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by

Marcus H. Miller

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Marcus H. Miller*
University of Warwick
Coventry CV47AL, England

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Abstract

This paper investigates optimal stabilization policy in a small open economy using a continuous time model in which inflation depends on future monetary policy as well as past inflation. The impact of monetary policy is assumed to operate via real interest rates and the real exchange rate and the setting of real interest rates is chosen so as to minimise quadratic costs of fluctuations in output and inflation, subject to varying expectations in the foreign exchange market. Analytical expressions and simulation results are presented for "time inconsistent" optimal policy, the "dynamic programming" solution, for policy which ignores the exchange rate effects when setting real interest rates, and for the "optimal linear feedback" rule.

"I do believe firmly that by monetary means exercised promptly and courageously we can greatly mitigate the worst evils of inflation and deflation ... one thing is certain: we will not obtain stability unless we work for it."

Marriner S. Eccles, Federal Reserve Board Chairman 1934-48.

Introduction

The purpose of this paper is to study the design of monetary stabilization policy in an open economy with a floating exchange rate, ignoring feedbacks from the rest of the world.[1] The impact of monetary policy on both output and inflation is assumed to operate via real interest rates and the real exchange rate: so the problem posed is that of choosing the optimal setting of real interest rates so as to minimise quadratic costs attached to fluctuations in output and inflation.

The model used here, cf. Buiter and Miller (1982), features a substantial autoregressive element in the inflationary process, as well as the rational expectations, asset-price approach to the determination of the exchange rate of Dornbusch (1976). Consequently the rate of inflation is influenced both by its own past history (via the autoregressive structure) and by current and expected future monetary policy (via the exchange rate).

The design of optimal open-loop monetary policy in such an economy has been studied by John Driffill (1982). He notes the systematic temptation upon the policy maker to depart from the plan first announced if ever policy can subsequently be reset; so the optimal trajectory is not "time consistent," to use the terminology suggested by Kydland and Prescott (1977) in their celebrated critique of optimal control techniques when agents form rational expectations.

The focus of attention here however, is, on alternative stabilisation policies which treat expectations in the foreign exchange market in two sharply contrasting ways, either by explicitly accomodating the effect that continuous policy reoptimisation will have on the market; or by ignoring the impact of interest rate policy on the market altogether. The former is achieved by "dynamic programming" using an elegant technique recently developed by Cohen and Michel (1984). The latter approach, suggested by William Buiter (1983), turns out here to produce a more activist policy. Both policies take the form of constant linear feedback rules for interest rate setting. So the question of how they compare with the optimal linear feedback rule is also addressed. (As a preliminary the policy problem in a closed economy briefly considered).

Oudiz and Sachs (1985) have already carried out such an investigation using a discrete-time model of inflation in a small open economy and a clear treatment of some of these issues is to be found in Turner (1984). In what follows, therefore, the intention is not to present novel results, but, by using a continuous time framework and applying the technique of Cohen and Michel, to make known results more analytically accessible and transparent.

In conclusion the case for choosing a simple rule with reference to more long run properties of the model is briefly discussed.

Closed Economy

The analysis of policy in an open economy with a floating exchange rate influenced by expected future policy may be assisted by focussing first on the nature of "optimal" stabilization policy in the absence of such forward-looking behaviour. That is the purpose of this section, cf. Turner (1984).

Consider for example a closed economy where the inflationary process is described by a Phillips curve "augmented" by a term representing backward-looking "core" inflation.[2] We write this as

$$i = \phi y + \pi$$

where

i is the rate of price inflation

y is the log of output (measured from its "natural" rate)

π is core inflation.

Let core inflation be an exponential moving average of past inflation, so

$$D\pi = \xi(i - \pi)$$

where $D = d/dt$ is the differential operator.

Then, by defining z as the integral of y, so z measures cumulated excess demand and

(1) $Dz = y$,

core inflation can be written as a multiple of z, namely

(2) $\pi = \xi\phi z$.

If the policy-maker penalizes the weighted but undiscounted squared deviations of core inflation[3] and output, then the infinite horizon policy problem can be expressed as that of minimizing V(o) with respect to y subject to equations (1) and (2) above, given z(o),

where $V(o) \equiv \frac{1}{2}\int_{o}^{\infty} \beta\pi^2(s) + y^2(s)\, ds$.

Details of the derivation are given in the Appendix. What emerges is that the optimal policy is to set

$$y = - p_z$$

where p_z is the "shadow cost" of z, cumulated excess demand. (This p_z measures the welfare cost imposed by a unit increase in z along an optimal trajectory.) This shadow cost is found to evolve according to $Dp_z = - \beta\xi\phi\pi$.

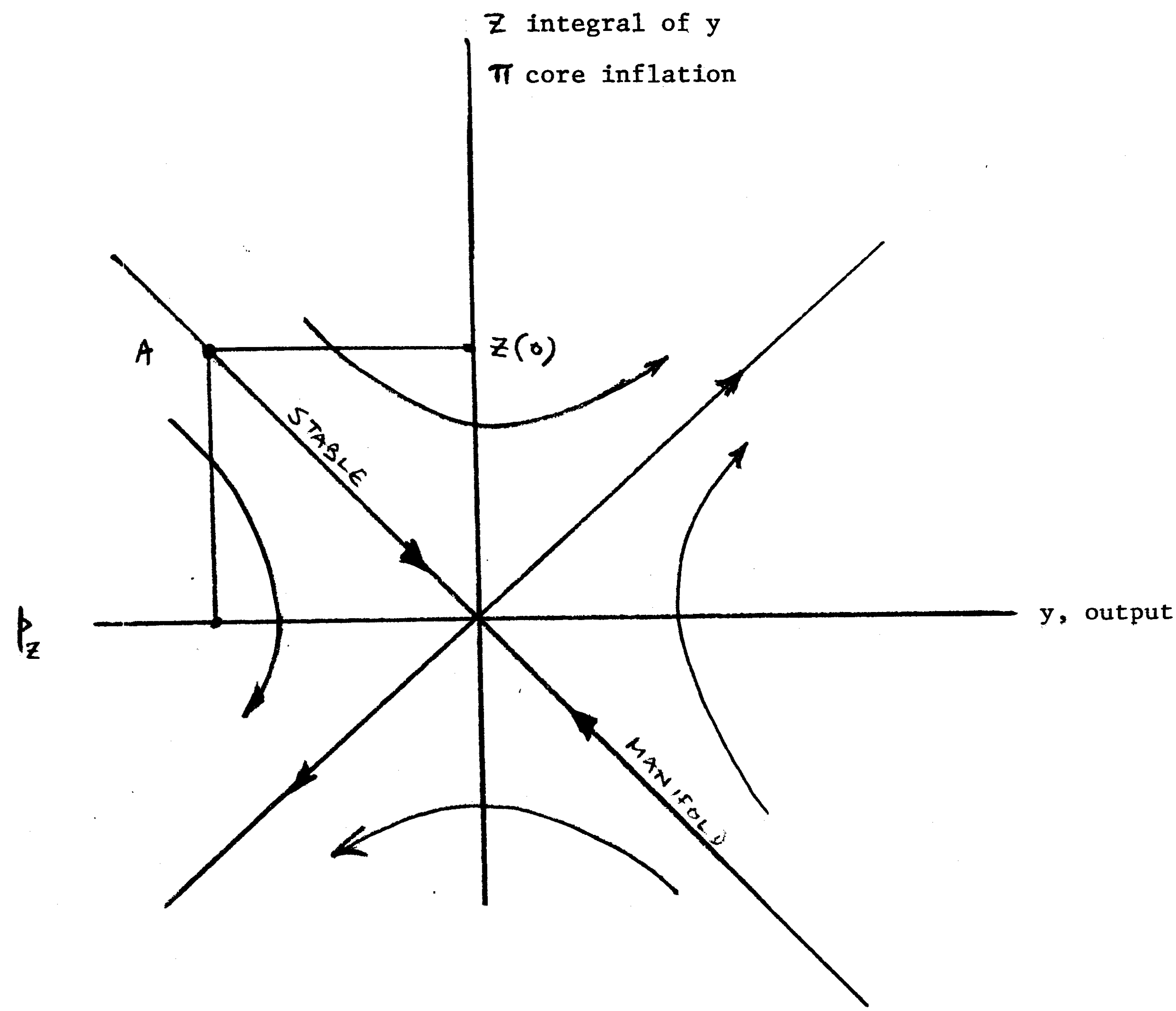


Figure 1. Optimum Policy in the Closed Economy

The nature of optimal policy can be shown by the saddlepoint phase diagram, figure 1. There the horizontal axis measures both output (increasing to the right) and the shadow cost p_z (increasing to the left), while the vertical axis measures both the integral of output, and the rate of core inflation, a positive multiple of z.

The dynamics of z are obvious as it is the integral of y. The adjustment of y over time follows from the first order conditions that $y = -p_z$ and that $Dp_z = -\beta\xi\phi\pi$; hence $Dy = \beta\xi\phi\pi = \beta\xi^2\phi^2 z$, so y rises for positive z and falls when z is negative. For the infinite horizon problem being treated here, the unique optimal trajectory is that lying along the "stable manifold" leading from A to the origin. For the case when $\beta = \xi = \phi = 1$ it turns out that the initial values are $y(o) = -p_z(o) = z(o)$ and the stable root is unity. Given the linear quadratic structure, the minimal integrated cost V^* is given by $0.5\, p_z z = 0.5z^2$.

Thus for an economy with a positive level of core inflation, $\pi > o$, the optimal policy is to initiate a recession followed by a gradual recovery towards the equilibrium (where inflation is zero and output at its non-inflationary equilibrium, $y = o$).

<u>Small Open Economy</u>

How is the analysis affected by the floating exchange rate? First of all, at the positive level, the movements in the (real) exchange rate alter the rate of inflation, so

$$i = \phi y + \pi + \sigma Dc$$

where c is the real exchange rate which we take to be a forward-looking variable, see (3) below. Thus, with the same exponential process for π, it turns out that, in the open economy,

$$(2)' \quad \pi = \xi\phi z + \xi\sigma c.$$

so core inflation is a weighted sum of the backward-looking variable, z, and the forward looking variable, c.

The idea that inflation is a centered moving-average of past and (expected) future movements in wages is of course a familiar one, cf. for example the overlapping-contracts model of John Taylor (1979). It is evident, however, that <u>openness</u> of the economy induces a forward-looking element into core inflation even when this was <u>not</u> present from the contractual structure (we began with a simple Phillips curve).

To proceed with the problem of policy, one needs to know more precisely how the real exchange is determined. As is common in theoretical analysis, we assume that expected movements of the real exchange rate match real interest differentials, so

$$(3) \quad Dc = r$$

where r is the real interest rate (measured as a deviation from the constant world level) and competitiveness, c, is measured as a deviation from equilibrium.

The model may be completed by specifying how output is determined. Specifically let output be determined by the demand side where the latter depends on the real interest rate and on the level of competitiveness (which is a forward-looking integral of future expected real interest rates) so

$$(4) \quad y = -\gamma r + \delta c.$$

The policy problem can now be formally expressed as that of minimizing V with respect to r given equations (1), (2)', (3) and (4) above. (Note that r may be eliminated by substitution, and the problem reduced to one of choosing an output path so as to minimize V.)

In choosing the path for real interest rates, however, the policy maker has to take a view as to what future path of real interest rates the market expects. One possibility, perhaps the most obvious, is that the market should presently expect that path of real interest rates which generates the minimum cost for the policy maker at the present time. This idea begins to lose its attractiveness when one observes that the policy which minimizes V at some later date will <u>not</u> in fact be a continuation of the policy chosen today - such policy design is said to be time inconsistent.

A second approach is to require that the Principle of Optimality be satisfied, so that what the market expects must be interest rate paths consistent with continous reoptimization. The techniques of dynamic programming can be used to find such a time consistent solution.

We begin with a third approach, proposed by William Buiter (1983) as a means of avoiding time inconsistent policy. The idea is that the policy-maker chooses the trajectory of real interest rates taking the path of the real exchange rate as exogenous. As the real exchange rate is a forward-looking variable, it will consequently anticipate a future in which policy is constrained by this "self-denying ordinance." The outcome of such a policy is easy to calculate -and we find that it dominates the dynamic programming solution! (Later we compare this with the optimal linear feedback rule).

(I) Policy when the exchange rate is treated as predetermined

Formally the policy-maker chooses r so as to minimize the integral of costs, V, subject to the equations of the model, (1), (2)', (3), (4), treating the path of c as exogenous. The formal derivation of the solution which ensures that the assumed path currently discounts future interest differentials is given in the Appendix. The numerical[4] outcomes (for parameter values $\phi = \xi = \beta = 1$, $\sigma = 0.1$) are given in table 1, column 2. Here we consider the changes from those obtained for the closed economy, shown in the first column of the table.

It is worth remarking ex ante that the integral of (net) lost output to reduce inflation to zero should not be affected by the openness of the economy, provided that the real exchange rate begins and ends at the same level, a point discussed in some detail in Buiter and Miller (1982). We observe, however, that for the open economy the initial size of the recession falls (from $z(o)$ to $0.95\, z(o)$) and the rate of recovery is slower (the stable root likewise falls from unity to 0.95 in absolute value).

In a setting such as this it is pretty obvious that the reduction of inflation requires a recession, but why should the policy maker in the open economy opt for a shallow initial cutback in output followed by a slower recovery? In order to induce the recession real interest rates have to be increased, but this necessarily means that the initial values of both competitiveness, c, and core inflation, π will be low (as the expected path of real rates lifts the external value of the currency): later, however, the recovery of competitiveness will put back into core inflation what this initial appreciation takes out. Without actually planning it this way, inflation in the open economy will be seen as less of a problem early on than more of a problem later on - which provides the rationale for having less of a recession now and more later.

It is evident that this "reshaping" of inflation and recession is attractive to the policy maker, since the minimized integral of costs, V^*, falls from $0.5\, z^2(o)$ to $0.45\, z^2(o)$, see Table 1.

(II) <u>The dynamic programming solution</u>

So far we have considered the outcome where policy-making is optimized subject to an "exogenous" path for the real exchange rate - a path which is solved to forecast correctly the continuation of such constrained policy optimization. Recently Cohen and Michel (1984) have shown that a similar, simple characterization of policy which satisfies the Principle of Optimality is available for a linear structure with quadratic costs.

To obtain the dynamic programming solution, Cohen and Michel show that policy makers must optimize subject to linear constraint which in this case is that the real exchange rate be a linear function of the state variable (i.e. $c = \theta z$). If one one finds the correct parameters (the right value for θ), so that the real exchange rate chosen is actually the integral of future real interest differentials, one will satisfy the Principle of Optimality: policy makers will be optimizing subject to a constraint which captures market expectations of continuous future optimization.

Formal details are given in the Appendix. We remark here that the solution for the infinite horizon case can be obtained by an iterative scheme involving the elements of the stable eigen vector. The stable eigen vector associated with the solution of the last section may be used to provide a first estimate of the value of θ: using that as a constraint one can solve to obtain a new stable eigen vector, and continue until convergence is achieved.

The numerical results for such time consistent optimal policy are shown in the third column of table 3. Compared to the second column, the initial recession is yet further reduced and the length of the recession correspondingly prolonged (the stable root falls to 0.905 for the dynamic programming solution). To see why this is so, it is worth considering how the Cohen/Michel constraint affects the "shadow cost of cumulated output," p_z, since the path of output is determined by this shadow cost ($y = - p_z$).

The constraint characterizing this dynamic programming solution is $c = -0.95\ z$; international competitiveness is negatively correlated with cumulated output when high real interest rates are used to check inflation. The shadow cost of a cumulated boom is consequently reduced since the inflationary consequences are seen to be damped by the counter-cyclical behaviour of competitiveness. Policy makers are thus less inclined to sacrifice output to check inherited inflation; so the size of an initial recession is lower and core inflation higher than under previous policy (both are $0.9050\ z(o)$).

While this behaviour seems perfectly reasonable, it is interesting to see that the integral of costs rises marginally relative to what was obtained under the previous self-denying policy of treating the exchange rate as given.

(III) Time inconsistent optimal policy

What trajectory of real interest rates would the policy maker choose at time zero, knowing that the announcement of these rates will influence the foreign exchange rate? Note that the so-called time inconsistent trajectory which (if credible) minimizes $V(o)$ will require some form of precommitment since subsequent reoptimization would not lead to a continuation of the announced policy. (In the absence of precommitment - or other factors like that of acquiring a reputation - the situation is likely to revert to the time consistent outcome obtained from dynamic programming.) How will the policy maker choose to use his or her influence over the foreign exchange market to alter the shape of an inflationary recession?

Formally the optimal trajectory can be found by setting up a shadow price for the real exchange rate, p_c, and assuming that $p_c(o) = o$, see Appendix. The ability to precommit oneself to future policy actions permits a more interesting convergence to equilibrium, an approach described by two eigen values. In this particular case the eigen values turn out to be complex conjugate so the convergence is cyclical, see table. Under this policy, the initial values for core inflation and recession are <u>both</u> reduced compared to the time consistent outcome, falling from 0.9050z(0) in column 3 to 0.9026z(o) in column 4.

How is this possible - to increase output while reducing inflation? The trick is to announce a reduced real interest rate spread for the early part of the period (cf. the initial values for r(o) shown in the table) but a wider spread for later. This shifts inflation and recession away from early period when the squared deviations are heavily penalized towards a later period when they are less "costly". The integral of costs consequently falls to the lowest value shown in the table.

These fluctuations over time in the real interest differential show up in the path followed by the real exchange rate, see figure 2, where the time inconsistent optimal trajectory is labelled (b). The other two trajectories have a constant ratio of competitiveness to cumulated output. This ratio (labelled θ) is, of course, explicitly taken into account in choosing the dynamic programming trajectory: but note that the outcome (a), when competitiveness is taken as predetermined, resembles the time inconsistent optimal trajectory more closely.

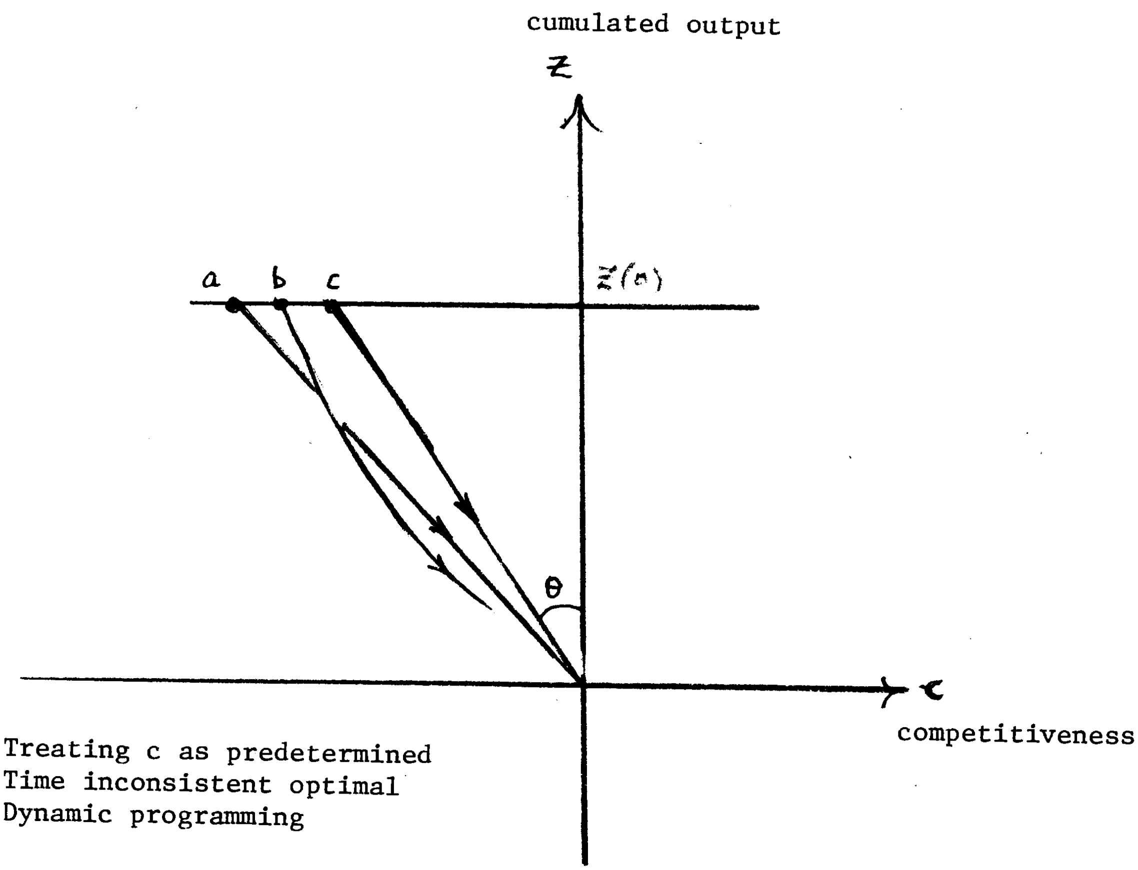
Table 1. Varieties of Stabilisation Policy

			Three Policies for the Open Economy		
		Closed Economy	Exogenous c	Dynamic Programming	Time Inconsistent
	Symbol	(1)	(2)	(3)	(4)
Stable eigen value	λ	-1	-0.950	-0.905	$-0.954 \pm 0.3i$
Stable eigen vector(1)	θ	n.a.	-0.9744	-0.95	Conjugate
	ψ	1	0.9500	0.905	Complex
Starting values for endogenous variables(2)	y(o)	-1	-0.9500	-0.9050	-0.9026
	π(o)	1	-0.9026	0.9050	0.9026
	c(o)	n.a.	-0.9744	-0.9500	-0.9737
	r(o)	n.a.	0.9257	0.8599	0.83151
Integrated costs(3)	V(o)	0.5 z^2(o)	0.4518 z^2(o)	0.4525 z^2(o)	0.4513 z^2(o)

Footnotes

(1) θ gives the ratio of c to z on the stable manifold.
ψ gives the ratio of p_z to z on the stable manifold.
(2) reported as multiple of z(o).
(3) measured as $p_z(o)\ z(o)/2$ for columns (1), (3) and (4) and as $(1 + 2\sigma\theta + \sigma^2\theta^2 + \psi^2)z^2(o)/4\ \lambda$ for column (2).

Parameters values used $\phi = \delta = \beta = 1$, $\gamma = \delta = 0.05$, $\sigma = 0.1$



(a) Treating c as predetermined
(b) Time inconsistent optimal
(c) Dynamic programming

Figure 2. Varieties of stabilisation policy

Summary

For a linear augmented Phillips curve, the underlying rate of inflation depends on the simple *integral* of output and not on the detailed trajectory (thus, for the closed economy, $\pi = \xi\phi z$ where z denotes the integral). Use of a quadratic penalty function allows one to discriminate between trajectories on welfare grounds, however, and "optimal" policies may then be derived. The openness of an economy helps to reduce the welfare costs of an anti-inflationary policy led by high real interest rates, as the early appreciation that this induces - even though it is reversed later - shifts inflation into the future.

Relative to the other policies examined, the optimal (but time inconsistent) trajectory of real interest rates for the open economy is a "low start" policy. This can be seen in Table 2 where the time inconsistent trajectory exhibits the lowest real interest rate at time zero but the highest levels over the next two years. Note that although real interest rates in column 3 are falling throughout, the *ratio* of r to z, shown in parentheses, does not. (Precommitment is of course required to add credibility to this threat -- that "tomorrow we will act tougher on inflation than we are acting today.") For the other two policies, however, the ratio of real interest rate to the integral of real output remains constant over time, so both imply fixed linear feedback rules, with dynamic programming having the smaller feedback coefficient as each government passes on some of the pain of fighting inflation to it successors.

Table 2. Real Interest Trajectories (percent points)

Time	Predetermined Path for c	Dynamic Programming	Time Inconsistent Optimal Policy	
	(1)	(2)	(3)	
0	9.3	8.6	8.3	(0.83)
1	3.6	3.5	3.7	(0.47)
2	1.4	1.4	1.5	(1.11)
3	0.5	0.6	0.6	(1.31)
4	0.2	0.2	0.2	(1.65)

Note (1) Real interest rate trajectories assuming z(o) - 10 percent points.

(2) The ratio of r to z is constant for (1) and (2), at 0.93 and 0.86 respectively. Values for r/z for (3) vary over time and are shown explicitly in parentheses.

One may be prompted to ask whether, if real interest policy were restricted to the class of linear feedback rules, one could improve on those already calculated. One can, of course, never do worse, and in general, as Cohen and Michel show, one can do better. Continued adherence to such an "optimal rule" would, doubtless, require some form of precommitment, though it seems much more likely that governments could credibly adopt such a simple feedback rule than precommit themselves to the time-varying trajectory required for the time inconsistent optimal policy earlier described.

For the particular coefficients and welfare weights used here, however, it so happens that the feedback coefficient of 0.926 obtained when the real exchange rate is taken as predetermined is approximately the optimal linear feedback policy. Table 3 which shows the integrated costs generated by various feedback rules, running from 0.80 to 1.0 (so real interest rates are set either just below, or equal to, inherited core inflation, measured by $\xi\phi z$ where $\xi = \phi = 1$). While, in this particular instance, taking account of future optimizing policy by dynamic programming leads to under-responding, it turns out that ignoring the exchange rate channel generates the optimal feedback policy! That this happy coincidence is not in general true, even for the given economic structure, is shown by doubling the welfare weight on inflation ($\beta = 2$). While the dynamic programming response is once again smaller than when one optimizes treating the exchange rate as given, the optimal feedback coefficient lies between the two, see Table.

Table 3. Welfare Costs of Various Linear Feedback Rules

Low weight on inflation (β =1)		High weight on inflation (β = 2)	
Feedback Coefficient	Integrated Cost	Feedback Coefficient	Integrated Cost
0.80	0.45423	1.40[a]	0.628351
0.86[a]	0.45250	1.45	0.627708
0.90	0.46197	1.49	0.627499
0.926[b,c]	0.45187	1.50[b]	0.627486
0.95	0.45194	1.51[c]	0.627489
1.00	0.45250	1.52	0.627506

Notes a Dynamic programming solution.
b Optimal feedback rule (approximate).
c Result of treating real exchange rate as predeterminent.
Feedback rule relates the real rate of interest r to z, integrated output.
Cost shown as multiple of z^2.

Conclusion

When high real interest rates are used to combat inflation, one can expect to find the exchange rate overvalued when inflation is high. Paradoxically, however, the time-consistent interest rate policy which recognises this, promptly chooses to underrespond in its choice of interest rates. (The perception that future policy makers will use high interest rates reduces the incentive to implement them now, since the rise in the exchange rate reduces the inflation problem in the first place!)

In the economic setting considered in this paper it happens that turning a blind eye to the exchange rate effects generates a more appropriate, activist response. Indeed for the initial set of parameters, such a Nelsonian policy resulted in the optimal linear feedback response! However, it is not in general true that the optimal linear feedback rule can be found by ignoring the effects of policy on forward-looking variables, so it will have to be determined by explicit minimisation of welfare costs, see Appendix.

Since it was not selected by dynamic programming, the viability of such an optimal rule depends on precommittment. As Sachs and Oudiz observe, it is easier to see policy makers adopting a simple rule than adopting a complicated trajectory for real interest rates. On the other hand, the independence of the rule and the initial conditions (characteristic of the scalar case) fails when there is more than one state variable (as Cohen and Michel have pointed out).

To avoid excessive dependence on particular initial conditions, one may alternatively consider the asymptotic properties of the system in choosing a simple rule (to minimise some function of the asymptotic variance-covariance matrix for example). This is the approach adopted by John Taylor (1985) who observes that: "By focussing on the stochastic equilibrium we are implicitly assuming an infinite time horizon for policy choice with no discounting. This seems appropriate for macroeconomic policy. We are also assuming that time consistency will not be a problem: once a policy rule is chosen we assume that it will remain in force with no attempt by the policy makers to exploit the past committments of economic agents" (p. 69).

As Kydland and Prescott forewarned, the search for simple and sustainable feedback rules in an environment where expectations are formed rationally appears to lead one away from explicit use of optimal control and dynamic programming techniques for short run stabilisation. Whether the alternative, longer-run focus of policy meets the criticisms levelled against floating (for the incentives it offers to use competitiveness for short-run stabilisation policy, cf. Williamson (1985)) is an issue worth further discussion.

Appendix

CLOSED ECONOMY

To minimize $\frac{1}{2}\int_{o}^{\infty} \beta\pi^2(s) + y^2(s)\, ds$ given $z(o)$ and
w.r.t. y

subject to (1) $Dz = y$ and (2) $\pi = \xi\phi z$, form the Hamiltonian

$H = \frac{\beta\pi^2 + y^2}{2} + p_z\, Dz$ with First Order Conditions for as minimum

follows:

$$\partial H/\partial y = y + p_z = 0$$

$$-Dp_z = \partial H/\partial z = \beta\xi\phi\pi.$$

The evolution of the state and costate variables $(z,\ p_z)$ is thus

$$\begin{bmatrix} Dz \\ Dp_z \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -\beta\xi^2\phi^2 & 0 \end{bmatrix} \begin{bmatrix} z \\ p_z \end{bmatrix}$$

which, when restricted to the stable manifold, implies

$$z(t) = e^{\lambda t}\, z(o) \qquad \lambda = -\xi\phi\sqrt{\beta}$$

$$y = -p_z = -\psi z \qquad \text{where} \quad \psi = -\lambda.$$

OPEN ECONOMY

To minimize $\frac{1}{2}\int_{o}^{\infty} \beta\pi(s)^2 + y^2(s)\, ds \equiv V(o)$
w.r.t.r

subject to (1) $Dz = y$ (2)' $\pi = \xi\phi z + \xi\sigma c$

(3) $Dc = r$ (4) $y = -\gamma r + \delta c$

given z(o), we consider three approaches.

I (c assumed predetermined) and II (assuming c = θz)

From the Hamiltonian $H = \frac{\beta\pi^2 + y^2}{2} + p_z\, Dz$ the First Order Conditions are

$$\partial H/\partial r = -\gamma y - \gamma p_z = o$$

$$-Dp_z = \partial H/\partial z = \beta\xi(\phi + \theta\sigma)\pi + \theta\delta(y + p_z)$$

which, together with the equations (1) to (4) imply

$$\begin{bmatrix} Dz \\ Dc \\ Dp_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & \gamma^{-1}\delta & \gamma^{-1} \\ -\phi h & -\sigma h & 0 \end{bmatrix} \begin{bmatrix} z \\ c \\ p_z \end{bmatrix} \quad \text{where } h \equiv \beta\xi^2(\phi+\theta\sigma).$$

For Case I, the solution is obtained by setting θ = o before computing the stable eigen value and vector. For Case II, however, θ must be determined so as to equal the second element of the stable eigen vector. This may be achieved by an iterative procedure, as discussed in the text.

III Time Inconsistent Optimal Policy

From the Hamiltonian $H = \frac{\beta\pi^2 + y^2}{2} + p_z\, Dz + p_c\, Dc$ the First Order Conditions are

$$\partial H/\partial r = -\gamma y - \gamma p_z + p_c = o$$

$$-Dp_z = \partial H/\partial z = \beta\xi\phi\pi$$

$$-Dp_c = \partial H/\partial c = \beta\xi\sigma\pi + \delta(y + p_z)$$

which, together with the equations of the model imply

$$\begin{bmatrix} Dz \\ Dc \\ Dp_z \\ Dp_c \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & \gamma^{-1}\delta & \gamma^{-1} & -1 \\ -\phi^2 k & -\phi\sigma k & 0 & 0 \\ -\phi\sigma k & -\sigma^2 k & 0 & -\delta\gamma^{-1} \end{bmatrix} \begin{bmatrix} z \\ c \\ p_z \\ p_c \end{bmatrix} \quad \text{where } k \equiv \beta\xi^2.$$

The unique solution is obtained by restricting the dynamic path to the stable manifold subject to the historically given value of z(o) and the optimality conditions that $p_c(o) = 0$, i.e. the shadow cost of the real exchange rate be minimized at time t = o.

The optimal linear feedback rule

The problem is to minimise V(o) with respect to ρ subject to (1), (2)', (3), (4) and (5) $r = \rho z$, given z(o). Under such a feedback rule, it follows that

$$\begin{bmatrix} Dz \\ Dc \end{bmatrix} = \begin{bmatrix} -\gamma\rho & \delta \\ \rho & o \end{bmatrix} \begin{bmatrix} z \\ c \end{bmatrix}$$

So $\lambda_s\lambda_u = -\rho\delta$, $\lambda_s + \lambda_u = -\gamma\rho$ and $\lambda_s\theta = \rho$, λ_s being the stable root, λ_u the unstable root and $c = \theta z$ on the stable manifold. Consequently $V(o) = \frac{1}{2}\int_o^\infty \beta\pi^2(s) + y^2(s)\, ds$

$$= \frac{z^2(o)}{4\lambda_s}\left\{\beta\left(\xi\phi + \xi\sigma\theta\right)^2 + \left(\delta\theta - \gamma\rho\right)^2\right\}.$$

Hence the First Order Condition to be satisfied, namely

$$\frac{dV(o)}{d\rho} = \frac{\partial V}{\partial\rho} + \frac{\partial V}{\partial\theta}\frac{\partial\theta}{\partial\rho} + \frac{\partial V}{\partial\lambda_s}\frac{\partial\lambda_s}{\partial\rho} = 0$$

(where $\frac{\partial\theta}{\partial\rho}$ and $\frac{\partial\lambda_s}{\partial\rho}$ follow from the relations given above) will be independent of z(o).

Footnotes

1/ These issues are discussed in a two-country, dynamic-game setting by Miller and Salmon (1985a,b) and Oudiz and Sachs (1985).

2/ Note, however, that the problem of designing policy in the face of rational expectations will not be avoided, even for a closed economy, where the wage level depends on averaging forward-looking contracts, as in Taylor (1979).

3/ For analytical simplicity π and not i appears in the welfare function. The analysis may, however, be repeated using i instead if this is thought desirable.

4/ All the numerical results reported here were obtained using "Saddlepoint", see Austin and Buiter (1982), to whom we are grateful for making this programme available.

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