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Abstract: This paper presents a general framework to address several issues that have arisen in recent work that investigates purchasing power parity (PPP) and other inter-regional relative price movements: (1) How can we model real exchange rate movements in a consistent manner, so that our model for the real exchange rate for country B relative to country C is commensurate with our models for country A/ country B and country A/ country C real exchange rates? For example, can things be modeled so that our tests do not depend on the "base country"? (2) How should we handle correlation across real exchange rates in panel tests of PPP? (3) Are speeds of adjustment toward PPP different for intra-national, cross-national and cross-continental real exchange rates? (4) Is the innovation variance different for intra-national, cross-national and cross-continental real exchange rates; and, if so, how does that influence how we model and test PPP?

Keywords: purchasing power parity, panel unit roots tests

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A large number of recent papers have employed panel data to study PPP, including Abuaf and Jorion (1990), Canzoneri, Cumby and Diba (1996), Cumby (1996), Frankel and Rose (1996), Jorion and Sweeney (1996), Liu and Maddala (1996), Lothian (1996), MacDonald (1996), O'Connell (1996), Oh (1996), Papell (1996), Wei and Parsley (1996), and Wu (1996). The motivation for using panel data is that it might increase the power of tests for PPP. It is well known that when the rate of convergence for a stationary series is very slow, long time series are needed to rule out the hypothesis that the series follows a random walk. Typically the length of time needed is much greater than the twenty-odd years that have passed since the decline of the Bretton Woods system and the move to floating exchange rates among major industrialized countries. One approach to solving the power problem is to use very long time series, but, as Frankel and Rose (1996) point out, those long time series encompass periods in which nominal exchange rate regimes shifted from floating to fixed and back again. The panel approach allows a different way of increasing power while only using post-Bretton Woods data, by pooling across many different real exchange rates.

This paper presents a general framework to address several issues that have arisen in recent work that investigates purchasing power parity (PPP) and other inter-regional relative price movements: (1) How can we model real exchange rate movements in a consistent manner, so that our model for the real exchange rate for country B relative to country C is commensurate with our models for country A/ country B and country A/ country C real exchange rates? For example, is there a way to avoid having our tests depend on which country is the "base country"? (2) How should we handle correlation across real exchange rates in panel tests of PPP? (3) Are speeds of adjustment toward PPP different for intra-national, cross-national and cross-continental real exchange rates? (4) Is the innovation variance different for intra-national, cross-national and cross-continental real exchange rates; and, if so, how does that influence how we model and test PPP? (5) What is the advantage of panel tests for stationarity of real exchange rates versus tests of cointegration of price levels expressed in a common currency?

One issue that has arisen is what the base currency should be in tests for PPP. Typically real exchange rates are calculated relative to the U.S. But, some studies have found that PPP holds better between European countries than between European countries and the U.S. (See, for example, Edison, Gagnon, and Melick (1994), Jorion and Sweeney (1996), Canzoneri, Cumby and Diba (1996) and Papell (1996)). This raises a question: if, for example, the German/Swiss, German/Canadian and German/U.S. real exchange rate panel is stationary, how could the U.S./Swiss, U.S./Canadian and U.S./German panel be non-stationary? With real exchange rates expressed in logs, the elements of the latter panel are just linear combinations of the former panel. If all elements of one panel are stationary, then all elements of the other panel must be stationary.[1]

Most of the panel studies of PPP have assumed that shocks to real exchange rates are uncorrelated across the different real exchange rates. O'Connell (1996) points out that this assumption is untenable.[2] Surely shocks to the U.S./German and U.S./Swiss real exchange rates are correlated. O'Connell proposes estimating the system by GLS. We address that issue here, and show how some reasonable assumptions may allow one to reduce the number of independent parameters in estimating the covariance matrix of real exchange rates.

The gain in power from using panel data comes when one assumes that the speed of adjustment is the same for a number of real exchange rates. The recent literature has typically assumed that the speed of adjustment is the same for all real exchange rates. Liu and Maddala (1996) question that assumption. Should we assume that the speed of adjustment is the same for

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[1] O'Connell (1996), following suggestions in an earlier version of this paper, employs GLS to perform a panel PPP test in a way in which the base currency is irrelevant.

[2] In an early paper, Hakkio (1984) considers cross-exchange rate correlations in a system estimation of PPP. His paper assumes, rather than tests, stationarity.

the U.S./Canadian and U.S./German real exchange rates? Davutyan and Pippenger (1990) produce evidence that PPP holds better for countries within a continent than for countries on separate continents. Parsley and Wei (1996) argue that convergence to PPP should be faster for locations within a country than for cross-country location pairs.

The recent literature also generally tends to assume homoskedasticity across real exchange rates. This assumption may also be untenable. For example, the U.S./Canadian real exchange rate appears to have a much lower innovation variance than the U.S./German real exchange rate. Also, if we compare locations within a country to locations across borders, Engel and Rogers (1996) provide evidence of large differences in variances of relative prices. So, we will want to take into account this heteroskedasticity in assessing PPP. We will also argue that the sample variances of innovations from our panel PPP tests may be better measures of the types of variances that Engel (1993) or Engel and Rogers (1995, 1996) calculate.

Finally, all of the recent literature that uses data from a large number of locations is presented in the form of panel tests of stationarity of the real exchange rate. An alternative approach would be to test for cointegration of price levels expressed in a common currency. We argue that under some plausible assumptions, the stationarity tests approach is likely to be the more useful and powerful approach.

We lay out our approach to these issues in section 1. Our exposition is in terms of an eight-location example: one in which there are two cities in each of four countries, with two countries and two different continents. We implement this example by using consumer price data from eight cities: New York, Los Angeles, Toronto, Vancouver, Zurich, Geneva, Frankfurt and Cologne. The data are monthly from September 1978 to September 1994. The results of our tests for PPP are presented in section 2.

While we find considerable differences in the variances of the real exchange rates, contrary to the conjecture of Parsley and Wei (1996), we find essentially no evidence that the speeds of adjustment are different for intra-national, intra-continental and inter-continental real exchange rates. We find that we cannot reject the null of the unit root. Apparently, as Papell (1996), O'Connell (1996) and Liu and Maddala (1996) argue, the gains in power from using panel data are limited.

1. Modeling multi-country PPP

Much of the recent literature on PPP assumes that the speed of adjustment is identical for all real exchange rates. We note that Liu and Maddala (1996) and Parsley and Wei (1996) question that assumption. But, if the speed of adjustment is not constant, an issue of modeling consistency arises.

Take a three location example. Each location has a log nominal price (all in the same currency): p_{At}, p_{Bt}, and p_{Ct}. Suppose we follow the practice in the literature of picking one location to be the "base" (location A), and estimate PPP adjustment equations for p_{At}-p_{Bt} and p_{At}-p_{Ct}. For simplicity, we will express these as first-order autoregressions (AR1s).

(1) $p_{At} - p_{Bt} = \alpha_1 + \beta_1\left(p_{At-1} - p_{Bt-1}\right) + u_{1t}$

(2) $p_{At} - p_{Ct} = \alpha_2 + \beta_2\left(p_{At-1} - p_{Ct-1}\right) + u_{2t}$

The problem with this model is that these equations imply a stochastic process for p_{Bt}-p_{Ct}, and it is not an AR1. Subtract (1) from (2):

$$\begin{aligned} p_{Bt} - p_{Ct} &= \alpha_2 - \alpha_1 + \beta_2\left(p_{At-1} - p_{Ct-1}\right) - \beta_1\left(p_{At-1} - p_{Bt-1}\right) + u_{2t} - u_{1t} \\ &= \alpha_2 - \alpha_1 + \beta_1\left(p_{Bt-1} - p_{Ct-1}\right) + \left(\beta_2 - \beta_1\right)\left(p_{At-1} - p_{Ct-1}\right) + u_{2t} - u_{1t}. \end{aligned}$$

Obviously, this is different than:

(3) $p_{Bt} - p_{Ct} = \alpha_3 + \beta_3\left(p_{Bt-1} - p_{Ct-1}\right) + u_{3t}$

That is, given that we thought equations (1) and (2) are a reasonable way to model p_{At}-p_{Bt} and p_{At}-p_{Ct}, then equation (3) should be a reasonable way to model p_{Bt}-p_{Ct}. But equations (1) and (2) do not give us an equation for p_{Bt}-p_{Ct} that is of the same form as equation (3) unless β_1 equals β_2.

A moment's thought will reveal that a consistent set of price adjustment equations must allow for each price to respond to all others. In this section, we will consider an eight location example. We will model the behavior of p_{UA} and p_{UB} (prices for two U.S. cities); p_{CA} and p_{CB} (prices for two Canadian cities); p_{GA} and p_{GB} (prices for two German cities); and, p_{SA} and p_{SB} (prices for two Swiss cities). Each of these prices is expressed in dollar terms. One of the issues we address below is how we can construct tests of PPP that do not depend in any way on the currency of denomination of our prices. We need to model at least this many prices to capture the differences in speeds of adjustment and covariances of price shocks that will occur for intra-national, intra-continental and intercontinental location pairs.

We begin by writing an eight-equation error correction model for these prices (we will suppress the intercept terms in all equations in this section, but will bring them back in our empirical work reported in section 2):

$$\begin{aligned}(4)\quad & p_{UAt} - p_{UAt-1} = \delta_{12}\left(p_{UBt-1} - p_{UAt-1}\right) + \delta_{13}\left(p_{CAt-1} - p_{UAt-1}\right) + \delta_{14}\left(p_{CBt-1} - p_{UAt-1}\right) \\ & + \delta_{15}\left(p_{GAt-1} - p_{UAt-1}\right) + \delta_{16}\left(p_{GBt-1} - p_{UAt-1}\right) + \delta_{17}\left(p_{SAt-1} - p_{UAt-1}\right) + \delta_{18}\left(p_{SBt-1} - p_{UAt-1}\right) + e_{1t}.\end{aligned}$$

$$\begin{aligned}(5)\quad & p_{UBt} - p_{UBt-1} = \delta_{21}\left(p_{UAt-1} - p_{UBt-1}\right) + \delta_{23}\left(p_{CAt-1} - p_{UBt-1}\right) + \delta_{24}\left(p_{CBt-1} - p_{UBt-1}\right) \\ & + \delta_{25}\left(p_{GAt-1} - p_{UBt-1}\right) + \delta_{26}\left(p_{GBt-1} - p_{UBt-1}\right) + \delta_{27}\left(p_{SAt-1} - p_{UBt-1}\right) + \delta_{28}\left(p_{SBt-1} - p_{UBt-1}\right) + e_{2t}.\end{aligned}$$

$$\begin{aligned}(6)\quad & p_{CAt} - p_{CAt-1} = \delta_{31}\left(p_{UAt-1} - p_{CAt-1}\right) + \delta_{32}\left(p_{UBt-1} - p_{CAt-1}\right) + \delta_{34}\left(p_{CBt-1} - p_{CAt-1}\right) \\ & + \delta_{35}\left(p_{GAt-1} - p_{CAt-1}\right) + \delta_{36}\left(p_{GBt-1} - p_{CAt-1}\right) + \delta_{37}\left(p_{SAt-1} - p_{CAt-1}\right) + \delta_{38}\left(p_{SBt-1} - p_{CAt-1}\right) + e_{3t}.\end{aligned}$$

$$(7)\quad \begin{aligned} & p_{CBt} - p_{CBt-1} = \delta_{41}\left(p_{UAt-1} - p_{CBt-1}\right) + \delta_{42}\left(p_{UBt-1} - p_{CBt-1}\right) + \delta_{43}\left(p_{CAt-1} - p_{CBt-1}\right) \\ & + \delta_{45}\left(p_{GAt-1} - p_{CBt-1}\right) + \delta_{46}\left(p_{GBt-1} - p_{CBt-1}\right) + \delta_{47}\left(p_{SAt-1} - p_{CBt-1}\right) + \delta_{48}\left(p_{SBt-1} - p_{CBt-1}\right) + e_{4t}. \end{aligned}$$

$$(8)\quad \begin{aligned} & p_{GAt} - p_{GAt-1} = \delta_{51}\left(p_{UAt-1} - p_{GAt-1}\right) + \delta_{52}\left(p_{UBt-1} - p_{GAt-1}\right) + \delta_{53}\left(p_{CAt-1} - p_{GAt-1}\right) \\ & + \delta_{54}\left(p_{CBt-1} - p_{GAt-1}\right) + \delta_{56}\left(p_{GBt-1} - p_{GAt-1}\right) + \delta_{57}\left(p_{SAt-1} - p_{GAt-1}\right) + \delta_{58}\left(p_{SBt-1} - p_{GAt-1}\right) + e_{5t}. \end{aligned}$$

$$(9)\quad \begin{aligned} & p_{GBt} - p_{GBt-1} = \delta_{61}\left(p_{UAt-1} - p_{GBt-1}\right) + \delta_{62}\left(p_{UBt-1} - p_{GBt-1}\right) + \delta_{63}\left(p_{CAt-1} - p_{GBt-1}\right) \\ & + \delta_{64}\left(p_{CBt-1} - p_{GBt-1}\right) + \delta_{65}\left(p_{GAt-1} - p_{GBt-1}\right) + \delta_{67}\left(p_{SAt-1} - p_{GBt-1}\right) + \delta_{68}\left(p_{SBt-1} - p_{GBt-1}\right) + e_{6t}. \end{aligned}$$

$$(10)\quad \begin{aligned} & p_{SAt} - p_{SAt-1} = \delta_{71}\left(p_{UAt-1} - p_{SAt-1}\right) + \delta_{72}\left(p_{UBt-1} - p_{SAt-1}\right) + \delta_{73}\left(p_{CAt-1} - p_{SAt-1}\right) \\ & + \delta_{74}\left(p_{CBt-1} - p_{SAt-1}\right) + \delta_{75}\left(p_{GAt-1} - p_{SAt-1}\right) + \delta_{76}\left(p_{GBt-1} - p_{SAt-1}\right) + \delta_{78}\left(p_{SBt-1} - p_{SAt-1}\right) + e_{7t}. \end{aligned}$$

$$(11)\quad \begin{aligned} & p_{SBt} - p_{SBt-1} = \delta_{81}\left(p_{UAt-1} - p_{SBt-1}\right) + \delta_{82}\left(p_{UBt-1} - p_{SBt-1}\right) + \delta_{83}\left(p_{CAt-1} - p_{SBt-1}\right) \\ & + \delta_{84}\left(p_{CBt-1} - p_{SBt-1}\right) + \delta_{85}\left(p_{GAt-1} - p_{SBt-1}\right) + \delta_{86}\left(p_{GBt-1} - p_{SBt-1}\right) + \delta_{87}\left(p_{SAt-1} - p_{SBt-1}\right) + e_{8t}. \end{aligned}$$

We assume that each of the disturbance terms, e_{it}, is stationary.

Note that we have written stochastic processes for the nominal prices rather than for the relative prices (as in equations (1)-(3)). We have also imposed the condition that, if the nominal prices are cointegrated, the cointegrating vectors are (1,-1). It is immediately apparent that if there are seven cointegrating vectors, each equal to (1,-1), that PPP holds for all possible relative prices. One seemingly simple way to test for PPP, then, would be to estimate the system (4)-(11), and investigate the cointegration properties of this system. The price of each location is treated symmetrically, so there can be no issue of inconsistency in the modeling of different relative prices.

We will not adopt this approach, however. In order to understand why, we first must discuss the structure of the error terms in these equations.

We will model each disturbance term as being the sum of four separate stochastic terms:

$$e_{1t} = l_{1t} + n_{1t} + c_{1t} + w_t ,$$

$$e_{2t} = l_{2t} + n_{1t} + c_{1t} + w_t \,,$$

$$e_{3t} = l_{3t} + n_{2t} + c_{1t} + w_t \,,$$

$$e_{4t} = l_{4t} + n_{2t} + c_{1t} + w_t \,,$$

$$e_{5t} = l_{5t} + n_{3t} + c_{2t} + w_t \,,$$

$$e_{6t} = l_{6t} + n_{3t} + c_{2t} + w_t \,,$$

$$e_{7t} = l_{7t} + n_{4t} + c_{2t} + w_t \,,$$

$$e_{8t} = l_{8t} + n_{4t} + c_{2t} + w_t \,.$$

The idea here is that the l's are local disturbances, the n's are national disturbances, the c's are continental disturbances, and w is a world disturbance. All of the disturbances are mutually independent.

The world shock, w_t, is the disturbance term that affects all dollar prices. It will be influenced by real events that affect all locations, and by dollar nominal disturbances. We allow w_t to follow a general stochastic process, as it is likely in particular that dollar nominal disturbances could have a quite complicated dynamic behavior.

Each of the other disturbance terms represent relative shocks. These are disturbances that alter the prices relative to the overall dollar index, w_t. Throughout this paper, we will maintain the assumption that these shocks are serially uncorrelated. In practice, this is probably a bad assumption.[3] We make it here for simplicity. In future work, we shall allow more general behavior for these shocks as well.

With this structure placed on the error terms, let us return to the dynamic system for nominal prices. Equations (4)-(11) can be used to derive these equations for relative prices:

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[3] We shall discuss some of the implications of this assumption in the conclusions.

$$
\begin{aligned}
&p_{UAt} - p_{UBt} = \left(1 - \delta_{12} - \delta_{21} - \delta_{23} - \delta_{24} - \delta_{25} - \delta_{26} - \delta_{27} - \delta_{28}\right)\left(p_{UAt-1} - p_{UBt-1}\right) + \\
&\left(\delta_{23} - \delta_{13}\right)\left(p_{UAt-1} - p_{CAt-1}\right) + \left(\delta_{24} - \delta_{14}\right)\left(p_{UAt-1} - p_{CBt-1}\right) + \left(\delta_{25} - \delta_{15}\right)\left(p_{UAt-1} - p_{GAt-1}\right) + \\
&\left(\delta_{26} - \delta_{16}\right)\left(p_{UAt-1} - p_{GBt-1}\right) + \left(\delta_{27} - \delta_{17}\right)\left(p_{UAt-1} - p_{SAt-1}\right) + \left(\delta_{28} - \delta_{18}\right)\left(p_{UAt-1} - p_{SBt-1}\right) + \\
&e_{1t} - e_{2t}.
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&p_{UAt} - p_{CAt} = \left(1 - \delta_{13} - \delta_{31} - \delta_{32} - \delta_{34} - \delta_{35} - \delta_{36} - \delta_{37} - \delta_{38}\right)\left(p_{UAt-1} - p_{CAt-1}\right) + \\
&\left(\delta_{32} - \delta_{12}\right)\left(p_{UAt-1} - p_{UBt-1}\right) + \left(\delta_{34} - \delta_{14}\right)\left(p_{UAt-1} - p_{CBt-1}\right) + \left(\delta_{35} - \delta_{15}\right)\left(p_{UAt-1} - p_{GAt-1}\right) + \\
&\left(\delta_{36} - \delta_{16}\right)\left(p_{UAt-1} - p_{GBt-1}\right) + \left(\delta_{37} - \delta_{17}\right)\left(p_{UAt-1} - p_{SAt-1}\right) + \left(\delta_{38} - \delta_{18}\right)\left(p_{UAt-1} - p_{SBt-1}\right) + \\
&e_{1t} - e_{3t}.
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
&p_{UAt} - p_{CBt} = \left(1 - \delta_{14} - \delta_{41} - \delta_{42} - \delta_{43} - \delta_{45} - \delta_{46} - \delta_{47} - \delta_{48}\right)\left(p_{UAt-1} - p_{CBt-1}\right) + \\
&\left(\delta_{42} - \delta_{12}\right)\left(p_{UAt-1} - p_{UBt-1}\right) + \left(\delta_{43} - \delta_{13}\right)\left(p_{UAt-1} - p_{CAt-1}\right) + \left(\delta_{45} - \delta_{15}\right)\left(p_{UAt-1} - p_{GAt-1}\right) + \\
&\left(\delta_{46} - \delta_{16}\right)\left(p_{UAt-1} - p_{GBt-1}\right) + \left(\delta_{47} - \delta_{17}\right)\left(p_{UAt-1} - p_{SAt-1}\right) + \left(\delta_{48} - \delta_{18}\right)\left(p_{UAt-1} - p_{SBt-1}\right) + \\
&e_{1t} - e_{4t}.
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
&p_{UAt} - p_{GAt} = \left(1 - \delta_{15} - \delta_{51} - \delta_{52} - \delta_{53} - \delta_{54} - \delta_{56} - \delta_{57} - \delta_{58}\right)\left(p_{UAt-1} - p_{GAt-1}\right) + \\
&\left(\delta_{52} - \delta_{12}\right)\left(p_{UAt-1} - p_{UBt-1}\right) + \left(\delta_{53} - \delta_{13}\right)\left(p_{UAt-1} - p_{CAt-1}\right) + \left(\delta_{54} - \delta_{14}\right)\left(p_{UAt-1} - p_{CBt-1}\right) + \\
&\left(\delta_{56} - \delta_{16}\right)\left(p_{UAt-1} - p_{GBt-1}\right) + \left(\delta_{57} - \delta_{17}\right)\left(p_{UAt-1} - p_{SAt-1}\right) + \left(\delta_{58} - \delta_{18}\right)\left(p_{UAt-1} - p_{SBt-1}\right) + \\
&e_{1t} - e_{5t}.
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
&p_{UAt} - p_{GBt} = \left(1 - \delta_{16} - \delta_{61} - \delta_{62} - \delta_{63} - \delta_{64} - \delta_{65} - \delta_{67} - \delta_{68}\right)\left(p_{UAt-1} - p_{GBt-1}\right) + \\
&\left(\delta_{62} - \delta_{12}\right)\left(p_{UAt-1} - p_{UBt-1}\right) + \left(\delta_{63} - \delta_{13}\right)\left(p_{UAt-1} - p_{CAt-1}\right) + \left(\delta_{64} - \delta_{14}\right)\left(p_{UAt-1} - p_{CBt-1}\right) + \\
&\left(\delta_{65} - \delta_{15}\right)\left(p_{UAt-1} - p_{GAt-1}\right) + \left(\delta_{67} - \delta_{17}\right)\left(p_{UAt-1} - p_{SAt-1}\right) + \left(\delta_{68} - \delta_{18}\right)\left(p_{UAt-1} - p_{SBt-1}\right) + \\
&e_{1t} - e_{6t}.
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
&p_{UAt} - p_{SAt} = \left(1 - \delta_{17} - \delta_{71} - \delta_{72} - \delta_{73} - \delta_{74} - \delta_{75} - \delta_{76} - \delta_{78}\right)\left(p_{UAt-1} - p_{SAt-1}\right) + \\
&\left(\delta_{72} - \delta_{12}\right)\left(p_{UAt-1} - p_{UBt-1}\right) + \left(\delta_{73} - \delta_{13}\right)\left(p_{UAt-1} - p_{CAt-1}\right) + \left(\delta_{74} - \delta_{14}\right)\left(p_{UAt-1} - p_{CBt-1}\right) + \\
&\left(\delta_{75} - \delta_{15}\right)\left(p_{UAt-1} - p_{GAt-1}\right) + \left(\delta_{76} - \delta_{16}\right)\left(p_{UAt-1} - p_{GBt-1}\right) + \left(\delta_{78} - \delta_{18}\right)\left(p_{UAt-1} - p_{SBt-1}\right) + \\
&e_{1t} - e_{7t}.
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
&p_{UAt} - p_{SBt} = \left(1 - \delta_{18} - \delta_{81} - \delta_{82} - \delta_{83} - \delta_{84} - \delta_{85} - \delta_{86} - \delta_{87}\right)\left(p_{UAt-1} - p_{SBt-1}\right) + \\
&\left(\delta_{82} - \delta_{12}\right)\left(p_{UAt-1} - p_{UBt-1}\right) + \left(\delta_{83} - \delta_{13}\right)\left(p_{UAt-1} - p_{CAt-1}\right) + \left(\delta_{84} - \delta_{14}\right)\left(p_{UAt-1} - p_{CBt-1}\right) + \\
&\left(\delta_{85} - \delta_{15}\right)\left(p_{UAt-1} - p_{GAt-1}\right) + \left(\delta_{86} - \delta_{16}\right)\left(p_{UAt-1} - p_{GBt-1}\right) + \left(\delta_{87} - \delta_{17}\right)\left(p_{UAt-1} - p_{SAt-1}\right) + \\
&e_{1t} - e_{8t}.
\end{aligned} \tag{18}
$$

The system of equations (12)-(18) is one for relative prices, while (4)-(11) represents a dynamic system for nominal prices. We will work with equations (12)-(18). This means we test for unit roots in real exchange rates using panel data, rather than testing for no cointegration of nominal prices. We adopt this approach because, in equations (12)-(18), the w_t disturbance is differenced out. We do not need to worry about the dynamic behavior of nominal shocks, because they do not appear in the system of real prices. If we were to attempt to estimate equations (4)-(11), we would need to model dollar nominal shocks. To the extent that our model did not capture that behavior completely, our tests would depend on the currency of denomination of the nominal prices.

Not all of the parameters of equations introduced in the system (4)-(11) are identified if (12)-(18) are estimated. However, there are some reasonable assumptions on equality of parameters that allow us to achieve identification.

An important aspect of our assumptions on the parameters is that they are symmetric across countries. That allows us to have consistent models of relative price movements that do not depend on which location is picked as the “base” location. For example, if we take the difference between equations (18) and (17), we get an equation for the relative price p_{SA}-p_{SB}. We would like the implied equation for p_{SA}-p_{SB} for the system where p_{UA} is the base price to be of exactly the same form as if we had picked p_{SA} or p_{SB} (or any of the other prices) as the base price.

We assume:

$\delta_{ij} = \delta_{ji}$ for all i,j: When there is a difference between two prices in the same country, both prices adjust equally toward the long run.

$\delta_{1k} = \delta_{2k}$, k = 3,4,…,8: The speed of adjustment in the two U.S. cities toward each foreign city is the same.

$\delta_{3k} = \delta_{4k}$, k = 1,2,5,6,7,8: The speed of adjustment in the two Canadian cities toward each foreign city is the same.

$\delta_{5k} = \delta_{6k}$, k = 1,2,3,4,7,8: The speed of adjustment in the two German cities toward each foreign city is the same.

$\delta_{7k} = \delta_{8k}$, k = 1,2,…,6: The speed of adjustment in the two Swiss cities toward each foreign city is the same.

$\delta_{13} = \delta_{14}$: The speed of adjustment for U.S. city A toward each of the two Canadian cities is the same. Note that, using the assumptions already made, this assumption implies $\delta_{23} = \delta_{24} = \delta_{31} = \delta_{41} = \delta_{32} = \delta_{42}$.

$\delta_{57} = \delta_{58}$: The speed of adjustment for German city A toward each of the two Swiss cities is the same. Note that, using the assumptions already made, this assumption implies $\delta_{67} = \delta_{68} = \delta_{75} = \delta_{85} = \delta_{76} = \delta_{86}$.

$\delta_{15} = \delta_{16}$: The speed of adjustment for U.S. city A toward each of the two German cities is the same. Note that, using the assumptions already made, this assumption implies $\delta_{25} = \delta_{26} = \delta_{51} = \delta_{61} = \delta_{52} = \delta_{62}$.

$\delta_{17} = \delta_{18}$: The speed of adjustment for U.S. city A toward each of the two Swiss cities is the same. Note that, using the assumptions already made, this assumption implies $\delta_{27} = \delta_{28} = \delta_{71} = \delta_{81} = \delta_{72} = \delta_{82}$.

$\delta_{35} = \delta_{36}$: The speed of adjustment for Canadian city A toward each of the two German cities is the same. Note that, using the assumptions already made, this assumption implies $\delta_{45} = \delta_{46} = \delta_{53} = \delta_{63} = \delta_{54} = \delta_{64}$.

$\delta_{37} = \delta_{38}$: The speed of adjustment for Canadian city A toward each of the two Swiss cities is the same. Note that, using the assumptions already made, this assumption implies $\delta_{47} = \delta_{48} = \delta_{73} = \delta_{83} = \delta_{74} = \delta_{84}$.

These assumptions reduce the number of different δ's to 10: $\delta_{12}, \delta_{34}, \delta_{56}, \delta_{78}, \delta_{13}, \delta_{15}, \delta_{17}, \delta_{35}, \delta_{37}$, and δ_{57}. One more set of assumptions reduces the number to three: $\delta_{12} = \delta_{34} = \delta_{56}, = \delta_{78}$; $\delta_{13} = \delta_{57}$; $\delta_{15} = \delta_{17} = \delta_{35} = \delta_{37}$. These assumptions are, first, that all within country speeds of adjustment are the same; that all adjustments across countries within the same continent are the same; and, that all cross-continent adjustments are the same.

Imposing all of the restrictions, equations (12)-(18) become:

$$p_{UAt} - p_{UBt} = (1 - 2\delta_{12} - 2\delta_{13} - 4\delta_{15})(p_{UAt-1} - p_{UBt-1}) + e_{1t} - e_{2t}. \tag{19}$$

$$\begin{aligned} p_{UAt} - p_{CAt} &= (1 - 3\delta_{13} - \delta_{12} - 4\delta_{15})(p_{UAt-1} - p_{CAt-1}) + (\delta_{13} - \delta_{12})(p_{UAt-1} - p_{UBt-1}) \\ &+ (\delta_{12} - \delta_{13})(p_{UAt-1} - p_{CBt-1}) + e_{1t} - e_{3t}. \end{aligned} \tag{20}$$

$$\begin{aligned} p_{UAt} - p_{CBt} &= (1 - 3\delta_{13} - \delta_{12} - 4\delta_{15})(p_{UAt-1} - p_{CBt-1}) + (\delta_{13} - \delta_{12})(p_{UAt-1} - p_{UBt-1}) \\ &+ (\delta_{12} - \delta_{13})(p_{UAt-1} - p_{CAt-1}) + e_{1t} - e_{4t}. \end{aligned} \tag{21}$$

$$\begin{aligned} p_{UAt} - p_{GAt} &= (1 - 5\delta_{15} - \delta_{12} - 2\delta_{13})(p_{UAt-1} - p_{GAt-1}) + (\delta_{15} - \delta_{12})(p_{UAt-1} - p_{UBt-1}) \\ &+ (\delta_{15} - \delta_{13})(p_{UAt-1} - p_{CAt-1}) + (\delta_{15} - \delta_{13})(p_{UAt-1} - p_{CBt-1}) + (\delta_{12} - \delta_{15})(p_{UAt-1} - p_{GBt-1}) \\ &+ (\delta_{13} - \delta_{15})(p_{UAt-1} - p_{SAt-1}) + (\delta_{13} - \delta_{15})(p_{UAt-1} - p_{SBt-1}) + e_{1t} - e_{5t}. \end{aligned} \tag{22}$$

$$
\begin{aligned}
(23)\quad & p_{UAt}-p_{GBt}=\left(1-5\delta_{15}-\delta_{12}-2\delta_{13}\right)\left(p_{UAt-1}-p_{GBt-1}\right)+\left(\delta_{15}-\delta_{12}\right)\left(p_{UAt-1}-p_{UBt-1}\right)\\
& +\left(\delta_{15}-\delta_{13}\right)\left(p_{UAt-1}-p_{CAt-1}\right)+\left(\delta_{15}-\delta_{13}\right)\left(p_{UAt-1}-p_{CBt-1}\right)+\left(\delta_{12}-\delta_{15}\right)\left(p_{UAt-1}-p_{GAt-1}\right)\\
& +\left(\delta_{13}-\delta_{15}\right)\left(p_{UAt-1}-p_{SAt-1}\right)+\left(\delta_{13}-\delta_{15}\right)\left(p_{UAt-1}-p_{SBt-1}\right)+e_{1t}-e_{6t}.
\end{aligned}
$$

$$
\begin{aligned}
(24)\quad & p_{UAt}-p_{SAt}=\left(1-5\delta_{15}-2\delta_{13}-\delta_{12}\right)\left(p_{UAt-1}-p_{SAt-1}\right)+\left(\delta_{15}-\delta_{12}\right)\left(p_{UAt-1}-p_{UBt-1}\right)\\
& +\left(\delta_{15}-\delta_{13}\right)\left(p_{UAt-1}-p_{CAt-1}\right)+\left(\delta_{15}-\delta_{13}\right)\left(p_{UAt-1}-p_{CBt-1}\right)+\left(\delta_{13}-\delta_{15}\right)\left(p_{UAt-1}-p_{GAt-1}\right)\\
& +\left(\delta_{13}-\delta_{15}\right)\left(p_{UAt-1}-p_{GBt-1}\right)+\left(\delta_{12}-\delta_{15}\right)\left(p_{UAt-1}-p_{SBt-1}\right)+e_{1t}-e_{7t}.
\end{aligned}
$$

$$
\begin{aligned}
(25)\quad & p_{UAt}-p_{SBt}=\left(1-5\delta_{15}-2\delta_{13}-\delta_{12}\right)\left(p_{UAt-1}-p_{SBt-1}\right)+\left(\delta_{15}-\delta_{12}\right)\left(p_{UAt-1}-p_{UBt-1}\right)\\
& +\left(\delta_{15}-\delta_{13}\right)\left(p_{UAt-1}-p_{CAt-1}\right)+\left(\delta_{15}-\delta_{13}\right)\left(p_{UAt-1}-p_{CBt-1}\right)+\left(\delta_{13}-\delta_{15}\right)\left(p_{UAt-1}-p_{GAt-1}\right)\\
& +\left(\delta_{13}-\delta_{15}\right)\left(p_{UAt-1}-p_{GBt-1}\right)+\left(\delta_{12}-\delta_{15}\right)\left(p_{UAt-1}-p_{SAt-1}\right)+e_{1t}-e_{8t}.
\end{aligned}
$$

Note that the consistency condition holds here: the implied real exchange rate adjustment equations for within country relative prices (p_{CA}-p_{CB}, p_{GA}-p_{GB}, p_{SA}-p_{SB}) all take the same form as equation (19). Equations (20) and (21) are symmetric equations for U.S./Canada relative prices. The implied equations for German/Swiss prices are also exactly like equations (20) and (21). Equations (22)-(25) are symmetric equations for U.S./European city pairs. The implied equations for all other cross-continental city pairs take this form.

It is helpful to write equations (19)-(25) in vector notation:

(26) $\mathbf{p}_t = \Gamma\,\mathbf{p}_{t-1} + \mathbf{e}_t$,

where $\mathbf{p}_t = \begin{bmatrix} p_{UAt}-p_{UBt} \\ p_{UAt}-p_{CAt} \\ p_{UAt}-p_{CBt} \\ p_{UAt}-p_{GAt} \\ p_{UAt}-p_{GBt} \\ p_{UAt}-p_{SAt} \\ p_{UAt}-p_{SBt} \end{bmatrix}$,

$$\mathbf{e}_t = \begin{bmatrix} e_{2t} - e_{1t} \\ e_{3t} - e_{1t} \\ e_{4t} - e_{1t} \\ e_{5t} - e_{1t} \\ e_{6t} - e_{1t} \\ e_{7t} - e_{1t} \\ e_{8t} - e_{1t} \end{bmatrix}, \text{ and,}$$

$$\Gamma = \begin{bmatrix} A & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta_{13} - \delta_{12} & B & \delta_{12} - \delta_{13} & 0 & 0 & 0 & 0 \\ \delta_{13} - \delta_{12} & \delta_{12} - \delta_{13} & B & 0 & 0 & 0 & 0 \\ \delta_{15} - \delta_{12} & \delta_{15} - \delta_{13} & \delta_{15} - \delta_{13} & C & \delta_{12} - \delta_{15} & \delta_{13} - \delta_{15} & \delta_{13} - \delta_{15} \\ \delta_{15} - \delta_{12} & \delta_{15} - \delta_{13} & \delta_{15} - \delta_{13} & \delta_{12} - \delta_{15} & C & \delta_{13} - \delta_{15} & \delta_{13} - \delta_{15} \\ \delta_{15} - \delta_{12} & \delta_{15} - \delta_{13} & \delta_{15} - \delta_{13} & \delta_{13} - \delta_{15} & \delta_{13} - \delta_{15} & C & \delta_{12} - \delta_{15} \\ \delta_{15} - \delta_{12} & \delta_{15} - \delta_{13} & \delta_{15} - \delta_{13} & \delta_{13} - \delta_{15} & \delta_{13} - \delta_{15} & \delta_{12} - \delta_{15} & C \end{bmatrix},$$

where $A = 1 - 2\delta_{12} - 2\delta_{13} - 4\delta_{15}$, $B = 1 - \delta_{12} - 3\delta_{13} - 4\delta_{15}$, $C = 1 - \delta_{12} - 2\delta_{13} - 5\delta_{15}$.

Let Λ be the diagonal matrix of eigenvalues for Γ:

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 \end{bmatrix},$$

where $\lambda_1 = 1 - 2\delta_{12} - 2\delta_{13} - 4\delta_{15}$, $\lambda_2 = 1 - 4\delta_{13} - 4\delta_{15}$, and $\lambda_3 = 1 - 8\delta_{15}$.

We can write $\Gamma = G^{-1}\Lambda G$, where G is the matrix of row eigenvectors. We have:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -.5 & .5 & .5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.5 & -.5 & .5 & .5 \\ -.25 & -.25 & -.25 & .25 & .25 & .25 & .25 \end{bmatrix}.$$

Of course, each of the rows of this matrix is defined only up to a constant of proportionality.

We can write the system as:

(27) $\mathbf{p}_t = G^{-1}\Lambda G\, \mathbf{p}_{t-1} + \mathbf{e}_t.$

Pre-multiply by G, and we get:

(28) $\mathbf{z}_t = \Lambda\, \mathbf{z}_{t-1} + \mathbf{u}_t,$

where $\mathbf{z}_t = G\, \mathbf{p}_t = \begin{bmatrix} p_{UAt} - p_{UBt} \\ p_{CAt} - p_{CBt} \\ p_{GAt} - p_{GBt} \\ p_{SAt} - p_{SBt} \\ .5(p_{UAt} + p_{UBt}) - .5(p_{CAt} + p_{CBt}) \\ .5(p_{GAt} + p_{GBt}) - .5(p_{SAt} + p_{SBt}) \\ .25(p_{UAt} + p_{UBt} + p_{CAt} + p_{CBt}) - .25(p_{GAt} + p_{GBt} + p_{SAt} + p_{SBt}) \end{bmatrix},$

and,

$\mathbf{u}_t = G\mathbf{e}_t = \begin{bmatrix} e_{2t} - e_{1t} \\ e_{4t} - e_{3t} \\ e_{6t} - e_{5t} \\ e_{8t} - e_{7t} \\ .5(e_{3t} + e_{4t}) - .5(e_{1t} + e_{2t}) \\ .5(e_{7t} + e_{8t}) - .5(e_{5t} + e_{6t}) \\ .25(e_{5t} + e_{6t} + e_{7t} + e_{8t}) - .25(e_{1t} + e_{2t} + e_{3t} + e_{4t}) \end{bmatrix}.$

Equation (28) shows that we can rewrite the system of equations as seven AR1 equations – one for each of the within-country relative prices; one for the average of the U.S. prices relative to the average of the Canadian prices; one for the average of the German prices relative to the average of the Swiss prices; and, one for the average of the North American prices relative to the average of the European prices.

The three distinct eigenvalues correspond to the three speeds of adjustment:[4] $2\delta_{12}+2\delta_{13}+4\delta_{15}$ $(=1-\lambda_1)$ is the speed of convergence within country borders; $4\delta_{13}+4\delta_{15}$ $(=1-\lambda_2)$ is the speed of convergence across country borders but within continents; and, $8\delta_{15}$ $(=1-\lambda_3)$ is the speed of convergence across continents. Parsley and Wei (1996) argue that within-country speeds of adjustment are likely to be higher than cross-country speeds, so we allow these to be different. Davutyan and Pippenger (1990) and Liu and Maddala (1996) argue that speeds of convergence may be different for countries within continents than for cross-country continent pairs. In our set-up, we account for this because we do not impose $\lambda_1=\lambda_2$.

The fact that we consider things in twos – two continents, two countries per continent, two cities per country – gives a particularly nice structure to the diagonalized system. Of course, any diagonalization will result in a system of AR1 equations. Our system has a simple interpretation, and there is also a simple structure to the error covariance matrix.

Given the structure we put on the stochastic disturbances, we can write:

$$\mathbf{u}_t=\begin{bmatrix} l_{2t}-l_{1t}\\ l_{4t}-l_{3t}\\ l_{6t}-l_{5t}\\ l_{8t}-l_{7t}\\ .5(l_{3t}+l_{4t})-.5(l_{1t}+l_{2t})+n_{2t}-n_{1t}\\ .5(l_{7t}+l_{8t})-.5(l_{5t}+l_{6t})+n_{4t}-n_{3t}\\ .25(l_{5t}+l_{6t}+l_{7t}+l_{8t})-.25(l_{1t}+l_{2t}+l_{3t}+l_{4t})+.5(n_{3t}+n_{4t}-n_{1t}-n_{2t})+c_{2t}-c_{1t}\end{bmatrix}.$$

Note that the first four elements of $\mathbf{u}_t$ are contemporaneously uncorrelated, as are the fifth and sixth elements.

[4] Cogley and Spiegel (1996) associate the speed of convergence in panel data to the eigenvalues of the system.

We now assume that the variances of the local shocks within countries are equal:

$Var(l_{2t}) = Var(l_{1t})$,

$Var(l_{4t}) = Var(l_{3t})$,

$Var(l_{6t}) = Var(l_{5t})$, and

$Var(l_{8t}) = Var(l_{7t})$.

With those assumptions, the first four elements are also uncorrelated with the fifth, sixth and seventh elements. So, the first four elements are each uncorrelated with all other elements. The only elements of $\mathbf{u}_t$ that are correlated are the fifth and the sixth with the seventh. The covariance matrix of $\mathbf{u}_t$, denoted Σ, can be written as:

$$(29)\quad \Sigma = \begin{bmatrix} \sigma_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{44} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{55} & 0 & \sigma_{57} \\ 0 & 0 & 0 & 0 & 0 & \sigma_{66} & \sigma_{67} \\ 0 & 0 & 0 & 0 & \sigma_{57} & \sigma_{67} & \sigma_{77} \end{bmatrix},$$

where σ_{ii} is the variance of the ith element of $\mathbf{u}_t$ and σ_{ij} is the covariance of the ith and jth elements. This structure on the covariance matrix greatly simplifies the GLS estimation performed in the next section, and increases the speed of the Monte Carlo simulations considerably.

Note that we allow heteroskedasticity in the errors. The disturbance in each of the first four equations is the relative local shock, which is allowed to have a different variance in each country. The fifth element of $\mathbf{u}_t$ is composed of the average local shocks plus the national shock in Canada less the average local shock plus the national shock in the U.S. Its variance is permitted to be different than that for the sixth element of $\mathbf{u}_t$: the average local shock plus the national shock in Switzerland less the average local shock plus the national shock in Germany. One would expect the seventh term to have the highest variance, since it involves shocks to relative cross-continent prices: the average local and national shocks plus the continental shock in Europe less the average local and national shocks added to the continental shock in North America.

A consistency condition holds for our covariance matrix, much like the one we describe as holding for equations (19)-(25). If we calculate the implied variance of any relative price that is not explicitly in the system we estimate, equation (28), (for example, the variance of p_{SAt}-p_{UAt}), or the implied covariances of any two relative prices that are not explicitly in our system, they will have characteristics which are consistent with the assumptions we have made on Σ. For example, as one would expect from the assumptions we have made, the implied variance of p_{SAt}-p_{UAt} equals the implied variance of p_{SB}-p_{UAt}.[5]

Since we model the dynamics of relative prices as a system, it is possible that we will estimate the innovation of these prices more accurately than using single equation methods. Engel (1993) and Engel and Rogers (1996) compare the variance of within country and cross-country relative prices, but take the sample variance of the innovations from univariate processes for the relative prices.

To summarize our answers to the questions posed in the introductory paragraph: (1) Our model of PPP adjustment treats all relative prices in a consistent way. We achieve this by writing a general structure of price adjustment and making symmetric restrictions. (2) We handle cross-correlation through GLS estimation. The covariance matrix of the diagonalized system is particularly simple. (3) We allow for different speeds of adjustment for within-country, within-

[5] O'Connell (1996) performs GLS estimation with an unrestricted covariance matrix, so his estimation satisfies these type of consistency conditions.

continent and cross-continent city pairs. (4) We allow for heteroskedastic errors. (5) We find that unit root tests of relative prices have the advantage of differencing out nominal shocks.

2. Empirical Results

We estimate the system given by equations (28) and (29) using monthly price data from September 1978 to September 1994, for eight cities: Los Angeles, New York, Toronto, Vancouver, Frankfurt, Cologne, Zurich and Geneva. The price data is converted into U.S. dollar terms, using monthly average exchange rates, before relative prices are computed. The data sources are described in the Data Appendix.

We estimate the parameters of (28) and (29) by generalized least squares (GLS). The first four equations of (28) are independent of the last three, and are treated separately. O'Connell (1996) demonstrates that in small samples, the GLS estimates allow much more powerful tests of the unit root null hypothesis than if we were to estimate system (28) by ordinary least squares.

As a first step, we estimate the first four equations as a panel, with different intercept coefficients but the same slope (l) using ordinary least squares (OLS). We use the residuals from the OLS regressions to construct the estimated Σ matrix for GLS estimation. Here, we constrain all the off-diagonal terms to be zero. We save the residuals from the GLS estimates to form a second round estimate of Σ. We iterate a total of five times.

The last three equations are estimated in essentially the same way as the first four. First, we take an OLS estimate of the panel, allowing different intercepts across the three equations, and constraining the slope coefficient on the two intra-continental relative price regressions (λ_2) to be equal. We use the residuals to construct an estimate of Σ, which we use for the GLS estimator. Here, we constrain the disturbances to the intra-continental price equations to be uncorrelated, but allow the other covariances to be non-zero (as in the bottom 3×3 block of equation (29)). We iterate five times.[6]

Table 1 reports the estimates of the slope coefficients λ_1, λ_2, and λ_3. Recall 1-λ_1 is the speed of adjustment for within-country relative prices; 1-λ_2 is the speed of adjustment for relative prices across countries within the same continent; and, 1-λ_3 is the speed of adjustment for relative prices across continents. The point estimates are all very similar. We shall return to the question of statistical significance momentarily, but the numerical similarity of these estimates certainly suggests that there is not much economic difference in these speeds of adjustment.

From the estimates of λ_1, λ_2, and λ_3, we can calculate δ_{12}, δ_{13}, and δ_{15} from equations (12)-(18). We find δ_{12} = 0.00107; δ_{13} = 0.00381; and δ_{15} = 0.00127.

Table 1 also reports the estimates of the (square roots of the) diagonal elements of Σ. These turn out as expected: the variances for within-country city pairs are the smallest; the variance for the average of North American prices relative to the average of European prices is the largest; and, the two intra-continental variances lie in between. There is clearly evidence of heteroskedasticity: the inter-continental variance is more than twice as large as the intra-continental variances. These, in turn, are both at least twice as large as each of the four intra-national variances. The differences in the volatility of these relative prices can easily be seen in Figures 1, 2 and 3. Figure 1 plots the average of North American prices relative to European prices (the "intra-planetary" relative price.) Figure 2 plots the U.S./Canada and

[6] In practice, we actually calculate the GLS estimate by a complicated weighted average least squares estimate which is computationally much faster than blindly applying the GLS formula.

Germany/Switzerland relative prices (the "intra-continental" relative prices.) The intra-national relative prices are plotted in Figure 3.

These variances tend to confirm the findings of Engel (1993) and Engel and Rogers (1996) -- intra-country relative prices are much less variable than cross-country relative prices. Those studies explain some of that difference by the fact that intra-country relative prices do not involve a nominal exchange rate. If nominal prices are sticky and the nominal exchange rate is highly variable, than cross-country prices will be more volatile than within country prices. Note, however, that the same pattern appears when comparing intra-continental to cross-continental relative price pairs: the intra-continental prices have lower variance.[7] Both of these sets of relative prices involve nominal exchange rates. Some authors (notably Barro (1993, 441-449)) have argued that this pattern is evidence of relative homogeneity of supply shocks within continents compared to across continents. While this is one potential explanation, we note that the within-continent nominal exchange rates are much less variable than the cross-continent nominal exchange rates, so the sticky-price story remains a viable alternative. Nonetheless, Engel and Rogers (1995) argue that the volatility of the nominal exchange rate cannot explain all of the difference between the variances of intra- and cross-continental relative prices.

We are interested in whether this data could have been generated by unit root processes. To answer this question, we perform Monte Carlo exercises. We construct 5000 replications of a 7-vector of unit root series, each with 193 observations as in our actual data. These matrices of unit root series are generated with $N(0, \Sigma)$ random errors, where Σ is given by our estimate of the covariance matrix. With each of these 5000 series we repeat our estimation procedure. We record the fraction of times the estimated λ_i from our artificial series is less than the estimated λ_i from the actual data to get p-values for each λ_i.

[7] See, for example, Engel and Rogers (1995).

These p-values are reported in Table 1. It is clear that we cannot reject the unit root hypothesis for any of the λ_i at conventional levels of significance.

The fact that there is so little evidence against $\lambda_i = 1$ for each of i = 1, 2 and 3 suggests that it is not possible to reject the hypothesis that the λ_i are all equal. Hence, we reestimate the seven-equation system (28), imposing that all the slope coefficients are equal, but allowing different intercepts in each equation. We follow the same GLS procedure as above, and use the same Monte Carlo procedure to test the null that $\lambda = 1$.

These results are also reported in Table 1. Not surprisingly, not much in our inference changes. The estimated λ is numerically very close to the estimated λ_is when we allowed λ_1, λ_2 and λ_3 to be different. The estimated variances are nearly unchanged. And, we still find virtually no evidence against the unit root null. The p-value is around 0.58. However, note that the estimate of λ is consistent with much of the recent literature on PPP, including the literature that rejects unit roots. The estimated half-life of the relative prices is about four years. Perhaps this is evidence that our tests are not exceptionally powerful -- they find the same point estimate of λ as the rest of the literature, but cannot reject the null that λ equals one.

3. Conclusions

We have attempted to resolve several outstanding issues in panel PPP tests. While we believe we have, at a theoretical level, found satisfactory solutions to the problems, our actual tests of PPP leave a major unanswered question: Why do we fail to reject unit roots in real exchange rates, when other tests have successfully rejected this null?

On the one hand, it is tempting to conclude that the other tests -- which did not pay sufficient attention to heteroskedasticity, to correlation of errors, and to the choice of base currency; and, which forced speeds of adjustment to be equal for all real exchange rates -- spuriously rejected unit roots because of these shortcomings of their tests. This is the implication of the criticisms raised by O'Connell (1996), Papell (1996) and Liu and Maddala (1996). Since our tests properly account for these issues, one could argue that our tests are more reliable.

On the other hand, we must weigh against this that our tests may be less powerful. We have a smaller cross-section than much of the other literature (such as Frankel and Rose (1996), Oh (1996), Parsley and Wei (1996) and Wu (1996).) That certainly diminishes the power of our tests. Also, as we mentioned earlier, we assume the error terms in equation (28) are white noise. But, Papell (1996) highlights the importance of the residual correlation for tests of unit roots in real exchange rates.[8]

Finally, the simple fact that the data is different could account for the different conclusions. This is consistent with the findings of Papell (1996) and Liu and Maddala (1996). They find that panels of real exchange rates for some groups of countries appear to be stationary, and others appear to have unit roots, even when the panels are the same size and length. It may be that this occurs because those authors have not sufficiently allowed for correlation of innovations, as we have attempted to do in this paper. But, Engel (1996) raises the issue that standard tests of PPP are more likely to falsely reject the presence of a unit root when there is a stationary component that dominates the dynamics of the movement of the real exchange rate in the short run. Within-country relative prices across cities are not very volatile. Perhaps with our

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[8] In fact, however, in his examples insufficient modeling of the serial correlation in the residuals biases the tests toward rejection of the unit root null.

city data this stationary component is not so prominent, and the data more nearly follow a pure random walk.

So, in the future, we hope to expand the panel to a larger number of cities, and to address the issue of higher order serial correlation. The present study should be considered an exploratory venture into the city-based approach to panel PPP testing.

Data Appendix

The price and exchange rate data used in the paper are monthly, spanning the period 9/78 -9/94. The exchange rate is a monthly average rate, and was obtained from the IMF's International Financial Statistics database.

The data were obtained from a variety of sources. The data for Los Angeles and New York are the monthly consumer price indexes from the U.S. Bureau of Labor Statistics. The data for Toronto and Vancouver are also the monthly consumer price indexes for those cities, and was obtained from Statistics Canada. The Swiss price data are the monthly CPIs for Zurich and Geneva. These data were obtained from the Swiss Federal Statistical Office. Finally, the German price data are cost-of-living indexes. These data are computed by Lander (State). Thus, the data for Frankfurt and Cologne are, respectively, the monthly cost-of-living index for Hesse and Northrhine-Westphalia. These data were obtained directly from the state statistical offices.

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Table 1

Generalized Least Squares Estimates and 5000 Draw Monte Carlo Simulation Results

System	λ_i	*p-value*	*s.d.'s*	*intercepts*	*price adjustment equation for:*
3-equation	0.989876	0.7870	0.029617	0.006986	N. America relative to Europe
	0.979688	0.4734	0.011127	0.007901	US relative to Canada
			0.012646	0.001729	Germany relative to Switzerland
4-equation	0.985164	0.6218	0.004869	-0.000204	Los Angeles relative to New York
			0.003989	0.000143	Toronto relative to Vancouver
			0.001413	0.000185	Frankfurt relative to Cologne
			0.003323	0.000027	Geneva relative to Zurich
7-equation	0.984674	0.5820	0.029595	0.010519	N. America relative to Europe
			0.011119	0.006123	US relative to Canada
			0.012652	0.001260	Germany relative to Switzerland
			0.004868	-0.000209	Los Angeles relative to New York
			0.003989	0.000143	Toronto relative to Vancouver
			0.001414	0.000188	Frankfurt relative to Cologne
			0.003323	0.000025	Geneva relative to Zurich

Figure 1 Average "Intra-Planetary" Relative Price

Deviation from Sample Mean

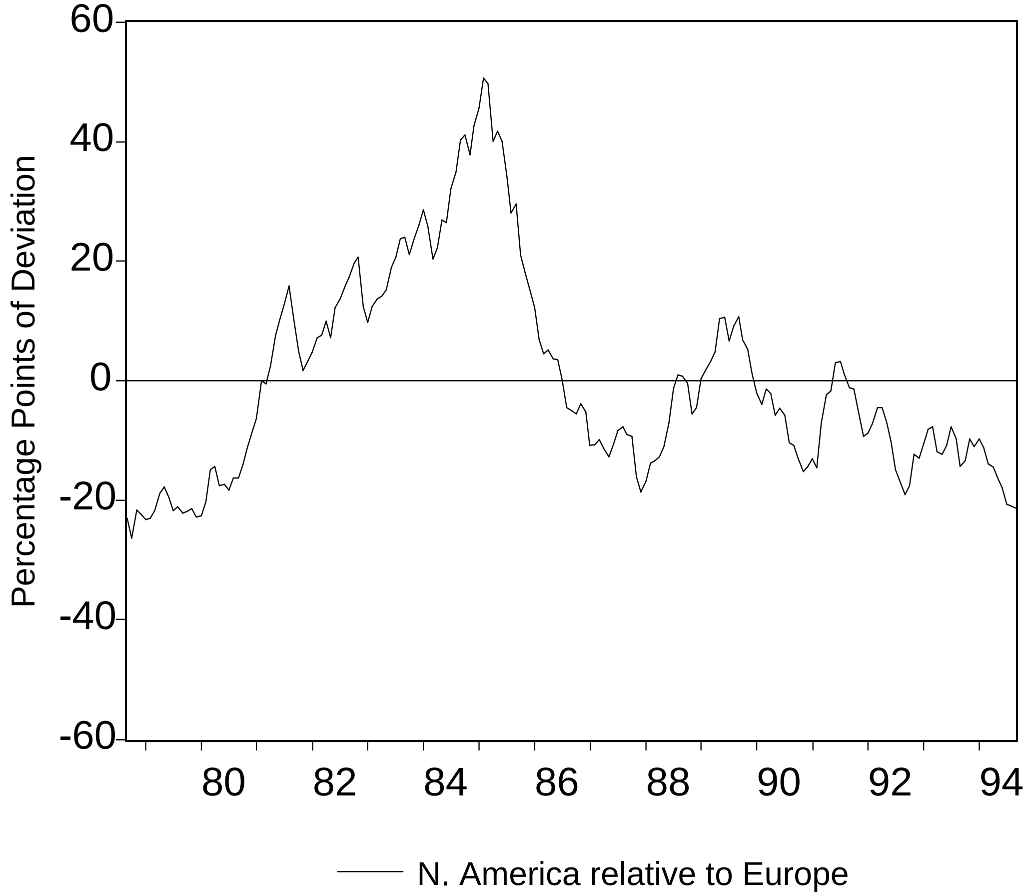

Figure 2   Average "Intra-Continental" Relative Prices

Deviations from Group Sample Mean

Figure 3 "Intra-National" Relative Prices

Deviations from Group Sample Mean