linearization of aggregate dynamics makes it just as easy to analyze a variety of monetary policy rules or shock processes as it would be in a standard, low-dimensional DSGE model.

Several closely related papers have also remarked that FMCs imply a counterfactual distribution of price adjustments, in which small changes never occur. They proposed some more complex pricing models to fix this problem, including sectoral heterogeneity in menu costs (Klenow and Kryvtsov, 2008), multiple products on the same "menu" combined with leptokurtic technology shocks (Midrigan, 2011), or a mix of flexible- and sticky-price firms plus a mix of two distributions of productivity shocks

(Dotsey *et al.*, 2008). This paper proposes a simpler approach: we just assume the probability of price adjustment increases with the value of adjustment, and treat the hazard function as a primitive of the model. A family of hazard functions with just three parameters suffices to match the distribution of price changes at least as well as the aforementioned papers do. Our setup can be interpreted as a stochastic menu cost (SMC) model, like Dotsey *et al.* (1999) or Caballero and Engel (1999); under this interpretation the hazard function corresponds to the *c.d.f.* of the menu cost.[2] Alternatively, our setup can be seen as a near-rational model, like Akerlof and Yellen (1985), in which firms are more likely to make mistakes when they are not very costly; in this case the hazard function corresponds to the distribution of error values.[3] Under either interpretation, the key point is that the adjustment hazard increases smoothly with the value of adjusting, in contrast with the discontinuous jump in probability implied by the FMC model. An appropriate calibration of the smoothness of the hazard function yields a smooth histogram of price changes consistent with microdata; this smoothness is the same property that eliminates the strong selection effect found by GL07. Thus, none of the additions Dotsey *et al.* and Midrigan make to the FMC framework are crucial for their most important finding: a state-dependent pricing model consistent with observed price changes implies nontrivial real effects of monetary shocks, similar to those found under the Calvo framework.

2. Model

This discrete-time model embeds state-dependent pricing by firms in an otherwise-standard New Keynesian general equilibrium framework based on GL07. Besides the firms, there is a representative household and a monetary authority that either implements a Taylor rule or follows an exogenous growth process for nominal money balances.

The aggregate state of the economy at time t, which will be specified in Section 2.3., is called Ω_t. Time subscripts on aggregate variables will indicate dependence, in equilibrium, on aggregate conditions Ω_t. For example, consumption is denoted by $C_t \equiv C(\Omega_t)$.

[2]For estimation purposes, Caballero and Engel usually assume that the probability of adjustment depends on the distance of the choice variable from some target level, but this is just an approximation to an underlying model in which the adjustment probability depends on the value of adjustment, as in our setup.

[3]The two interpretations imply slightly different Bellman equations: in the first case, but not in the second, a flow of menu costs is subtracted out of the firm's flow of profits.

2.1. Household

The household's period utility function is $\frac{1}{1-\gamma}C_t^{1-\gamma} - \chi N_t + \nu \log(M_t/P_t)$, where C_t is consumption, N_t is labor supply, and M_t/P_t is real money balances. Utility is discounted by factor β per period. Consumption is a CES aggregate of differentiated products C_{it}, with elasticity of substitution ϵ:

$$C_t = \left\{ \int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

The household's nominal period budget constraint is

$$\int_0^1 P_{it}C_{it}di + M_t + R_t^{-1}B_t = W_tN_t + M_{t-1} + T_t + B_{t-1} + U_t, \tag{2}$$

where $\int_0^1 P_{it}C_{it}di$ is total nominal consumption. B_t is nominal bond holdings purchased at t, paying interest rate $R_t - 1$ at time $t+1$. T_t is a nominal lump-sum transfer consisting of seignorage revenues from the central bank plus dividend payments from the firms.

Households choose $\{C_{it}, N_t, B_t, M_t\}_{t=0}^{\infty}$ to maximize expected discounted utility, subject to the budget constraint (2). Optimal consumption across the differentiated goods implies

$$C_{it} = (P_t/P_{it})^{\epsilon} C_t, \tag{3}$$

where P_t is the price index

$$P_t \equiv \left\{ \int_0^1 {P_{it}}^{1-\epsilon} di \right\}^{\frac{1}{1-\epsilon}}, \tag{4}$$

so total nominal spending can be written as $P_tC_t = \int_0^1 P_{it}C_{it}di$.

Optimal labor supply, consumption, and money use imply the following first-order conditions:

$$\chi = C_t^{-\gamma} W_t/P_t, \tag{5}$$

$$R_t^{-1} = \beta E_t \left(\frac{P_t C_{t+1}^{-\gamma}}{P_{t+1} C_t^{-\gamma}} \right), \tag{6}$$

$$1 - \frac{\nu P_t}{M_t C_t^{-\gamma}} = \beta E_t \left(\frac{P_t C_{t+1}^{-\gamma}}{P_{t+1} C_t^{-\gamma}} \right). \tag{7}$$

2.2. Monopolistic firms

Each firm i produces output Y_{it} using labor N_{it} as its only input, under a constant returns technology: $Y_{it} = A_{it}N_{it}$. A_{it} is an idiosyncratic productivity process that is AR(1) in logs:

$$\log A_{it} = \rho \log A_{it-1} + \varepsilon^a_{it}, \tag{8}$$

where $0 \leq \rho < 1$ and $\epsilon^a_{it} \sim i.i.d.N(0, \sigma^2_a)$. Firm i is a monopolistic competitor that sets a price P_{it}, facing the demand curve $Y_{it} = C_t P_t^{\epsilon} P_{it}^{-\epsilon}$, and must fulfill all demand at its chosen price. It hires in a competitive labor markets at wage rate W_t, generating profits

$$U_{it} = P_{it}Y_{it} - W_t N_{it} = \left(P_{it} - \frac{W_t}{A_{it}}\right) C_t P_t^{\epsilon} P_{it}^{-\epsilon} \equiv U(P_{it}, A_{it}, \Omega_t) \tag{9}$$

per period. Firms are owned by the household, so they discount nominal income between times t and $t+1$ at the rate $\beta \frac{P(\Omega_t)C(\Omega_{t+1})^{-\gamma}}{P(\Omega_{t+1})C(\Omega_t)^{-\gamma}}$, consistent with the household's marginal rate of substitution.

Let $V(P_{it}, A_{it}, \Omega_t)$ denote the nominal value of a firm at time t that produces with productivity A_{it} and sells at nominal price P_{it}. Prices are sticky, so P_{it} may or may not be optimal. However, we assume that whenever a firm adjusts its price, it chooses the optimal price conditional on its current productivity, keeping in mind that it will sometimes be unable to adjust in the future. Hence, the value function of an adjusting firm, after netting out any costs that may be required to make the adjustment, is $V^*(A_{it}, \Omega_t) \equiv \max_P V(P, A_{it}, \Omega_t)$. For clarity, it helps to distinguish the firm's beginning-of-period price, $\widetilde{P}_{it} \equiv P_{i,t-1}$, from the end-of-period price at which it sells at time t, P_{it}, which may or may not be the same. The distributions of prices and productivities across firms at the beginning and end of t will be denoted $\widetilde{\Phi}_t(\widetilde{P}, A)$ and $\Phi_t(P, A)$, respectively.

The gain from adjusting at the beginning of t is:

$$D(\widetilde{P}_{it}, A_{it}, \Omega_t) \equiv \max_P V(P, A_{it}, \Omega_t) - V(\widetilde{P}_{it}, A_{it}, \Omega_t). \tag{10}$$

The main assumption of our framework is that the probability of price adjustment increases with the gain from adjustment. The weakly increasing function λ that governs this probability is taken as a primitive of the model. Invariance of λ requires that its argument, the gain from adjustment, be written in appropriate units. As was mentioned in the introduction, this setup can be interpreted as a stochastic menu cost model, or as a model of near-rational price decisions. In the SMC case, the labor effort of changing price tags or menus is likely to be a large component of the cost; in the near-rational case, the adjustment probability should depend on the labor effort required to obtain new information or to recompute the optimal price. Under either interpretation, the most natural units for the argument of the λ function are units of labor time. Thus, the probability of adjustment will be defined as $\lambda\left(L\left(\widetilde{P}_{it}, A_{it}, \Omega_t\right)\right)$, where $L\left(\widetilde{P}_{it}, A_{it}, \Omega_t\right) = \frac{D(\widetilde{P}_{it}, A_{it}, \Omega_t)}{W(\Omega_t)}$ expresses the adjustment gain in time units by dividing by the wage. The functional form for λ will be specified in Sec. 2.2.1.

The value of selling at any given price equals current profits plus the expected value of future production, which may or may not be sold at a new, adjusted price. Given the firm's idiosyncratic state variables (P, A) and the aggregate state Ω, and denoting next period's variables with primes, the Bellman equation under the near-rational interpretation of the model is

$$V(P,A,\Omega) = \left(P - \frac{W(\Omega)}{A}\right) C(\Omega) P(\Omega)^{\epsilon} P^{-\epsilon} + \tag{11}$$

$$\beta E \left\{ \tfrac{P(\Omega) C(\Omega')^{-\gamma}}{P(\Omega') C(\Omega)^{-\gamma}} \left[\left(1 - \lambda\left(\tfrac{D(P,A',\Omega')}{W(\Omega')}\right)\right) V(P,A',\Omega') \; + \; \lambda\left(\tfrac{D(P,A',\Omega')}{W(\Omega')}\right) \max_{P'} V\left(P',A',\Omega'\right) \right] \middle| A, \Omega \right\}.$$

Here the expectation refers to the distribution of A' and Ω' conditional on A and Ω. Note that on the left-hand side of the Bellman equation, and in the term representing current profits, P refers to a given firm i's price P_{it} at the end of t, when transactions occur. In the expectation on the right, P represents the price $\widetilde{P}_{i,t+1}$ at the beginning of $t+1$, which may (probability λ) or may not $(1-\lambda)$ be adjusted prior to time $t+1$ transactions to a new value P'.

The right-hand side of the Bellman equation can be simplified by using the notation from (9), and the rearrangement $(1-\lambda)V + \lambda \max V = V + \lambda(\max V - V)$:

$$V(P,A,\Omega) \;=\; U(P,A,\Omega) + \beta E \left\{ \tfrac{P(\Omega) C(\Omega')^{-\gamma}}{P(\Omega') C(\Omega)^{-\gamma}} \left[V(P,A',\Omega') + G(P,A',\Omega') \right] \middle| A, \Omega \right\}, \tag{12}$$

where

$$G(P,A',\Omega') \equiv \lambda\left(\frac{D(P,A',\Omega')}{W(\Omega')}\right) D(P,A',\Omega'). \tag{13}$$

The terms inside the expectation in the Bellman equation represent the value V of continuing without adjustment, plus the flow of expected gains G due to adjustment. Since the firm sets the optimal price whenever it adjusts, the price process associated with (12) is

$$P_{it} = \begin{cases} P^*(A_{it}, \Omega_t) \equiv \arg\max_P V(P, A_{it}, \Omega_t) & \text{with probability } \lambda\left(\frac{D(\widetilde{P}_{it}, A_{it}, \Omega_t)}{W(\Omega_t)}\right) \\ \widetilde{P}_{it} \equiv P_{i,t-1} & \text{with probability } 1 - \lambda\left(\frac{D(\widetilde{P}_{it}, A_{it}, \Omega_t)}{W(\Omega_t)}\right) \end{cases} . \tag{14}$$

Equation (14) is written with time subscripts for additional clarity.

2.2.1. *Alternative sticky price frameworks*

Our assumptions require the function λ to be weakly increasing and to lie between zero and one. The paper focuses primarily on the following functional form:

$$\lambda(L) \equiv \frac{\overline{\lambda}}{\overline{\lambda} + (1 - \overline{\lambda})\left(\frac{\alpha}{L}\right)^{\xi}} \tag{15}$$

with α and ξ positive, and $\overline{\lambda} \in [0, 1]$. This function equals $\overline{\lambda}$ when $L = \alpha$, and is concave for $\xi \leq 1$ and S-shaped for $\xi > 1$ (see the second panel of Fig. 1). The parameter ξ effectively controls the degree of state dependence. In the limit $\xi = 0$, (15) nests Calvo (1983), with $\lambda(L) = \overline{\lambda}$, making the adjustment hazard literally independent of the relevant state variable, which is L. At the opposite extreme, as $\xi \to \infty$, $\lambda(L)$ becomes the indicator $\mathbf{1}\{L \geq \alpha\}$, which equals 1 whenever $L \geq \alpha$ and is zero otherwise. This implies very strong state dependence, in the sense that the adjustment probability jumps from 0 to 1 when the state L passes the threshold level α. For all intermediate values $0 < \xi < \infty$, the hazard increases smoothly with the state L. In this sense, choosing ξ to match microdata means finding the degree of state dependence most consistent with firms' observed pricing behavior.

The combination of Bellman equation (12) with (13) is based on a near-rational interpretation of our setup; for $0 < \xi < \infty$ this version of the model will be called "SSDP", for "smoothly state-dependent pricing". However, as Table 1 shows, (12) nests several other models under appropriate choices of the gains function G and the hazard function λ. Subtracting a flow of menu costs $E(\kappa|\kappa < L) \equiv \int_0^L \kappa\lambda(d\kappa)$ out of the gains G converts the SSDP model into the SMC model. The FMC model sets the adjustment probability to a step function, subtracting a constant menu cost α out of G; it is the limit of the SMC model as $\xi \to \infty$. The Calvo model is derived both from SSDP and from SMC as $\xi \to 0$.[4] An alternative hazard function derived from Woodford (2009) is also considered.

[4]In the limit of SMC as $\xi \to 0$, the menu cost is zero with probability $\overline{\lambda}$ and infinite with probability $1 - \overline{\lambda}$, which is when firms do not adjust. The flow of menu costs paid is therefore zero.

2.3. Monetary policy and aggregate consistency

Two specifications for monetary policy are compared: a money growth rule and a Taylor rule. In both cases the systematic component of monetary policy is perturbed by an AR(1) process z,

$$z_t = \phi_z z_{t-1} + \epsilon_t^z, \tag{16}$$

where $0 \leq \phi_z < 1$ and $\epsilon_t^z \sim i.i.d.N(0, \sigma_z^2)$. Under the money growth rule, which is analyzed first to build intuition and for comparison with previous studies, z affects money supply growth:

$$M_t/M_{t-1} \equiv \mu_t = \mu^* \exp(z_t). \tag{17}$$

Alternatively, under a Taylor interest rate rule, which is a better approximation to actual monetary policy, the nominal interest rate follows

$$\frac{R_t}{R^*} = \exp(-z_t)\left(\left(\frac{P_t/P_{t-1}}{\Pi^*}\right)^{\phi_\pi}\left(\frac{C_t}{C^*}\right)^{\phi_c}\right)^{1-\phi_R}\left(\frac{R_{t-1}}{R^*}\right)^{\phi_R}, \tag{18}$$

where $\phi_c \geq 0$, $\phi_\pi > 1$, and $0 < \phi_R < 1$, so that when inflation P_t/P_{t-1} exceeds its target Π^* or consumption C_t exceeds its target C^*, R_t tends to rise above its target $R^* \equiv \Pi^*/\beta$. For comparability between the two monetary regimes, the inflation target is set to $\Pi^* \equiv \mu^*$, and the rules are specified so that in both cases, a positive z represents an expansive shock.

Seigniorage revenues are paid to the household as a lump-sum transfer, and the government budget is balanced each period. Therefore total nominal transfers to the household, including dividend payments, are

$$T_t = M_t - M_{t-1} + \int_0^1 U_{it} di. \tag{19}$$

Bond market clearing is simply $B_t = 0$. When supply equals demand for each good i, total labor supply and demand satisfy

$$N_t = \int_0^1 \frac{C_{it}}{A_{it}} di = P_t^\epsilon C_t \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di \equiv \Delta_t^p C_t. \tag{20}$$

Equation (20) also defines a measure of price dispersion, $\Delta_t^p \equiv P_t^\epsilon \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di$, weighted to allow for heterogeneous productivity. As in Yun (2005), an increase in Δ_t^p decreases the goods produced per unit of labor, effectively acting like a negative aggregate productivity factor.

At this point, all equilibrium conditions have been spelled out, so an appropriate aggregate state variable Ω_t can be identified. At time t, the lagged distribution of transaction prices $\Phi_{t-1}(P, A)$ is predetermined. Knowing Φ_{t-1}, the lagged price level can be substituted out of the Taylor rule, using $P_{t-1} = \left[\iint P^{1-\epsilon}\Phi_{t-1}(dP, dA)\right]^{1/(1-\epsilon)}$. It can thus be seen that $\Omega_t \equiv (z_t, R_{t-1}, \Phi_{t-1})$ suffices to define the aggregate state. Given Ω_t, equations (4), (5), (6), (8), (9), (10), (12), (13), (14), (16), (18), and (20) together give enough conditions to determine the distributions $\widetilde{\Phi}_t$ and Φ_t, the price level P_t, the functions $U(P, A, \Omega_t)$, $V(P, A, \Omega_t)$, $D(P, A, \Omega_t)$, and $G(P, A, \Omega_t)$, and the variables R_t, C_t, N_t, W_t, and z_{t+1}. Thus they determine the next state, $\Omega_{t+1} \equiv (z_{t+1}, R_t, \Phi_t)$.

Under a money growth rule, the time t state can instead be defined as $\Omega_t \equiv (z_t, M_{t-1}, \Phi_{t-1})$. Substituting (7) for (6) and (17) for (18), knowing $\Omega_t \equiv (z_t, M_{t-1}, \Phi_{t-1})$ suffices to determine $\widetilde{\Phi}_t$, Φ_t, $U(P, A, \Omega_t)$, $V(P, A, \Omega_t)$, $D(P, A, \Omega_t)$, and $G(P, A, \Omega_t)$, as well as P_t, C_t, N_t, W_t, z_{t+1}, and M_t. Thus the next state, $\Omega_{t+1} \equiv (z_{t+1}, M_t, \Phi_t)$, can be calculated.

3. Computation

The fact that this model's state variable includes the distribution Φ, an infinite-dimensional object, makes computing equilibrium a challenge. The popularity of the Calvo model reflects its implication that general equilibrium can be solved up to a first-order approximation by keeping track of the average price only. Unfortunately, this result typically fails to hold if pricing is state-dependent; instead, computation requires tracking the whole distribution Φ.

Equilibrium will be computed here following the two-step algorithm of Reiter (2009), which is intended for contexts, like this model, with relatively large idiosyncratic shocks and also relatively small aggregate shocks. In the first step, the aggregate steady state of the model is computed on a finite grid, using backwards induction.[5] Second, the stochastic aggregate dynamics are computed by linearization, grid point by grid point. In other words, the Bellman equation is treated as a large system of expectational difference equations, instead of as a functional equation.

[5]Actually, Reiter's algorithm permits calculation of the aggregate steady state using a variety of finite-element methods; we choose backwards induction on a grid since it is a familiar and transparent procedure.

3.1. *Detrending*

Calculating a steady state requires detrending to make the economy stationary. Here it suffices to scale all nominal variables by the aggregate price level, defining the real wage and money supply $w_t = W_t/P_t$ and $m_t \equiv M_t/P_t$, and the real prices at the beginning and end of t, $\widetilde{p}_{it} \equiv \widetilde{P}_{it}/P_t$ and $p_{it} \equiv P_{it}/P_t$. The beginning-of-t and end-of-t price distributions will be written as $\widetilde{\Psi}_t(\widetilde{p}_{it}, A_{it})$ and $\Psi_t(p_{it}, A_{it})$, respectively. At the end of t, when goods are sold, the real price level is one by definition:

$$1 = \left\{ \iint p^{1-\epsilon} \Psi_t(dp, dA) \right\}^{1/(1-\epsilon)}. \tag{21}$$

For this detrending to make sense, the nominal price level P_t must be irrelevant for real quantities, which must instead be functions of a real state variable Ξ_t that is independent of nominal prices and the nominal money supply. A time subscript on any aggregate variable must now denote dependence on the *real* state, implying for example $w_t = w(\Xi_t) = W(\Omega_t)/P(\Omega_t)$ and $C_t = C(\Xi_t) = C(\Omega_t)$. While the price level will cancel out, inflation $\Pi_t \equiv P_t/P_{t-1}$ will still appear in the model. It must be determined by real variables, satisfying $\Pi_t = \Pi(\Xi_{t-1}, \Xi_t) = P(\Omega_t)/P(\Omega_{t-1})$.

A similar property applies to the value function and profits, which must be homogeneous of degree one in prices. Thus, define real profits u and real value v as follows:

$$u\,(p, A, \Xi) \;=\; u\,(P/P(\Omega), A, \Xi) \equiv P(\Omega)^{-1} U(P, A, \Omega), \tag{22}$$

$$v\,(p, A, \Xi) \;=\; v\,(P/P(\Omega), A, \Xi) \equiv P(\Omega)^{-1} V(P, A, \Omega). \tag{23}$$

To verify homogeneity, divide through the nominal Bellman equation (12) by $P(\Omega)$ to obtain

$$v(p, A, \Xi) = u(p, A, \Xi) + \beta E \left\{ \frac{C(\Xi')^{-\gamma}}{C(\Xi)^{-\gamma}} \left[v\left(\frac{p}{\Pi(\Xi, \Xi')}, A', \Xi' \right) + g\left(\frac{p}{\Pi(\Xi, \Xi')}, A', \Xi' \right) \right] \middle| A, \Xi \right\}, \tag{24}$$

using the definitions

$$g\,(\widetilde{p}, A, \Xi) \;\equiv\; \lambda \left(\frac{d\,(\widetilde{p}, A, \Xi)}{w(\Xi)} \right) d\,(\widetilde{p}, A, \Xi), \tag{25}$$

$$d\,(\widetilde{p}, A, \Xi) \;\equiv\; \max_p v(p, A, \Xi) - v(\widetilde{p}, A, \Xi), \tag{26}$$

which satisfy $g\left(\widetilde{p}, A, \Xi\right) = G(P(\Omega)\widetilde{p}, A, \Omega)/P(\Omega)$ and $d\left(\widetilde{p}, A, \Xi\right) = D(P(\Omega)\widetilde{p}, A, \Omega)/P(\Omega)$.[6] This detrending implies that when a firm's nominal price remains unadjusted at time t, its real price is deflated by factor Π_t. Therefore the real price process is

$$p_{it} = \begin{cases} p^*(A_{it}, \Xi_t) \equiv \arg\max_p v(p, A_{it}, \Xi_t) & \text{with probability } \lambda\left(\frac{d\left(\Pi_t^{-1}p_{i,t-1}, A_{it}, \Xi_t\right)}{w(\Xi_t)}\right) \\ \Pi_t^{-1}p_{i,t-1} & \text{with probability } 1-\lambda\left(\frac{d\left(\Pi_t^{-1}p_{i,t-1}, A_{it}, \Xi_t\right)}{w(\Xi_t)}\right) \end{cases}. \tag{27}$$

To see that these definitions of real quantities suffice to detrend the model, define the real state as $\Xi_t \equiv (z_t, R_{t-1}, \Psi_{t-1})$. Knowing Ξ_t, in the case of a Taylor rule, equations (5), (6), (8), (16), (18), (20), (21), (22), (24), (25), (26), and (27), with substitutions of real for nominal variables where necessary, suffice to determine the distributions $\widetilde{\Psi}_t$ and Ψ_t, inflation Π_t, the functions $u(p, A, \Xi_t)$, $v(p, A, \Xi_t)$, $d(p, A, \Xi_t)$, and $g(p, A, \Xi_t)$, and the variables C_t, w_t, N_t, R_t, and z_{t+1}. For a money growth rule, the real state can be defined as $\Xi_t \equiv (z_t, m_{t-1}, \Psi_{t-1})$, and equation (18) is replaced by (7) and by

$$m_t = \mu^* \exp(z_t) m_{t-1}/\Pi_t, \tag{28}$$

which together determine R_t and m_t. Thus next period's state Ξ_{t+1} can be calculated if Ξ_t is known.

3.2. *Discretization*

Price process (27) is defined over a continuum of possible values, but to solve the model numerically, the idiosyncratic states must be restricted to a finite-dimensional support. Hence, the continuous model will be approximated on a two-dimensional grid $\Gamma \equiv \Gamma^p \times \Gamma^a$, where $\Gamma^p \equiv \{p^1, p^2, ...p^{\#^p}\}$ and $\Gamma^a \equiv \{a^1, a^2, ...a^{\#^a}\}$ are logarithmically-spaced grids of possible values of of p_{it} and A_{it}. Thus the time-varying distributions will be treated as matrices $\widetilde{\mathbf{\Psi}}_t$ and $\mathbf{\Psi}_t$ of size $\#^p \times \#^a$, in which the row j, column k elements, called $\widetilde{\Psi}_t^{jk}$ and Ψ_t^{jk}, represent the fraction of firms in state (p^j, a^k) before and after price adjustments in period t, respectively. From here on, bold face is used to identify matrices and superscripts are used to identify notation related to grids.

Similarly, the value function is written as a $\#^p \times \#^a$ matrix $\mathbf{V}_t$ of values $v_t^{jk} \equiv v(p^j, a^k, \Xi_t)$ associated with the prices and productivities $\left(p^j, a^k\right) \in \Gamma$. The time subscript indicates the fact that the value

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[6] In deriving (24) from (12), initially a term of the form $\frac{C(\Omega')^{-\gamma}}{P(\Omega')C(\Omega)^{-\gamma}} V\left(P, A', \Omega'\right)$ appears on the right-hand side; using (23) this reduces to $\frac{C(\Xi')^{-\gamma}}{C(\Xi)^{-\gamma}} v\left(\frac{p}{\Pi(\Xi',\Xi)}, A', \Xi'\right)$. Reducing the G term in the same way yields (24).

function shifts due to changes in the aggregate state Ξ_t. When necessary, the value is evaluated using splines at points $p \notin \Gamma^p$ off the price grid. In particular, the policy function

$$p^*(A, \Xi_t) \equiv \arg\max_{p\in\mathcal{R}} v(p, A, \Xi_t) \tag{29}$$

is selected from the reals ($p \in \mathcal{R}$) instead of being chosen from the grid ($p \in \Gamma^p$), because the solution method will require policies to vary continuously with their arguments. The policies at the productivity grid points $a^k \in \Gamma^a$ are written as a row vector $\mathbf{p}_t^* \equiv \left\{p_t^{*1}...p_t^{*\#^a}\right\} \equiv \left\{p^*(a^1, \Xi_t)...p^*(a^{\#^a}, \Xi_t)\right\}$. Various other equilibrium functions are also treated as $\#^p \times \#^a$ matrices. The adjustment values $\mathbf{D}_t$, the probabilities $\mathbf{\Lambda}_t$, and the expected gains $\mathbf{G}_t$ have (j, k) elements given by[7]

$$d_t^{jk} \equiv \max_{p\in\mathcal{R}} v(p, a^k, \Xi_t) - v_t^{jk}, \tag{30}$$

$$\lambda_t^{jk} \equiv \lambda\left(d_t^{jk}/w_t\right), \tag{31}$$

$$g_t^{jk} \equiv \lambda_t^{jk} d_t^{jk}. \tag{32}$$

Given this discrete representation, the distributional dynamics can be written in a more explicit way. First, to keep productivity A on the grid Γ^a, it is assumed to follow a Markov chain defined by a matrix $\mathbf{S}$ of size $\#^a \times \#^a$. The row m, column k element of $\mathbf{S}$ represents the probability

$$S^{mk} = prob(A_{it} = a^m | A_{i,t-1} = a^k). \tag{33}$$

Also, beginning-of-t real prices must be adjusted for inflation. Ignoring grids, the time $t-1$ price $p_{i,t-1}$ would be deflated to $\widetilde{p}_{it} \equiv p_{i,t-1}/\Pi_t$ at the beginning of t. Prices are forced to remain on the grid by a $\#^p \times \#^p$ Markov matrix $\mathbf{R}_t$ in which the row m, column l element represents

$$R_t^{ml} \equiv prob(\widetilde{p}_{it} = p^m | p_{i,t-1} = p^l, \Pi_t = \Pi(\Xi_t, \Xi_{t-1})). \tag{34}$$

When the deflated price $p_{i,t-1}/\Pi_t$ falls between two grid points, $\mathbf{R}_t$ rounds it up or down stochastically without changing its mean. Also, if $p_{i,t-1}/\Pi_t$ drifts up or down past the largest or smallest grid points,

[7]The max in (30), like the arg max in (29), ignores the grid Γ^p so that d_t^{jk} varies continuously in response to any change in the value function.

then $\mathbf{R}_t$ rounds it down or up to keep prices on the grid. Thus the transition probabilities are

$$R_t^{ml} = \begin{cases} 1 & \text{if } p^l/\Pi_t \leq p^1 = p^m \\ \frac{p^l/\Pi_t - p^{m-1}}{p^m - p^{m-1}} & \text{if } p^1 < p^m = \min\{p \in \Gamma^p : p \geq p^l/\Pi_t\} < p^{\#^p} \\ \frac{p^{m+1} - p^l/\Pi_t}{p^{m+1} - p^m} & \text{if } p^1 \leq p^m = \max\{p \in \Gamma^p : p < p^l/\Pi_t\} < p^{\#^p} \\ 1 & \text{if } p^l/\Pi_t > p^{\#^p} = p^m \\ 0 & \text{otherwise} \end{cases} . \tag{35}$$

Combining the adjustments of prices and productivities, the beginning-of-t distribution $\widetilde{\mathbf{\Psi}}_t$ can be calculated from the lagged distribution $\mathbf{\Psi}_{t-1}$ as follows:

$$\widetilde{\mathbf{\Psi}}_t = \mathbf{R}_t * \mathbf{\Psi}_{t-1} * \mathbf{S}', \tag{36}$$

where the operator $*$ represents matrix multiplication. Two facts explain the simplicity of this equation. First, the exogenous shocks to A_{it} are independent of the inflation adjustment linking $\widetilde{p}_{it}$ with $p_{i,t-1}$. Second, productivity is arranged from left to right in the matrix $\mathbf{\Psi}_{t-1}$, so productivity transitions are represented by right multiplication, while prices are arranged vertically, so price transitions are represented by left multiplication.

Next, a firm with beginning-of-t state $(\widetilde{p}_{it}, A_{it}) = (p^j, a^k) \in \Gamma$ will adjust its price to $p_{it} = p_t^{*k}$ with probability λ_t^{jk}, and otherwise leave it unchanged. If adjustment occurs, prices are kept on the grid by rounding p_t^{*k} up or down stochastically to the nearest grid points, without changing the mean. To be precise, let Γ^p be wide enough so that $p^1 < p_t^{*k} < p^{\#^p}$ for all $k \in \{1, 2, ...\#^a\}$. For each k, define $l_t(k)$ so that $p^{l_t(k)} = \min\{p \in \Gamma^p : p \geq p_t^{*k}\}$. Then the following $\#^p \times \#^a$ matrix governs the rounding:

$$\mathbf{P}_t \equiv \begin{cases} \frac{p^{l_t(k)} - p_t^{*k}}{p^{l_t(k)} - p^{l_t(k)-1}} & \text{in column } k, \text{ row } l_t(k) - 1 \\ \frac{p_t^{*k} - p^{l_t(k)-1}}{p^{l_t(k)} - p^{l_t(k)-1}} & \text{in column } k, \text{ row } l_t(k) \\ 0 & \text{elsewhere} \end{cases} . \tag{37}$$

Now let $\mathbf{E}_{pp}$ and $\mathbf{E}_{pa}$ be matrices of ones of size $\#^p \times \#^p$ and $\#^p \times \#^a$, respectively, and (as in MATLAB) let the operator $.*$ represent element-by-element multiplication. Then the end-of-t distribution $\mathbf{\Psi}_t$ can be calculated from $\widetilde{\mathbf{\Psi}}_t$ as follows:

$$\mathbf{\Psi}_t = (\mathbf{E}_{pa} - \mathbf{\Lambda}_t) \;.*\; \widetilde{\mathbf{\Psi}}_t + \mathbf{P}_t \;.*\; (\mathbf{E}_{pp} * (\mathbf{\Lambda}_t \;.*\; \widetilde{\mathbf{\Psi}}_t)). \tag{38}$$

The same transition matrices show up when the Bellman equation is written in matrix form. Let $\mathbf{U}_t$ be the $\#^p \times \#^a$ matrix of current profits, with elements

$$u_t^{jk} = u(p^j, a^k, \Xi_t) = \left(p^j - \frac{w_t}{a^k}\right) C_t p_j^{-\epsilon} \tag{39}$$

for $\left(p^j, a^k\right) \in \Gamma$. Then the Bellman equation is simply

$$\mathbf{V}_t = \mathbf{U}_t + \beta E_t \left\{ \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \mathbf{R}'_{t+1} * (\mathbf{V}_{t+1} + \mathbf{G}_{t+1}) * \mathbf{S} \right\}, \tag{40}$$

where $\mathbf{G}_{t+1} = \mathbf{\Lambda}_{t+1} \ .* \ \mathbf{D}_{t+1}$ was defined by (32). Several comments may help clarify this Bellman equation. Note that the expectation E_t refers only to the effects of the time $t+1$ aggregate shock z_{t+1}, because multiplying by $\mathbf{R}'_{t+1}$ and $\mathbf{S}$ fully describes the expectation over the idiosyncratic state $(p^j, a^k) \in \Gamma$. $\mathbf{S}$ has no time subscript, since the Markov productivity process is not subject to aggregate shocks, whereas the inflation adjustment represented by $\mathbf{R}'_{t+1}$ varies with the policy shock. Also, while the distributional dynamics iterate forward in time, with transitions governed by $\mathbf{R}$ and $\mathbf{S}'$, the Bellman equation iterates backwards, so its transitions are described by $\mathbf{R}'$ and $\mathbf{S}$.

3.3. Computation: steady state

In an aggregate steady state, policy shocks z are zero, and transaction prices converge to an ergodic distribution $\mathbf{\Psi}$, so the aggregate state of the economy is constant: $\Xi_t = (z_t, R_{t-1}, \mathbf{\Psi}_{t-1}) = (0, R, \mathbf{\Psi}) \equiv \Xi$ under the Taylor rule, or $\Xi_t = (z_t, m_{t-1}, \mathbf{\Psi}_{t-1}) = (0, m, \mathbf{\Psi}) \equiv \Xi$ under a money growth rule. The steady state of any aggregate equilibrium object is indicated by dropping the subscript t.

The steady-state calculation nests the firm's backwards induction problem inside a loop that determines the steady-state real wage w. Note first that $\Pi = \mu^* = \beta R$ in steady state; hence the matrix $\mathbf{R}$ is known. Then, given w, (5) determines C, so all elements u^{jk} of $\mathbf{U}$ can be calculated from (39). Then, backwards induction on the grid Γ can solve the Bellman equation

$$\mathbf{V} = \mathbf{U} + \beta \mathbf{R}' * (\mathbf{V} + \mathbf{G}) * \mathbf{S}. \tag{41}$$

Solving (41) involves finding the matrices $\mathbf{V}$, $\mathbf{D}$, $\mathbf{\Lambda}$, and $\mathbf{G}$, so the matrix $\mathbf{P}$ can also be calculated from (37). Then (36) and (38) can be used to find the distributions $\widetilde{\mathbf{\Psi}}$ and $\mathbf{\Psi}$, and finally (4) serves to check the guessed value of w. In discretized notation, equation (4) becomes

$$1 = \sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} \Psi_t^{jk} \left(p^j\right)^{1-\epsilon}. \tag{42}$$

If (42) holds at the ergodic distribution $\mathbf{\Psi}_t = \mathbf{\Psi}$, then a steady-state equilibrium has been found.

3.4. Computation: dynamics

Bellman equation (40) and distribution dynamics (36)-(38) are usually viewed as functional equations. However, under the discretization of Sec. 3.2, they can also be seen as two long lists of difference equations describing the values and probabilities at all grid points. Thus, Reiter (2009) proposes linearizing these equations around their steady state, calculated in Sec. 3.3. To do so, it first helps to reduce the number of variables by eliminating simple intratemporal relationships. Under a money growth rule, the model can be described by the following vector of endogenous variables:

$$\overrightarrow{X}_t \equiv \left(vec\left(\mathbf{V}_t\right)', \quad C_t, \quad \Pi_t, \quad vec\left(\mathbf{\Psi}_{t-1}\right)', \quad m_{t-1}\right)' \tag{43}$$

Vector $\overrightarrow{X}_t$, together with the shock process z_t, consists of $2\#^p\#^a + 4$ variables determined by the following system of $2\#^p\#^a + 4$ equations: (40), (7), (42), (38), (28), and (16). Under a Taylor rule, m_{t-1} is replaced by R_{t-1}, and (7) and (28) are replaced by (6) and (18). Thus the difference equations governing dynamic equilibrium constitute a first-order system of the form

$$E_t\mathcal{F}\left(\overrightarrow{X}_{t+1}, \overrightarrow{X}_t, z_{t+1}, z_t\right) = 0, \tag{44}$$

where E_t is an expectation conditional on z_t and all previous shocks.[8] Next, system $\mathcal{F}$ can be linearized numerically to construct the Jacobian matrices $\mathcal{A} \equiv D_{\overrightarrow{X}_{t+1}}\mathcal{F}$, $\mathcal{B} \equiv D_{\overrightarrow{X}_t}\mathcal{F}$, $\mathcal{C} \equiv D_{z_{t+1}}\mathcal{F}$, and $\mathcal{D} \equiv D_{z_t}\mathcal{F}$. This yields the following first-order linear expectational difference equation system:

$$E_t\mathcal{A}\Delta\overrightarrow{X}_{t+1} + \mathcal{B}\Delta\overrightarrow{X}_t + E_t\mathcal{C}z_{t+1} + \mathcal{D}z_t = 0, \tag{45}$$

where Δ represents a deviation from steady state. This system has the form considered by Klein (2000), so it will be solved using his QZ decomposition method, though other linear rational expectations solvers would be applicable as well.

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[8]Given $(\overrightarrow{X}_{t+1}, \overrightarrow{X}_t, z_{t+1}, z_t)$, all other variables appearing in (40), (7), (42), (38), (28), and (16) can be substituted out using intratemporal equations. Given Π_t and Π_{t+1}, $\mathbf{R}_t$ and $\mathbf{R}_{t+1}$ are known; thus $\widetilde{\mathbf{\Psi}}_t = \mathbf{R}_t * \mathbf{\Psi}_{t-1} * \mathbf{S}'$ can be calculated too. The wage is given by (5), so $\mathbf{U}_t$ can be constructed. Finally, given $\mathbf{V}_t$ and $\mathbf{V}_{t+1}$ one can construct $\mathbf{P}_t$, $\mathbf{D}_t$, and $\mathbf{D}_{t+1}$, and thus $\mathbf{\Lambda}_t$ and $\mathbf{G}_{t+1}$. Therefore the arguments of $\mathcal{F}$ suffice to evaluate the system (44).

The virtue of Reiter's method is that it combines linearity and nonlinearity in a way appropriate for the context of price adjustment, where idiosyncratic shocks are larger and more economically important to individual firms than aggregate shocks. To deal with large idiosyncratic shocks, it treats functions of idiosyncratic states nonlinearly (calculating them on a grid). But in linearizing each equation at each grid point, it recognizes that aggregate changes (policy shocks z, or shifts of the distribution $\mathbf{\Psi}$) are unlikely to affect individual value functions in a strongly nonlinear way. On the other hand, it makes no assumption of approximate aggregation like that of Krusell and Smith (1998).

4. Results

4.1. Parameterization

Our calibration seeks price adjustment and productivity processes consistent with microdata on price changes, like those in Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), and Midrigan (2011). Since utility parameters are not the main focus, these are set to the values used by GL07. The discount factor is set to $\beta = 1.04^{-1}$ per year; the coefficient of relative risk aversion of consumption is set at $\gamma = 2$. The coefficients on labor disutility and the utility of money are $\chi = 6$ and $\nu = 1$, respectively, and the elasticity of substitution in the consumption aggregator is $\epsilon = 7$.

The main price data that will serve as an empirical benchmark are the monthly AC Nielsen data reported by Midrigan (2011).[9] Therefore, the model will be simulated at monthly frequency, with a zero steady state money growth rate, consistent with the zero average price change in that dataset. Midrigan reports the data after removing price changes attributable to temporary "sales", so our simulation results should be interpreted as a model of "regular" price changes unrelated to sales. Conditional on these specification choices, the adjustment parameters ($\bar{\lambda}$, α, and ξ) and productivity parameters (ρ and σ_ε^2) are chosen to minimize a distance criterion between the data and the model's steady state.[10] The criterion sums two terms, scaled for comparability: the first is the absolute

[9]However, we fit the model to Nakamura and Steinsson's (2008) measure of the median frequency of price adjustments. This is lower, but presumably more robust, than the mean adjustment frequency reported by Midrigan.

[10]The productivity process (8) is approximated on the grid Γ^a using Tauchen's method; we thank Elmar Mertens for making his software available. The productivity grid has 25 points, and the price grid Γ^p has 31 points. Both grids are logarithmically spaced; steps in Γ^p represent 4% changes. Results are not sensitive to the use of this coarse grid, since the average absolute price adjustment is much larger (around 10%).

difference between the adjustment frequencies in the data and the simulation, while the second is the Euclidean distance between the frequency vectors associated with the histograms of nonzero price adjustments in the data and the simulation.

Table 2 summarizes the steady-state behavior of the model under the estimated parameters, together with evidence from four empirical studies. The baseline specification, in which $\overline{\lambda}$, α, and ξ are all estimated, is labelled SSDP. The table also reports Calvo ($\overline{\lambda}$ estimated, $\xi \equiv 0$, and α undefined) and FMC specifications (α estimated, $\xi \equiv \infty$, and $\overline{\lambda}$ undefined), as well as a version based on Woodford's (2009) adjustment function and an SMC specification. All versions of the model match the target adjustment frequency of 10% per month. But the extreme cases of the model (Calvo and FMC) are less successful in fitting the size distribution of price adjustments than are the intermediate cases; the Calvo model understates the average size and standard deviation of price adjustments, whereas the FMC model overstates both.

The trouble with the FMC model, as Fig. 1 shows, is that it only produces price changes lying just outside the (S,s) bands, whereas the adjustments observed in the data are very diverse.[11] Thus the FMC model that best fits the data produces adjustments that are too large on average; no adjustments in the model are less than 5%, whereas one quarter of all adjustments are below the 5% threshold in the AC Nielsen data. The Calvo model errs in the opposite direction, with too many small price adjustments, though its fit statistics are better than those of the FMC model. In contrast, all three specifications with a smoothly increasing adjustment hazard (SSDP, SMC, and Woodford) match the data well, since they permit large and small price adjustments to coexist. In fact, there is so little difference between these models that only SSDP will be discussed from here on.[12]

Our estimates imply fairly strong frictions impeding price adjustment. The estimated function λ (see Fig. 1, right panel) rises quickly at zero but is thereafter very flat. It equals 10% per month at a loss of $L = 0.0235$ (6% of monthly labor input) and only reaches 30% per month at the highest loss that occurs with nonzero probability in the steady state equilibrium, which is $L = 7.91$, roughly 21

[11] Klenow and Kryvtsov (2008) document that large and small price changes coexist even within narrow product categories, and that the FMC model performs poorly even when menu costs are allowed to differ across sectors.

[12] Our companion paper, Costain and Nakov (2011), shows that the SSDP model performs somewhat better than Woodford's specification at high (*e.g.* 70% annually) inflation rates. But at low inflation rates, the responses to monetary shocks (available from the authors) are indistinguishable across the SSDP, SMC, and Woodford specifications.

months' worth of labor input. Of course, the λ function is flat in the Calvo model by construction. Our estimate of Woodford's specification implies $\lambda \approx 1$ at the most extreme losses that occur in equilibrium, but it is also very flat over the range of losses that occur frequently. For example, the cross-sectional standard deviation of λ is roughly 4% in SSDP and 3% in the Woodford setup, whereas it is 30% in the FMC model.

Thus, in the models considered, firms do not adjust instantly even when faced with very large losses. Nonetheless, typical losses in equilibrium are more moderate, since firms usually adjust before reaching extreme situations. Across specifications, the decrease in average profits due to price stickiness (see Table 2) ranges from 1.5% of average revenues in the FMC case to 5.3% of average revenues in the Woodford specification. (The differences look larger when expressed as a fraction of average profits, since profits are a small fraction of revenues.) Clearly, these estimated adjustment frictions are nontrivial; whether they seem unrealistically large may depend on whether they are conceived literally as "menu costs" or as costs of managerial decision making, along the lines of Zbaracki *et al.* (2004).

4.2. Effects of monetary policy shocks

Since all specifications are calibrated to the same observed adjustment frequency, the fact that only large, valuable price changes occur in the FMC model, whereas some changes in the SSDP and Calvo frameworks are trivial, has important implications for monetary transmission. Fig. 2 compares responses to several types of monetary shocks across these three adjustment specifications. All simulations assume the same utility parameters, and zero baseline inflation, and are calculated starting from the steady-state distribution associated with the corresponding specification. The first two rows show impulse responses to one percentage point money growth shocks, comparing the *i.i.d.* case with that of monthly autocorrelation $\phi_z = 0.8$. The third row shows the responses to an *i.i.d.* interest rate shock under a Taylor rule.

In all three models, an increase in money growth stimulates consumption. The fact that some prices fail to adjust immediately means expected inflation rises, decreasing the *ex ante* real interest rate; it also means households' real money balances increase; both of these effects raise consumption demand. However, as GL07 emphasized, the average price level adjusts rapidly in the FMC specification (lines with circles), with a large, short-lived spike in inflation. This makes changes in real variables small and transitory, approaching the monetary neutrality associated with full price flexibility. At the opposite extreme, prices adjust gradually in the Calvo specification (lines with squares), leading to a large, persistent increase in output. The response of the SSDP model (lines with dots) mostly lies between the other two, but is generally closer to that of the Calvo model.

Comparing the first and second rows of Fig. 2 shows that while the shape of the inflation and output responses differs substantially across models, it is qualitatively similar under *iid* and autocorrelated money growth processes. In the FMC model inflation spikes immediately regardless of the persistence of money growth. However, with autocorrelated money growth the initial spike exceeds 1% as firms anticipate that money growth will remain positive for some time. The rise in inflation is smaller but more persistent in the SSDP and Calvo cases. Note that the persistence of inflation does not differ noticeably depending on the autocorrelation of money growth, but instead appears to be determined primarily by the degree of state dependence. Thus the big difference between the impulse responses in the first and second rows is one of size, not of shape.

The third row of Fig. 2 shows responses under a Taylor rule, assuming that the underlying shock z is *i.i.d.*, and that the rule has inflation and output coefficients $\phi_\pi = 2$ and $\phi_c = 0.5$, and smoothing coefficient $\phi_R = 0.9$. While money growth shocks are small, incremental changes to the *level* of the nominal money supply, Taylor rule shocks involve large fluctuations in the level of nominal money. Nonetheless, the two types of monetary-policy shocks have similar real effects, and moreover, the finding that a micro-calibrated model of state-dependent pricing implies substantial monetary non-neutrality is strengthened in several ways by considering a Taylor rule. First, under the Taylor rule, the SSDP and Calvo impulse responses of inflation and consumption are even closer together than they were under the money growth rule. In fact, for consumption, both SSDP *and* FMC imply virtually the same effect on impact as that occurring in the Calvo model, though the effect is less persistent in the FMC case.

Recall, though, that the Taylor rule responses in Fig. 2 suppose an initial drop in the nominal interest rate of 25 basis points. Since the interest rate is endogenous, the required underlying shock ϵ^z varies across models, and it is particularly large in the FMC case. Therefore, it is useful to consider additional ways of comparing the degree of monetary nonneutrality across models. Thus, Table 3 compares monetary policies that imply the same inflation variability, as in Sec. VI of GL07. The calculation asks the following question: if monetary-policy shocks were the only source of observed US inflation volatility, how much output variation would they cause? Under the SSDP specification, money growth shocks alone would explain 65% of observed output fluctuations; the figure rises to 116% under the Calvo specification, and falls to 15% in the FMC case.[13] Assuming a Taylor rule, the differences across models are even stronger, and the monetary nonneutrality of the SSDP and Calvo specifications is even greater. Taylor rule shocks alone would explain 110% of US output fluctuation under the SSDP specification, rising to 306% in the Calvo case. The table also reports a "Phillips curve" coefficient, calculated by regressing log output on inflation, instrumented by the exogenous monetary policy shock. The conclusions are similar: the SSDP model implies large real effects of monetary shocks, closer to the Calvo specification than to the FMC specification, and the differences across models are larger under a Taylor rule than they are under a money growth rule.

Next, Fig. 3 plots the response of price dispersion, Δ_t^p, defined in (20). In our model, one reason prices vary is that firms face different productivities. But additional price dispersion, caused by failure to adjust when necessary, implies inefficient variation in demand across goods that implies a decrease in aggregate productivity: $C_t = N_t/\Delta_t^p$. In a representative agent model near a zero-inflation steady state, Δ_t^p is negligible because it is roughly proportional to the cross-sectional variance of prices, a quantity of second order in the inflation rate.[14] But cross-sectional price variance is not second order when large idiosyncratic shocks are present, so the dispersion wedge Δ_t^p may be quantitatively important, especially since $\epsilon = 7$ magnifies variations in the ratio P_{it}/P_t. The first row of Fig. 3 shows that for SSDP and Calvo, increased money growth throws firms' prices further out of line with fundamentals, increasing dispersion; raising consumption therefore requires a larger increase in labor in these specifications. In contrast, the variation in Δ_t^p is smaller in the FMC case, because all firms with severe price misalignments do in fact adjust. Interestingly, since the Taylor rule leans against inflationary shocks, there is much less variation in the price level for the SSDP and Calvo cases

[13] The table considers autocorrelated money growth shocks. The results for *i.i.d.* money growth are very similar, since, as demonstrated in Figure 2, correlation mostly changes the scale of the impulse responses, rather than their shape.

[14] See for example Galí (2008), p. 46 and Appendix 3.3.

in our Taylor rule simulation than there is under autocorrelated money growth. Thus, in all three specifications, a Taylor rule shock causes little variation in Δ^p_t.

4.3. Inflation decompositions

Several decompositions can help illustrate the inflation dynamics implied by this model. To a first-order approximation, inflation can be calculated as an average of log nominal price changes. Using our grid-based notation, and starting from the beginning-of-period distribution $\widetilde{\boldsymbol{\Psi}}_t$,

$$\pi_t = \log \Pi_t = \sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} x_t^{jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}, \tag{46}$$

where $x_t^{jk} \equiv \log\left(\frac{p^*(a^k, \Xi_t)}{p^j}\right)$ is the *desired* log price adjustment of a firm with price p^j and productivity a^k. Klenow and Kryvtsov (2008) rewrite (46) as the product of the average log price adjustment $\overline{x}_t$ times the frequency of price adjustment $\overline{\lambda}_t$:

$$\pi_t = \overline{x}_t \overline{\lambda}_t, \qquad \overline{x}_t \equiv \frac{\sum_{j,k} x_t^{jk} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}}{\sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}}, \qquad \overline{\lambda}_t \equiv \sum_{j,k} \lambda_t^{jk} \widetilde{\Psi}_t^{jk}. \tag{47}$$

Dropping higher-order terms, this implies the following inflation decomposition:

$$\Delta \pi_t = \overline{\lambda} \Delta \overline{x}_t + \overline{x} \Delta \overline{\lambda}_t, \tag{48}$$

where variables without time subscripts represent steady states, and Δ represents a deviation from steady state.[15] Klenow and Kryvtsov's "intensive margin", $\mathcal{I}_t^{KK} \equiv \overline{\lambda} \Delta \overline{x}_t$, is the part of inflation attributable to changes in the average price adjustment; their "extensive margin", $\mathcal{E}_t^{KK} \equiv \overline{x} \Delta \overline{\lambda}_t$, is the part due to changes in the frequency of adjustment.

Unfortunately, this decomposition does not reveal whether a rise in the average log price adjustment $\overline{x}_t$ is caused by a rise in all firms' desired adjustments, or by a reallocation of adjustment opportunities from firms desiring small or negative price changes to others wanting large price increases. That is, $\mathcal{I}_t^{KK}$ mixes changes in *desired* adjustments (the only relevant changes in the Calvo model) with the "selection effect" emphasized by GL07. To distinguish between these last two effects, inflation can instead be broken into three terms: an intensive margin capturing changes in the average *desired* log

[15] Actually, Klenow and Kryvtsov (2008) propose a time series variance decomposition, whereas (47) is a decomposition of each period's inflation realization. But the logic of (47) is the same as that in their paper.

price change, an extensive margin capturing changes in *how many* firms adjust, and a selection effect capturing changes in *who* adjusts. These three effects are distinguished by rewriting (46) as

$$\pi_t = \overline{x}^*_t \overline{\lambda}_t + \sum_{j,k} x^{jk}_t \left(\lambda^{jk}_t - \overline{\lambda}_t \right) \widetilde{\Psi}^{jk}_t, \qquad \overline{x}^*_t \equiv \sum_{j,k} x^{jk}_t \widetilde{\Psi}^{jk}_t. \tag{49}$$

Note that in (49), $\overline{x}^*_t$ is the average *desired* log price change, and *not* an average of the *actual* log price changes of those firms that do adjust (as was the case in 47). Thus, (49) says that inflation equals the mean desired adjustment times the adjustment frequency plus a selection term $\sum_{j,k} x^{jk}_t \left(\lambda^{jk}_t - \overline{\lambda}_t \right) \widetilde{\Psi}^{jk}_t = \sum_{j,k} \lambda^{jk}_t \left(x^{jk}_t - \overline{x}^*_t \right) \widetilde{\Psi}^{jk}_t$ that can be nonzero if some changes x^{jk}_t are more or less likely than the mean adjustment probability $\overline{\lambda}_t$, or (equivalently) if firms with different probabilities of adjustment λ^{jk}_t tend to prefer adjustments that differ from the mean desired change $\overline{x}^*_t$.

Equation (49) leads to the following inflation decomposition:

$$\Delta\pi_t = \overline{\lambda}\Delta\overline{x}^*_t + \overline{x}^*\Delta\overline{\lambda}_t + \Delta \sum_{j,k} x^{jk}_t \left(\lambda^{jk}_t - \overline{\lambda}_t \right) \widetilde{\Psi}^{jk}_t. \tag{50}$$

Our intensive margin effect, $\mathcal{I}_t \equiv \overline{\lambda}\Delta\overline{x}^*_t$, is the effect of changing all firms' desired adjustment by the same amount (or more generally, changing the mean preferred adjustment in a way that is uncorrelated with the adjustment probability). $\mathcal{I}_t$ is the only nonzero term in the Calvo model, where $\lambda^{jk}_t = \overline{\lambda}$ for all j, k, t. Our extensive margin effect, $\mathcal{E}_t \equiv \overline{x}^*\Delta\overline{\lambda}_t$, is the effect of changing the fraction of firms that adjust, assuming the new adjusters are selected randomly. Our selection effect, $\mathcal{S}_t \equiv \Delta \sum_{j,k} x^{jk}_t \left(\lambda^{jk}_t - \overline{\lambda}_t \right) \widetilde{\Psi}^{jk}_t$, is the effect of redistributing adjustment opportunities across firms with different desired changes x^{jk}_t, while fixing the overall fraction that adjust.

An alternative decomposition, proposed by Caballero and Engel (2007), also differences (46):

$$\Delta\pi_t = \sum_{j,k} \Delta x^{jk}_t \lambda^{jk} \widetilde{\Psi}^{jk} + \sum_{j,k} x^{jk} \Delta\lambda^{jk}_t \widetilde{\Psi}^{jk} + \sum_{j,k} x^{jk} \lambda^{jk} \Delta\widetilde{\Psi}^{jk}_t \tag{51}$$

They further simplify this to

$$\Delta\pi_t = \overline{\lambda}\Delta\mu_t + \sum_{j,k} x^{jk} \Delta\lambda^{jk}_t \widetilde{\Psi}^{jk} \tag{52}$$

under the assumption that all desired price adjustments change by $\Delta x^{jk}_t = \Delta\mu_t$ when money growth increases by $\Delta\mu_t$, and by taking an ergodic average so that the last term drops out.[16] Their first

[16] Our equation (50) is intended to decompose each period's inflation realization, so it allows for shifts in the current distribution $\widetilde{\Psi}^{jk}_t$. Caballero and Engel instead propose a decomposition (see their eq. 17) of the *average* impact of a

term, $\mathcal{I}_t^{CE} \equiv \Delta\mu_t\overline{\lambda}$, is the same as our intensive margin $\mathcal{I}_t$, if their assumption that all desired price adjustments change by $\Delta\mu_t$ is correct. But therefore, their "extensive margin" term $\mathcal{E}_t^{CE} \equiv \sum_{j,k} x^{jk}\Delta\lambda_t^{jk}\widetilde{\Psi}^{jk}$, confounds the question of *how many* firms adjust (our extensive margin $\mathcal{E}_t$) with the question of *who* adjusts (our selection effect $\mathcal{S}_t$), which is the mechanism stressed by GL07.

The importance of identifying the selection effect separately becomes clear in Fig. 4, which illustrates our decomposition of the inflation impulse response to monetary shocks. The three components of inflation, $\mathcal{I}_t$, $\mathcal{E}_t$, and $\mathcal{S}_t$, are shown to the same scale for better comparison. The graphs demonstrate (in contrast to Caballero and Engel's claim) that the short, sharp rise in inflation observed in the FMC specification results from the selection effect. This is true both under Taylor rule shocks, where inflation spikes to 1.5% on impact, of which 1.25% is the selection component, and under (autocorrelated) money growth shocks, where inflation spikes to 2.8%, with 2.25% due to selection. In contrast, inflation in the Calvo model is caused by the intensive margin only; in SSDP there is a nontrivial selection effect but it still only accounts for around one-third of the inflation response.

On the other hand, the extensive margin $\mathcal{E}_t \equiv \overline{x}^*\Delta\overline{\lambda}_t$ plays a negligible role in the inflation response. This makes sense, because the simulation assumes a steady state with zero inflation, so steady state price adjustments are responses to idiosyncratic shocks only, and the average desired adjustment $\overline{x}^*$ is very close to zero. Therefore $\mathcal{E}_t$ is negligible even though the adjustment frequency $\overline{\lambda}_t$ itself does vary.[17] The extensive margin only becomes important when there is high trend inflation, so that the average desired adjustment $\overline{x}^*$ is large and positive.

As for the intensive margin, its initial effect after a money growth shock is similar across all adjustment specifications, but it is more persistent in the Calvo and SSDP cases than in the FMC case. The scale of the intensive margin depends on the autocorrelation of money growth: the mean desired price change rises roughly one-for-one after an *i.i.d.* money growth shock (not shown), and rises by roughly five percentage points when money growth has autocorrelation $\phi_z = 0.8$ (first row of Fig. 4). Thus, in the autocorrelated case, the intensive margin is initially $\mathcal{I}_1 \equiv \overline{\lambda}\Delta\overline{x}_1^* \approx 0.5\%$.

monetary shock. Therefore they evaluate their decomposition at the ergodic distribution (the time average over all cross-sectional distributions, called $f_A(x)$ in their paper). Since this is a fixed starting point of their calculation, they do not need to include a $\Delta f_A(x)$ term.

[17]The fact that the steady state has exactly zero inflation is not crucial here; $\mathcal{E}_t$ is quantitatively insignificant compared to the other inflation components at any typical OECD inflation rate.

In other words, firms wish to "frontload" price adjustment by approximately the same amount in all three specifications; but many of these changes occur immediately in the FMC case (showing up as a redistribution of adjustment opportunities, *i.e.*, a selection effect), whereas they are realized gradually in the other specifications. Under a Taylor rule, the intuition is similar, bearing in mind that Fig. 4 is scaled to give an initial decline of 25 basis points in the nominal interest rate. This requires a larger underlying shock z in the FMC specification than in the other cases; thus the effect on the intensive margin is larger (but less persistent) for FMC than it is for Calvo and SSDP.

4.4. Comparing effects of sector-specific and aggregate shocks

Another issue of intererest in recent empirical literature is how prices respond to sector-specific shocks. In particular, Boivin, Giannoni, and Mihov (2007) and Mackowiak, Moench, and Wiederholt (2009, henceforth MMW09) present evidence that sector-specific prices respond much more quickly to sector-specific shocks than they do to aggregate shocks. This is important, since it suggests that a Calvo model with a single adjustment rate may be inappropriate. Indeed, it might be interpreted as evidence for state dependence, and it suggests that the present model might be tested by assessing its ability to reproduce these empirical observations.

To address these questions, this section investigates "sector-specific" shocks in our model, applying the estimation routines of MMW09 to artificial panel data produced by simulating the SSDP calibration under a Taylor rule. The data cover the price levels in 79 sectors over 245 months, as in the dataset of MMW09. Of course, the model defined here does not actually have a sectoral structure. Nonetheless, for a fixed integer $\#^s > 0$, one can simulate a panel of $79\#^s$ firms (each producing one specialized product), and call each block of $\#^s$ consecutive firms a "sector". Productivity innovations remain *i.i.d.* across firms, as they are elsewhere in the paper. However, since the number of firms per sector is finite, sampling error will cause average productivity to differ across sectors at each time. An innovation to average productivity in any artificially-defined sector can thus be regarded as a sector-specific shock. Two questions are then relevant. First, can empirical findings like those of MMW09 be reproduced by applying their methods to the model-generated data? Second, do their estimation methods correctly identify the effects of sector-specific shocks? The answers are yes and no, respectively.

The MMW09 statistical framework breaks inflation into aggregate and sector-specific components:

$$\pi_{n,t} = \mu_n + A_n(L)u_t + B_n(L)v_{n,t} \tag{53}$$

where $\pi_{n,t}$ is the inflation rate in sector n at time t, μ_n is a sector-specific constant, $A_n(L)$ and $B_n(L)$ are sector-specific lag polynomials, u_t is an aggregate shock, and $v_{n,t}$ is a sector-specific shock. Inflation $\pi_{n,t}$ and the shocks u_t and $v_{n,t}$ are all scaled to have unit variance. Fig. 5 shows estimated impulse responses of sector-specific price levels to $v_{n,t}$ (left column) and u_t (right column), identified by applying the Bayesian estimation programs of MMW09 to sectoral panel simulations from the SSDP model (one- and two-standard-error bands are shown too). All the impulse responses in the left column are consistent with the main finding of MMW09 (see Fig. 1 of their paper): the identified sector-specific shocks cause an immediate, permanent rise in prices, with little change thereafter. In other words, the inflation associated with sector-specific shocks is essentially white noise. In contrast, the right column shows that the reaction to aggregate shocks is more gradual.

MMW09 also find that the impact of an sectoral shock on sectoral inflation is almost one-for-one, and that sectoral shocks account for around 90% of sectoral inflation variance. Fig. 5 shows that if simulated sectors are small (8 or 64 equally-weighted products), then the impact of a sectoral shock is indeed almost one-for-one; moreover, in these cases sectoral shocks explain 80% to 90% of sectoral inflation (see Fig. 6). However, if sectors consist of 512 equally-weighted products, then a one-standard deviation sectoral shock creates only 0.7 standard deviations of inflation, and sectoral shocks explain less than half of sectoral inflation variance. This is a consequence of the law of large numbers: with more firms per sector, sector-specific inflation stays closer to its conditional expectation, so aggregate shocks must explain a larger part of sectoral inflation (see the right-hand column of Fig. 5). Typical observations of sectoral inflation in MMW09's CPI data involve several hundred individual price quotes, so sectors with 8 or 64 products are unrealistically small.[18] But on the other hand, CPI weights vary greatly across products in each sector (Leaver and Folk, 2004). Therefore the fourth row of the figure reports a simulation with 512 products per sector, in which the CPI weights on the products are

[18]The BLS collects approximately 80000 prices each month to calculate the CPI; roughly 70% of these prices correspond to the 79 sectors included in the MMW09 estimates. So the typical observation of sectoral inflation averages several hundred individual prices. We thank an anonymous referee for providing these details.

distributed according to Zipf's law.[19] This increases the importance of the highest-weighted goods, so 512 products with heterogeneous weights act like a much smaller number of equally-weighted products, with a contribution of sectoral shocks to sectoral inflation variance exceeding 85% (see Fig. 6.)

Thus, running the estimation programs of MMW09 on simulated data from our model with 512 firms of heterogeneous size largely reproduces their empirical findings.[20] However, this is rather puzzling, because the estimation results are *inconsistent* with the *known* properties of the simulated model. In the model, prices only adjust once in ten months on average, and the degree of state-dependence is low, so the true response of prices to any idiosyncratic or aggregate shock must be fairly slow. Moreover, all sector-specific behavior in the model is mean-reverting, whereas the estimates in Fig. 5 show a permanent effect of a sector-specific shock on prices. To demonstrate these facts numerically, we consider the sector-specific shock $\bar{\epsilon}^a_{n,t}$, defined as the weighted average of firm-specific productivity shocks $\epsilon^a_{i,t}$ across firms in sector n. We assume inflation can be written as a moving average of these sectoral shocks and the aggregate monetary shock ϵ^z_t:

$$\pi_{n,t} = \mu_n + A_n(L)\epsilon^z_t + B_n(L)\bar{\epsilon}^a_{n,t} + \epsilon^\pi_{n,t} \tag{54}$$

The notation is the same here as in (53). However, sector-specific inflation will not generally equal its predicted value conditional on the underlying shocks, so this specification must allow for a sector-specific inflation residual $\epsilon^\pi_{n,t}$. In contrast, (53) attributes any inflation unexplained by the aggregate shock to the sector-specific shock, by construction.

Fig. 7 reports the responses to the sector-specific shock $\bar{\epsilon}^a_{n,t}$ (left column) and the aggregate shock ϵ^z_t, using the same simulated datasets analyzed in Fig. 5 (for accuracy, the length of each dataset is extended to 5400 months.) Responses are estimated by OLS on a sector-by-sector basis; all sectoral estimates are shown in the same graph. For all four datasets, responses to sector-specific and aggregate shocks occur with a lag. The peak response to a sector-specific shock occurs five to ten months after the time of the shock; the sectoral price level thereafter reverts to steady state. The time of reaction to an aggregate shock is similar, but the effects are permanent. The only effect of increasing the number

[19]That is, the weight of the jth-largest firm in sector n in that sector's CPI is proportional to j^{-1}.

[20]Fig. 2 of MMW09 also reports "speed of response" statistics showing that sector-specific inflation reacts more quickly sector-specific shocks. The same result obtains in our simulations; the graphs are available from the authors.

of firms per sector is that aggregate shocks become more important for sectoral inflation, relative to sectoral shocks, for the reasons discussed previously.

Why does the MMW09 estimation routine find effects of sectoral shocks so different from the response to the true sectoral shock, shown in Fig. 7? The problem is that true shocks in microdata are unknown to an econometrician, so Mackowiak *et al.* must identify sectoral shocks as residual price increases not explained by aggregate shocks. In the SSDP model, individual prices typically respond with a lag to true productivity shocks, so sectoral price levels do too. But in the MMW09 decomposition, the moment of the shock *corresponds by assumption* to the moment of the price increase, so the response is estimated to be immediate. Mean reversion occurs by individual stochastic price jumps in the model, whereas MMW09 assumes past shocks decay deterministically (component Bv in their eq. 1). Hence their method interprets price movements back to the mean as a sequence of new sectoral shocks that happen to go in the opposite direction (which is why the initial shock is interpreted as permanent). Thus, results from their procedure (or others that identify sectoral shocks as inflation residuals, *e.g.* Boivin *et al.*, 2007) should be treated with caution. Our Monte Carlo exercise shows that, at least in some cases, the procedure may exaggerate the speed of response to sectoral shocks, suggesting stronger state dependence than the data actually warrant.

5. Conclusions

This paper has computed the impact of monetary policy shocks in a quantitative macroeconomic model of state-dependent pricing. It has calibrated the model for consistency with microeconomic data on firms' pricing behavior, estimating how the probability of price adjustment depends on the value of adjustment. Given the estimated adjustment function, the paper has characterized the dynamics of the distribution of prices and productivities in general equilibrium.

The calibrated model implies that prices rise gradually after a monetary stimulus, causing a large, persistent rise in consumption and labor. Looking across specifications, the main factor determining how monetary shocks propagate through the economy is the degree of state dependence. That is, raising the autocorrelation of money growth shocks just makes their effects proportionally larger, without any notable change in the shape or persistence of the impulse responses. In contrast, decreasing state dependence from the extreme of fixed menu costs (FMC) to the opposite Calvo (1983) extreme strongly damps the initial inflation spike caused by a money shock and increases its effect on real variables. The parameterization most consistent with microdata (labelled "SSDP" throughout the paper) is fairly close to the Calvo model in terms of its quantitative effects. The conclusions are similar if the monetary authority follows a Taylor rule instead of a money growth rule, except that the degree of monetary nonneutrality differs more across adjustment specifications; in particular, the nonneutrality of the SSDP specification is increased.

This paper also decomposes the impulse response of inflation into an intensive margin effect relating to the average desired price change, an extensive margin effect relating to the number of firms adjusting, and a selection effect relating to the relative frequencies of small and large or negative and positive adjustments. Under the preferred (SSDP) calibration, about two-thirds of the effect of a monetary shock comes through the intensive margin, and most of the rest through the selection effect. The extensive margin is negligible unless the economy starts from a high baseline inflation rate. Under the FMC specification, a monetary shock instead causes a quick increase in inflation, driven by the selection effect, which eliminates most of its effects on real variables.

Since the selection effect represents changes in the adjustment probability across firms, its strength depends directly on the degree of state dependence. We say that state dependence is strong in the FMC model because it makes λ a step function: at the threshold, a tiny increase in the value of adjustment raises the adjustment probability from 0 to 1. Therefore the histogram of price changes consists of two spikes: there are no small changes, and firms change their prices as soon as they pass the adjustment thresholds. Hence, in steady state, those firms that might react to monetary policy are all near the two thresholds; a monetary stimulus decreases λ from 1 to 0 for some firms desiring a price decrease, while increasing λ from 0 to 1 for others preferring an increase, making the inflation response quick and intense. That is, the same property that makes money nearly neutral in the FMC model is the one which makes that model inconsistent with price microdata. A model in which adjustment depends more smoothly on the value of adjusting fits microdata better and yields larger real effects of monetary policy. Our two other smooth specifications (SMC, and Woodford's hazard function) yield results similar to the SSDP setup that was our main focus.

Low state dependence might seem inconsistent, at first, with recent empirical claims that prices react more quickly to sectoral shocks than to aggregate shocks. Indeed, our calibrated model implies that prices react only gradually to sectoral shocks. However, this paper demonstrates that the empirical methods of Mackowiak *et al.* (2009) may attribute an immediate, permanent impact to sectoral shocks even in a dataset where the true sectoral shocks have a lagged, temporary effect. The problem is that by treating any sector-specific change in inflation as a sectoral shock, they may confound sampling error in the timing of price adjustments with fundamental shocks. Applying the MMW09 estimation routines to simulated data from our model suggests that this problem may suffice to explain their empirical results, calling into question the evidence for price flexibility at the sectoral level.

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6. Appendix: notation

Table N1: Exogenous parameters

Symbol	Definition	Where
Preferences of household		
β	Utility discount factor	Sec. 2.1.
γ	Coefficient of relative risk aversion	Sec. 2.1.
χ	Disutility of labor	Sec. 2.1.
ν	Coefficient on utility of money	Sec. 2.1.
ϵ	Elasticity of substitution across differentiated goods	Sec. 2.1.
Technology of firms		
ρ	Persistence of firm-specific productivity	Sec. 2.2.
σ_a^2	Variance of firm-specific productivity shock	Sec. 2.2.
$\bar{\lambda}$	Adjustment probability parameter	Sec. 2.2.1.
α	Adjustment probability parameter	Sec. 2.2.1.
ξ	Adjustment probability parameter	Sec. 2.2.1.
Monetary policy		
ϕ_z	Persistence of monetary policy process	Sec. 2.3.
σ_z^2	Variance of monetary policy shock	Sec. 2.3.
μ^*	Money growth target	Sec. 2.3.
Π^*	Inflation target in Taylor rule	Sec. 2.3.
C^*	Output target in Taylor rule	Sec. 2.3.
R^*	Interest rate target in Taylor rule	Sec. 2.3.
ϕ_R	Interest smoothing parameter in Taylor rule	Sec. 2.3.
ϕ_π	Inflation weighting parameter in Taylor rule	Sec. 2.3.
ϕ_c	Output weighting parameter in Taylor rule	Sec. 2.3.

Table N2: Endogenous variables, nominal representation

Symbol	Definition	Where
Aggregate state		
Ω_t	Nominal aggregate state	Sec. 2.1. and 2.3.
Note: In the nominal representation, any aggregate variable indexed by t is determined, in equilibrium, as a function of the time t state Ω_t. Sometimes this functional relationship will be written explicitly, *e.g.* $W_t = W(\Omega_t)$.		
Variables appearing in household's problem		
C_t	Real household consumption	Sec. 2.1.
N_t	Labor supply	Sec. 2.1.
M_t	Nominal money supply	Sec. 2.1.
P_t	Nominal price level	Sec. 2.1.
W_t	Nominal wage	Sec. 2.1.
R_t	Nominal interest factor from t to $t+1$	Sec. 2.1.
B_t	Nominal bonds held at t to pay off in $t+1$	Sec. 2.1.
T_t	Nominal lump sum transfer to household at time t	Sec. 2.1.
C_{it}	Consumption of good produced by firm i	Sec. 2.1.
P_{it}	Price of good produced by firm i	Sec. 2.1.
Other aggregate variables		
$\widetilde{\Phi}_t$	Distribution of productivities and nominal prices at beginning of t	Sec. 2.2.
Φ_t	Distribution of productivities and nominal prices at end of t	Sec. 2.2.
z_t	Stochastic process driving monetary policy	Sec. 2.3.
ϵ_t^z	Monetary policy shock	Sec. 2.3.
μ_t	Monetary growth factor from $t-1$ to t	Sec. 2.3.
Δ_t^p	Price dispersion statistic	Sec. 2.3.

Table N3: Endogenous variables, nominal representation – continued

Symbol	Definition	Where
Variables specific to firm i		
Y_{it}	Real output of firm i at time t	Sec. 2.2.
A_{it}	Productivity of firm i at time t	Sec. 2.2.
ϵ^a_{it}	Productivity shock to firm i at time t	Sec. 2.2.
N_{it}	Labor input to firm i at time t	Sec. 2.2.
U_{it}	Nominal profits of firm i at time t	Sec. 2.2.
$\widetilde{P}_{it}$	Nominal price of output of firm i at beginning of t	Sec. 2.2.
Note: prior to selling (at end of period t), price $\widetilde{P}_{it}$ may or may not be adjusted.		
The nominal price at which firm i sells in period t is called P_{it}		
(hence P_{it} is the price that appears in the household's problem.)		
Functions describing firm behavior in equilibrium		
$U(P, A, \Omega)$	Nominal profits of firm with productivity A that sells at price P in state Ω	Sec. 2.2.
$V(P, A, \Omega)$	Nominal value of firm with productivity A that sells at price P in state Ω	Sec. 2.2.
$V^*(A, \Omega)$	Optimal value of firm with productivity A in state Ω	Sec. 2.2.
$P^*(A, \Omega)$	Optimal nominal price of firm with productivity A in state Ω	Sec. 2.2.
$D(\widetilde{P}, A, \Omega)$	Nominal gain from adjusting, given beginning-of-period nominal price $\widetilde{P}$	Sec. 2.2.
$L(\widetilde{P}, A, \Omega)$	Real gain from adjusting, given beginning-of-period nominal price $\widetilde{P}$	Sec. 2.2.
$\lambda(L)$	Probability of price adjustment, given real gain L from adjusting	Sec. 2.2.
$G(\widetilde{P}, A, \Omega)$	Expected nominal gains from stochastic adjustment in current period	Sec. 2.2.

Table N4: Endogenous variables, real representation

Symbol	Definition	Where
Aggregate state		
Ξ_t	Real aggregate state	Sec. 3.1.
Note: In the real representation, any aggregate variable indexed by t is determined, in equilibrium, as a function of the time t state Ξ_t.		
Aggregate variables		
m_t	Real money supply	Sec. 3.1.
w_t	Real wage	Sec. 3.1.
Π_t	Inflation factor from $t-1$ to t	Sec. 3.1.
$\widetilde{\Psi}_t$	Distribution of productivities and real prices at beginning of t	Sec. 3.1.
Ψ_t	Distribution of productivities and real prices at end of t	Sec. 3.1.
Note: C_t, N_t, R_t, z_t have the same meaning in the real and nominal representations. Some variables defined in the nominal representation are not mentioned in the real representation.		
Variables specific to firm i		
$\widetilde{p}_{it}$	Real price of firm i at beginning of period t	Sec. 3.1.
p_{it}	Real price of firm i at end of period t	Sec. 3.1.
Functions describing firm behavior in equilibrium		
$u(P, A, \Xi)$	Real profits of firm with productivity A that sells at real price p in state Ξ	Sec. 3.1.
$v(P, A, \Xi)$	Real value of firm with productivity A that sells at real price p in state Ξ	Sec. 3.1.
$p^*(A, \Xi)$	Optimal real price of firm with productivity A in state Ξ	Sec. 3.1.
$d(\widetilde{p}, A, \Xi)$	Real gain from adjusting, given beginning-of-period real price $\widetilde{p}$	Sec. 3.1.
$g(\widetilde{p}, A, \Xi)$	Expected real gains from stochastic adjustment in current period	Sec. 3.1.
Note: Function λ has the same meaning in the real and nominal representations.		

Table N5: Discretized real representation

Symbol	Definition	Where
Discretization		
Note: In the discretized real representation, superscripts indicate notation related to grids, and bold face indicates matrices and vectors.		
$\mathcal{R}$	The real numbers	Sec. 3.2.
Γ^a	Finite grid of possible values of productivity	Sec. 3.2.
a^k	Element k of grid Γ^a	Sec. 3.2.
$\#^a$	Number of elements of grid Γ^a	Sec. 3.2.
Γ^p	Finite grid of possible values of real price	Sec. 3.2.
p^j	Element j of grid Γ^p	Sec. 3.2.
$\#^p$	Number of elements of grid Γ^p	Sec. 3.2.
Γ	Two dimensional grid of prices and productivities, $\Gamma = \Gamma^p \times \Gamma^a$	Sec. 3.2.
Endogenous variables		
Note: Firm-specific variables A_{it}, $\widetilde{p}_{it}$, and p_{it} have the same meanings as in previous representations. Aggregate variables Ξ_t, C_t, N_t, Π_t, R_t, m_t, and z_t have the same meanings as in previous representations. Steady states of aggregate variables are indicated by dropping time subscripts.		
Matrix notation describing discretized problem of firm		
$\mathbf{U}_t$	Profits matrix, with elements $u_t^{jk} \equiv u(p^j, a^k, \Xi_t)$	Sec. 3.2.
$\mathbf{V}_t$	Value matrix, with elements $v_t^{jk} \equiv v(p^j, a^k, \Xi_t)$	Sec. 3.2.
$\mathbf{p}_t^*$	Policy vector, with elements $p_t^{*k} \equiv \arg\max_{p\in\mathcal{R}} v(p, a^k, \Xi_t)$	Sec. 3.2.
$\mathbf{D}_t$	Adjustment gains matrix, with elements $d_t^{jk} \equiv \max_{p\in\mathcal{R}} v(p, a^k, \Xi_t) - v_t^{jk}$	Sec. 3.2.
$\mathbf{\Lambda}_t$	Adjustment probabilities matrix, with elements $\lambda_t^{jk} \equiv \lambda(d_t^{jk}/w_t)$	Sec. 3.2.
$\mathbf{G}_t$	Matrix of expected gains from adjustment, with elements $g_t^{jk} \equiv \lambda_t^{jk} d_t^{jk}$	Sec. 3.2.
Note: Function λ has the same meaning it had in previous representations. Steady states of these matrices are indicated by dropping time subscripts.		

Table N6: Discretized real representation, continued

Symbol	Definition	Where
Matrix notation describing distributional dynamics		
$\widetilde{\boldsymbol{\Psi}}_t$	Beginning-of-period distribution matrix, with elements $\widetilde{\Psi}_t^{jk} \equiv \widetilde{\Psi}_t(p^j, a^k)$	Sec. 3.2.
$\boldsymbol{\Psi}_t$	End-of-period distribution matrix, with elements $\Psi_t^{jk} \equiv \Psi_t(p^j, a^k)$	Sec. 3.2.
$\mathbf{E}_{pp}$, $\mathbf{E}_{pa}$	Matrices of ones, of sizes $\#^p \times \#^p$ and $\#^p \times \#^a$, respectively	Sec. 3.2.
$\mathbf{S}$	Markov productivity matrix, with elements $S^{mk} \equiv prob(A_{it} = a^m \vert A_{i,t-1} = a^k)$	Sec. 3.2.
$\mathbf{R}_t$	Markov matrix for inflation adjustment and stochastic rounding to grid Γ^p, with elements $R_t^{ml} \equiv prob(\widetilde{p}_{it} = p^m \vert p_{i,t-1} = p^l)$, conditional on inflation $\Pi_t = \Pi(\Xi_t, \Xi_{t-1})$	Sec. 3.2.
$l_t(k)$	Index of least grid element above preferred real price: $p^{l_t(k)} \equiv \min\{p \in \Gamma^p : p \geq p_t^{*k}\}$	Sec. 3.2.
$\mathbf{P}_t$	Matrix allocating newly adjusted prices to optimum value p_t^{*k}, with mean-preserving stochastic rounding to grid Γ^p	Sec. 3.2.
Note: Steady states of these objects are indicated by dropping time subscripts.		
Linearization of dynamics		
$\overrightarrow{X}_t$	Vector of variables in dynamic computation	Sec. 3.4.
$\mathcal{F}$	Equation system linearized for dynamic computation	Sec. 3.4.
$\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$	Jacobian matrices appearing in linearized equation system	Sec. 3.4.
Note: Deviation between time t value and steady state is denoted by Δ.		

Table N7: Inflation decomposition and sectoral shocks

Symbol	Definition	Where
Inflation decomposition		
π_t	Inflation rate: $\pi_t = \log \Pi_t$	Sec. 4.3.
$\overline{\lambda}_t$	Fraction of firms adjusting prices at time t	Sec. 4.3.
x_t^{jk}	Desired log price adjustment, given real price p^j and productivity a^k	Sec. 4.3.
$\overline{x}_t^*$	Average desired log price adjustment at time t (across all firms)	Sec. 4.3.
$\overline{x}_t$	Average log price adjustment at time t (across firms that adjust)	Sec. 4.3.
$\mathcal{I}_t$, $\mathcal{E}_t$, $\mathcal{S}_t$	Intensive, extensive, and selection margins of inflation deviation	Sec. 4.3.
$\mathcal{I}_t^{KK}$, $\mathcal{E}_t^{KK}$	Intensive and extensive margins (Klenow and Kryvstov definition)	Sec. 4.3.
$\mathcal{I}_t^{CE}$, $\mathcal{E}_t^{CE}$	Intensive and extensive margins (Caballero and Engel definition)	Sec. 4.3.
Note: Steady states of these objects are indicated by dropping time subscripts.		
Deviation between time t value and steady state is denoted by Δ.		
Sector-specific shocks		
$\pi_{n,t}$	Inflation rate in sector n	Sec. 4.4.
μ_n	Sector-specific mean inflation	Sec. 4.4.
$A_n(L)$	Sector-specific lag polynomial on aggregate shocks	Sec. 4.4.
$B_n(L)$	Sector-specific lag polynomial on sectoral shocks	Sec. 4.4.
u_t	Aggregate inflation shock identified by MMW09 methodology	Sec. 4.4.
$v_{n,t}$	Aggregate inflation shock identified by MMW09 methodology	Sec. 4.4.
ϵ_t^z	True shock to Taylor rule	Sec. 2.3.
$A_{n,t}$	Shock to average productivity in sector n	Sec. 4.4.
$\epsilon_{n,t}$	Unexplained residual inflation in sector n	Sec. 4.4.

Tables for "Distributional Dynamics with Smoothly State-Dependent Pricing"

James Costain[a], Anton Nakov[b]

[a] Banco de España; [b] Federal Reserve Board

Table 1: Adjustment specifications

Specification	Adjustment probability $\lambda(L)$	Mean gains, in units of time: $G(P,A,\Omega)/W(\Omega)$
Calvo	$\bar{\lambda}$	$\bar{\lambda}L(P,A,\Omega)$
Fixed MC	$\mathbf{1}\{L \geq \alpha\}$	$\lambda(L(P,A,\Omega))\,[L(P,A,\Omega)-\alpha]$
Woodford	$\bar{\lambda}/[\bar{\lambda}+\left(1-\bar{\lambda}\right)\exp(\xi(\alpha-L))]$	$\lambda(L(P,A,\Omega))\,L(P,A,\Omega)$
Stoch. MC	$\bar{\lambda}/[\bar{\lambda}+\left(1-\bar{\lambda}\right)(\alpha/L)^{\xi}]$	$\lambda(L(P,A,\Omega))\,[L(P,A,\Omega)-E\left(\kappa\vert\kappa<\lambda\left(L(P,A,\Omega)\right)\right)]$
SSDP	$\bar{\lambda}/[\bar{\lambda}+\left(1-\bar{\lambda}\right)(\alpha/L)^{\xi}]$	$\lambda(L(P,A,\Omega))\,L(P,A,\Omega)$

Note: $\lambda(L)$ is the probability of price adjustment; L is the real loss from failure to adjust, as a function of firm's price P and productivity A, and aggregate conditions Ω. G represents mean nominal gains from adjustment; dividing by the nominal wage W converts gains to real terms. $\bar{\lambda}$, α and ξ are parameters to be estimated.

Table 2. Steady-state simulated moments for alternative estimated models and evidence

Model	Productivity parameters See eq. (8) for definitions	Adjustment parameters See Table 1 for definitions
Calvo	$(\sigma_\varepsilon, \rho) = (0.0850, 0.8540)$	$\bar{\lambda} = 0.10$
Fixed MC	$(\sigma_\varepsilon, \rho) = (0.0771, 0.8280)$	$\alpha = 0.0665$
Woodford	$(\sigma_\varepsilon, \rho) = (0.0924, 0.8575)$	$(\bar{\lambda}, \alpha, \xi) = (0.0945, 0.0611, 1.3335)$
Stochastic MC	$(\sigma_\varepsilon, \rho) = (0.0676, 0.9003)$	$(\bar{\lambda}, \alpha, \xi) = (0.1100, 0.0373, 0.2351)$
SSDP	$(\sigma_\varepsilon, \rho) = (0.0677, 0.9002)$	$(\bar{\lambda}, \alpha, \xi) = (0.1101, 0.0372, 0.2346)$

Moments	Calvo	FMC	Wdfd	SMC	SSDP	MAC	MD	NS	KK
Frequency of price changes	10.0	10.0	10.0	10.0	10.0	20.5	19.2	**10**	13.9
Mean absolute price change	6.4	17.9	10.3	10.0	10.1	**10.5**	7.7		11.3
Std of price changes	8.2	18.4	13.6	12.2	12.2	**13.2**	10.4		
Kurtosis of price changes	3.5	1.3	4.0	2.9	2.9	**3.5**	5.4		
% price changes $\leq$5% in abs value	47.9	0.0	37.0	26.3	26.3	**25**	47		44
Mean loss in % of frictionless profit	36.8	10.6	37.4	25.6	25.6				
Mean loss in % of frictionless revenue	5.2	1.5	5.3	3.6	3.6				
Fit: Kolmogorov-Smirnov statistic	0.111	0.356	0.038	0.024	0.025				
Fit: Euclidean distance	0.159	0.409	0.072	0.060	0.056				

Note: Price statistics refer to non-sale consumer price changes and are stated in percent. The last four columns report statistics from Midrigan (2011) for AC Nielsen (MAC) and Dominick's (MD), Nakamura and Steinsson (2008) (NS), and Klenow and Kryvtsov (2008) (KK). To calibrate the productivity parameters ρ and σ_ε^2, together with the adjustment parameters $\bar{\lambda}$, α and ξ, we minimize a distance criterion with two terms, (1) the difference between the median frequency of price changes in the model (fr) and in the data, and (2) the distance between the histogram of log price changes in the model ($histM$) and the data ($histD$): $\min(25\,\|fr - 0.10\| + \|histM - histD\|)$.

Table 3. Variance decomposition and Phillips curves of alternative models

	Data	SSDP model	Calvo model	FMC model
Std of quarterly inflation (×100)	0.246	0.246	0.246	0.246
% explained by nominal shock		100	100	100
Money growth rule (see eq. 16-17)				
Std of money growth shock (×100)		0.174	0.224	0.111
Std of detrended output (×100)	0.909	0.586	1.053	0.121
% explained by money growth shock		64.5	115.9	13.3
Slope coeff. of the Phillips curve		0.598	1.069	0.134
Standard error		0.004	0.039	0.005
Taylor rule (see eq. 18)				
Std of Taylor rule shock (×100)		0.393	0.918	0.129
Std of detrended output (×100)	0.909	0.995	2.741	0.134
% explained by Taylor rule shock		109.6	301.6	14.7
Slope coeff. of the Phillips curve		1.055	2.785	0.126
Standard error		0.093	0.290	0.006

Note: for each monetary regime (Taylor or money growth rule) and each pricing model, the nominal shock is scaled to account for 100% of the standard deviation of inflation. The volatility of output in the data is measured as the standard deviation of HP-filtered quarterly log real GDP. The "slope coefficients" are the estimates of β_2 in a 2SLS regression of (log) consumption on inflation, instrumented by the exogenous nominal shock. The first stage regression is $\pi_t^q = \alpha_1 + \alpha_2 \mu_t^q + \epsilon_t$, and the second stage is $c_t^q = \beta_1 + \beta_2 (4\hat{\pi}_t^q) + \varepsilon_t$, where $\hat{\pi}_t^q$ is the prediction for inflation from the first-stage and the superscript q denotes conversion to quarterly frequency.

Figures for “Distributional Dynamics with Smoothly State-Dependent Pricing”

James Costain[a], Anton Nakov[b]

[a] Banco de España; [b] Federal Reserve Board

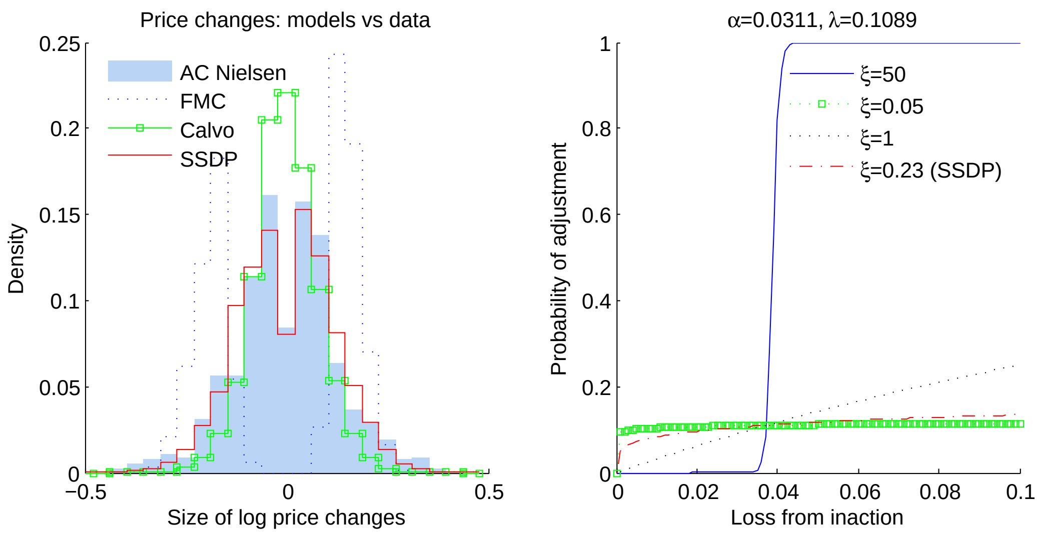

Fig. 1. Price change distributions and adjustment function

Note: size distribution of changes in log prices: data vs. models (left panel). Adjustment function λ for alternative values of state dependence ξ (right panel).

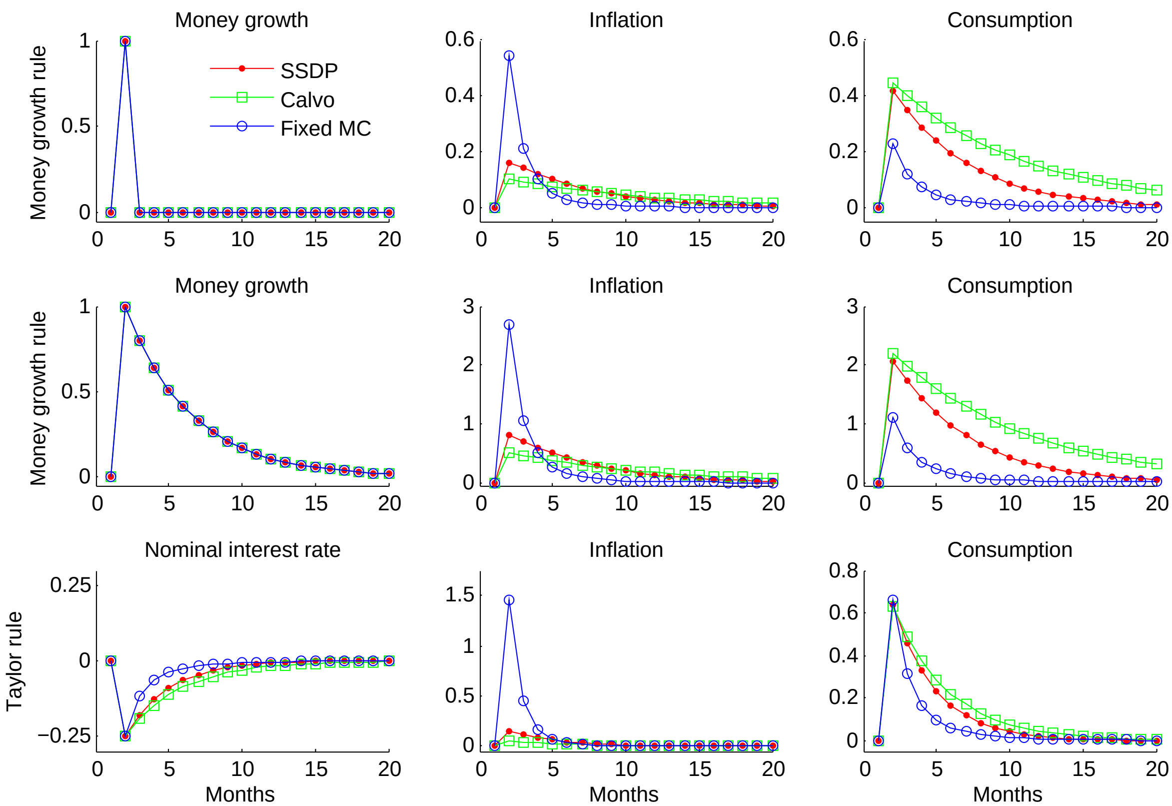

Fig. 2. The real effects of nominal shocks across models

Note: responses of inflation and consumption to an iid money growth shock (top row); responses to a correlated money growth shock (middle row); responses to a Taylor rule shock (bottom row). Inflation responses are in percentage points; consumption responses are in percent deviation from steady-state. Lines with dots - benchmark SSDP model; lines with squares - Calvo; lines with circles - fixed menu costs.

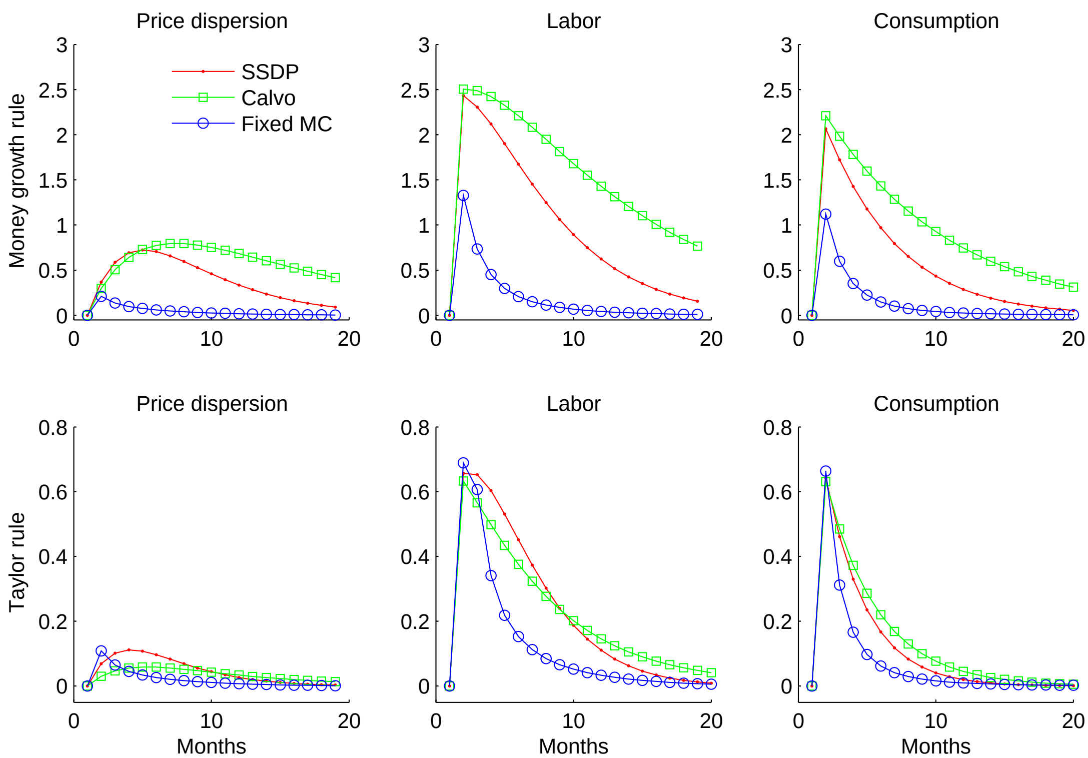

Fig. 3. Price dispersion across models

Note: responses to a correlated money growth shock (top row); responses to a Taylor rule shock (bottom row). The responses are in percent deviation from steady-state. Lines with dots - benchmark SSDP model; lines with squares - Calvo; lines with circles - fixed menu costs.

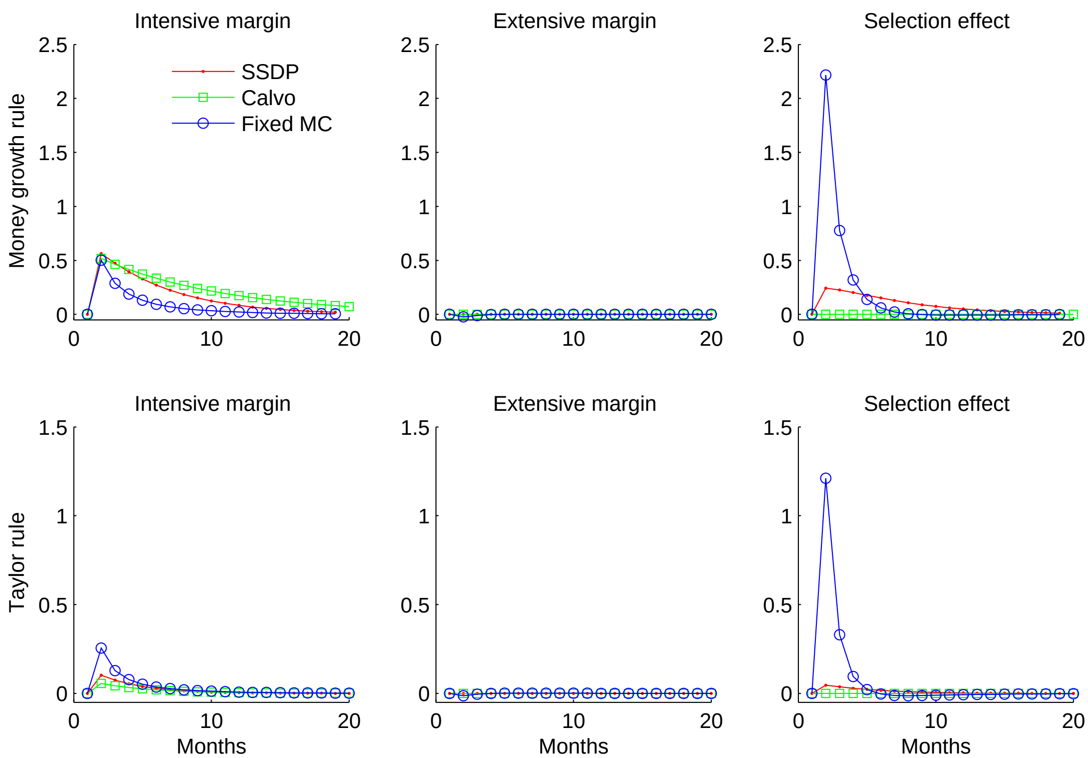

Fig. 4. Inflation decomposition across models

Note: decomposition of the inflation response into an intensive margin, extensive margin, and selection effect (see eq. 54). Top row: responses to a correlated money growth shock. Bottom row: responses to a Taylor rule shock. The responses are in percentage points and sum up to the total inflation response shown in Fig. 2. Lines with dots - benchmark SSDP model; lines with squares - Calvo; lines with circles - fixed menu costs.

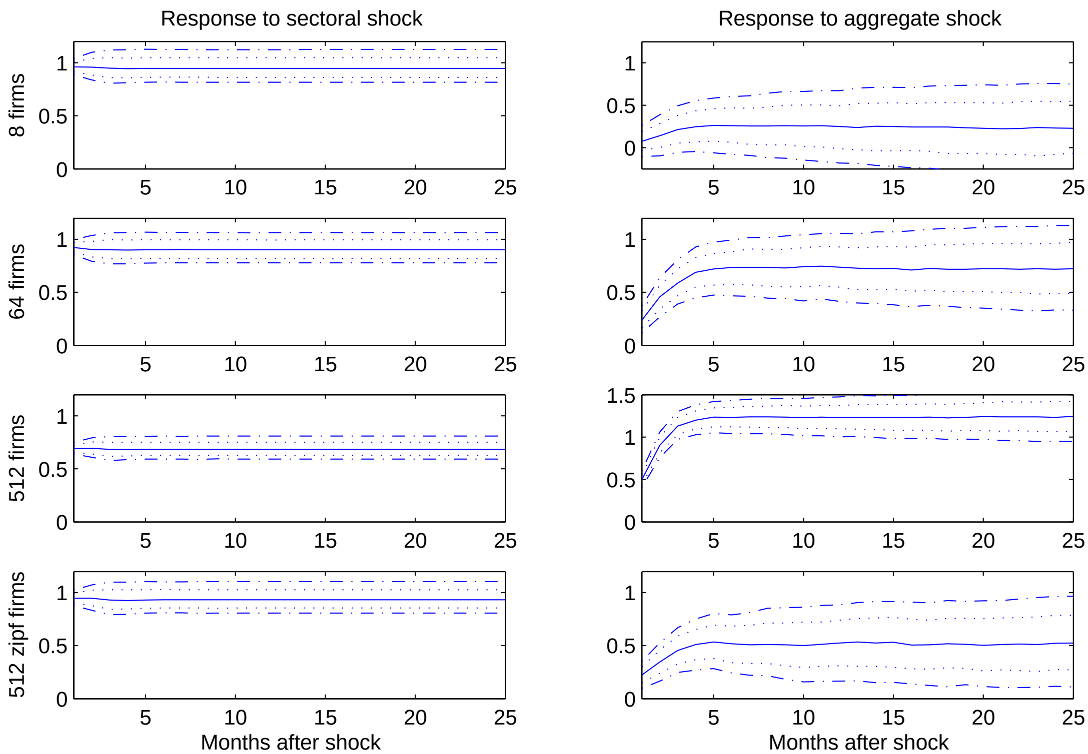

Fig. 5. Sectoral price responses to shocks identified from model-generated data

Note: responses of sector-specific prices to "sector-specific" shocks (left column) and to "aggregate" shocks (right column), estimated from SSDP model-generated data. One- and two-standard-error bands shown. Simulated economy consists of 79 sectors with 8, 64, or 512 firms with equally-weighted products (top three rows), or 512 firms with product weights satisfying Zipf's law (fourth row). Simulated economy is subject to aggregate Taylor rule shocks and firm-specific productivity shocks that are uncorrelated across firms; sectors are defined as fixed sets of unrelated firms. Shocks are identified by applying the procedure of Mackowiak, Moench, and Wiederholt (2009).

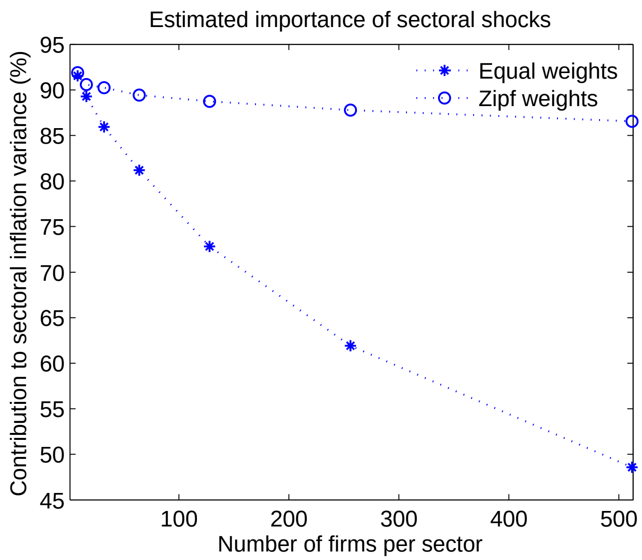

Fig. 6. Inflation variance contribution of sector-specific shocks identified from model-generated data

Note: share of variance of sector-specific prices explained by "sector-specific" shocks, as a function of number of firms per sector, estimated from SSDP model-generated data. Simulated economy consists of 79 sectors with equally-weighted products (line with stars), or with product weights satisfying Zipf's law (line with circles). Simulated economy is subject to aggregate Taylor rule shocks and firm-specific productivity shocks that are uncorrelated across firms; sectors are defined as fixed sets of unrelated firms. Shocks are identified by applying the procedure of Mackowiak, Moench, and Wiederholt (2009).

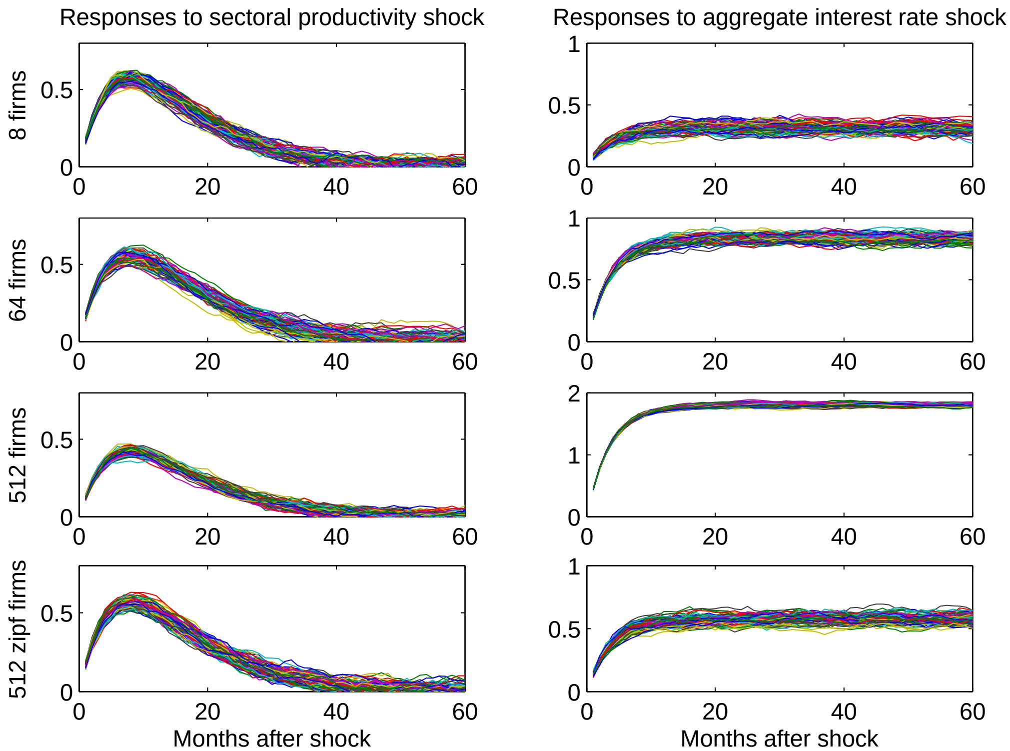

Fig. 7. Sectoral price responses to true shocks in model-generated data

Note: responses of sector-specific prices to "sector-specific" shocks (left column) and to Taylor rule shocks (right column), estimated from SSDP model-generated data. Simulated economy consists of 79 sectors with 8, 64, or 512 firms with equally-weighted products (top three rows), or 512 firms with product weights satisfying Zipf's law (fourth row). Simulated economy is subject to aggregate Taylor rule shocks and firm-specific shocks that are uncorrelated across firms; sectors are defined as fixed sets of unrelated firms. "Sector-specific" shock is the change in sector-specific weighted average productivity.