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Abstract

We examine the optimal taxation of capital in a Ramsey setting of a general-equilibrium heterogeneous-agent economy with uninsurable idiosyncratic investment or capital-income risk. We prove that the ex ante optimal tax, evaluated at steady state, maximizes human wealth, namely the present discounted value of agents' income from sources that are not subject to capital risk. Furthermore, when the amount of idiosyncratic risk in the economy is higher than a minimum lower bound, the optimal tax is positive and it is precisely the tax that maximizes the economy-wide aggregates, such as the capital stock and output. By contrast, when the amount of risk is exogenously very low, the social planner finds it optimal to increase social risk taking by subsidizing investment in capital.

1 Introduction

We study optimal capital taxation in a Ramsey framework and in an environment where agents face uninsurable idiosyncratic investment or capital-income risk. Such risk is empirically important for all investment decision makers, whether they are entrepreneurs and private business owners or managers of publicly traded firms. In this context, capital taxation raises an interesting trade-off: On the one hand, it comes at the usual cost, as it distorts agents' saving decisions. On the other hand, it has benefits, as it provides agents with partial insurance against idiosyncratic capital-income risk. Moreover, this insurance aspect of capital taxation may even lead to higher capital accumulation in general equilibrium, contrary to what happens in models of complete markets or uninsurable labor income risk. Therefore, a positive tax on capital income could be welfare improving, and more so than in other Ramsey environments.

Our modelling framework builds on Angeletos (2007), who develops a variant of the neoclassical growth model that allows for idiosyncratic investment risk, and studies the effects of such risk on macroeconomic aggregates. Agents own privately-held businesses that operate under constant returns to scale. These private businesses are subject to idiosyncratic risk that the agents cannot diversify away. However, agents are not exposed to labor-income risk, and they can also freely borrow and lend in a riskless bond. Abstracting from borrowing constraints, labor-income risk, and other market frictions, isolates the impact of the idiosyncratic investment risk, and preserves tractability of the model. There is a government, imposing a proportional tax on capital income, along with a non-contingent lump-sum tax or transfer. In a similar framework, Panousi (2012) performs comparative statics with respect to the capital income tax and finds that an increase in capital taxation may actually stimulate capital accumulation, due to a general equilibrium insurance aspect of the interest rate or safe rate adjustment.

We fully characterize the optimal taxation problem, where the social planner maximizes ex ante welfare, taking into account the entire transitional dynamics of the economy toward the steady state with the optimal tax. We prove that the ex ante optimal tax, evaluated at steady state, maximizes human wealth, namely the present discounted value of agents' future safe income, namely of income from wages or government transfers (or any other sources that are not subject to idiosyncratic risk). Furthermore, when the amount of idiosyncratic risk in the economy is higher than a minimum lower bound, the optimal tax is positive and it is precisely the tax that maximizes the economy-wide aggregates, such as the capital stock and output. By contrast, when the amount of risk is exogenously very low, then aggregate capital and output are falling with the tax at the optimum, and the social planner finds it optimal to increase social risk taking by subsidizing investment in capital.

As a simple and illustrative example, we provide the case of an AK economy. There, if the interest rate is endogenous, the optimal tax is always constant, even off steady state, it maximizes the risk-adjusted growth rate of wealth (the difference between the risk-adjusted return to saving and the marginal propensity to consume), and it is equal to $1 - A/\sigma^2$. Therefore, the optimal tax is positive when the mean return to the risky asset (capital), A, is lower than the variance of idiosyncratic capital-income risk, σ^2. If the interest rate is exogenous, then the optimal tax is actually always exactly equal to zero.

Our paper belongs in the optimal taxation literature in the Ramsey tradition, though it also allows for lump-sum transfers, and it shows that the rationale for positive capital income taxation in the long run does not necessarily extend to the case where markets are incomplete due to the presence of idiosyncratic capital income risk. However, regardless of whether the ex ante optimal tax is positive or negative, the planner's motivation for setting the optimal tax is always related to ensuring a sufficient amount of social risk taking. When the exogenous risk in the economy is sufficiently high, then the optimal capital tax is positive, so that the ensuing general equilibrium adjustment of the interest rate endogenously provides agents with the insurance they need to undertake risky capital investment, despite the fact that the tax tends to reduce the mean return to saving. When the exogenous risk in the economy is sufficiently low, then the optimal capital tax is negative, so that the capital subsidy encourages capital accumulation directly, by increasing the mean return to capital, despite the fact that the subsidy tends to reduce the interest rate and therefore wealth accumulation. This is because risk was low enough to begin with, and therefore the agents can tolerate the increase in the variance of risk resulting from the subsidy, both directly and indirectly through its general equilibrium effects.

2   Related Literature

We focus on an environment with idiosyncratic investment risk, because such risk is in fact empirically relevant for all investment decision makers, even in a financially developed country like the United States. First, Moskowitz and Vissing-Jorgensen (2002), among others, find that about 80 percent of all *private equity* in the US is owned by agents who are actively involved in the management of their own firm, and for whom such investment constitutes at least half of their total net worth. It seems plausible, then, that entrepreneurial risk must be even more prevalent in less developed economies. Second, Panousi and Papanikolaou (2012) show that the negative relationship between idiosyncratic risk and the investment of *publicly-traded* firms in the US is stronger in firms where the mangers hold a larger fraction of the firm's shares. Combined, these findings strengthen the empirical applicability of our model setup, because they demonstrate that a large fraction of total investment in the US is sensitive to idiosyncratic risk, essentially through the risk aversion of the agents making the investment decisions.

This paper relates to the macroeconomic literature on optimal taxation. Most of this literature has focused either on complete markets or on incomplete markets with labor-income risk. Starting with the Ramsey literature of exogenously given market structure and exogenous policy instruments, Chamley (1986), Judd (1985), and Atkeson, Chari and Kehoe (1999) establish the result of zero optimal capital taxation when markets are complete. Correia (1996) shows that, in the neoclassical model, if there are restrictions on the taxation of production factors, then the tax rate on capital income in steady state is different from zero. Aiyagari (1995) extends the complete-markets framework to include uninsurable labor-income risk and borrowing constraints, and finds that the optimal capital tax is positive in the long run.[1] Chamley (2001) argues that it might be best to think of the reasoning behind

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[1]A related but different normative exercise to that in Aiyagari (1995) is conducted by Davila, Hong,

Aiyagari's positive optimal capital tax result as related to the ex-ante insurance or ex-post redistribution aspect of the tax: the planner taxes agents with high income realizations, and subsidizes agents with low income realizations, thereby equalizing consumption across different types of agents. Our paper contributes to the literature on optimal taxation in the Ramsey tradition, though also allowing for lump-sum transfers, and shows that, when markets are incomplete due to the presence of uninsurable capital-income risk, then the optimal capital tax will be different from zero, and will be positive or negative depending on general equilibrium insurance considerations.

Moving to the Mirrlees literature of endogenous market incompleteness and endogenous policy instruments, Albanesi (2006) considers optimal taxation in a two-period model of entrepreneurial activity in a constrained-efficiency setting. Shourideh (2011) also studies the optimal taxation of entrepreneurial income. In his model, as in our paper, the intertemporal wedge determining the tax on wealth cannot be unambiguously signed. However, the incentive constraint seems to create a force towards a wealth subsidy, since increasing capital tends to loosen the incentive compatibility constraint in the future. In general, however, the extensive theoretical work on taxation originating from the Mirrlees tradition focuses on labor-income risk. This literature shows that, if insurance is limited due to the presence of asymmetric information, then it may be best to restrict free access to savings. This result has in turn been interpreted as a justification for capital taxation. Some additional examples include Diamond and Mirrlees (1978), Golosov, Kocherlakota, and Tsyvinski (2003), Albanesi and Sleet (2006), and Golosov, Troshkin, Tsyvinsky and Weinzierl (2010). Farhi and Werning (2010) study optimal nonlinear taxation of labor and capital in a political economy model with heterogeneous agents, where policies are chosen sequentially over time, without commitment, as the outcome of democratic elections. They find that credible policies show a concern for future inequality and that capital taxation emerges as an efficient redistributive tool for this purpose.

The overlapping-generations literature has often found support for positive optimal capital taxation. Conesa, Kitao and Krueger (2009) quantitatively characterize the optimal capital and labor income taxes in an overlapping generations model with idiosyncratic uninsurable income shocks and permanent productivity differences across households. They find that the optimal capital-income tax rate is significantly positive at 36 percent, mainly driven by the life-cycle structure of the model.[2] Erosa and Gervais (2002), in an overlapping-generations economy where agents' productivity varies over time, find that positive capital taxes may be optimal when labor taxes cannot be conditioned on age. Garriga (2003), and Peterman (2011) also find similar results.

A strand of the public finance literature has examined the effects of capital taxation on risk taking, mostly in a partial equilibrium framework. Some examples include Domar and Musgrave (1944), Stiglitz (1969), Ahsan (1974), Sandmo (1977), and Kanbur (1981). These authors argued that, by effectively reducing the variance of capital income, the capital tax allowed for increased social risk taking, leading to an increase in investment in the

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Krusell and Rios-Rull (2005), in the spirit of Geanakoplos-Polemarchakis.

[2]Domeij and Heathcote (2004) perform a similar exercise. Uhlig and Yanagawa (1996) show that higher capital income taxes lead to faster growth in an overlapping generations economy with endogeneous growth.

risky asset (capital). Our results are of similar flavor, though reflecting general equilibrium considerations that this literature cannot capture, even in the case where the optimal tax turns out to be negative. As it turns out, a capital subsidy enhances risk taking in cases where the amount of exogenous risk in the economy was too low to begin with. Our paper is also related to Varian (1980), who assumes that differences in observed income are due to exogenous differences in luck. In a two-period model of endogenous saving, he finds that the optimal capital-income tax is positive, due to the trade-off it involves between the distortion in the saving decision and the provision of social insurance through redistribution.

Finally, Angeletos and Panousi (2009) use a model similar to the one in the present paper to examine the effects of government consumption on steady state aggregates, for the case where government spending is financed solely through lump-sum taxes.

3 The basic model

Time is continuous and indexed by $t \in [0, \infty)$. There is a continuum of infinitely lived households distributed uniformly over $[0, 1]$. Each household consists of a worker and an entrepreneur. The worker is endowed with one unit of labor, supplied inelastically in a competitive labor market. The entrepreneur owns and runs a privately-held firm. Each firm employs labor in the competitive labor market, but can only use the capital stock invested by the particular household. Each firm is hit by idiosyncratic shocks, which the household can only partially diversify, as it cannot invest in other households' firms. However, each household can freely save or borrow in a riskless bond (up to a natural borrowing constraint), which is in zero net supply. In terms of timing for the firm's problem, first capital is installed, then the idiosyncratic shock is realized, and lastly the labor choice is made. All uncertainty is purely idiosyncratic, and therefore aggregates are deterministic. Finally, the government imposes proportional taxes on savings and labor income, and balances the budget by giving back to agents, in the form of lump-sum transfers, the proceeds of taxation minus any government spending. Throughout the paper, for any variable y, the notation y_t is used as short-hand notation for $y(t)$, where t is time.

3.1 Households, firms, and idiosyncratic risk

Preferences are logarithmic over consumption, c:

$$U_t = E_t \int_t^\infty e^{-\beta s} \, ln(c_s) \, , ds \tag{1}$$

where $\beta > 0$ is the discount rate.

The financial wealth of a household i, denoted by a_t^i, is the sum of its asset holdings in private capital, k_t^i, and in the riskless bond, b_t^i, so that $a_t^i = k_t^i + b_t^i$. The evolution of a_t^i is given by the household budget:

$$da_t^i = (1 - \tau_t) \, d\pi_t^i + [\, (1 - \tau_t) \, R_t \, b_t^i + \, w_t + T_t - c_t^i \,] \, dt \,, \tag{2}$$

where $d\pi_t^i$ are the profits from the firm the household operates or the household's capital income, R_t is the the safe rate or the interest rate on the riskless bond, w_t is the wage rate in the aggregate economy, c_t^i is consumption, τ_t is the proportional savings or capital-income tax, applied to the income from the capital and the bond alike, and T_t are non-contingent lump-sum transfers received from the government. A no-Ponzi-game condition is also imposed.

Firm profits are subject to undiversified idiosyncratic risk:

$$d\pi_t^i = [\, F(k_t^i, l_t^i) - w_t\, l_t^i - \delta\, k_t^i\,]\, dt \,+\, \sigma\, k_t^i\, dz_t^i\,. \tag{3}$$

Here, F is a constant-returns-to-scale neoclassical production function, assumed to be Cobb-Douglas for simplicity, namely $F(k, l) = k^\alpha l^{1-\alpha}$ with $\alpha \in (0, 1)$, where l_t^i is the amount of labor the firms hires in the competitive labor market. In addition, δ is the mean depreciation rate in the aggregate economy. Idiosyncratic risk is introduced through dz_t^i, a standard Wiener process that is i.i.d. across agents and across time. Literally taken, dz_t^i represents a stochastic depreciation shock. However, these shocks can also be interpreted as stochastic productivity shocks. The scalar σ measures the amount of undiversified idiosyncratic risk, and it is an index of market incompleteness, with higher σ corresponding to a lower degree of risk-sharing, and with $\sigma = 0$ corresponding to complete markets.

3.2   Government

At each point in time the government taxes capital income and bond income at the rate τ_t. The government also does some government spending at the rate G_t, where G_t does not enter any production or utility functions. The proceeds of taxation, minus any government consumption, are then distributed back to the households in the form of non-contingent lump-sum transfers, T_t. The government budget constraint is therefore:

$$0 = [\, \tau_t\, (\, F_{K_t}(\int_i k_t^i, 1) - \delta\,)\, \int_i k_t^i - G_t - T_t\,]\, dt\,, \tag{4}$$

where $F_{K_t}(\int_i k_t^i, 1)$ is the marginal product of capital in the aggregate economy, and $\int_i l_t^i = 1$. For simplicity, we will henceforth set $G_t = 0$ for all t.[3]

4   Equilibrium and steady state

This section characterizes the equilibrium of the economy. First, it solves for households' optimal plans, given the sequences of prices and policies. It then aggregates across households to derive the general equilibrium dynamics.

4.1   Individual behavior

Entrepreneurs choose employment after their capital stock has been installed and their idiosyncratic shock has been observed. Hence, since their production function, F, exhibits

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[3]None of our theoretical results hinge on this assumption.

constant returns to scale, optimal firm employment and profits are linear in own capital:

$$l_t^i = l(w_t)\, k_t^i \qquad and \qquad d\pi_t^i = r(w_t)\, k_t^i\, dt + \sigma\,(1 - \tau_t)k_t^i\, dz_t^i\,, \tag{5}$$

where $l(w_t) \equiv \arg\max_l [\, F(1, l) - w_t\, l\,]$ and $r(w_t) \equiv \max_l [\, F(1, l) - w_t\, l\,] - \delta$. Here, $r_t \equiv r(w_t)$ is an entrepreneur's expectation of the return to his capital prior to the realization of his idiosyncratic shock, as well as the mean of the realized returns in the cross section of firms, since there is no aggregate uncertainty. As in Angeletos (2007), the key result here is that entrepreneurs face linear, albeit risky, returns to their investment. To see how this translates to linearity of wealth in assets, let h_t denote a household's human wealth, namely the present discounted value of net-of-taxes labor endowment plus government transfers:

$$h_t = \int_t^\infty e^{-\int_t^s (1 - \tau_j) R_j\ dj} (w_s + T_s\,)\, ds\,. \tag{6}$$

Next, define total effective wealth, x_t^i, as the sum of financial wealth and human wealth:

$$x_t^i \equiv a_t^i + h_t = k_t^i + b_t^i + h_t\,. \tag{7}$$

Total effective wealth is then the only state variable relevant for the household's optimization problem. The only constrained imposed is that consumption is non-negative, which implies non-negativity of total effective wealth, so that:

$$x_t^i \geqslant 0 \qquad \Leftrightarrow \qquad x_t^i \geqslant -h_t\,.$$

In other words, there is no ad hoc borrowing constraint, and agents can freely borrow and lend up to the natural borrowing limit. The evolution of total effective wealth is then described by:

$$dx_t^i = [\,(1 - \tau_t)\, r_t\, k_t^i + (1 - \tau_t)\, R_t\, (b_t^i + h_t) - c_t^i\,]\, dt + \sigma\,(1 - \tau_t)\, k_t^i\, dz_t^i\,. \tag{8}$$

The first term in 8 captures the expected rate of growth of effective wealth, and it shows that wealth grows when entrepreneurial saving exceeds consumption expenditures. The second term captures the effect of idiosyncratic risk. This linearity of wealth in assets, together with the homotheticity of preferences, ensures that the household's consumption-saving problem reduces to a tractable homothetic optimization problem, as in Samuelson's and Merton's classic portfolio analysis. Therefore, the optimal individual policy rules will be linear in total effective wealth, for given prices and government policies, as the next proposition shows.

Proposition 1. *Let $\{w_t, R_t, r_t\}_{t\in[0,\infty)}$ and $\{\tau_t, T_t\}_{t\in[0,\infty)}$ be equilibrium price and policy sequences. The household maximizes preferences as described in (1) subject to the total effective wealth evolution constraint (8). The optimal consumption, investment, and bond holding choices, respectively, are given by:*

$$c_t^i = m_t\, x_t^i, \quad k_t^i = \phi_t\, x_t^i, \quad b_t^i = (1 - \phi_t)\, x_t^i - h_t\,, \tag{9}$$

where the fraction of effective wealth invested in capital, ϕ_t, is given by:

$$\phi_t = \frac{(1 - \tau_t)\, r_t - (1 - \tau_t)\, R_t}{\sigma^2\,(1 - \tau_t)^2}\,, \tag{10}$$

and the marginal propensity to consume, m_t, is constant and equal to the discount rate in preferences, i.e. $m_t = \beta$ for all t.

The fact that investment is subject to undiversifiable idiosyncratic risk introduces a wage between the marginal product of capital and the risk-free rate, so that $(1 - \tau_t)R_t < (1 - \tau_t)r_t$. In other words, it has to be that, in equilibrium, the mean return to the risky asset (capital) exceeds the mean return to the safe asset (bond) by an amount equal to the positive (private) risk premium agents require as compensation for undertaking risky investment. The fraction of wealth invested in the risky asset, ϕ_t, is then increasing in this risk premium, and decreasing in the effective variance of risk, $\sigma(1 - \tau_t)$. Furthermore, it is the same across all entrepreneurs and it does not depend on the level of wealth. In equation (9), optimal consumption is a linear function of total effective wealth, where the marginal propensity to consume, m_t, is also independent of wealth. Moreover, because preferences are logarithmic, the marginal propensity to consume is constant over time and equal to the discount rate in preferences, β.

The wealth evolution constraint, incorporating bond market clearing and individual optimization, is:

$$dx_t^i = [(1 - \tau_t)r_t\phi_t + (1 - \tau_t)R_t(1 - \phi_t) - \beta]x_t^i dt + \sigma(1 - \tau_t)\phi_t x_t^i dz_t^i. \tag{11}$$

Using (11) and Proposition 1, we get the following characterization for individual consumption dynamics.

Lemma 1. *The evolution of individual consumption, investment, and wealth is given by:*

$$\frac{dc_t^i}{c_t^i} = \frac{dx_t^i}{x_t^i} = (\rho_t - \beta)\,dt + \sigma\,(1 - \tau_t)\,\phi_t dz_t^i, \tag{12}$$

where $\rho_t = (1 - \tau_t)\,\phi_t\,r_t + (1 - \tau_t)\,(1 - \phi_t)\,R_t$ is the total return to saving. Solving for c_t^i, the evolution of individual consumption is:

$$c_t^i = c_0^i \cdot exp\{ \int_0^t (\hat{\rho}_t - \beta)\,ds + \int_0^t \sigma(1 - \tau_s)\phi_s\,dz_s^i \}, \tag{13}$$

where $\hat{\rho}_t = \rho_t - \frac{1}{2}\sigma^2(1 - \tau_t)^2\phi_t^2$ is the risk-adjusted return to saving.

The proof of equation (13) follows from Ito's lemma. Here, ρ_t is the mean return to saving, namely the total or overall portfolio return for the household. In other words, the total return to saving is a weighted average of the (net-of-tax) marginal product of capital and the (net-of-tax) risk-free rate. Then, the risk-adjusted return to saving, $\hat{\rho}_t$, is the certainty equivalent of the overall portfolio return, and is lower than ρ_t because agents are risk averse and face risk in their consumption stream. It follows that, in equilibrium, it will have to be $(1 - \tau_t)\,R_t < \hat{\rho}_t < \rho_t < (1 - \tau_t)\,r_t$.

4.2   General equilibrium

The initial position of the economy is given by the cross sectional distribution of (k_0^i, b_0^i) across households. Households choose plans $\{c_t^i, l_t^i, k_t^i, b_t^i, x_t^i\}_{t \in [0,\infty)}$ for $i \in [0, 1]$, contingent on the history of their idiosyncratic shocks, and given the price sequence and the government

policy, so as to maximize their lifetime utility. Idiosyncratic risk washes out in the aggregate. An equilibrium is then defined as a deterministic sequence of prices $\{w_t, R_t, r_t\}_{t \in [0,\infty)}$, policies $\{\tau_t, T_t\}_{t \in [0,\infty)}$, and macroeconomic variables $\{C_t, K_t, Y_t, L_t, X_t\}_{t \in [0,\infty)}$, along with a collection of individual contingent plans $\{c_t^i, l_t^i, k_t^i, b_t^i, x_t^i\}_{t \in [0,\infty)}$ for $i \in [0,1]$, such that the following conditions hold: (i) given the sequences of prices and policies, the plans are optimal for the households; (ii) the labor market clears, $\int_i l_t^i = 1$, in all t; (iii) the bond market clears, $\int_t b_t^i = 0$, in all t; (iv) the government budget constraint (4) is satisfied in all t; and (v) the aggregates are consistent with individual behavior, $C_t = \int_i c_t^i$, $L_t = \int_i l_t^i = 1$, $K_t = \int_i k_t^i$, $Y_t = \int_i F(k_t^i, l_t^i) = F(\int_i k_t^i, 1)$, $X_t = \int_i x_t^i$, in all t. Note that the aggregates do not depend on the extend of wealth inequality, because individual policies are linear in wealth.

Define $f(K) \equiv F(K, 1) = K^\alpha$. From Proposition 1, the equilibrium ratio of capital to effective wealth and the equilibrium mean return to savings are identical across agents and can be expressed as functions of the capital stock and risk-free rate: $\phi_t \equiv \phi(K_t, R_t)$ and $\rho_t \equiv \rho(K_t, R_t)$. Similarly, the wage is $w_t \equiv w(K_t) = f(K_t) - f'(K_t)K_t = (1-\alpha)f(K_t)$. Using this, aggregating the policy rules across agents, and imposing bond market clearing, we arrive at the following characterization of the general equilibrium.

Proposition 2. *In equilibrium, the aggregate dynamics satisfy:*

$$\frac{\dot{X}_t}{X_t} = \rho_t - \beta \tag{14}$$

$$\dot{H}_t = (1 - \tau_t)R_t H_t - w_t - \tau_t \left(F_{K_t} - \delta \right) K_t \tag{15}$$

$$K_t = \frac{\phi_t}{1 - \phi_t} H_t \, , \tag{16}$$

along with $m_t = \beta$.

Condition (14) follows from aggregating the individual wealth evolution constraints across agents, and using (9). It captures the evolution of total effective wealth, and shows that wealth grows when the mean net-of-tax return to saving, ρ_t, exceeds the marginal propensity to consume, β. Condition (15) is the evolution of human wealth, combined with factor market clearing and with the intertemporal government budget. Condition (16) represents clearing in the bond market and ensures that the bond is in zero net supply in the aggregate.

From this point on, and for simplicity, we let $\delta = 0$ without loss of generality.[4] The aggregate resource constraint of the economy is then:

$$dK_t = [K_t^\alpha - C_t]dt \, . \tag{17}$$

4.3   Steady state

The steady state is the fixed point of the dynamic system described in Proposition 2. The following proposition characterizes the steady state.

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[4]None of the theoretical results hinge on this assumption.

Proposition 3. *The steady state always exists, and it is the unique solution to the following system of two equations in the capital stock, K, and the interest rate, R:*

$$r(K) = R + \sqrt{\sigma^2(\beta - (1-\tau)R)} \tag{18}$$

$$K = \frac{\phi(K,R)}{1 - \phi(K,R)} \frac{w(K) + \tau r(K)K}{(1-\tau)R}, \tag{19}$$

where $\phi(K,R) \equiv (r(K) - R)/(\sigma^2(1-\tau))$, $r(K) \equiv f'(K) = \alpha K^{\alpha-1}$, and $w(K) \equiv (1-\alpha)K^\alpha$.

Equation (18) combines stationarity of wealth from (14) with the definition of the mean return to saving, $\rho = (1-\tau)r\phi + (1-\tau)R(1-\phi)$. Note from (14) that wealth stationarity requires $\rho = \beta$, namely that the mean return to saving is equal to the marginal propensity to consume. But since $(1-\tau)R < \rho$, it follows that $(1-\tau)R < \beta$ in steady state. This is a manifestation of the precautionary saving motive, with a rationale similar to that in Aiyagari (1994).[5] Equation (19) follows from stationarity of human wealth in (15), bond market clearing in (16), and the government budget constraint.

Lastly, we note that, in this model, as shown in Angeletos (2007), steady state capital is below complete markets, if and only if the elasticity of intertemporal substitution is higher than the ratio $\phi/(2-\phi)$, where ϕ is the fraction of total effective wealth invested in capital. Here, because the elasticity of intertemporal substitution is 1 with log preferences, and because $\phi < 1$, the steady state aggregates will always be below the corresponding ones in complete markets.

5   The ex ante optimal tax

The optimal tax in our framework is the one maximizing ex ante welfare. In particular, assume that at time $t = 0$ the economy rests at an arbitrary steady state, and that the social planner is considering implementing an unexpected policy change at some future time $t = u$. The question posed from an ex ante perspective is: What is the impact of that policy change on agents' welfare at time $t = 0$, taking into account the entire transitional dynamics of the economy toward the new steady state resulting from the policy change? The tax maximizing agents' utility at time $t = 0$ is then the (ex ante) optimal tax. We will first characterize the planner's problem of maximizing ex ante welfare, and then we will evaluate the solution of the problem in steady state.[6]

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[5]In particular, agents have a precautionary saving motive, because the idiosyncratic investment risk generates risk in their consumption stream. Therefore, if the net interest rate were higher than the discount rate in preferences, savings and wealth would explode, which violates the notion of steady state. In fact, the net interest rate has to be lower than the discount rate by exactly as much as is needed for the corresponding substitution effect of a lower saving return to exactly offset the precautionary saving motive.

[6]Note however, that we are taking the transition to the new steady state explicitly into account, and that we are not imposing a constant tax along the transition.

5.1 The planner's problem

From Proposition 1, it follows that the value function for an agent with initial wealth x_0^i at $t = 0$, given the tax sequence $\{\tau_t\}_{t=0}^{\infty}$, is the solution to the problem:

$$V(x_0^i; \{\tau_t\}_{t=0}^{\infty}) = max_{c^i,\phi^i} \; E_{-1} \int_0^{\infty} e^{-\beta t} \, ln(c_t^i) \, dt \,, \tag{20}$$

where the maximands are the optimal policy functions described in Proposition 1.

The social planner's objective is to choose the tax sequence $\{\tau_t\}_{t=0}^{\infty}$ that maximizes ex ante expected utility, subject to the conditions for individual optimization and general equilibrium in section 4. The planner's objective function, $\tilde{W}$, is then weighted sum of agents' value functions, V^i, where the weights, $\psi(x_0^i; \tau_t)$, depend on the initial wealth of each agent:

$$\tilde{W}(x_0^i; \{\tau_t\}_{t=0}^{\infty}) = max_{\{\tau_t\}_{t=0}^{\infty}} \int_i V(x_0^i; \tau_t) \, \psi(x_0^i; \tau_t) \, dx_0^i \,. \tag{21}$$

Without loss of generality, we will assume that at $t = 0$ the wealth distribution is concentrated at one point, so that all agents hold the same amount of capital, equal to the economy-wide aggregate capital stock, and therefore receive the same weight in the planner's objective, so that $\psi(x_0^i; \tau_t) = 1$. The following proposition characterizes the planner's problem of maximizing ex ante welfare.

Proposition 4. *The planner chooses the sequence of taxes $\{\tau_t\}_{t=0}^{\infty}$ to maximize the objective function:*

$$\tilde{W}(x_0^i; \{\tau_t\}_{t=0}^{\infty}) = \int_0^{\infty} e^{-\beta t} \{ln(\beta x_0^i) + \int_0^t [(1-\tau_s)r_s\phi_s + (1-\tau_s)R_s(1-\phi_s) - \beta - \frac{1}{2}\sigma^2(1-\tau_s)^2\phi_s^2]ds\}dt \,, \tag{22}$$

subject to the following constraints:

$$\phi_t = \frac{r_t - R_t}{\sigma^2 (1 - \tau_t)} \,, \tag{23}$$

$$dK_t = [K_t^{\alpha} - C_t]dt \,, \tag{24}$$

$$dX_t/X_t = [(1 - \tau_t)r_t\phi_t + (1 - \tau_t)R_t(1 - \phi_t) - \beta]dt \,, \tag{25}$$

$$r_t = \alpha K_t^{\alpha-1} \,, \quad w_t = (1 - \alpha)K_t^{\alpha} \,, \tag{26}$$

$$T_t = \tau_t r_t K_t \,. \tag{27}$$

Hence, at time $t = 0$, the first order condition with respect to a change in the capital tax in period $t = u$ is:

$$\frac{d\tilde{W}}{d\tau_u} = \frac{\partial \tilde{W}}{\partial x_0} \frac{dx_0}{d\tau_u} + \frac{\partial \tilde{W}}{\partial \phi_u} \frac{d\phi}{d\tau_u} + \frac{\partial \tilde{W}}{\partial r_u} \frac{dr}{d\tau_u} + \frac{\partial \tilde{W}}{\partial R_u} \frac{dR}{d\tau_u} + \frac{\partial \tilde{W}}{\partial \tau_u} = 0 \,. \tag{28}$$

The planner's objective follows from equations (9) and (13), and the fact that all uncertainty is idiosyncratic, namely that $E_i \int_0^t \sigma(1-\tau_s)\phi_s dz_s^i = 0$. Equation (23) describes the optimal portfolio allocation from Proposition 1, equation (24) is the aggregate resource constraint in the economy, equation (25) is the aggregate wealth evolution constraint using the definition of the mean return to saving ρ_t, equation (26) is market clearing for the factors of production, and equation (27) is the government budget constraint.

Therefore, the planner maximizes an objective consisting of two terms. The first captures the effect of the entire path of future prices and tax policies on effective wealth at time zero, x_0. This is because effective wealth is the sum of asset holdings, $a_0 = k_0 + b_0$, which are historically given, and human wealth, h_0, which is the present discounted value of future wages and transfers (i.e. of future safe income), and therefore depends on the entire future path for the tax, the wage, the return to capital, and the interest rate. The second term in the planner's objective captures both the direct effects of the capital tax on the mean return to saving and on the effective volatility of risk, as well as the indirect effects of the tax through the corresponding adjustment of the risky return, r_t, the risk-free rate, R_t, and portfolio allocation, ϕ_t. Note that the term $(1-\tau_t)r_t\phi_t + (1-\tau_t)R_t(1-\phi_t) - \frac{1}{2}\sigma^2(1-\tau_t)^2\phi_t^2$ in the planner's objective is actually the risk-adjusted return to saving, $\hat{\rho}_t$.

In other words, the planner has to weigh two considerations when choosing the optimal tax sequence. First, how the path of the taxes will affect the paths of wages and prices, thereby possibly maximizing time-zero wealth, x_0, through h_0. Second, whether the path of the taxes will maximize the difference between the risk-adjusted return to saving, $\hat{\rho}_t$, and the marginal propensity to consume, β, i.e. the risk-adjusted rate of growth of household consumption and wealth (see equation (13)).

We can also see from (28) that, if x_0 did not include human wealth, h_0, then $x_0 = k_0 + b_0$ would be historically given and therefore not relevant for the planner's maximization problem. In turn, absence of human wealth means that there is only risky income in the economy, as in an AK version of our model. In that case, the planner chooses the path of taxes to maximize $\hat{\rho}_t - \beta$, which is exactly the difference between the risk-adjusted return to saving and the marginal propensity to consume, or the risk-adjusted growth rate of individual consumption and wealth.

The next step is now to characterize each one of the derivatives in equation (28) in turn. Simple differentiation of the objective function yields:

$$\frac{\partial \tilde{W}}{\partial x_0} = \frac{1}{x_0} \int_0^\infty e^{-\beta s} ds \tag{29}$$

The calculation of the derivatives $\partial \tilde{W}/\partial \phi_u$, $\partial \tilde{W}/\partial r_u$, $\partial \tilde{W}/\partial R_u$, and $\partial \tilde{W}/\partial \tau_u$ is more complicated. Intuitively, these derivatives indicate the effect on the objective at time $t=0$ of a change in the functions ϕ_t, r_t, and R_t at time $t=u$, due to a tax change at that point in time. Hence, the calculations use the definition of a functional derivative, with the Dirac delta function as the appropriate test function. In turn, the Dirac delta function allows for an impulse change in the functions at time $t=u$, due to a change in the tax at that time,

while the functions remain unchanged at all other points in time. The outcome is:

$$\frac{\partial \tilde{W}}{\partial \phi_u} = [(1 - \tau_u)r_u - (1 - \tau_u)R_u - \sigma^2(1 - \tau_u)^2\phi_u] \int_u^\infty e^{-\beta s} ds \tag{30}$$

$$\frac{\partial \tilde{W}}{\partial r_u} = (1 - \tau_u)\phi_u \int_u^\infty e^{-\beta s} ds \tag{31}$$

$$\frac{\partial \tilde{W}}{\partial R_u} = (1 - \tau_u)(1 - \phi_u) \int_u^\infty e^{-\beta s} ds \tag{32}$$

$$\frac{\partial \tilde{W}}{\partial \tau_u} = [-r_u\phi_u - R_u(1 - \phi_u) + \sigma^2(1 - \tau_u)\phi_u^2] \int_u^\infty e^{-\beta s} ds . \tag{33}$$

From (30), and using portfolio allocation from (10), we can see that $\partial \tilde{W}/\partial \phi_u = 0$. This is because the optimal choice of ϕ actually maximizes the risk-adjusted return to saving, $\hat{\rho}$. Equations (31) and (32) capture indirect effects of the tax on welfare operating through asset returns, for the risky (capital) and the riskless (bond) asset, respectively. The weights on these terms, which depend on ϕ_u, capture the importance of each asset on the overall portfolio. Equation (33) captures the direct effect of the tax on welfare and consists of two terms. The first term is $-[r_u\phi_u + R_u(1 - \phi_u)]$, and is negative, reflecting the standard result that the tax lowers the mean return to saving. The second term is $\sigma^2(1 - \tau_u)\phi_t^u$, and is positive, reflecting the fact that the tax directly provides some insurance by lowering the effective variance of risk, $\sigma(1 - \tau_u)$. Using portfolio allocation from (10), we get that the term in brackets in (33) is equal to $-R_u$.

In order to calculate the derivative $dx_0/d\tau_u$, note first that we can write x_0 as:

$$x_0(\{\tau_t\}_{t=0}^\infty) = k_0 + b_0 + \int_0^\infty e^{-\int_0^s p(\tau_j)dj} q(\tau_s)ds , \tag{34}$$

where $a_0 \equiv k_0 + b_0$ is historically given and where:

$$p(\tau_t) \equiv (1 - \tau_t)R(\tau_t) , \tag{35}$$

$$q(\tau_t) \equiv w_t + T_t = [(1 - \alpha)\alpha^{\alpha/(1-\alpha)} + \tau_t\alpha^{1/(1-\alpha)}]r(\tau_t)^{\alpha/(\alpha-1)} , \tag{36}$$

are, respectively, the equilibrium after-tax interest rate and the equilibrium safe income from wages and transfers, both as a function of prices only. The latter is given by the sum of $w_t = (1 - \alpha)(r_t/\alpha)^{\alpha/(\alpha-1)}$ and $T_t = \tau_t r_t^{\alpha/(\alpha-1)}\alpha^{1/(1-\alpha)}$, using equations (26) and (27). Then, using the definition of a functional derivative, the Dirac delta function, and the series expansion for the exponential, we get:

$$\frac{dx_0}{d\tau_u} = \frac{dh_0}{d\tau_u} = e^{-\int_0^u p(\tau_s)ds}\frac{dq}{d\tau_u} - \frac{dp}{d\tau_u}\int_u^\infty q_t e^{-\int_0^t p(\tau_s)ds}dt , \tag{37}$$

where

$$\frac{dp}{d\tau_u} = -R(\tau_u) + (1 - \tau_u)\frac{dR}{d\tau_u} , \tag{38}$$

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$$\frac{dq}{d\tau_u} = \alpha^{1/(1-\alpha)} r(\tau_u)^{\alpha/(\alpha-1)} + [(1-\alpha)\alpha^{\alpha/(1-\alpha)} + \tau_u \alpha^{1/(1-\alpha)}]\alpha(\alpha-1)^{-1} r(\tau_u)^{1/(\alpha-1)} \frac{dr}{d\tau_u} . \quad (39)$$

This leaves us with the derivatives $d\phi/d\tau_u$, $dr/d\tau_u$, and $dR/d\tau_u$. Unfortunately, we cannot explicitly characterize these derivatives along the transition. Therefore, in the next section, we will proceed to evaluate them in steady state.

5.2 Evaluating the ex ante optimal tax in steady state

In this section, we will evaluate the ex ante optimal policy in steady state.[7] In steady state, and using the fact that $rK = \alpha K^\alpha$, $C = \beta X$, and $K = \phi X$, the aggregate resource constraint (17) yields:

$$r\phi = \alpha\beta . \quad (40)$$

Substituting this as well as portfolio allocation from (10) into the aggregate wealth evolution constraint (25) in steady state yields:

$$(1-\tau)(1-\phi)(\frac{\alpha\beta}{\phi} - \phi\sigma^2(1-\tau)) - \beta[1-(1-\tau)\alpha] = 0 .$$

Define the left-hand-side of this equation as:

$$G(\phi,\tau) \equiv (1-\tau)(\frac{\alpha\beta}{\phi} - \phi\sigma^2(1-\tau))(1-\phi) - \beta[1-(1-\tau)\alpha] \quad (41)$$

From the implicity function theorem it follows that:

$$\frac{d\phi}{d\tau} = -\frac{\partial G/\partial\tau}{\partial G/\partial\phi} \quad (42)$$

where

$$\frac{\partial G}{\partial\tau} = -(\frac{\alpha\beta}{\phi} - \phi\sigma^2(1-\tau))(1-\phi) + (1-\tau)(1-\phi)\phi\sigma^2 - \alpha\beta , \quad (43)$$

$$\frac{\partial G}{\partial\phi} = (1-\tau)(1-\phi)(-\frac{\alpha\beta}{\phi^2} - \sigma^2(1-\tau)) - (1-\tau)(\frac{\alpha\beta}{\phi} - \phi\sigma^2(1-\tau)) . \quad (44)$$

Hence, $d\phi/d\tau$ could be either positive or negative. Using portfolio allocation (10) and (40), we then get respectively:

$$\frac{dR}{d\tau} = \frac{dr}{d\tau} - \frac{d\phi}{d\tau}\sigma^2(1-\tau) + \phi\sigma^2 , \quad (45)$$

$$\frac{dr}{d\tau} = -\frac{\alpha\beta}{\phi^2}\frac{d\phi}{d\tau} . \quad (46)$$

At this point, note also that, since $r = \alpha K^{\alpha-1}$, it follows:

$$\frac{dr}{d\tau} = \alpha(\alpha-1)K^{\alpha-2}\frac{dK}{d\tau} \quad (47)$$

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[7]In steady state, the variables are not a function of time, as time goes to infinity.

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Therefore, when $dK/d\tau > 0$, then $dr/d\tau < 0$ and also $d\phi/d\tau > 0$ from (46). In other words, K and ϕ always have the same monotonicity with respect to the tax. The possibility that the capital stock might be increasing with the capital tax over some range of taxes has been studied in Panousi (2012). Here note that K and ϕ will be maximized for the same value of the capital tax. However, this need not be the case for human wealth, which in steady state is $H = q/p$. Combining 16, (46), and (47), we get that:

$$\frac{dH}{d\tau} = \frac{d\phi}{d\tau} \frac{K}{\phi^2} \frac{\alpha - \phi}{1 - \alpha} \ . \tag{48}$$

We can show that $\phi < (1 - \tau)\alpha$ in steady state. If the tax is positive, then $\phi < \alpha$ always, and therefore H has the same monotonicity as K and ϕ. If the tax is negative, though, then $\alpha - \phi$ might be either positive or negative, and it may change sign endogenously as the tax, and therefore ϕ, changes. Hence, for positive taxes, H gets maximized at the same point as K and ϕ. However, for negative taxes, if $\phi > \alpha$, it is possible that H increases, even while K falls. We will return to this point in what follows.

This completes the derivation of all the equations needed to evaluate the planner's first order condition (28) in steady state. Plugging everything we have derived so far into (28), imposing steady state, doing some algebra, and using the notation y_τ to denote the derivative of a variable y with respect to the tax in steady state, we get that the optimal tax in steady state solves:

$$\lim_{u \to \infty} \{(q_\tau - p_\tau \frac{q}{p}) + x_0[(1 - \tau)\phi r_\tau + (1 - \tau)(1 - \phi)R_\tau - R]\frac{e^{-\beta u}}{e^{-pu}}\} = 0 \tag{49}$$

where the second term is zero as $u \to \infty$, because $p = (1 - \tau)R < \beta$. Therefore, the following proposition characterizes the optimal tax in steady state.

Proposition 5. *The ex ante optimal tax, evaluated at steady state, solves:*

$$\frac{dq}{d\tau}p(\tau) - \frac{dp}{d\tau}q(\tau) = 0 \ , \tag{50}$$

where $p(\tau) = (1 - \tau)R(\tau)$ is the steady state after-tax interest rate, $q(\tau) = [(1 - \alpha)\alpha^{\alpha/(1-\alpha)} + \tau\alpha^{1/(1-\alpha)}]r(\tau)^{\alpha/(\alpha-1)}$ is the steady state sum of wages and transfers (safe income), and $dp/d\tau$ and $dq/d\tau$ are given by (38) and (39) evaluated at steady state.

Since $H = q/p$ is the steady state value of human wealth, (50) can be equivalently written as:

$$\frac{dH}{d\tau} = 0 \ . \tag{51}$$

In other words, conditions (50) or (51) show that, evaluated at steady state, the ex ante optimal tax maximizes human wealth, H, i.e. the present discounted value of wage and government transfer income. However, the optimal tax cannot be unambiguously signed, and could therefore be either positive or negative.

If the optimal tax turns out to be positive, then it also maximizes aggregate capital, K, and aggregate consumption, C, in addition to also maximizing human wealth, H. This case

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is illustrated in Figure 1. The top left panel plots the planner's first order condition (28), evaluated at steady state, against the capital tax. The optimal tax is determined at the point where the first order condition curve intersects the horizontal axis, which in this example occurs at $\tau = 0.33$. At that point, $d\phi/d\tau = 0$, as shown in the top right panel, which means that the share of wealth invested in risky capital is at a maximum. Furthermore, the bottom left and rights panels show that aggregate capital, K, and human wealth, H, also attain their maximum at the same tax.

If the optimal tax is negative, then it will not maximize capital and, for example, capital may be falling at the optimum. This case is illustrated in Figure 2. The top left panel again plots the planner's first order condition (28), evaluated at steady state, and shows that the optimal tax is actually a subsidy at $\tau = -0.13$. The top right panel show that, at the optimal tax, $d\phi/d\tau < 0$, which means that the share of capital in wealth is falling at the optimum. The bottom left panel captures the fact that, at the optimum, human wealth is at a maximum, as shown in Proposition 5. The bottom right panel shows that, as is the case with ϕ, capital is also falling at the optimum. Note that this is a case where $\sigma = 0.1$, whereas the rest of the parameters remain as in Figure 1. We could also obtain results of similar flavor if we set $\alpha = 0.1$, while keeping the rest of the parameters as in Figure 1. In other words, when risk in the economy is below a minimum lower bound, as captured either by low volatility of risk or low returns to the risky asset in the production, then the planner finds it optimal to subsidize capital. This increases the effective variance of risk, but it also increases capital accumulation, as capital is a decreasing function of the tax. The agents can in fact undertake this increase in risk endogenously, because the exogenous risk in the economy was too low to begin with.

Hence, as already discussed, the optimal tax is always the one that maximizes the steady state value of human wealth, H. However, at the optimum, capital and the rest of the aggregates will either be at a maximum or they will be falling with the tax. Calibrations show that the latter will be the case when the variance of idiosyncratic risk, σ^2 is very low, see Figure 3, panel (b). This is a case where basically there is not enough risk taking in the economy. By subsidizing capital, the planner increases the effective variance of risk, $(1-\tau)^2\sigma^2$, and also levies a lump sum tax on agents, thereby reducing their human wealth. In addition, because capital is falling all the way with the tax, the subsidy increases capital accumulation, which translates into endogenously higher risk taking in the economy. Agents are actually able to undertake this endogenous increase in risk, as the exogenous variance of risk, σ^2, was very low to begin with.

5.3 A simple example: the AK model

The AK version of the present model is the one where there is no safe income in the economy, such as income from wages or government transfers. In this case, human wealth is zero, $h_0 = 0$. Hence, as already mentioned in section 5 and indicated in Proposition 4, the planner then maximizes the difference between the risk-adjusted return to saving, $\hat\rho$, and the marginal propensity to consume, β, i.e. the risk-adjusted rate of growth of consumption and wealth.

In that case, the planner's first order condition becomes:

$$\frac{d\tilde{W}}{d\tau_t} = \frac{\partial \tilde{W}}{\partial \phi_t}\frac{d\phi_t}{d\tau_t} + \frac{\partial \tilde{W}}{\partial r_t}\frac{dr_t}{d\tau_t} + \frac{\partial \tilde{W}}{\partial R_t}\frac{dR_t}{d\tau_t} + \frac{\partial \tilde{W}}{\partial \tau_t} = 0 \ . \tag{52}$$

Note that in the AK model, $r_t = A$ for all t and hence $dr_t/d\tau_t = 0$. In addition, since all income is risky, $\phi_t = 1$ for all t, which from (10) implies that $R = A - \sigma^2(1 - \tau)$ and also that $dR/d\tau = \sigma^2$. In addition, $\partial \tilde{W}/\partial R_t = 0$, and again $\partial \tilde{W}/\partial \phi_t = 0$. Putting all of this together, the planner's first order condition yields:

$$-R = 0 \quad \Leftrightarrow \quad \tau = 1 - \frac{A}{\sigma^2} \tag{53}$$

Note that this is the case even off the steady state (we have not imposed any steady state restrictions here), and therefore the optimal tax in the AK model is always constant, even along the transition. The optimal tax is positive when $A < \sigma^2$, i.e. when the mean return to the risky asset is lower than the variance of idiosyncratic risk, a condition likely to be satisfied in the data. By contrast, if the variance of risk is very low, then the optimal tax will actually be negative, reflecting the planner's desire for increased risk taking in the economy.

6   Conclusions

We study the optimal taxation of capital in a Ramsey setting of a general-equilibrium heterogeneous-agent economy with uninsurable idiosyncratic investment or capital-income risk. We prove that the ex ante optimal tax, evaluated at steady state, maximizes human wealth, namely the present discounted value of agents' income from sources that are not subject to capital risk. Furthermore, when the amount of idiosyncratic risk in the economy is higher than a minimum lower bound, the optimal tax is positive and it is precisely the tax that maximizes the economy-wide aggregates, such as the capital stock and output. By contrast, when the amount of risk is exogenously very low, the social planner finds it optimal to increase social risk taking by subsidizing investment in capital.

Our paper contributes to the optimal taxation literature in the Ramsey tradition, though also allowing for lump-sum transfers, and it shows that the rationale for positive capital income taxation in the long run does not necessarily extend to the case where markets are incomplete due to the presence of idiosyncratic capital income risk. However, regardless of whether the ex ante optimal tax is positive or negative, the planner's motivation for setting the optimal tax is always related to ensuring a sufficient amount of social risk taking. When the exogenous risk in the economy is sufficiently high, then the optimal capital tax is positive, so that the ensuing general equilibrium adjustment of the interest rate endogenously provides agents with the insurance they need to undertake risky capital investment, despite the fact that the tax tends to reduce the mean return to saving. When the exogenous risk in the economy is sufficiently low, then the optimal capital tax is negative, so that the capital subsidy encourages capital accumulation directly, by increasing the mean return to capital, despite the fact that the subsidy tends to reduce the interest rate and therefore wealth accumulation. This is because risk was low enough to begin with, and therefore the agents

can tolerate the increase in the variance of risk resulting from the subsidy, both directly and indirectly through its general equilibrium effects. In both cases, the outcome of the optimal tax is therefore higher investment in the risky asset, higher capital accumulation, and a higher amount of social risk taking in the economy.

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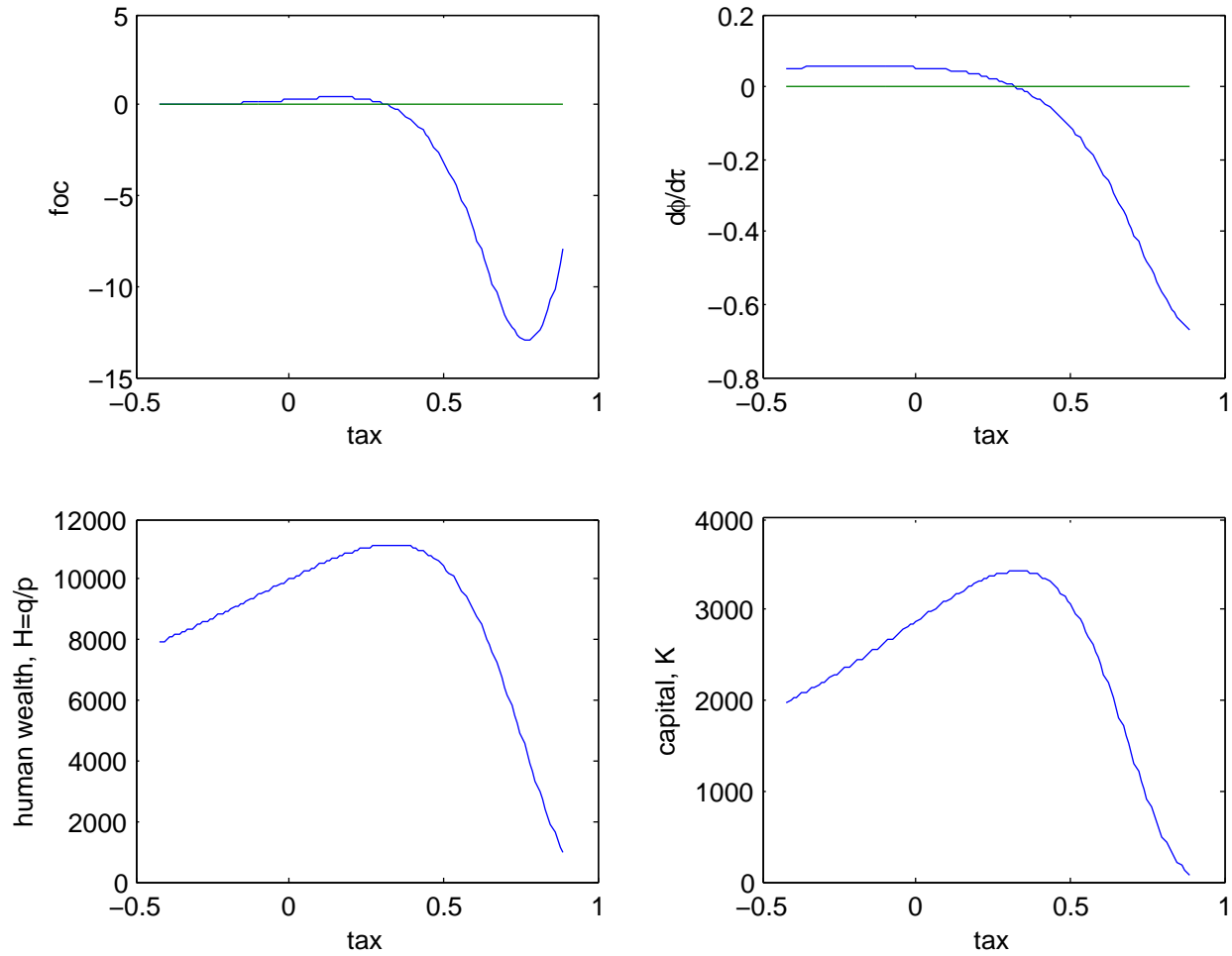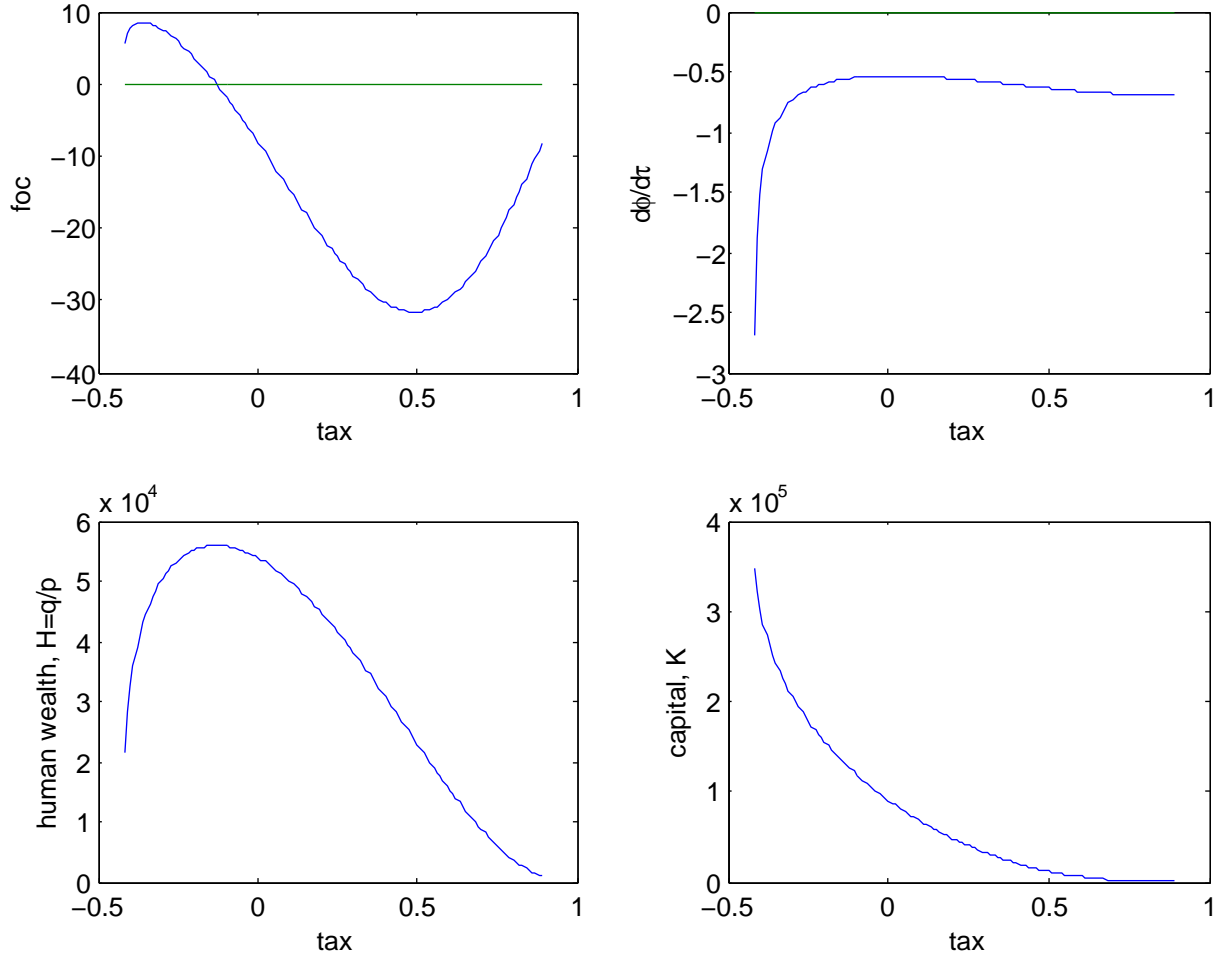## Figure 1: Ex ante welfare 1: optimal tax maximizes capital

Figure 1 uses parameter values $\alpha = 0.7$, $\sigma = 0.5$, $\beta = 0.98$. The top left panel plots the first order condition (28) against the capital tax. The line cuts the horizontal axis at the ex ante optimal tax, which is positive at $\tau = 0.33$. The top right panel plots the derivative of ϕ against the tax. This derivative is zero at the optimal tax. The bottom left panel plots human wealth, H, against the tax. Human wealth is maximized at the optimal tax, $\tau = 0.33$. The bottom right panel plots aggregate capital, K, against the tax. Capital is maximizes at the optimal tax, $\tau = 0.33$.

**Figure 2: Ex ante welfare 2: optimal tax does not maximize capital**

Figure 2 uses parameter values $\alpha = 0.7$, $\sigma = 0.1$, $\beta = 0.98$. The top left panel plots the first order condition (28) against the capital tax. The line cuts the horizontal axis at the ex ante optimal tax, which is negative at $\tau = -0.13$. The top right panel plots the derivative of ϕ against the tax. This derivative is always negative, indicating that ϕ is falling at the optimum. The bottom left panel plots human wealth, H, against the tax. Human wealth is maximized at the optimal tax, $\tau = -0.13$. The bottom right panel plots aggregate capital, K, against the tax. Capital is falling at the optimum.

Figure 3: Comparative statics
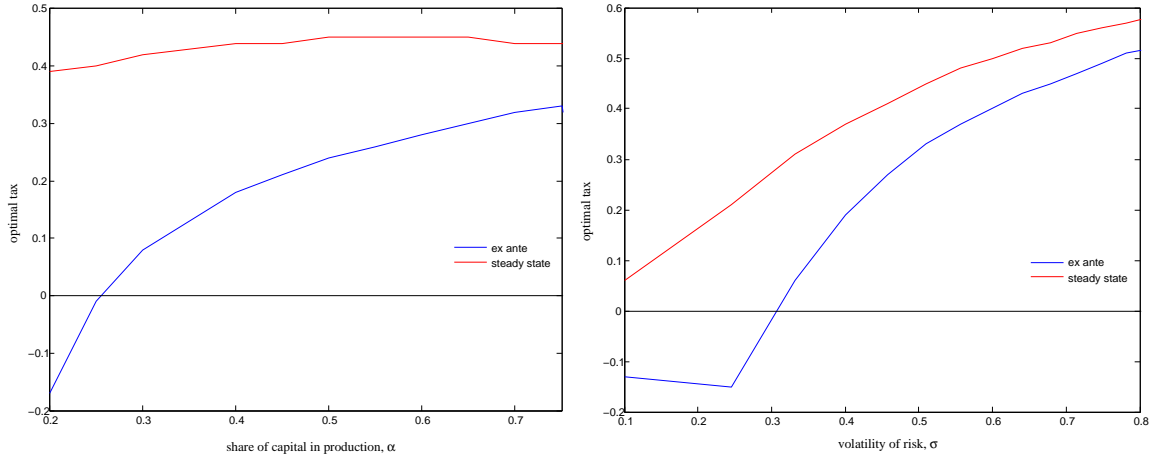
(a) Share of Capital

(b) Volatility of Risk
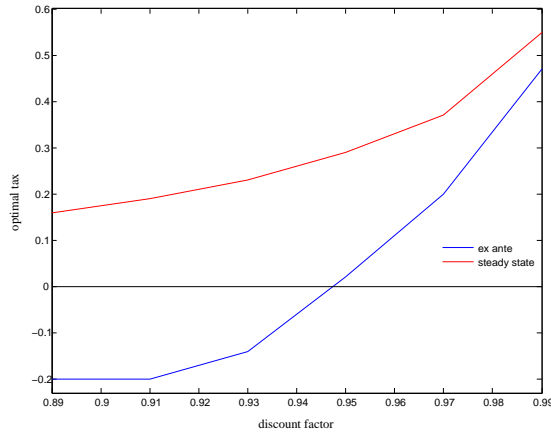
(c) Discount Factor

Figure 3 performs robustness tests with respect to the main model parameters. On the horizontal axis are the various values of each parameter. On the vertical axis is the optimal capital tax. The blue line shows the ex ante optimal tax for each parameter value. The red line shows the steady state optimal tax for each parameter value. Panel (a) uses $\sigma = 0.5$, $\beta = 0.98$, and varies the values of α. Panel (b) uses $\alpha = 0.7$, $\beta = 0.98$, and varies the values of σ. Panel (c) uses $\alpha = 0.7$, $\sigma = 0.5$, and varies the values of β.