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Financing Constraints, Firm Dynamics, and International Trade

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Abstract

There is growing empirical support for the conjecture that access to credit is an important determinant of firms' export decisions. We study a multi-country general equilibrium economy in which entrepreneurs and lenders engage in long-term credit relationships. Financial constraints arise as a consequence of financial contracts that are optimal under private information. Consistent with empirical regularities, the model implies that older and larger firms have lower average and more stable growth rates, and are more likely to survive. Exporters are larger, their survival in international markets increases with the time spent exporting, and the sales of older exporters are larger and more stable.

Keywords: private information, long-term financial contracts, exporter dynamics, international trade, financial intermediation

JEL classifications: F10, D82, L14

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1 Introduction

Empirical studies of firms and industries reveal that most exports are produced by a small number of very large firms. In the United States, for example, less than 5 percent of all firms exported some of their production in 2010. More than 97 percent of these exporters were small- and medium-sized firms (500 employees or less), which accounted for about 34 percent of total exports.[1] In contrast to large exporters, there is substantial year-to-year transition in and out of export markets for smaller firms. New exporters are typically small relative to the average exporter and frequently stop exporting after one year, while continuing exporters are more likely to expand rapidly in export markets (Bernard and Jensen (2004)).

Does access to credit affect a firm's decision to export? How does the nature of the credit relationship between banks and firms shape the growth of new exporters? Small- and medium-sized firms tend to be more reliant on external financing, which is mostly debt as equity is typically owned by proprietors. There is substantial empirical evidence that the financial conditions faced by small and young firms play an important role in shaping their growth, which is widely interpreted as indirect evidence of frictional financial markets.[2] Hysteresis in export markets suggests the presence of a fixed cost of entry (e.g., Das, Roberts, and Tybout (2007), Paravisini, Rappoport, Schnabl, and Wolfenzon (2011)), thereby suggesting that participation in international trade requires greater access to financing. This in turn could imply that the export decisions of small and medium-sized firms to export is sensitive to the availability of credit. Minetti and Zhu (2011) find evidence supporting this hypothesis by showing that the probability of exporting and the intensity of export

[1] http://www.census.gov/foreign-trade/Press-Release/edb/2010/text.txt

[2] See Hubbard (1998) and Stein (2003) for surveys.

is significantly lower for credit rationed firms in Italy.[3]

Economic theory posits that small and young firms are generally more opaque to external scrutiny. This opaqueness creates an informational asymmetry between lenders and entrepreneurs leading to adverse selection and moral hazard problems. As a result, competitive banks may choose to either ration the supply of credit to young and small firms instead of increasing the price of credit to clear the market (e.g., Stiglitz and Weiss (1981)), or to reduce the private information through repeated interaction and monitoring of firms (e.g., Diamond (1984, 1991), Rajan (1992), and Allen and Gale (1999)). Clementi and Hopenhayn (2006) show that long-term financial contracts that are constrained efficient under private information can help account for some of the empirical regularities on firm dynamics – firm entry and exit, and the mean and variance of firm growth. Economic theory is, however, relatively uninformative regarding how private information and long-term credit relationships may affect firms' export decisions and shape their growth in international markets. We propose to fill this gap by studying a general equilibrium multi-country model economy in which entrepreneurs and lenders enter into multi-period credit relationships that are constrained efficient under informational asymmetry.

In the model, entrepreneurs are born with a blueprint to start a long-lived firm. A firm requires an initial fixed investment to start, and working capital to pay for factor input and the trade cost before production takes place. New entrepreneurs do not have wealth to start a firm, and must seek financing from competitive financial inter-

[3]Several empirical studies find a positive association between firm financial health and export status (e.g., Greenaway and Kneller (2007), Mulls (2008) and Chor and Manova (2010), and Bellone, Musso, Nesta, and Schiavo (2010)). However, difficulties in directly measuring the extent of firm financial constraints and separating the effect of credit supply from credit demand on export decisions make it difficult to interpret these correlations. For example, it could also be the case that firms' export status has an important impact on their financial health by giving them access to a larger market and further risk diversification.

mediaries. Financial frictions arise because financial intermediaries cannot directly observe the revenue generated by the firms they are financing, and must instead rely on reports from creditor entrepreneurs. Financial intermediaries mitigate the moral hazard by offering new entrepreneurs a long-term financing contract designed to induce truthful reporting. The financial arrangement in our model is closely related to that in Clementi and Hopenhayn (2006), and a financing constraint emerges as an outcome of the optimal contract.

In equilibrium, new firms operate below their efficient level, and the financing constraint is relaxed as the entrepreneur's claim to future cash-flows increases. Firms that are able to service their debt for a sufficiently long time may borrow enough to pay the trade costs and expand into international markets. New exporters are less financially constrained than domestic firms, but their growth continues to depend on their performance each period until they become fully unconstrained. Financial intermediaries actively engage in maturity and risk transformation in a competitive financial market using workers' and entrepreneurs' short-term deposits to fund a portfolio of long-term risky projects.

Consistent with empirical regularities on firms dynamics (e.g, Cooley and Quadrini (2001)), the model implies that older and larger firms have lower average and more stable growth rates, and are more likely to survive; and that smaller and younger firms pay fewer dividends, borrow and invest more, and that the investment of small firms is more sensitive to cash flows, even after controlling for their future profitability. Consistent with with empirical studies on exporters (e.g., Eaton, Eslava, Kugler, and Tybout (2007) and Ruhl and Willis (2008)), the model implies that new exporters account only for a small share of total exports, and that a large fraction of new exporters does not continue to export in the following year. Furthermore, continuing exporters are less likely to exit export markets as the number of years exporting increases, have larger and more stable sales, and generally reach their efficient size in a few years.

This paper contributes to the theoretical literature exploring the dynamics of firm export decisions. Research in this direction has modeled firm export dynamics as the outcome of learning as in Eaton, Eslava, Jinkins, Krizanc, and Tybout (2012), investment in risky R&D as in Atkeson and Burstein (2010), persistent idiosyncratic shocks to productivity (e.g., Ruhl and Willis (2008), Arkolakis (2011) and Alessandria and Choi (2011), and Kohn, Leibovici, and Szkup (2011)).[4] A key difference is that selection into export markets does not depend on a firm's (expected) productivity (e.g., Melitz (2003)), which is constant in our model. Rather, selection into export market depends on a firm's present value of expected discounted cash flow, whose evolution is governed by the financial contract and its performance. Furthermore, our general equilibrium framework proposes a novel link between industry dynamics, the balance sheets of lenders, and aggregate conditions, thereby relating financial intermediation to international trade.

The rest of the paper is organized as follows: Section 2 presents the model and Section 3 describes the financial arrangement between investors and entrepreneurs and derives the properties of the optimal contract. Section 4 defines the general equilibrium, and section 5.1 analyzes the model numerically. Concluding comments are contained in Section 6; proofs of propositions and derivations are relegated to the Appendix.

[4]To a lesser degree, our paper is also related to research on the impact of financing constraints on the welfare gains of international trade. Financing constraints in these models arise because of exogenous collateral requirement (e.g., Chaney (2005), Manova (2008)), one-period contracts that are optimal under private information (Feenstra, Li, and Yu (2011)), or long-term contracts that are optimal under limited enforcement (Wang (2010), Brooks and Dovis (2011)).

2 Model

2.1 Workers

Workers are born without wealth, survive into the next period with exogenous probability $(1-\gamma_w)$, and are instantly replaced by new ones when deceased. Workers discount the future at rate $(1-\gamma_w)\hat{\beta}$ and are endowed with one unit of time each period, which they allocate between labor h_t and leisure. Labor is paid at wage w_t, and workers use their income to either buy the numéraire consumption good c_t, or to purchase contingent claims d_{t+1} at price p_t^a that pay $(1+r_t)$ units of consumption in the next period if the agent is alive, and zero otherwise. Agents do not value bequests, and will thus place all their savings in these claims. Workers assess their consumption-leisure decision according to

$$E_0 \sum_{t=0}^{\infty} [(1-\gamma_w)\hat{\beta}]^t u(c_t, 1-h_t), \tag{1}$$

which they maximize subject to a budget constraint

$$c_t + p_t^a d_{t+1} \leq d_t(1+r_t) + w_t h_t \ , \tag{2}$$

and $d_{t+1} \geq \epsilon$, where ϵ is the workers' natural borrowing limit.[5]

2.2 Entrepreneurs

New entrepreneurs are born with a blueprint to produce an intermediate good $\omega \in \widetilde{\Omega}$. Entrepreneurs, like workers, are born without wealth, survive into the next period with probability $(1-\gamma_e)$, and are instantly replaced upon death. Entrepreneurs

[5] We show in Appendix C that the natural borrowing limit is never binding in our calibration.

are risk-neutral, and discount the future at the rate $(1-\gamma_e)/(1+r)$.[6] We assume entrepreneurs do not make labor-leisure decisions, and instead devote a fixed fraction of their time to supervise their firm. Entrepreneurs assess their consumption decision according to

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t, \tag{3}$$

where $\beta = \left(\frac{1-\gamma_e}{1+r}\right)$. Entrepreneurs do not take part in the annuity market, and consume all their period income.

2.3 Financial intermediaries

Financial intermediaries are risk-neutral and discounts the future at the same rate as entrepreneurs. They raise short-term deposits from workers via the annuity market, and can offer long-term financing to the entrepreneurs. The assumptions on worker characteristics imply stationary demographics of workers so that annuities can be offered without risk. Deposits from workers and entreprenuers in period t are used to fund entrepreneurs' risky projects in period $t+1$. Repayments from entrepreneurs to the intermediary are used to repay the deposits with interest. Perfect competition and constant returns to scale in financial intermediation implies that we can focus on a representative financial intermediary.

2.4 Final good firms

The final good is assembled by a large number of firms using domestically produced and imported intermediate goods and a constant elasticity of substitution aggregator. Intermediate goods are imperfect substitutes, and final good producers maximize

[6] We are anticipating that the risk free rate r_t is constant in the stationary steady state equilibrium.

their profit

$$\left(\int_{\Omega} y(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\Omega_f} y(\omega_f)^{\frac{\sigma-1}{\sigma}} d\omega_f\right)^{\frac{\sigma}{\sigma-1}} - \int_{\Omega} y(\omega)p(\omega)d\omega - X\int_{\Omega_f} y(\omega_f)p(\omega_f)d\omega_f \tag{4}$$

by choosing quantities $y(\omega)$ and $y(\omega_f)$ taking prices $p(\omega)$ and $p(\omega_f)$ as given, where Ω and Ω_f are the set of goods available from domestic and foreign producers, respectively, $\sigma > 1$ is the elasticity of substitution between varieties, and X is the exchange rate. Constant returns to scale and perfect competition in the final goods market imply zero-profits, which lets us concentrate on a representative final good producer.

2.5 Intermediate goods firms

Producing an intermediate good $\omega \in \widetilde{\Omega}$ requires an initial investment I_0 that is sunk, and per period working resources R_t to hire labor and to be used as capital. The ω-th firm produces the ω-th good according to a neo-classical production function $G(k_t, n_t)$, where k_t is capital input and n_t is labor input. We assume that the capital used in production is fully depreciated at the end of the period. An entrepreneur wishing to export must pay a fixed export cost I_E before production begins, and chooses the quantity q_t and q_t^* of goods to sell domestically and abroad, respectively.[7] It follows that the allocation of period working resources R must satisfy:

$$k + nw + \mathbf{1}(q^* > 0)I_E \leq R \ , \tag{5}$$

where $\mathbf{1}(q^* > 0)$ is an indicator function that is equal to 1 when $q^* > 0$. To sell one unit of goods abroad, a firm must ship $(1 + I_T)$ units of this good. This is a

[7] In what follows, variables marked with an asterisks denote exported goods' quantities and prices.

standard iceberg cost, and it implies that the allocation of output between domestic and export sales must satisfy:

$$q + q^*(1 + I_T) \le G(k, n). \tag{6}$$

The ω-th firm is a monopolist for its differentiated product, and takes the inverse demand function for its product $p(q)$ – price as a function of quantity – as given. Project returns of all firms $\omega \in \widetilde{\Omega}$ are subject to a sequence of independent and identically distributed idiosyncratic revenue shocks $(\theta_t)_{t\ge 0}$, where $Pr(\theta_t = 1) = 1 - Pr(\theta_t = 0) = \pi$. Firm status is indexed by $i \in \{D, E\}$, where D and E indicate that a firm sells to the domestic market only, or to both the domestic and export market, respectively. The maximum revenue a firm can generate with resources R is:[8]

$$\begin{aligned} \theta F_i(R) = \theta \max\nolimits_{q,q^*,k,n} \quad & p(q)q + \mathbf{1}(i = E)p^*(q^*) \\ \text{s.t.} \quad & q + q^*(1 + I_T) \le G(k, n) \\ & k + nw + \mathbf{1}(q^* > 0)I_E \le R \ . \end{aligned} \tag{7}$$

A firm is terminated when the entrepreneur dies.[9]

---

[8] We assume there exists a unique level of resources past which a firm can only maximize its revenue by exporting some of its production. That is, we assume that there exist R_{dx} such that $F_D(R) > F_E(R)$ for all $R < R_{dx}$ and $F_D(R) < F_E(R)$ for all $R > R_{dx}$. If no such level exists, trade is never profitable and the two countries do not trade. In the appendix, we show that if R_{dx} exists, then the crossing point of $F_D(R)$ and $F_E(R)$ is unique.

[9] This assumption is convenient to capture other sources of exit not modeled explicitly and is necessary to obtained a stationary distribution of firms. See for instance Cooley and Quadrini (2001), Cooley, Marimon, and Quadrini (2004), and Smith and Wang (2006).

3 Information and long-term credit relationship

Revenues are only observed by entrepreneurs, so lenders can only learn about the firm's performance and the realizations of the revenue shocks θ_t through entrepreneurs' reports, $\hat{\theta}_t$. We denote the history of reports up to period t by $h^t = (\hat{\theta}_1, \ldots, \hat{\theta}_t)$. A contract is a set of decision rules $\kappa_t = \{\ell_t(h^{t-1}), e_t(h^{t-1}), Q_t(h^{t-1}), R_t(h^{t-1}), \tau_t(h^{t-1}, \hat{\theta}_t)\}$. Conditional on surviving, a firm is either liquidated, $\ell_t(h^{t-1})$, in which case the entrepreneur receives $Q_t(h^{t-1})$ and the financial intermediary receives $S - Q(h^{t-1})$, where $S \leq I_0$ is the salvage value, or it remains in operation. If a firm is kept in operation, the contract specifies whether or not the firm exports, $e_t(h^{t-1})$, and the size of the loan, $R_t(h^{t-1})$. After production takes place and revenues are realized, an entrepreneur makes a repayment $\tau(h^{t-1}, \hat{\theta}_t)$ to the financial intermediary conditional on his ex-post report $\hat{\theta}_t$. Figure 1 summarizes the timing of events within one period.

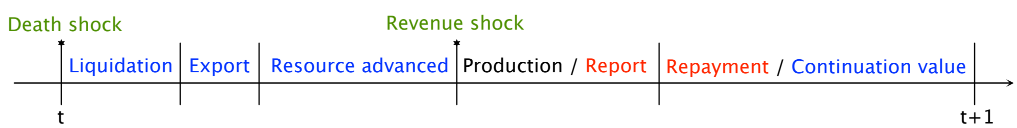


Figure 1: Timing

3.1 The optimal financial contract

A reporting strategy for an entrepreneur is a sequence of reports $\underline{\hat{\theta}} = \{\hat{\theta}_t(\theta^t)\}$, where $\theta^t = (\theta_1, \ldots, \theta_t)$ is the true history of realizations of revenue shocks. After every history h^{t-1}, the pair $(\kappa, \underline{\hat{\theta}})$ implies an expected discounted cash flow $V_t(\kappa, \underline{\hat{\theta}}, h^{t-1})$ and $B_t(\kappa, \underline{\hat{\theta}}, h^{t-1})$ for the entrepreneur and the financial intermediary, respectively. A feasible and incentive compatible contract is optimal if it maximizes $B_t(V_t)$ for every possible V_t. Following Clementi and Hopenhayn (2006), we refer to V_t and B_t as equity and debt, respectively, so that the joint surplus $W(V) = B(V) + V$ is the value of the firm.

Using the method of Abreu, Pearce, and Stacchetti (1990), the optimal contract can be written recursively by using V_t as a state variable and by defining V_t^H and V_t^L as promised continuation values. It follows that equity must satisfy the following accounting identity:

$$V = \pi(F_i(R) - \tau) + \beta[\pi V^H + (1-\pi)V^L] \ , \tag{8}$$

which states that current period equity is equal to the expected net cash flow this period plus the discounted expected equity next period. In order to induce truthful reporting, incentive compatibility constraints are required. Since entrepreneurs who receive a low shock do not have an incentive to report a high shock, there is only one binding such constraint:

$$F_i(R) - \tau + \beta V^H \geq F_i(R) + \beta V^L \ . \tag{9}$$

Limited liability requires entrepreneurs' dividends to be non-negative,[10] so that

$$\tau \leq F_i(R) \ . \tag{10}$$

[10]Let $\tau_t = \tau_t(h^{t-1}, \hat{\theta}_t = 1)$ since limited liability implies that repayments are 0 when an entrepreneur reports a low shock and negative repayments are not optimal under risk-neutrality.

Conditional on surviving, the value of an i-type firm is given by,

$$
\begin{aligned}
\widehat{W}_i(V) = \max_{\tau, R, V^H, V^L} \quad & \pi F_i(R) - (1+r)R + \beta \mathbb{E} W(V') \\
\text{s.t.} \quad & (8),\ (9),\ \text{and}\ (10) \\
& V^H, V^L \geq 0.
\end{aligned} \tag{11}
$$

There exists a region $[V_D, V_E]$ within which a greater value of the joint surplus can be reached by allowing for a lottery on the export decision:[11]

$$
\begin{aligned}
\widehat{W}(V) = \max_{\delta \in [0,1], V_D, V_E} \quad & \delta \widehat{W}_E(V_E) + (1-\delta)\widehat{W}_D(V_D) \\
\text{s.t.} \quad & \delta V_E + (1-\delta) V_D = V \\
& V_D, V_E \geq 0
\end{aligned} \tag{12}
$$

where δ is the probability of becoming an exporter and V_D and V_E are the respective continuation values (within the same period) if the firm sells purely domestically or also exports.[12] Furthermore, $W(V)$ takes into account the option of liquidating the firm

$$
\begin{aligned}
W(V) = \max_{\alpha \in [0,1], Q, V_r} \quad & \alpha S + (1-\alpha)\widehat{W}(V_r) \\
\text{s.t.} \quad & \alpha Q + (1-\alpha) V_r = V \\
& Q, V_r \geq 0
\end{aligned} \tag{13}
$$

where α is the probability of liquidation and V_r is the continuation value when the firm is not liquidated.[13] Proposition 3.1 summarizes the basic properties of the value

[11] See proof in the appendix. Note that we implicitly assume that this region does not overlap with the liquidation region, that is $V_D > V_r$.

[12] Whenever a firm reaches a size $V \in [V_D, V_E]$, it is offered an export lottery and becomes an exporter of size V_E with probability $\delta = (V - V_D)/(V_E - V_D)$, or a domestic seller of size V_D with probability $(1-\delta)$. The boundaries of the export region $[V_D, V_E]$ are determined such that the tangent of $\widehat{W}_D(V)$ at V_D is equal to the tangent of $\widehat{W}_E(V)$ at V_E.

[13] This states that whenever V falls below V_r, the financial intermediary offers the entrepreneur a lottery where the project is either liquidated with probability $\alpha = (V_r - V)/V_r$ in which case the

function.

Proposition 3.1 *The value function $W(V)$ is increasing and concave. There exist values $0 < V_r < V_D < V_E < \widetilde{V}$ such that:*

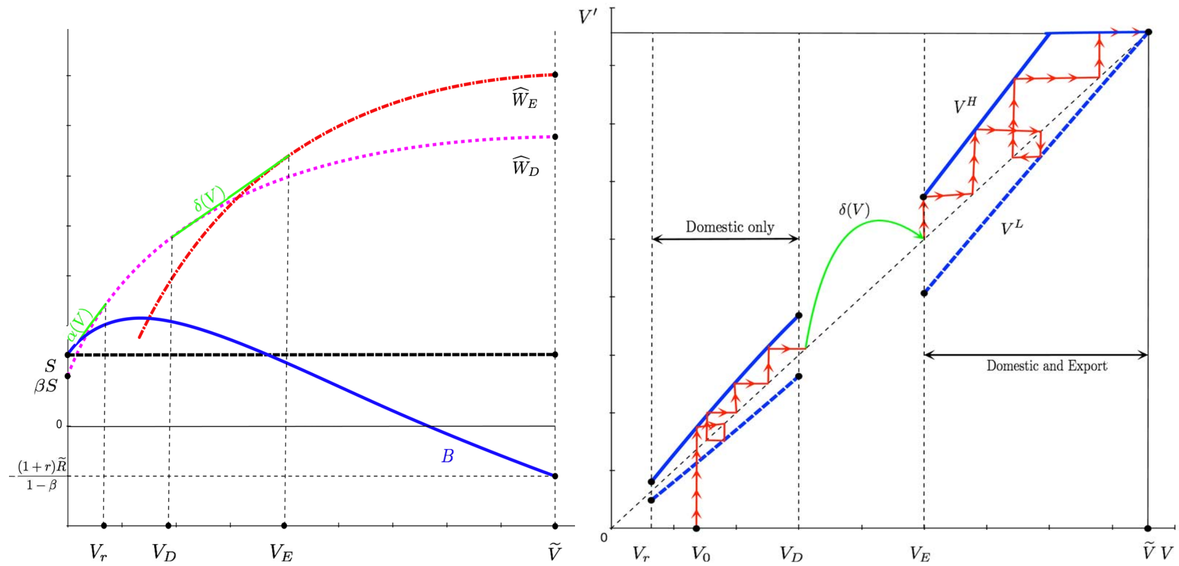
- *$W(V)$ is linear for $V \in [0, V_r] \cup [V_D, V_E]$, equal to $\widetilde{W}$ when $V = \widetilde{V}$ and strictly increasing when $V \in [V_r, V_D] \cup [V_E, \widetilde{V})$*
- *The firm is liquidated with probability $\alpha(V) = (V_r - V)/V_r$ if $V \in [0, V_r)$*
- *The firm exports with probability one when $V \in [V_E, \widetilde{V}]$, with probability $\delta(V) = (V_E - V)/(V_E - V_D)$ when $V \in (V_D, V_E)$, and zero otherwise*

3.2 Properties of the financial contract

Panel (a) of Figure 2 plots the optimal value of the firm, $W(V)$, and the value to the intermediary, $B(V)$, as a function of equity, V. A firm faces a binding borrowing constraint whenever its equity is below $\widetilde{V} = \pi F_E(\widetilde{R})/(1-\beta)$, where $\widetilde{R}$ is the unconstrained level of resources. That is, $\widetilde{R}$ is the level of resources that solves the static profit maximization of the firm such that $\widetilde{R} = \text{argmax}_R\{\pi F_E(R) - R(1+r)\}$. New firms start at $V_0 \leq \widetilde{V}$, so that expected profits of the intermediary $B(V_0)$ cover the cost of the initial investment $I_0(1+r)$. Smaller firms take on more debt than larger firms, and firms with equity less than V_D cannot borrow enough to pay the trade costs.

Firms' access to credit and growth are determined by the evolution of their capital structure. Using constraints (8), (9) and (10), and solving for next period's equity

entrepreneur receives Q from the intermediary, or kept in operation with probability $1-\alpha$ and is awarded V_r. Optimally, $V_r = \sup\{V : \widehat{W}(V) - S = V\widehat{W}'(V)\}$ and $Q = 0$.



(a) Value function (b) Continuation values

Figure 2: Financial contract

conditional on the revenue report yields the following law of motion for equity:

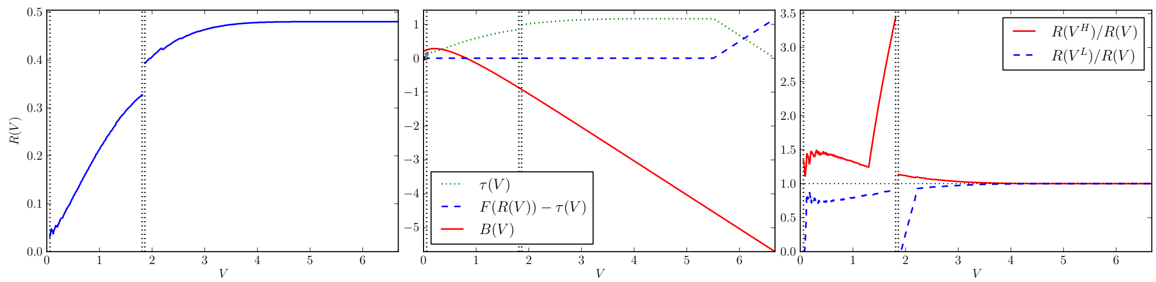
$$V^L(V) = \begin{cases} \frac{V - \pi F_D(R(V))}{\beta} & \text{if } V \in [V_r, V_D] \\ \frac{V - \pi F_E(R(V))}{\beta} & \text{if } V \in [V_E, \widetilde{V}) \end{cases} , \tag{14}$$

and

$$V^H(V) = \begin{cases} \frac{V + (1-\pi) F_D(R(V))}{\beta} & \text{if } V \in [V_r, V_D] \\ \min\left\{\widetilde{V}, \frac{V + (1-\pi) F_E(R(V))}{\beta}\right\} & \text{if } V \in [V_E, \widetilde{V}) \end{cases} . \tag{15}$$

Panel (b) of Figure 2 summarizes the evolution of firm equity following a particular sequence of revenue shocks. Small domestic firm may start exporting after receiving a finite sequence of positive revenue shocks. New exporters are less financially constrained than domestic firms, but their growth continues to be shaped by the optimal contract and the revenue shocks as long as $V_E < \widetilde{V}$. And from Proposition 3.2, firms grow on average.

Proposition 3.2 *Conditional on surviving, a firm grows on average. That is* $\{V'\}_{t\geq 0}$ *is a sub-martingale so that* $E(V'|V) \geq V$*.*



(a) Period loan (b) debt and dividend (c)Conditional investment

Figure 3: Financial characteristics of the contract

Figure 3 plots the decision rules for loans, repayments, and dividends as a function of equity. Due to risk-neutrality, joint surplus is maximized when equity grows fastest, so dividends to the entrepreneur are optimally zero until the firm can no longer grow faster by postponing dividends, which is when $V^H(V) = \widetilde{V}$. This implies that it is optimal for the financial intermediary to set the entrepreneur's repayments to $\tau(V) = F_i(R(V))$ for $i = \{D, E\}$ whenever $V^H(V) < \widetilde{V}$ as it allows for the fastest accumulation of equity toward the unconstrained level. Furthermore, the optimization problem takes place on the convex set $[0, \widetilde{V}]$, which implies $V^H(V) = \widetilde{V}$ whenever $(V + (1-\pi)F_i(R(V))/\beta > \widetilde{V}$. From constraints (8) and (9):

$$\tau(V) = \begin{cases} F_i(R(V)) & \text{if } V^H(V) < \widetilde{V} \\ \beta(\widetilde{V} - V^L(V)) & \text{if } V^H(V) = \widetilde{V} \end{cases}, \tag{16}$$

which implies that conditional on a high revenue shock, resource advancement $R(V)$ and repayment $\tau(V)$ increase with firm equity up until $V^H(V) = \widetilde{V}$. Past this threshold, repayments start declining and dividend payments start increasing until they eventually reach 0 and $F(R)$, respectively. At this size, firm equity no longer changes, and the borrowing constraint ceases forever. The value of an unconstrained firm is

$$W(\widetilde{V}) = \widetilde{V} + B(\widetilde{V}) = \frac{\pi F_E(\widetilde{R})}{1-\beta} - \frac{\widetilde{R}(1+r)}{1-\beta}. \tag{17}$$

Therefore, a firm is financially unconstrained when its entrepreneur has accumulated enough capital through its repayments to the financial intermediary to finance the firm operation at full scale in every period and under all contingencies at the current interest rate.

Panel (c) of Figure 3 plots the investment rate conditional on receiving a high and low revenue shock as a function of equity. Investment by constrained firms is always positive after receiving a high shock, and always negative after recieving a low shock for constrained firms. The investment of small firm, and therefore cashflow, is also more sensitive to revenue shocks than that of larger firms. Furhermore, there is a large increase in investment once the firm becomes an exporter, with subsequent very high possible disinvestment should the firm receive a low shock and exit export markets.

4 General equilibrium

Perfect competition in the financial sector implies that annuities are priced at the workers' survival rate $(1-\gamma_w)$.[14] The assumptions on worker characteristics ensure that there exists a stationary demographic with constant aggregate deposits and labor supply. Let d_j and h_j be the deposits and hours worked of a j-period old

[14] In what follows, we focus on the domestic economy, but analogous conditions must also hold in the foreign economy.

worker. Setting the mass of workers to one, it follows that aggregate deposits by workers each period are given by

$$D = \gamma_w \sum_{j=1}^{\infty} (1 - \gamma_w)^j d_j. \tag{18}$$

Similarly, the aggregate labor supply in each period is

$$H = \gamma_w \sum_{j=0}^{\infty} (1 - \gamma_w)^j h_j \ . \tag{19}$$

Perfect competition in the financial sector also implies that financial intermediaries break even on new contracts with entrepreneurs, or that $W(V_0) - V_0 = B(V_0) = I_0(1 + r)$ in equilibrium. As discussed in the previous section, firm equity evolves according to the conditional continuation values specified in the contract, $V^H(V)$ and $V^L(V)$, and the sequence of revenue shocks. Let $\mathbf{M}$ be the state space for firm equity so that $V \in \mathbf{M}$, $\mathcal{M}(V)$ be the Borel $\sigma-$algebra generated by $\mathbf{M}$, and μ the measure defined over $(V, \mathcal{M}(V))$. Proposition 4.1 follows:

Proposition 4.1 *There exists a unique stationary distribution of firms that is ergodic.*

The intermediary uses the capital it has accumulated through entreprenuers' repayment, Z, and workers' deposits, D, to finance the initial set-up cost and the wage and capital expenditures of all firms before production takes place. It follows that the capital market clears when

$$\int R d\mu + \Gamma I_0 = D + Z \ , \tag{20}$$

where $\Gamma = \Gamma_b + \gamma_e$ is the fraction of new born firms, which is equal to the share of liquidated firms Γ_b plus exogenously exiting ones.[15] Equation 20 characterizes the balance sheet of the representative financial intermediary, and we refer to $Z = -\int B(V)d\mu$ as banking capital.[16]

Furthermore, the intermediary's budget must be balanced each period. That is, the intermediary's receipts from entrepreneurs plus the scrap value from liquidating poorly performing firms and the return on their own equity must be large enough to finance the cost of borrowing funds on the capital market. A stationary distribution of firms implies that $Z = Z'$ in equilibrium, and it follows that:

$$rZ = (1+r)\left(\int R d\mu + \Gamma I_0\right) - \left(\pi \int \tau d\mu + \Gamma_b S\right) \ , \tag{21}$$

Equation 21 shows that the interest earned on bank capital is equal to the difference between aggregate receipts from, and disbursements to, entrepreneurs in the financial intermediary's portfolio.

Labor market clearing requires that the labor supply from workers is equal to the demand for labor by firms, so that:

$$H = \int n d\mu \ . \tag{22}$$

Clearing of the intermediate goods markets requires that the demand from the final

[15]We use the shorthand $\int R d\mu$ for $\int_{V\in[0,\widetilde{V}]} R(V)d\mu(V)$ and similarly for other expressions.

[16]As we discussed in the previous section, new firms start with a positive level of debt B. Young entreprenuers increase their stake in the future discounted present value of their firm by making positive net-payment to the intermediaries. Unconstrained entreprenuers can have accumulted enough euquity through repayment to finance their firm using the return on their equity. It follows that aggregate entrepreneur savings can be positive, negative, or zero; and the same is true for aggregate worker savings. Total deposits must be to be positive however, or no p[rodcution would takle place otherwise.

good producer is equal to the supply of intermediate goods:

$$y(\omega) = q(\omega) \quad \forall \omega \in \Omega \text{ , and } y(\omega_f) = q(\omega_f) \quad \forall \omega_f \in \Omega_f \text{ ,} \tag{23}$$

where Ω and Ω_f are the sets of all domestically produced and imported goods respectively, available in an economy to be converted into the final good.[17] Trade must be balanced, so that the total value of imports is equal to the total value of exports times the exchange rate X:[18]

$$\int_{\Omega_f} y(\omega_f)p(\omega_f)d\omega_f = X \int_{\Omega^*} y(\omega^*)p(\omega^*)d\omega^*. \tag{24}$$

The final good market clears when total production equals aggregate consumption of workers and entrepreneurs plus investment.[19] That is,

$$Y = C_w + C_e + K, \tag{25}$$

where total capital expenditure K, aggregate consumption by workers, C_w, and by

[17]Similar conditions must also hold abroad and thus $y(\omega^*) = q(\omega^*) \, \forall \omega^* \in \Omega^*$ where Ω^* is the set of exported goods.

[18]The exchange rate is one when countries are symmetric. Furthermore, note that a condition concerning arbitrage between the home and foreign final good is not necessary as the final good cannot be traded.

[19]See the appendix for a proof that Walras's law indeed holds in this economy.

entrepreneurs, C_e, are given by

$$K = \int k d\mu + \Gamma_e I_E + \Gamma I_0 - \Gamma_b S \tag{26}$$

$$C_w = \gamma_w \sum_{j=0}^{\infty} (1-\gamma_w)^j c_{wj} = Dr + Hw \tag{27}$$

$$C_e = \pi \int F(R) d\mu - \pi \int \tau d\mu \ . \tag{28}$$

The definition of the worldwide stationary equilibrium follows:

Definition 4.2 (Worldwide Stationary Equilibrium) *A stationary equilibrium consists of decision rules for labor supply h, consumption c_w, and deposits d' for workers in each country; a contract policy in each country, consisting of: promised values $V^H(V)$ and $V^L(V)$, period resource advancements $R(V)$, liquidation lottery $\alpha(V)$, export lottery $\delta(V)$, and repayments $\tau(V)$; an initial contract state V_0 in each country; wages $\{w, w^*\}$ and interest rates $\{r, r^*\}$; prices $\{p(\omega), p_f(\omega)\}$ and $\{p(\omega^*), p_f(\omega^*)\}$ for intermediate goods; and an exchange rate X, such that*

1. *the labor and consumption function maximize the workers' value function $U(d)$ in each country*

2. *the contract policy maximizes the value of the firm $W(V)$ in each country*

3. *the initial state V_0 is such that the intermediary in each country breaks even on a new contract – i.e. $V_0 = \sup_V \{B(V) = (1+r)I_0\}$*

4. *the intermediary's budget in each country is balanced every period*

5. *the labor and capital markets clear in each country*

6. *the domestic and imported intermediary goods market clear in each country*

7. *trade between the two countries is balanced.*

Proposition 4.3 *There exists a worldwide stationary equilibrium.*

5 Numerical analysis

The contract needs to be solved numerically.[20] Once the value of the firm $W(V)$ and the decision rule for loan size $R(V)$ are known, the remaining decision rules can be expressed in closed form as functions of $R(V)$. Given the initial firm size V_0 and the law of motion for V, we can simulate the life-cycle of a large number of firms to estimate the stationary distribution of firms.[21]

5.1 Parametrization and parameter values

Let the instantaneous utility function for the workers be $u(c,h) = \ln(c) + \lambda \ln(1 - h)$.[22] We simplify the analysis by considering the case of symmetric countries and a constant returns to scale Cobb-Douglas production technology for intermediate goods: $G(k,n) = k^{\eta_k} n^{\eta_n}$, with $\eta_l = 1 - \eta_k$. The final good is produced according to a constant elasticity of substituion (CES) production function with constant returns to scale.

A period in the model is 1 year. We begin by fixing five parameters: The worker death rate γ_w is chosen so that the average life of workers is 50 years. The iceberg

[20] The code to solve and simulate the model is written in object oriented Python using the Scipy library, and is available from the authors.

[21] It is computationally efficient to estimate the stationary distribution of firms by simulating the evolution of a single perpetually regenerating firm. We show in the appendix that this stochastic process is a Markov chain and derive its stability properties.

[22] Smith and Wang (2006) show that this functional form implies closed-form solutions for the aggregate supply of labor and aggregate deposits given the workers' demographic assumption, which simplifies the numerical implementation and reduces the computational burden.

cost of exporting is set to 40 percent, which is in line with previous studies such as Anderson and van Wincoop (2004). The probability of a high revenue shock π is 0.5, which produces investment volatility roughly in line with studies of firm such as Cooper and Haltiwanger (2006); and the salvage value S is set to 80 percent of the set-up cost I_0. We set the elasticity of substitution between intermediates to $\sigma = 6$, which is consistent with Broda and Weinstein (2006).[23]

	Parameters	
σ	Elasticity of substitution	6
$\hat{\beta}$	Worker's discount rate	0.959
λ	Elasticity of leisure	2.304
γ_w	Workers' death probability	0.02
η_k	Capital share	0.20
I_0	Setup investment	0.26
S	Salvage value	$0.8 \times I_0$
I_T	Iceberg cost	0.40
I_E	Fixed export cost	0.012
π	Probability of high/low shock	0.5
γ_e	Firm exogenous exit rate	0.051

Table 1: Parameter values

Given the above, the six remaining parameters are jointly chosen to match the following six moments:[24] a labor income share of 60 percent, an average working time of 35 percent, an interest rate of 4 percent, an exit and entry rate of 6.3 percent (in line with Lee and Mukoyama (2008)), a share of exporters of 27 percent in line

[23]The study by Simonovska and Waugh (2010) suggests a lower value for the elasticity of substitution. In the appendix, we investigate how a lower elasticity by considering another economy with $\sigma = 4$ and calibrated to the same set of moments as the economy with $\sigma = 6$. We find comparable qualitative results.

[24]The parameters are chosen so that the distance between the targeted moments and the model simulated moments is minimized.

with Bernard, Jensen, Redding, and Schott (2007), and we require that new firms start at a size that is 15 percent of the unconstrained firm size.[25] Table 1 summarizes the calibration. After solving the model, we simulate the life of 60,000 firms from which we compute the statistics reported in Table 2 and the figures discussed in the next sub-section.

Targeted:		**Not targeted:**	
Interest rate	4	Consumption/Output	79.2
Hours worked	35	Investment/Output	20.8
Labor income share	60	Export/Output	8.7
Entry/exit rate	6.3	Entry rate in export market	3.3
Relative size of entrants	15	New firm size relative to incumbents	33.5
Share of exporters	27	Exit rate from export market after 1 year	32.0
		Domestic firm size relative to exporters	24.2
		Fraction of unconstrained firms	14.8

Table 2: Steady state moments (in percent)

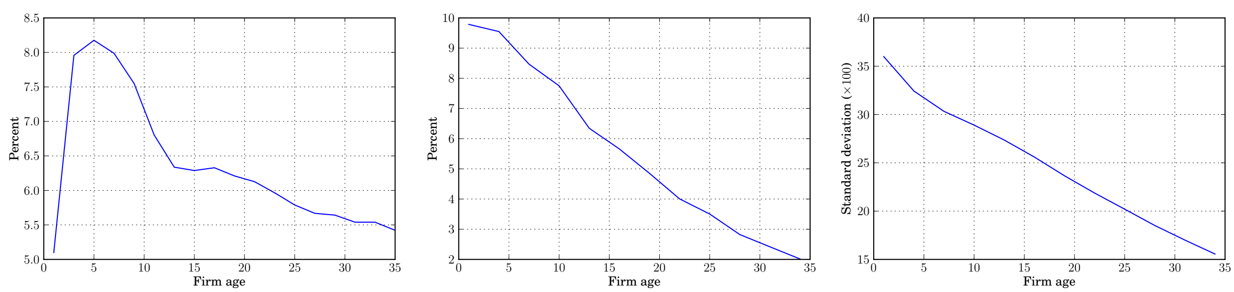
Table 2 shows that, in the aggregate, the consumption-to-output and investment-to-output ratios are roughly in line with data, which principally follows from targeting the labor income share and labor hours. The export-to-output ratio is 8.7 percent, which is in line with the US over the last four decades.[26]

[25] The last target is somewhat arbitrary as it does not have a clear counterpart in the data. However, holding the size of new firms relative to unconstrained ones constant becomes useful when studying the effect of competition (changes in σ) in the market for intermediate goods. See the appendix for more details.

[26] The export share is significantly affected by the elasticity of substitution for intermediate goods. Using the same parametrization targets, we obtain an export to output ratio of 13.8 percent for an economy with $\sigma = 4$. See appendix for details.

5.2 Firm export dynamics

Our results on firm dynamics are consistent with the empirical regularities reported in Cooley and Quadrini (2001) and similar to those in Clementi and Hopenhayn (2006). Panel (a) of Figure 4 shows that the hazard rate of exit first increases for new firms and then decreases with firm age. On average, 1.2 percent of all firms are liquidated every period, which accounts for about 20 percent of all exiting firms. Panels (b) and (c) of Figure 4 plots the mean and standard deviation for firms of a given age, respectively, and show that younger firms experience a faster albeit more volatile growth than older ones.[27]



(a) Hazard rate of exit (b) Mean investment (c) Std. Dev. investment

Figure 4: Firm dynamics

Comparing exporters to domestic firms, Table 2 shows that the average exporter is four times larger (in terms of labor and capital) than the average domestic firm. On one hand, the contract requires that the entrepreneur have sufficient stake into the firm (by accumulating equity) to obtain a loan that is sufficiently large to pay the export costs and generate the additional revenue. On the other hand, the financial position of a firm improves and its access to credit increases after it begins

[27]Given the full depreciation assumption, we define firm investment as the change in loan size from one period to the next, R_t/R_{t-1}.

exporting (Figure 3). Thus, while less financially constrained firms are able to export, the financial health of exporters is substantially higher than that of domestic firms because of their activities. This observation highlights the great difficulty of disentangling these two effects in the data.

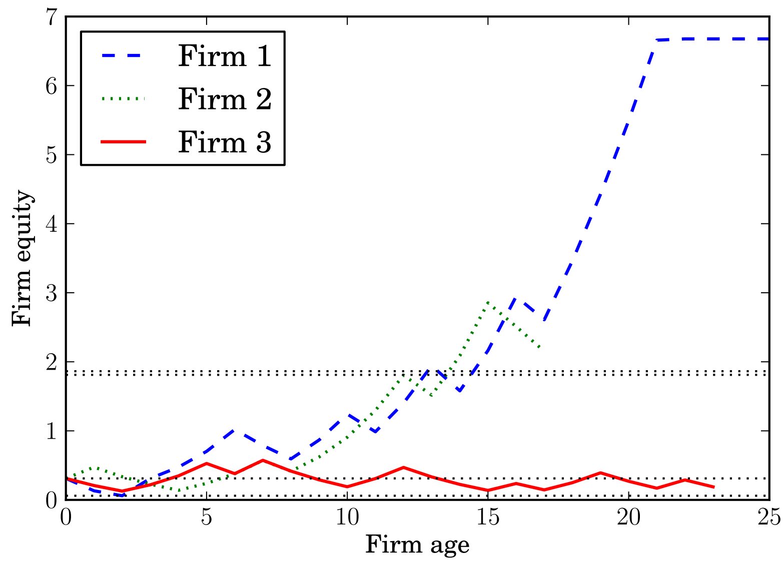
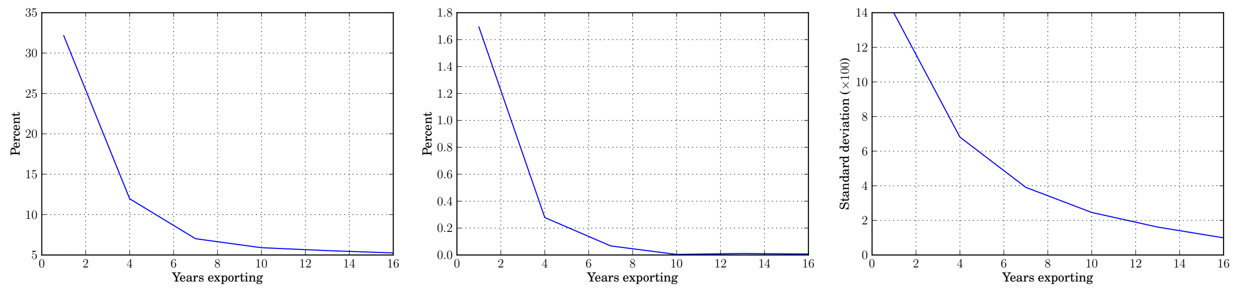


Figure 5: Life cycle of three firms

Let us begin our discussion of exporter dynamics with an example. Figure 5 plots the life-cycle of three firms taken from our sample of simulated firms. It takes 13 years for Firm 1 to accumulate enough equity to beginning exporting, but it exits export markets after its first year. Firm 1 gains access again to export markets at age 15, from which time it continues to grow until it reaches its efficient size at age 21 and finally exits at age 25. Firm 2 reaches the export lottery region after 12 years but initially fails to secure funding to export. Firm 2 successfully become an exporter 2 years later and continue to export until it exits at age 17. Firm 3 is the least successful of our three firms, and was never able to grow nearly large enough to export, and was liquidated by the bank at age 23.[28]

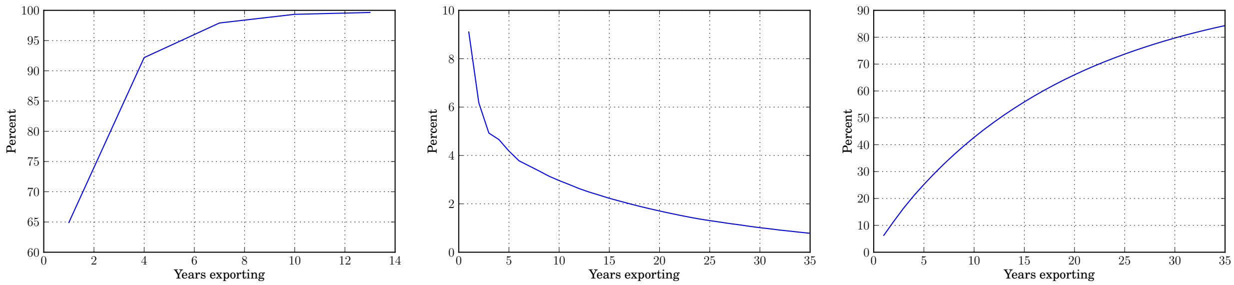


(a) Hazard rate of exit (b) Investment mean (c) Investment std. Dev.

Figure 6: Exporter dynamics

The long-term financial contract plays an important role in shaping both the extensive as well as the intensive margin of trade. New exporters face a high probability of exit from export markets during their first year. Panel (a) of Figure 6 plots the hazard rate of exit from export markets. A third of new exporters exit after their first year. Continuing exporters become less likely to exit as their export spells increase, until they only face the exogenous exit rate of 5 percent. To see this, note that new exporters start out with equity that is close to the export lottery region, so that a low shock leads to exit from export markets if the firm loses the lottery. But since firms grow on average, older exporters, who have more equity, are further away from the export lottery region; and unconstrained exporters only cease to export when the entrepreneur dies.

[28]There is widespread empirical evidence that exporters are more productive than domestic firms. Our model focuses on the effect of financing constraints on export dynamics, and we assume firms have the same expected productivity throughout their lifetime. If we were to interpret the revenue as a productivity shock, an interesting implication of the model is that exporters are more productive than domestic firms in the same age cohort just before they start exporting. Indeed, a new exporter of age j has by definition received a greater number of high shocks than a domestic firm of the same age.



(a) Size of exporters (b) Fraction of exporters (c) Share of total exports

Figure 7: New versus established exporters

Young exporters grow faster than established ones, but their growth is more volatile. Panels (b) and (c) of of Figure 6 plot the mean and standard deviation of investment of exporter conditional on the length of their export spell. The average growth rate of a two year old exporter is 1.2 percent, and is close to 0 after ten years. The standard deviation of investment, however, is about five times higher for a two year old exporter than a ten year old one.

Few firms start exporting every year, and continuing exporters expand rapidly. Panel (a) of Figure 7 shows that new exporters start with about two thirds of the resources used by an unconstrained firm, and operate close to their unconstrained size (95 percent) after exporting for five years (on average).

New exporters are small, and most exports are produced by very large firms. Only 3.3 percent of domestic firms start exporting every period, and this cohort accounts for about 9 percent of all exporters (panel (b) of Figure 7). Exporters that have been exporting for up to five years account only for approximately 25 percent of all exports (panel (c) of Figure 7). Therefore, the model predicts that the bulk of all exports is produced by established firms that have been exporting for five years or more.

6 Conclusion

There is widespread empirical evidence that financial fictions play an important role in shaping the growth of small and young firms. There is also growing empirical evidence that the export decisions of firms are sensitive to the availability of credit. This paper investigates how private information and long-term credit relationships may affect firms' export decisions and shape their growth in international markets. We propose and analyze a general equilibrium multi-country model economy in which entrepreneurs and lenders enter into multi-period credit relationships subject to an informational asymmetry.

We show that the model is consistent with empirical regularities on firms dynamics from the industrial organization literature, and with the models proposed to account for them. Furthermore, in line with recent empirical studies on firm export dynamics, our model predicts that new exporters account only for a small share of total exports, and that a large fraction of new exporters does not continue to export in the following year. Continuing exporters, are less likely to exit export markets as the export spell increases; moreover, continuing exporters experience faster and more volatile growth, and generally reach their efficient size in a few years.

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A Demand for intermediate goods

The first-order condition for the maximization of equation (4) with respect to variety $\hat{\omega} \in \Omega$ yields

$$y(\hat{\omega})^{-1/\sigma} Y^{1/\sigma} = p(\hat{\omega}). \tag{29}$$

Dividing by the equivalent expression of another variety $\omega \in \Omega$ leads to

$$\frac{y(\hat{\omega})}{y(\omega)} = \left(\frac{p(\hat{\omega})}{p(\omega)}\right)^{-\sigma} \tag{30}$$

and similarly for imported goods to

$$\frac{y(\hat{\omega_f})}{y(\omega)} = \left(\frac{Xp(\hat{\omega_f})}{p(\omega)}\right)^{-\sigma}. \tag{31}$$

Multiplying by $p(\hat{\omega})$ (or $Xp(\hat{\omega_f})$ for imported goods) and $y(\omega)$ and summing over all varieties $\hat{\omega} \in \Omega + \Omega_f$, we obtain

$$\int_{\Omega} p(\hat{\omega}) y(\hat{\omega}) d\hat{\omega} + X \int_{\Omega_f} p(\hat{\omega_f}) y(\hat{\omega_f}) d\hat{\omega}_f = \tag{32}$$
$$y(\omega) p(\omega)^{\sigma} \left(\int_{\Omega} p(\hat{\omega})^{(1-\sigma)} d\hat{\omega} + X^{1-\sigma} \int_{\Omega_f} p(\hat{\omega_f})^{(1-\sigma)} d\hat{\omega}_f \right).$$

Since the final goods producer generates zero profits, the left-hand side is simply equal to total output Y:

$$Y = y(\omega) p(\omega)^{\sigma} \left(\int_{\Omega} p(\hat{\omega})^{(1-\sigma)} d\hat{\omega} + X^{1-\sigma} \int_{\Omega_f} p(\hat{\omega_f})^{(1-\sigma)} d\hat{\omega}_f \right). \tag{33}$$

Let $P = \left(\int_{\Omega} p(\hat{\omega})^{1-\sigma} d\hat{\omega}\right)^{\frac{1}{1-\sigma}}$ be the price index of domestic intermediate goods, and P_f the equivalent price index of imported goods. Then we can define the inverse demand function for domestic intermediate inputs as

$$p(\omega) = y(\omega)^{-1/\sigma} Y^{1/\sigma} (P^{1-\sigma} + X^{1-\sigma} {P_f}^{1-\sigma})^{-1/\sigma}, \tag{34}$$

and equivalently $p(\omega_f)$. From analogue functions abroad, we can infer the inverse demand functions for exported goods $p(\omega^*)$:

$$p(\omega^*) = y(\omega^*)^{-1/\sigma} X Y^{*1/\sigma} (P_f^{*1-\sigma} + X^{\sigma-1} P^{*1-\sigma})^{-1/\sigma}. \tag{35}$$

These inverse demand functions are used in the intermediate goods producers' maximization problems. Since each entrepreneur is atomistic, he cannot affect the demand functions and thus takes them as given. Note that Ω and Ω^* denote the set of firms producing and selling their goods, so that $y(\omega) > 0$ and $y(\omega^*) > 0$.

B Optimal revenue function for intermediate good producers

Let us redefine the inverse demand functions as $p = q^{-1/\sigma} A$ and similarly $p^* = q^{*-1/\sigma} A^*$, where

$$A = Y^{1/\sigma} (P^{1-\sigma} + X^{1-\sigma} P_f{}^{1-\sigma})^{-1/\sigma} \text{ , and} \tag{36}$$

$$A^* = Y^{*1/\sigma} X (P_f^{*1-\sigma} + X^{\sigma-1} P^{*1-\sigma})^{-1/\sigma} \text{ .} \tag{37}$$

Our analysis thus far has made no assumption on the form of the production function, and the results of this paper are robust to a wide choice of production technology. Consider now the case of a Cobb-Douglas production function such that $G(k,n) = k^{\eta_k} n^{\eta_n}$, which is what we use to solve the model numerically. The cost minimization then implies that the quantity sold by a domestic firm is

$$q_D = R^{\eta_k+\eta_n} (1 + \eta_n/\eta_k)^{-\eta_k} [w(1 + \eta_k/\eta_n)]^{-\eta_n} = R^{\nu} x. \tag{38}$$

where $\nu = \eta_k + \eta_n$ is the returns to scale parameter of the production function $G(k,n)$ and $x = (1 + \eta_n/\eta_k)^{-\eta_k} [w(1 + (1 + \eta_k/\eta_n)]^{-\eta_n}$ reflects the impact of wages and the shares of capital and labor on production. When a firm maximizes its cash-flow $F_i(R) - R(1+r)$, the optimal amount of resources used by a non-exporting firm is then

$$\widetilde{R}_D = \left[\pi \frac{A x^{(1-1/\sigma)} \nu (1 - 1/\sigma)}{1+r} \right]^{\frac{1}{1-\nu(1-1/\sigma)}} \text{ .} \tag{39}$$

From the underlying first-order condition one can deduce that $\nu(1 - 1/\sigma) < 1$ must hold, or otherwise there would not be a finite optimum. This means that if the production $G(k,n)$ exhibits increasing returns to scale, the reduction in marginal cost by expanding capacity must not outpace the reduction in marginal revenue. Analogously, we can derive the expressions for goods sold domestically and abroad by an exporter

$$q_E = \frac{(R - I_E)^\nu x}{1 + (A^*/A)^\sigma (1 + I_T)^{1-\sigma}} = (R - I_E)^\nu B^{\sigma/(\sigma-1)} \text{ , and} \tag{40}$$

$$q_E^* = \frac{(R - I_E)^\nu x (A^*/A)^\sigma (1 + I_T)^{-\sigma}}{1 + (A^*/A)^\sigma (1 + I_T)^{1-\sigma}} = (R - I_E)^\nu B^{*\sigma/(\sigma-1)} \text{ .} \tag{41}$$

where

$$B = \left(\frac{x}{1 + (A^*/A)^\sigma (1 + I_T)^{1-\sigma}} \right)^{1-1/\sigma} \text{ , and} \tag{42}$$

$$B^* = \left(\frac{x (A^*/A)^\sigma (1 + I_T)^{-1\sigma}}{1 + (A^*/A)^\sigma (1 + I_T)^{1-\sigma}} \right)^{1-1/\sigma} \text{ .} \tag{43}$$

The quantity sold domestically depends positively on the (endogenous) domestic demand parameter A and negatively on the foreign demand parameter A^*. The higher the transportation cost, the more goods an exporter sells at home. The reverse applies to goods sold abroad. A firm operating at full scale when it is profitable to trade requires period resources

$$\widetilde{R}_E = \left[\pi \frac{(AB + A^* B^*) \nu (1 - 1/\sigma)}{1 + r} \right]^{\frac{1}{1-\nu(1-1/\sigma)}} + I_E \text{ .} \tag{44}$$

In this case, the markup is constant, as is common in the literature with monopolistic competition. Finally, we can infer the amount of resources above which it pays off to incur the fixed export cost, i.e. the R_{dx} such that $F_D(R) = F_E(R)$:

$$R_{dx} = \frac{I_E}{1 - \left(\frac{x^{(1-1/\sigma)} A}{AB + A^* B^*} \right)^{\frac{1}{\nu(1-1/\sigma)}}} = \frac{I_E}{1 - \phi} \text{ ,} \tag{45}$$

where

$$\phi = \left(\frac{x^{1-1/\sigma} A}{AB + A^* B^*} \right)^{\frac{1}{\nu(1-1/\sigma)}} \text{ .} \tag{46}$$

It follows that the necessary condition for exports to be profitable is $\phi > 1$. When $\phi \leq 1$ the fixed costs of exporting cannot be compensated by the access to a new market.

C Workers' decision rules

The workers problem can be written recursively as

$$
\begin{aligned}
U(d) = \max_{d',c,h} \quad & u(c,h) + (1-\gamma_w)\hat{\beta}\mathbb{E}U(d') \\
\text{s.t.} \quad & c + p^a d' = d(1+r) + wh \\
& d' \geq -\epsilon
\end{aligned}
\tag{47}
$$

We assume that the instantaneous utility function for workers is

$$
u(c_w, 1-h) = \log(c_w) + \lambda \log(1-h) \ , \tag{48}
$$

where $\lambda > 0$ is the elasticity of leisure. The optimal decision rules for saving and labor are:

$$
d' = \frac{w[(1+r)\hat{\beta} - 1]}{(1+r) - (1-\gamma_w)} + (1+r)\hat{\beta}d \tag{49}
$$

$$
\begin{aligned}
h = {} & \frac{(1+r) + \lambda(1-\gamma_w)\hat{\beta}(1+r) - (1-\gamma_w)(1+\lambda)}{(1+\lambda)[(1+r) - (1-\gamma_w)]} \\
& - \frac{\lambda(1+r)(1-\hat{\beta}(1-\gamma_w))}{w(1+\lambda)} d.
\end{aligned}
\tag{50}
$$

It follows that workers' deposits increase with age as long as the interest rate plus the principal is greater than the inverse of the workers' discount factor (i.e., $(1+r) > 1/\hat{\beta}$). Otherwise workers' debt level increases up to a point where $d' = d = -w/(r + \gamma_w)$, which is always less than the maximum amount of debt a worker can service in perpetuity working full time if he worked full time ($d_{\min} = w/r$). Deposits depend positively on the interest rate ($\partial d'/\partial r > 0$), and on wage. Labor supply depends positively on the interest rate for younger workers with few deposits and negatively when $d > w(1 + r - (1-\gamma_w))^{-2}$. It always depends positively on wages.[29] It follows

[29] Labor supply decreases linearly in deposits, which implies that workers may choose negative working hours if they accumulate enough deposits. Given our calibration, only a very small fraction of very old workers choose negative hours. We follow Smith and Wang (2006) an interpret negative working hours as purchasing household services from others workers.

that aggregate deposits deposits and labor supply are

$$D = \frac{w(1-\gamma_w)((1+r)\hat{\beta}-1)}{(1-(1-\gamma_w)(1+r)\hat{\beta})((1+r)-(1-\gamma_w))} \tag{51}$$

$$H = \frac{(1+r)+\lambda(1-\gamma_w)\hat{\beta}(1+r)-(1-\gamma_w)(1+\lambda)}{(1+\lambda)[(1+r)-(1-\gamma_w)]} - \frac{\lambda(1+r)(1-\hat{\beta}(1-\gamma_w))(1-\gamma_w)((1+r)\hat{\beta}-1}{(1+\lambda)(1-(1-\gamma_w)(1+r)\hat{\beta})((1+r)-(1-\gamma_w))}. \tag{52}$$

D Final good market clearing

To show this condition holds, we start from the zero profit condition for final goods producers and invoke the market clearing condition for intermediate goods (Equation (23)):

$$Y = \int py d\omega + X \int p_f y_f d\omega_f = \int pq d\omega + X \int p_f q_f d\omega_f \tag{53}$$

where we omit the argument ω in $p(\omega)$ for notational simplicity. Given the balanced trade condition equation (24), the market clearing for exported goods and the definition of revenues, it follows that

$$Y = \int pq d\omega + X \int p_f q_f d\omega_f = \int pq d\omega + \int p^* q^* d\omega^* = \pi \int F(R) d\mu \ . \tag{54}$$

Using the definition of entrepreneurial consumption, $C_e = \pi \int F(R)d\mu - \pi \int \tau d\mu$ and Equation (21) for the intermediaries' budget yields

$$Y = \pi \int F(R) d\mu = C_e + \pi \int \tau d\mu = C_e + (1+r) \int R d\mu + (1+r)\Gamma I_0 - rZ - \Gamma_b S \ . \tag{55}$$

The clearing of the capital market (Equation (20)) allows to substitute for rZ:

$$Y = C_e + (1+r) \int R d\mu + (1+r)\Gamma I_0 - rZ - \Gamma_b S = C_e + \int R d\mu + \Gamma I_0 - \Gamma_b S + rD \ . \tag{56}$$

Plugging in for the use of resource advancements yields

$$Y = C_e + \int R d\mu + \Gamma I_0 - \Gamma_b S + rD = C_e + \int k d\mu + \Gamma_E I_E + \Gamma I_0 - \Gamma_b S + \int nwd\mu + rD \ . \tag{57}$$

The definition of total capital expenditures allows us to rewrite this as

$$Y = C_e + \int k d\mu + \Gamma_E I_E + \Gamma I_0 - \Gamma_b S + \int nwd\mu + rD = C_e + K + \int nwd\mu + Dr \ . \tag{58}$$

Finally, the labor market clearing condition (Equation (22)), and the aggregate budget constraint for workers $C_w + D = wH + D(1+r)$, complete the proof:

$$Y = C_e + K + \int nwd\mu + Dr = C_e + K + wH + Dr = C_e + C_w + K \ . \tag{59}$$

E Other proofs

Proposition E.1 *There exists a point V_{dx} such that $\widehat{W}_D(V) < \widehat{W}_E(V)$ for all $V \in [0, V_{dx})$, and $\widehat{W}_D(V) > \widehat{W}_E(V)$ for all $V \in (V_{dx}, \widetilde{V}]$.*

Proof It is optimal to reach the unconstrained value $\widetilde{V}$ in the shortest time possible because the joint surplus is maximized there and both the entrepreneur and the financial intermediary are risk-neutral and share the same discount factor. Hence, repayments should be set equal to revenues $\tau = \pi F_i(R)$ as long as $V^H < \widetilde{V}$. This follows the argument set forth in Clementi and Hopenhayn (2006). Let us thus rewrite the value of a firm with a given export status $i \in \{D, E\}$ as

$$\begin{aligned} \widehat{W}_i(V) = \quad & \max_{R,V^H,V^L}\{\pi F_i(R) - R(1+r) + \beta[\pi W(V^H) + (1-\pi)W(V^L)]\} \\ \text{s.t.} \quad & V = \beta(\pi V_H + (1-\pi)V_L) \\ & F_i(R) \leq \beta(V^H - V^L) \\ & V^H, V^L \geq 0 \\ & R_E \geq I_X . \end{aligned} \tag{60}$$

Since $F_i(R)$ is a strictly increasing, strictly concave function, then the expected cash flows $\pi F_i(R) - R(1+r)$ for $R_i \leq \widetilde{R}_i$ are too. Let $\widehat{W}_D(V) = \widehat{W}_D(\widetilde{V}) \forall V > \widetilde{V}$. Therefore, the function $\widehat{W}_i(V)$ for $V \in [0, \widetilde{V}_i]$ inherits the same properties, where $\widetilde{V}_i$ is the equity of an unconstrained firm of type i. We have to show that the function $\widehat{W}_E(V)$ is lower than $\widehat{W}_D(V)$ for all $V < V_{dx}$ and vice versa – i.e. that there is a unique crossing point V_{dx}. It is easy to see that the unconstrained value of an exporting firm is higher than that of a purely domestic firm, $\max \widehat{W}_E(V) > \max \widehat{W}_D(V)$, since $\max\{F_E(R) - R(1+r)\} > \max\{F_D(R) - R(1+r)\}$.

When the equity of a domestic firm goes to zero, its value goes to βS: the first constraint, together with the fact that continuation values have to be non-negative, forces continuation values to go to zero, as equity V approaches zero. It follows that the spread between V_H and V_L goes to zero, so $F_i(R)$ has to go to zero to maintain incentive compatibility. Therefore, the optimal resource advancement of a domestic firm will approach zero and thus its value $\widehat{W}_D(V)$ will go to the discounted scrap value. In the case of an exporting firm, the logic is very similar, except that the resource advancement approaches the fixed cost of exporting I_X, which cannot be seized by the entrepreneur. The value of an exporting firm with equity zero will hence be the discounted scrap value minus the cost of paying the export cost, $\widehat{W}_E(0) = -I_X + \beta S$. As both firm value functions are increasing and concave, and strictly so for $\widetilde{W}_E(V), V < \widetilde{V}$, the fact that $\widehat{W}_E(0) < \widehat{W}_D(0)$ and $\widehat{W}_E(\widetilde{V}_E) > \widehat{W}_D(\widetilde{V}_D)$ implies a unique crossing. ∎

Proposition E.2 *The function* $\max\{\widehat{W}_D, \widehat{W}_E\}$ *contains an interval* $[V_D, V_E] \subset (0, \widetilde{V})$ *on which it is not concave. This implies together with risk neutrality that it is optimal to use an export lottery.*

Proof As shown in the previous proposition, there exists a unique equity value V_{dx} where the two value functions cross. For any given $V < \widetilde{V}$, the slope of the exporting firm's value function is steeper than the slope of the non-exporting firm, i.e. $W_D'(V) < W_E'(V) \quad \forall V < \widetilde{V}$. This follows from the fact that the same is true for the underlying revenue functions. Therefore, by continuity of the value functions, $\widehat{W}_D'(V_{DX} - \epsilon) < \widehat{W}_E'(V_{DX} + \epsilon)$. This implies that the function $\max\{\widehat{W}_D, \widehat{W}_E\}$ is not concave on some interval $[V_D, V_E]$. Since the marginal profit of an extra unit of resources goes to infinity as R approaches zero, $V_D > 0$, and as the slope of the exporting value function is zero at the unconstrained level, $V_E < \widetilde{V}$. ∎

Proof of Proposition 3.1 As outlined above, $\widehat{W}_i(V)$ is increasing and concave for $i \in \{D, E\}$, so any convex combination of the two functions and S is too. Since $\widehat{W}(0) < S$ and $\widehat{W}(V) > S$ for some $V < \widetilde{V}$ (otherwise it would not pay to finance firms at all), $V_r > 0$ and by assumption, $V_D > V_r$. We know that $\widehat{W}_E'(V) > \widehat{W}_D'(V) \forall V < \widetilde{V}$, so by concavity of the functions $\widehat{W}_i(V)$, it follows that $V_E > V_D$. Finally, because $\widehat{W}_E'(\widetilde{V}) = 0$, it has to be that $V_E < \widetilde{V}$.

The first point follows immediately from the fact that the expected value of the liquidation lottery is equal to the equity with which an entrepreneur enters it, thus pinning down the probabilities of liquidation and survival. To prove the second point, we have to show that $V_{dx} \in (V_D, V_E)$. From the above, it follows that the interval is non-empty. By definition, $\widehat{W}_E(V_{dx}) = \widehat{W}_D(V_{dx})$, which implies together with concavity that $V_D < V_{dx} < V_E$. This means that no company with $V \leq V_D$ finds it profitable to export, and all companies with $V \geq V_E$ do. A company with equity $V \in (V_D, V_E)$ is offered a lottery with expected value equal to the equity the entrepreneur had before. The probabilities of getting V_D and V_E are determined thereby. An entrepreneur wins the lottery with probability $\delta(V)$, receiving V_E and thus exporting; with probability $(1 - \delta(V))$, he gets V_D and will hence not export.

Concerning the third point, it is clear that $W(V)$ is linear in the two lottery regions. When $V = \widetilde{V}$, the value of the firm does not change anymore, so it will stay constant at $\widetilde{W}$. The functions $\widehat{W}_i(V)$ are strictly increasing as long as $V < \widetilde{V}$, since $F_E(R) - (1+r)R$ is strictly increasing in that region. Therefore the firm value function $W(V)$ is strictly increasing for all $V < \widetilde{V}$. ∎

Proof of Proposition 3.2 Partition the domain of the contract $[0, \widetilde{V}]$ in five parts $[0, V_r) \cup [V_r, V_D] \cup (V_D, V_E) \cup [V_E, \widetilde{V}) \cup \{\widetilde{V}\}$. From the above $E(V'|V) = V/\beta$ when $V \in [V_r, V_D]$ and when $V \in [V_E, \widetilde{V})$.[30] When $V = \widetilde{V}$, the firm is unconstrained and there is no need to provide any incentives to report the truth (as all revenues will go to the entrepreneur), and hence $V^L = V^H = \widetilde{V}$, so $E(V'|\widetilde{V}) = \widetilde{V}$. Whenever $V^H \in (V_D, V_E)$ or $V^L \in (V_D, V_E)$, the firm enters a lottery and will either end up with V_D or V_E, with the expected value of the lottery being exactly equal to the promised value V^L or V^H. The expected next period equity for each of these is V^L/β and V^H/β, so that $E(V'|V \in (V_D, V_E)) = V/\beta$. Similarly, when $V^L \in [0, V_r)$, the lottery for liquidation yields the expected payoff V^L, and the expected equity for next period is then just V^L/β. ∎

F Stationary equilibrium

In the stationary steady state, given interest rate r and wage w, perfect competition in the financial market implies financial intermediaries earn zero-profit on a new contract. This implies new entrepreneurs receive an initial equity $V_0 = \sup_V \{W(V) - V - (1+r)I_0 = 0\}$, for which the lender earns just enough to break even. The initial equity V_0 is endogenous in the general equilibrium.

[30]Here we assume that repayments τ are equal to all revenues $F_i(R(V))$ for all $V < \widetilde{V}$. In our numerical analysis we have $\tau < F_i(R(V))$ for some V with $V^H(V) = \widetilde{V}$, but the sub-martingale property still holds.

Consider the sequence $(X_t)_{t\geq 0}$ of equity levels from a single firm indefinitely replaced by a new one when it dies. It is clear $(X_t)_{t\geq 0}$ is a sequence of random variables, and its evolution depends on the properties of the contracts and on the sequence of shocks – productivity, death, export, and liquidation. In what follows, we show that $\mathbf{X} = (X_t)_{t\geq 0}$ is a time-homogeneous Markov chain such that

$$X_{t+1} = T_\omega(X_t, \epsilon_t), (\epsilon_t)_{t\geq 0} \sim \phi_\omega \in \mathscr{P}(Z), X_0 = V_0 \in S \tag{61}$$

where $T_\omega : S \times Z \to S$ is a collection of measurable functions indexed by $(r, w) = \omega \in \Omega$ the parameter space, $(\epsilon_t)_{t=1}^{\infty}$ is a sequence of independent random shocks with (joint) distribution ϕ_ω, and S and Z are the state space and the probability space respectively. The existence of an invariant distribution of firms follows if $\mathbf{X}$ has a unique and ergodic invariant distribution. Existence of stationary equilibrium follows if the stationary distribution is continuous in prices.

Proposition F.1 (Stationary distribution) X *is a time-homogeneous Markov chain on a general state space and is globally stable.*

Equip the state space S with a boundedly compact, separable, metrizable topology $\mathscr{B}(S)$. Let $(Z, \mathscr{Z})$ be the measure space for the shocks. Let A be any subset of $\mathscr{B}(S)$. It follows for any $x \in \{x : V_r < x < V_D \text{ and } V^L(x) < V_r\}$

$$P(x, A) = \begin{cases} (1-\gamma)(1-\pi)\alpha(V^L(x)) + \gamma & \text{if } A = \{V_0\} \\ (1-\gamma)(1-\pi)(1-\alpha(V^L(x)) & \text{if } A = \{V_r\} \\ (1-\gamma)\pi & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \tag{62}$$

For any $x \in \{x : V_r < x < V_D \text{ and } V_r \leq V^L(x) \leq V_D \text{ and } V^H(x) \leq V_D\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1-\gamma)(1-\pi) & \text{if } A = \{V^L(x)\} \\ (1-\gamma)\pi & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \tag{63}$$

For any $x \in \{x : V_r < x < V_D \text{ and } V_r \leq V^L(x) \leq V_D \text{ and } V^H(x) \geq V_D\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1-\gamma)(1-\pi) & \text{if } A = \{V^L(x)\} \\ (1-\gamma)\pi\delta(V^H(x)) & \text{if } A = \{V_E\} \\ (1-\gamma)\pi(1-\delta(V^H(x))) & \text{if } A = \{V_D\} \\ 0 & \text{otherwise} \end{cases} \tag{64}$$

For any $x \in \{x : V_E < x < \widetilde{V} \text{ and } V^L(x) \leq V_E \text{ and } V_E \leq V^H(x) < \widetilde{V}\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1-\gamma)(1-\pi)\delta(V^L(x)) & \text{if } A = \{V_E\} \\ (1-\gamma)(1-\pi)(1-\delta(V^L(x))) & \text{if } A = \{V_D\} \\ (1-\gamma)\pi & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \tag{65}$$

For any $x \in \{x : V_E < x < \widetilde{V} \text{ and } V_E \leq V^L(x) \leq \widetilde{V} \text{ and } V^H(x) \geq \widetilde{V}\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1-\gamma)(1-\pi) & \text{if } A = \{V^L(x)\} \\ (1-\gamma)\pi & \text{if } A = \{\widetilde{V}\} \\ 0 & \text{otherwise} \end{cases} \tag{66}$$

And for $x = \{\widetilde{V}\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1-\gamma) & \text{if } A = \{\widetilde{V}\} \\ 0 & \text{otherwise} \end{cases} \tag{67}$$

For each $A \in \mathscr{B}(S)$, $P(\cdot, A)$ is a non-negative function on $\mathscr{B}(S)$, and for each $x \in S$, $P(x, \cdot)$ is a probability measure on $\mathscr{B}(S)$. Therefore, for any initial distribution ψ, the stochastic process $\mathbf{X}$ defined on S^∞ is a time-homogeneous Markov chain. Let $\mathbf{M}$ denote the corresponding Markov operator, and let $\mathscr{P}(S)$ denote the collection of firms distribution generated by $\mathbf{M}$ for a given initial distribution.[31]

Write the stochastic kernel P with the density representation p so that $P(x, dy) = p(x, y)dy$ for all $x \in S$. The Dobrushin coefficient $\alpha(p)$ of a stochastic kernel p is defined by

$$\alpha(p) := \min \left\{ \int p(x, y) \wedge p(x', y)dy : (x, x') \in S \times S \right\} \tag{68}$$

$(\mathscr{P}(S), \mathbf{M})$ is globally stable if $(\psi \mathbf{M}^t)_{t \geq 0} \to \psi^* \mathbf{M}$ where $\psi^* \in \mathscr{P}(S)$ is the unique fixed point of $(\mathscr{P}(S), \mathbf{M})$. This occurs if the Markov operator is a uniform contraction of modulus $1 - \alpha(p)$ on $\mathscr{P}(S)$ whenever $\alpha(p) > 0$. A firm dies with a fixed, exogenous and independent probability γ each period, and is instantaneously replaced by a new

[31] Note that Stokey, Lucas, and Prescott (1989, Theorem 12.12) fails to apply in this case because the stochastic kernel is not monotone near the randomization region. See Verani (2011) for more details.

one of size V_0. Therefore,

$$P(x, \{V_0\}) \geq 0 \;\forall\; x \in S. \tag{69}$$

Equation (11.15) and Exercise (11.2.24) in Stachurski (2009) yield $\alpha(p) > \gamma$. By Stachurski (2009, Th. 11.2.21), this implies

$$||\psi\mathbf{M} - \psi'\mathbf{M}||_{TV} \leq (1-\gamma)||\psi - \psi'|| \tag{70}$$

for every pair ψ, ψ' in $\mathscr{P}(Z)$, and where ${}_{TV}$ indicates the total variation norm. ∎

Proposition F.2 (Existence of a stationary equilibrium) *The unique and ergodic invariant distribution of* $\mathbf{X}$ *is continuous in prices.*

The result follows if the conditions of LeVan and Stachurski (2007, Proposition2) are satisfied and the proof is similar to the one in Verani (2011). ∎

G A higher elasticity of substitution

We investigate the effect of the elasticity of substitution between intermediate goods firms on firm dynamics and the aggregate by considering another world economy with $\sigma = 4$. To help with the comparison, we calibrate the economy with $\sigma = 6$ to the same moments as the economy with $\sigma = 6$. Table 3 summarizes the value of the parameters used for each economy.

Table 4 reports the results for the two economies. Given the calibration, the wage rate, aggregate output, consumption and investment are roughly the same in the two model economies so that, from a macroeconomic point of view, the two world economies are comparable. However, a higher σ reduces the market power of intermediate goods producers at home and abroad. This translates into lower prices for all goods, with a comparatively greater decrease in the price of internationally traded goods. Furthermore, a decrease in firms' market power leads to a substantial reduction of the export share of aggregate output. A greater fraction of domestic firms begins exporting every period, and the hazard rate of exit for new exporters after one year is also higher. Last, a reduction in market power also increases the size differential between domestic and exporting firms, while the share of unconstrained firms becomes smaller.

The results for exports are driven by how the calibration for the economy with $\sigma = 6$ affects the fixed and variable trade costs, I_E and I_T. We keep the variable trade costs I_T constant across the two economies, which implies that the ratio of

Table 3: Parameter values

		$\sigma = 4$	$\sigma = 6$
$\hat{\beta}$	Worker's discount rate	0.959	0.959
λ	Elasticity of leisure	2.300	2.304
γ_w	Workers' death probability	0.02	0.02
η_k	Capital share	0.137	0.2
I_0	Setup investment	0.452	0.26
S	Salvage value	$0.8 \times I_0$	$0.8 \times I_0$
I_T	Iceberg cost	0.4	0.4
I_E	Fixed export cost	0.033	0.012
π	Probability of high/low shock	0.5	0.5
γ_e	Firm exogenous exit rate	0.047	0.05

exports to domestic sales decreases as market power decreases for each exporter.[32] It follows that the fixed cost of exporting must be lower in an economy with a higher σ to keep the number exporters constant. This implies that the export lottery region is smaller making it easier for firms to enter and exit export markets.

Furthermore, unconstrained firms with lower market power use more resources and sell higher quantities of goods. Since large firms are always exporters, the relative size of exporters is also higher. A lower market power implies that firm profit is also lower, thereby reducing the speed of firm growth and leading to a smaller fraction of unconstrained firms.[33] To see this, note that the incentive compatibility constraint binds for constrained firms, which implies that $F_i(V) = \beta(V_H - V_L)$. It follows that smaller revenue implies that the spread between the continuation values is smaller too; and the number of steps needed to reach the unconstrained level is higher.

[32] In a symmetric world, this ratio reduces to $q_E^*/q_E = 1 + I_T^{\sigma}$.

[33] The lower profits are also reflected in the much lower initial set-up cost I_0. Since all fixed costs are lower when market power is reduced, the capital share in the production function is higher in order to reach the target labor share of income.

Table 4: Steady state equilibrium.

Targeted:	$\sigma = 4$	$\sigma = 6$
Interest rate	0.040	0.040
Hours worked	0.350	0.349
Labor income share	0.597	0.602
Entry/exit rate	0.066	0.063
Relative size of entrants	0.158	0.153
Share of exporters	0.273	0.269
Not targeted:		
Wage rate	0.493	0.501
Domestic goods price index	3.272	3.079
Imported goods price index	6.033	4.925
Output	0.289	0.291
Consumption/Output	0.796	0.792
Investment/Output	0.204	0.208
Export/Output	0.138	0.087
Entry rate in export market	0.028	0.033
Exit rate from export market after 1 year	0.256	0.320
New firm size relative to incumbents	0.321	0.335
Domestic firm size relative to exporters	0.269	0.242
Share of unconstrained firms	0.177	0.148