Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 369

December 1989

EXCHANGE RATE VARIABILITY AND THE LEVEL OF INTERNATIONAL TRADE

Joseph E. Gagnon

ABSTRACT

There have been numerous theoretical and empirical studies of the effect of exchange rate variability on the level of international trade. Most theoretical studies have concluded that under reasonable assumptions exchange rate variability ought to depress the level of trade. Empirical studies generally have not identified a significant effect of exchange rate variability on trade flows. This paper builds a theoretical model in which exchange rate variability has a negative effect on the level of trade. The model is calibrated to observed trade flows and real exchange rates. Simulation of the model demonstrates that the effect of increasing exchange rate variability on trade flows is very small. These results are not sensitive to a wide range of parameter values. Moreover, reasonable extensions of the model only serve to minimize further the effect of exchange rate variability on trade flows.

EXCHANGE RATE VARIABILITY AND THE LEVEL OF INTERNATIONAL TRADE

Joseph E. Gagnon[1]

One of the most often researched topics in international economics is the effect of exchange rate uncertainty on international trade. A resolution of this issue has obvious implications for the choice of an international monetary system. For example, if large fluctuations in relative prices under the floating exchange rate regime have depressed the level of trade worldwide and reduced global welfare, there would be a strong case for a return to fixed exchange rates.[2]

Theoretical studies generally predict that increased uncertainty about exchange rates will lead to a lower level of trade under the assumption that traders are risk averse. For example, Clark [1973] demonstrates that exchange rate uncertainty is likely to have a negative effect on trade in a simple static model. Subsequent work has yielded similar conclusions using somewhat different models and assumptions, although risk aversion is a common theme. Farrell, DeRosa, and McCown [1983] present a review of the theoretical literature on the effects of exchange rate uncertainty.

---

1. I would like to thank Tam Bayoumi, Marc Dudey, Dale Henderson, Cathy Mann, and Andrew Rose for useful discussions. Seminar participants at the Board of Governors, the World Bank, and Georgetown University have also provided valuable comments. Special thanks to W. John Coleman for helping me to use his FORTRAN program to compute nonlinear decision rules. This paper represents the views of the author and should not be interpreted as reflecting those of the Board of Governors of the Federal Reserve System or other members of its staff.

2. Any argument for fixed exchange rates would have to address the issue of uncertainty associated with potential currency realignments. If monetary policies were credibly committed to avoiding realignments, there might be welfare losses due to the inability to use monetary policy for domestic stabilization.

Motivated by these theoretical results, numerous empirical researchers have tested for a significantly negative effect of exchange rate variability on trade. (Under reasonable assumptions, exchange rate variability is closely related to exchange rate uncertainty.) By and large, these researchers have not found evidence for the hypothesized effects of exchange rate variability. According to the International Monetary Fund [1984], "The large majority of empirical studies on the impact of exchange rate variability on the volume of international trade are unable to establish a systematically significant link between measured exchange rate variability and the volume of international trade." The conclusion of the IMF study is supported by a more recent survey of the results of empirical tests in the literature by Edison and Melvin [1988].

This paper extends previous theoretical research by constructing a dynamic optimizing model of risk averse traders that is characterized by adjustment costs and rational expectations. This class of models has been popularized by Lucas and Prescott [1971] and Hansen and Sargent [1980]. The model is analyzed both under the assumption of perfect competition and under the assumption of monopoly. Static models are seen as special cases with no adjustment costs. The connection between exchange rate variability and exchange rate uncertainty is modeled explicitly. Some generalizations about the effect of incorporating inventories and futures markets can be drawn.

The model is used to explore the theoretical implications for trade of an increase in exchange rate variability. The model replicates earlier theoretical findings that exchange rate variability tends to depress the level of trade. To make the analysis empirically more relevant, the paper considers the effect of an increase in exchange rate variability similar to that which accompanied the breakdown of the Bretton Woods regime in the early 1970s. For any plausible parameterization of the model the effect of the observed increase in exchange rate variability on trade flows is too small to be statistically detectable. Some intuition for this result is derived by studying the optimization problem graphically in a simple case.

I. A Theoretical Framework

In this paper, the trader is modeled as an international arbitrageur who buys goods from producers in one country and sells these goods to consumers in another country. The trader's objective is to maximize the infinite sum of discounted expected future period-by-period levels of utility. The trader is averse to risk; his period-by-period level of utility is a concave function of his period-by-period level of profits. The analysis considers the case of the trader-exporter who values his profits in the currency of the exporting country. The basic conclusions would be unaffected by assuming instead that profits are valued in the currency of the importing country.[3]

The trader faces a demand curve that is either downward-sloping or horizontal in terms of the importer's currency.[4] This demand curve is given by equation (1) in which P^* is the foreign currency price of the good and X is the volume of goods sold. The trader also faces a supply curve for

---

3. There are two offsetting effects associated with the analysis of a trader-importer. First, the trader-importer is subject to exchange risk only on his variable cost, whereas the trader-exporter is subject to exchange risk on his total revenue. Thus, a given amount of exchange rate variability should have a smaller effect on the trader-importer. On the other hand, the expected value of the exchange rate increases with its variance, thereby raising the trader-exporter's expected profits and lowering the trader-importer's expected profits. This latter effect tends to increase the impact of exchange rate variability on the trader-importer. Simulations of the model with a trader-importer demonstrated no significant differences from the simulations reported in this paper.

4. The results of this paper are not substantially altered by positing a nonlinear constant-elasticity demand curve instead of a simple linear demand curve.

the good that is either horizontal, upward-sloping, or downward-sloping.[5] The supply curve is given by equation (2) in which P is the domestic currency price of the good. The cost of transportation is assumed to be included in the supply curve. Because of contracting costs and marketing costs, the trader also faces a convex cost of adjusting the level of trade. For simplicity, the adjustment cost is modeled as quadratic in the period-by-period change in trade flow.[6] The adjustment cost is assumed to be split equally between the two currencies. The one-period profit, Π, of the trader is thus given by equation (3).

(1) $P_t^* = a - bX_t.$

(2) $P_t = c + dX_t.$

(3) $\Pi_t = R_t P_t^* X_t - P_t X_t - e\left(\frac{1 + R_t}{2}\right)\left(X_t - X_{t-1}\right)^2.$

R is the exchange rate, defined in terms of the exporter's currency. The prices and quantities of this model are all expressed in real terms, so the exchange rate is a real exchange rate. The parameters a, b, c, and e are all nonnegative. The slope parameter of the supply curve, d, may be positive, negative, or zero. In order to assure a well-behaved solution, it is assumed that $b > -d$.

5. In the case of increasing returns to scale production technology, a large trader might be able to negotiate a downward-sloping supply curve for the good. The case then becomes one of natural monopoly in the importing market.

6. The next section discusses the implications of assuming different functional forms of the adjustment cost.

Because of transportation lags, the trader chooses the quantity of goods to ship in the period before the settlement period, and he is financed by free trade credit for one period.[7] Assuming that the exchange rate is perceived to be exogenous, the trader's optimization problem is given by (4).

$$(4)\quad \underset{X_t}{\text{Max}}\ E_{t-1} \sum_{i=0}^{\infty} \theta^i U\left[\Pi_{t+i}\right].$$

θ is a constant real discount factor. U[ ] is the period-by-period utility function. E_s denotes the mathematical expectation conditional on information dated in period s or earlier.

Equation (5) is the Euler equation that represents the first order condition for (4). The transversality condition for (5) to be an optimal rule is that the sequence of one-period utilities does not grow at an exponential rate greater than $\theta^{-1/2}$.

$$(5)\quad E_{t-1}\left\{U'\left[\Pi_t\right]\left[aR_t - 2bX_tR_t - c - 2dX_t - \left(2eX_t - 2eX_{t-1}\right)\left(\frac{1 + R_t}{2}\right)\right]\right\}$$

$$= \theta E_{t-1}\left\{U'\left[\Pi_{t+1}\right]\left(2eX_t - 2eX_{t+1}\right)\left(\frac{1 + R_{t+1}}{2}\right)\right\}.$$

In order to analyze the effect of exchange rate variability on trade, one must first specify the stochastic process followed by the exchange rate.

---

7. The trader cannot set the price in advance because transportation lags preclude instantaneous adjustment of the volume supplied. If the trader were allowed to hold inventories in the destination market he could set both price and volume in advance. The implications of incorporating inventories into the model are discussed in the next section.

One also must specify the functional form of the utility function. These relations are given by equations (6) and (7).

(6) $\log(R_t) = \rho\log(R_{t-1}) + \epsilon_t, \quad \epsilon \text{ iid } N(0,\sigma^2).$

(7) $U[\Pi] = \frac{\exp(-\kappa\Pi)}{-\kappa}.$

The connection between exchange rate variability and exchange rate uncertainty can be described readily in the context of equation (6). Uncertainty about the next period's exchange rate (in logarithms) is indexed by its conditional variance σ^2. The variability of the exchange rate (in logarithms) is captured by its unconditional variance $\sigma^2/(1-\rho^2)$. Thus, persistence in the response of the exchange rate to shocks will increase the variability of the exchange rate without affecting its uncertainty. If there were no costs of adjustment in trade, only uncertainty would affect the trader's decision. With adjustment costs, both uncertainty and variability have an effect on trade flows.[8]

The utility function in equation (7) is characterized by a constant Arrow-Pratt risk aversion equal to κ for all levels of Π. (This is the class of constant absolute risk aversion, or CARA, utility functions.) In

8. The unconditionally expected value of R is $\exp[0.5\sigma^2/(1-\rho^2)]$. In the absence of any disturbances ($\sigma = 0$) the equilibrium value of the exchange rate is unity. In the case of uncertainty ($\sigma > 0$) the expected value of R is greater than unity. Thus, increasing the variance of the exchange rate process does not induce a mean-preserving spread in the distribution of exchange rates. At first glance this property seems unappealing for the study of the effect of uncertainty on trade. In fact, it is impossible in any model of exchange rates to design a mean-preserving spread in the distribution of the exchange rate that is invariant to the choice of the numeraire currency. The exchange rate process embodied in equation (6) does have the attractive characteristic that it is invariant to the choice of which currency is the numeraire.

studies of utility maximization in individuals, it is standard to assume that risk aversion decreases proportionally with increases in consumption. (This is the class of constant relative risk aversion, or CRRA, utility functions.) However, the CRRA utility function is characterized by a lower bound of permissible consumption, typically zero, below which utility is undefined. While a lower bound on consumption is reasonable for individuals, a lower bound on firm profits is less attractive. The CARA utility function given in (7) is more appealing for the study of risk averse firms because it allows for the possibility of losses. Nevertheless, exploratory simulations of the model with both CRRA utility and quadratic utility do not contradict the conclusions of this paper.

By studying equations (1)-(3) and equations (5)-(7) it is clear that the trader's choice of X_t depends on the state variables X_{t-1} and R_{t-1} as well as all of the parameters: a, b, c, d, e, θ, κ, ρ, and σ. Unfortunately, there does not exist an analytic solution for X_t in terms of information available in period t-1. It is possible, though, to obtain numerical decision rules for X_t in terms of X_{t-1} and R_{t-1} given particular values of the parameters. These numerical decision rules can be used to ascertain the effect of increasing uncertainty and variability of the exchange rate on the average volume of trade.

Before proceeding to the next section it is useful to establish the concept of the "deterministic steady state" of equations (5)-(7). The deterministic steady state is the combination of X_t and R_t that eventually would prevail in the absence of uncertainty ($\sigma = 0$). These values are designated X_{ss} and R_{ss}. By inspection of (6) it is clear that if $\rho < 1$, $R_{ss} = 1$. Given a perfectly foreseeable and constant exchange rate, the deterministic steady state for X is solved by substituting R_{ss} for R_t and

R_{t+1} in equation (5) and by setting $X_{t-1} = X_t = X_{t+1} = X_{ss}$. Since $U'[\Pi_t]$ now equals $U'[\Pi_{t+1}]$ it is easy to solve for X_{ss}:

$$(8) \quad X_{ss} = \left(\frac{a - c}{2(b + d)}\right) R_{ss}, \quad \text{where} \quad R_{ss} = 1.$$

II. Model Simulation

The numerical results of this paper are calibrated to quarterly models of trade and exchange rates. The assumption of a one-period transportation lag seems reasonable for overseas trade on a quarterly basis.[9] The discount rate is assumed to be 2 percent per quarter (θ=0.98) or roughly 8 percent per annum. To motivate the analysis of an increase in exchange rate variability, special attention is paid to the transition from the Bretton Woods regime to the floating exchange rate regime.

The parameters of the exchange rate process were calibrated using real exchange rates between the United States and five of the next six largest industrial countries taken individually.[10] For example, the U.S. GNP deflator is multiplied by the mark-dollar exchange rate and divided by the German GNP deflator. The logarithm of this real exchange rate is then demeaned and regressed on its own lagged value. The results of these

9. Magee [1974, pp. 132-133] calculates that the median lag between the decision to export a good and its receipt by a U.S. purchaser is 84 days for German goods and 116 days for Japanese goods.

10. The U.S.-Canadian real exchange rate was not included because Canada did not maintain a fixed exchange rate during the Bretton Woods period. Also, the Canadian dollar has been much less volatile than other currencies relative to the U.S. dollar during the floating rate period. Finally, U.S.-Canadian trade is characterized by a large amount of border trade and commodity trade, for which the differentiated product and costly adjustment model of this paper is less appropriate.

regressions are presented in Table 1 for the Bretton Woods period (1960-71) and the floating exchange rate period (1972-88).

The average value of $\hat{\rho}$ in Table 1 is .962 during 1960-71 and .956 during 1972-88. Most of the analysis in this paper will consider the case in which ρ = .96. Because a unit root in the exchange rate process cannot be rejected by standard tests, and because of the extreme effect a unit root would have on exchange rate variability, some attention also is devoted to the case in which $\rho = 1$.

The average value of $\hat{\sigma}$ is .016 during 1960-71 and .052 during 1972-88. In the analysis that follows, the case of low uncertainty assumes that σ = .02, the case of moderate uncertainty assumes that σ = .05, and the case of high uncertainty assumes that σ = .08. Thus, one may associate the case of low uncertainty with the experience of the Bretton Woods regime, the case of moderate uncertainty with recent experience, and the case of high uncertainty with a hypothetical increase in exchange rate variability.[11] To put these standard deviations in perspective, the case of moderate uncertainty implies a mean quarterly forecast error of 5 percent. When ρ = .96 and σ = .05, Monte Carlo simulations of the real exchange rate typically generate large and persistent cycles that deviate 20-25 percent from the deterministic steady state. These randomly generated paths look very much like the observed behavior of real exchange rates since 1972.

As a guide to calibration of the remaining parameters, Table 2 presents the estimated coefficients of a reduced form regression of quarterly bilateral real trade flows on one lag each of trade and the real

11. The case of high uncertainty also may be appropriate for real exchange rates with developing countries that have high and variable inflation rates.

Table 1

Estimated Real Exchange Rate Parameters

Exchange Rate	1960-1971 $\hat{\rho}$ (s.e.)	1960-1971 $\hat{\sigma}$	1972-1988 $\hat{\rho}$ (s.e.)	1972-1988 $\hat{\sigma}$
U.S./France	0.95 (0.05)	.015	0.95 (0.04)	.052
U.S./Germany	1.01 (0.05)	.019	0.95 (0.04)	.055
U.S./Italy	0.95 (0.03)	.010	0.96 (0.04)	.046
U.S./Japan	0.96 (0.03)	.015	0.97 (0.04)	.055
U.S./U.K.	0.94 (0.05)	.020	0.95 (0.04)	.054

Note: The quarterly average exchange rates were obtained from national sources. The GNP deflators were obtained from the OECD's *Quarterly National Accounts*.

Table 2

Estimated Real Trade Flow Parameters, 1972-87

Exporter/Importer	Coefficient Estimate and (Standard Error) on X_{t-1}	R_{t-1}	Long-Run Exchange Rate Elasticity
U.S./France	0.45 (0.10)	-0.26 (0.07)	-0.47
U.S./Germany	0.39 (0.10)	-0.23 (0.06)	-0.38
U.S./Italy	0.59 (0.10)	-0.28 (0.10)	-0.68
U.S./Japan	0.63 (0.09)	-0.09 (0.06)	-0.24
U.S./U.K.	0.63 (0.08)	-0.26 (0.09)	-0.70
France/U.S.	0.33 (0.11)	0.45 (0.09)	0.67
Germany/U.S.	0.55 (0.09)	0.35 (0.08)	0.78
Italy/U.S.	0.61 (0.09)	0.48 (0.12)	1.23
Japan/U.S.	0.76 (0.10)	0.25 (0.10)	1.04
U.K./U.S.	0.70 (0.10)	0.05 (0.08)	0.17

Note: The real exchange rate data are the same as in Table 1. The real trade flows are obtained by dividing nominal bilateral exports by each country's aggregate export deflator. The bilateral trade data are from the IMF's *Direction of Trade Statistics*. Real aggregate demands and export deflators are from the OECD's *Quarterly National Accounts*.

exchange rate.[12] The regression also included a constant and one lag of aggregate demand in the importing country, but these coefficients are not reported. Because of the existence of long-run growth in the aggregate trade data, the data were expressed in logarithms. If the model of the previous section is correct the regression of Table 2 is misspecified, but it may be considered a log-linear approximation to the true reduced-form decision rule. In Table 2 the average coefficient on lagged trade is 0.57 and the average long-run elasticity of trade to the exchange rate is 0.64. It is important to recognize that the long-run elasticities reported in Table 2 are neither the demand nor the supply elasticities faced by the trader. Rather, they are reduced-form statistics that reflect the interaction of the trader's behavior with conditions in both the purchase and the sales market.

The lagged adjustment of trade is primarily determined by the parameter e. Higher values of e slow the process of trade adjustment to shocks. In the basic model, the value of e is set to yield a lag coefficient of about 0.57 in the simulated approximate decision rule. This parameter has no effect on the volume of trade in deterministic steady state, as can be seen from equation (8).

The parameters a, b, c, and d jointly determine such properties as the responsiveness of trade to exchange rate movements and the average ratio of variable cost to total revenue. Of these four parameters, there are actually only three degrees of freedom in calibration because the choice of

12. The theoretical model of this paper is one of partial equilibrium in which traders do not perceive their actions to affect the exchange rate. The use of aggregate data for calibration raises the issue of general equilibrium feedbacks from trade flows to the real exchange rate. Gagnon [1989] finds no evidence for Granger-causality of bilateral trade flows on real exchange rates over this sample, so the use of aggregate data is not likely to bias the reduced-form estimates.

units of trade volume is arbitrary. In the following simulations the parameter a is always chosen to yield $X_{ss} = 10$. In the basic model the remaining parameters were chosen to yield a long-run elasticity of trade to the exchange rate of close to 0.64 in the simulated decision rule. Various combinations of b, c, and d were consistent with this behavior, but the simulated effects of exchange rate variability were not sensitive to the particular combination employed.

Finally, the parameter κ is the trader's level of absolute risk aversion. In the simulations that follow, κ is chosen such that the trader is indifferent between a standard deviation in profit equal to 10 percent of steady state revenue, and a certain reduction in profit equal to 2 percent of steady state revenue. It follows from the functional form of utility that in place of a 5 percent of revenue standard deviation in profit the trader would accept a 0.5 percent of revenue reduction in profit. And he would accept an 8 percent of revenue reduction in profit rather than face a 20 percent of revenue standard deviation in profit. These indifference tradeoffs represent a fairly high degree of risk aversion for a firm.

Recall that for the trader-exporter, total revenues are scaled up or down by the exchange rate. Thus, a 10 percent movement in the exchange rate will change the level of profit by 10 percent of total revenue. In the period after an exchange rate shock, the trader can adjust the volume of trade to restore a fraction of his lost profit, but this adjustment carries its own cost. When trade is very costly to adjust, the trader will respond very slowly to exchange rate movements and he will be exposed to risk not just from uncertainty about next period's exchange rate, but also from uncertainty about exchange rates far into the future. The unconditional standard deviation of the exchange rate is one measure of uncertainty about exchange rates in the distant future. In the case of moderate uncertainty, the unconditional standard deviation of the exchange rate is approximately 18 percent. The unconditional standard deviation of the exchange rate is approximately 29 percent in the case of high uncertainty. Thus, in the cases of moderate and high uncertainty, the trader perceives a very substantial exposure to risk when adjustment is costly.

Table 3 presents the decision rules for X_t in the basic model as functions of the state variables X_{t-1} and R_{t-1} given the three different cases of 1) low uncertainty; 2) moderate uncertainty; and 3) high uncertainty.[13] In each case the parameters a, b, c, d, e, κ, and ρ are fixed at the values shown at the top of the table.[14] Since ρ is fixed, exchange rate uncertainty is proportional to exchange rate variability. The entries in the decision rule matrices represent the optimal choice of X_t when the trader observes the values of X_{t-1} and R_{t-1} associated with the particular row and column. If exchange rate variability depresses the level of trade, then the entries of the decision rule matrix ought to decline as uncertainty increases. In Table 3 these entries never change by more than 1 percent as one moves from low to high uncertainty.

Another way to assess the effect of increasing exchange rate variability on trade is to draw random shocks to the exchange rate process and to generate artificial time series for R_t and X_t. This Monte Carlo approach allows one to use standard statistical techniques to analyze the model's behavior. In a Monte Carlo analysis it is also possible to determine under what conditions the effect of exchange rate variability on trade is statistically detectable.

---

13. For a description of the algorithm used to compute the numerical decision rules, see Coleman [1989].

14. The demand elasticity of trade implied by these parameters is 3 at the deterministic steady state. The supply elasticity is 3.5.

Table 3. Decision Rules for the Basic Model

Parameters: a = 18 b = .45 c = 5 d = .2 e = 2 κ = .0309 ρ = .96

Case 1: Low Uncertainty (σ = .02)

X_{t-1} \ R_{t-1}	0.85	0.90	0.95	1.00	1.05	1.10	1.15
8.5	8.68	8.84	8.99	9.13	9.26	9.38	9.50
9.0	8.97	9.13	9.28	9.42	9.55	9.67	9.79
9.5	9.26	9.42	9.57	9.71	9.84	9.96	10.08
10.0	9.56	9.71	9.86	10.00	10.13	10.25	10.37
10.5	9.85	10.00	10.15	10.28	10.41	10.53	10.65
11.0	10.15	10.30	10.44	10.57	10.70	10.82	10.93
11.5	10.45	10.60	10.73	10.86	10.99	11.10	11.22

Case 2: Moderate Uncertainty (σ = .05)

X_{t-1} \ R_{t-1}	0.85	0.90	0.95	1.00	1.05	1.10	1.15
8.5	8.66	8.82	8.97	9.11	9.24	9.36	9.47
9.0	8.95	9.11	9.26	9.40	9.53	9.65	9.77
9.5	9.24	9.40	9.55	9.68	9.82	9.94	10.06
10.0	9.54	9.69	9.83	9.97	10.10	10.22	10.34
10.5	9.83	9 98	10.12	10.26	10.39	10.51	10.62
11.0	10.13	10.28	10.41	10.55	10.67	10.79	10.90
11.5	10.43	10.58	10.71	10.84	10.96	11.08	11.19

Case 3: High Uncertainty (σ = .08)

X_{t-1} \ R_{t-1}	0.85	0.90	0.95	1.00	1.05	1.10	1.15
8.5	8.63	8.78	8.93	9.07	9.20	9.32	9.43
9.0	8.92	9.07	9.22	9.36	9.49	9.61	9.72
9.5	9.21	9.36	9.51	9.64	9.77	9.89	10.01
10.0	9.50	9.65	9.79	9.93	10.06	10.18	10.29
10.5	9.80	9 94	10.08	10.21	10.34	10.46	10.57
11.0	10.10	10.24	10.37	10.50	10.62	10.74	10.85
11.5	10.40	10.54	10.67	10.79	10.91	11.02	11.13

For the basic model of Table 3, 10,000 observations of R_t and X_t were drawn for each of the three levels of uncertainty. The mean values of trade computed from the Monte Carlo simulations are presented in Table 4. The standard deviation of trade about its mean is also presented for each of the three cases. According to Table 4, the mean level of trade drops by about 1 percent going from the case of low uncertainty to the case of moderate uncertainty. The mean level of trade drops by another 1 percent going from the case of moderate uncertainty to the case of high uncertainty.

Not only is the effect of increasing exchange rate uncertainty on the level of trade quite small, but the effect would not be statistically significant in the sample sizes typically available to researchers. For example, with 25 years of quarterly data there are 100 observations on the level of trade. The standard deviation of the sample mean with 100 observations is equal to the standard deviation of the individual observations divided by the square root of 100. In Table 4, the standard deviation of the mean of a sample of size 100 would be .04 in the case of low uncertainty, .10 in the case of moderate uncertainty, and .16 in the case of high uncertainty. Thus, the mean level of trade in the case of low uncertainty is well within two standard deviations of the expected value of the mean under moderate or high uncertainty, assuming a sample of 100 observations. Moreover, one would have to control for the effects of long-run growth, tariff changes, and cyclical fluctuations, to name but a few factors that complicate the analysis. These extraneous factors probably serve to reduce the precision of empirical estimates of the effect of exchange rate uncertainty, making it all the more unlikely to find a statistically significant effect.

Table 4 also presents the mean values of the coefficients of reduced-form regressions of trade on lagged trade and the lagged exchange rate (in

Table 4. Monte Carlo Results for the Basic Model

Case 1: **Low Uncertainty (σ = .02)**

Mean Level of Trade: 9.96

Standard Deviation: 0.44

Mean Estimated Reduced Form:

$$X_t = \underset{\substack{(0.16)\\ [0.15]}}{0.99} + \underset{\substack{(0.07)\\ [0.06]}}{0.57}X_{t-1} + \underset{\substack{(0.04)\\ [0.04]}}{0.25}R_{t-1}$$

Implied Long-Run Elasticity: 0.58

Case 2: **Moderate Uncertainty (σ = .05)**

Mean Level of Trade: 9.83

Standard Deviation: 1.02

Mean Estimated Reduced Form:

$$X_t = \underset{\substack{(0.15)\\ [0.17]}}{0.95} + \underset{\substack{(0.07)\\ [0.08]}}{0.59}X_{t-1} + \underset{\substack{(0.04)\\ [0.04]}}{0.24}R_{t-1}$$

Implied Long-Run Elasticity: 0.59

Case 3: **High Uncertainty (σ = .08)**

Mean Level of Trade: 9.75

Standard Deviation: 1.65

Mean Estimated Reduced Form:

$$X_t = \underset{\substack{(0.15)\\ [0.18]}}{0.87} + \underset{\substack{(0.07)\\ [0.08]}}{0.62}X_{t-1} + \underset{\substack{(0.04)\\ [0.06]}}{0.22}R_{t-1}$$

Implied Long-Run Elasticity: 0.58

logarithms). These regressions were conducted on 100 samples of 100 observations each, created by partitioning the Monte Carlo time series. Below each coefficient in parentheses is the mean of the standard errors of the estimated coefficients. In brackets is the standard deviation of the estimated coefficients about their mean. If the reduced-form regression were correctly specified, the statistics in parentheses and the statistics in brackets would have the same expected value. However, the reduced form is not correct because the true model is nonlinear. The standard errors in parentheses represent what empirical researchers are likely to observe and to use in conducting statistical inference about trade. The standard errors in brackets represent the extent to which the estimated reduced form can change across samples of size 100. In Table 4 both estimates of the coefficient standard errors are quite similar.

There is some evidence in Table 4 that increasing exchange rate uncertainty does affect the coefficient estimates of a simple trade regression. In particular, the constant term declines and the lag coefficient increases. However, the changes in the estimated coefficients are clearly not statistically significant in samples of 100 observations.

In order to explore the robustness of the results for the basic model, decision rules were computed over a wide range of the parameters b, c, d, and e. For each combination of parameter values the effect of increasing variability in the exchange rate was explored. In all cases the qualitative results are the same: Increasing exchange rate variability tends to reduce the equilibrium level of trade. However, different parameter values do have different implications for the quantitative effects of uncertainty. Increases in the parameters b and d tend to decrease the elasticity of trade with respect to the exchange rate, and they tend to reduce the effect of uncertainty on trade.[15] Increases in the parameter c tend to increase the exchange rate elasticity of trade, and they tend to increase the effect of uncertainty on trade. Increases in the parameter e tend to reduce the adjustment speed of trade, and they tend to increase the effect of uncertainty on trade.

Table 5 presents decision rules for trade under an extreme combination of b, c, d, and e designed to yield the maximum plausible effect of uncertainty on trade.[16] The greatest change in any of the elements of the decision rule matrix between the case of low uncertainty and that of high uncertainty is 1.2 percent. Table 6 presents Monte Carlo results for this extreme model.[17] Note that the coefficient on X_{t-1} in the mean estimated reduced form is greater than any of those estimated in Table 2. Also, the approximate long-run elasticity of trade with respect to the exchange rate is at the high end of the estimates in Table 2. Despite these extreme assumptions, the effect of increasing exchange rate variability from $\sigma = .02$ to $\sigma = .05$ is to lower the equilibrium level of trade by only 3.4 percent.

15. The case of perfect competition is modeled by setting $b = 0$. As long as the supply curve is upward-sloping ($d > 0$) there exists a stationary decision rule for trade when $b = 0$. The effects of uncertainty on trade in this case are not significantly different from the case of imperfect competition.

16. The demand elasticity implied by these parameters is 5 at the deterministic steady state. The supply elasticity is 15.

17. In the case of high uncertainty, the Monte Carlo draws yielded 17 observations on trade that were negative. These negative values were included in the calculation of the sample mean and standard deviation. These observations were excluded from the reduced-form regression in logarithms.

A negative value of trade implies that the demand curve for the good extends through the vertical axis to become a supply curve at very high prices. In addition, the supply curve extends through the vertical axis to become a demand curve at very low prices. An alternative assumption would have been to place a lower bound of zero on both supply and demand. The results are not substantially affected by such an assumption.

Table 5. Decision Rules for the Extreme Model

Parameters: a = 12 b = .2 c = 7 d = .05 e = 20 κ = .0417 ρ = .96

<u>Case 1</u>: No Uncertainty (σ = 0)

X_{t-1} \ R_{t-1}	0.85	0.90	0.95	1.00	1.05	1.10	1.15
8.5	8.45	8.52	8.58	8.64	8.69	8.75	8.80
9.0	8.91	8.97	9.03	9.09	9.15	9.20	9.25
9.5	9.36	9.42	9.48	9.54	9.60	9.65	9.71
10.0	9.82	9.88	9.94	9.99	10.05	10.10	10.16
10.5	10.28	10.34	10.39	10.45	10.50	10.55	10.61
11.0	10.74	10.79	10.85	10.90	10.95	11.00	11.05
11.5	11.20	11.25	11.30	11.35	11.40	11.45	11.50

<u>Case 2</u>: Moderate Uncertainty (σ = .05)

X_{t-1} \ R_{t-1}	0.85	0.90	0.95	1.00	1.05	1.10	1.15
8.5	8.43	8.49	8.55	8.61	8.66	8.72	8.77
9.0	8.88	8.94	9.00	9.06	9.12	9.17	9.22
9.5	9.34	9.40	9.45	9.51	9.57	9.62	9.67
10.0	9.80	9.85	9.91	9.96	10.01	10.07	10.12
10.5	10.26	10 31	10.36	10.41	10.46	10.51	10.56
11.0	10.72	10.76	10.81	10.86	10.91	10.96	11.01
11.5	11.18	11.22	11.27	11.31	11.36	11.41	11.45

<u>Case 3</u>: High Uncertainty (σ = .08)

X_{t-1} \ R_{t-1}	0.85	0.90	0.95	1.00	1.05	1.10	1.15
8.5	8.38	8.44	8.50	8.56	8.61	8.66	8.71
9.0	8.84	8.89	8.95	9.00	9.06	9.11	9.16
9.5	9.29	9.35	9.40	9.45	9.50	9.55	9.60
10.0	9.75	9.80	9.85	9.90	9.95	10.00	10.05
10.5	10.21	10 26	10.30	10.35	10.40	10.44	10.49
11.0	10.67	10.72	10.76	10.80	10.84	10.89	10.93
11.5	11.14	11.17	11.21	11.25	11.29	11.33	11.37

Table 6. Monte Carlo Results for the Extreme Model

Case 1: Low Uncertainty (σ = .02)

Mean Level of Trade: 9.86

Standard Deviation: 0.70

Mean Estimated Reduced Form:

$$X_t = \underset{\substack{(0.02) \\ [0.02]}}{0.23} + \underset{\substack{(0.01) \\ [0.01]}}{0.90X_{t-1}} + \underset{\substack{(0.01) \\ [0.01]}}{0.11R_{t-1}}$$

Implied Long-Run Elasticity: 1.07

Case 2: Moderate Uncertainty (σ = .05)

Mean Level of Trade: 9.52

Standard Deviation: 1.70

Mean Estimated Reduced Form:

$$X_t = \underset{\substack{(0.02) \\ [0.03]}}{0.23} + \underset{\substack{(0.01) \\ [0.01]}}{0.90X_{t-1}} + \underset{\substack{(0.01) \\ [0.02]}}{0.11R_{t-1}}$$

Implied Long-Run Elasticity: 1.10

Case 3: High Uncertainty (σ = .08)

Mean Level of Trade: 8.63

Standard Deviation: 2.45

Mean Estimated Reduced Form:

$$X_t = \underset{\substack{(0.02) \\ [0.07]}}{0.23} + \underset{\substack{(0.01) \\ [0.02]}}{0.90X_{t-1}} + \underset{\substack{(0.01) \\ [0.12]}}{0.13R_{t-1}}$$

Implied Long-Run Elasticity: 1.25

In a sample of 100 observations with $\sigma = .05$, it is likely that one could not reject at the 5 percent level that the expected value of trade is the same as when $\sigma = .02$. Moving from $\sigma = .05$ to $\sigma = .08$ further reduces the volume of trade by 9.3 percent. In this latter case, one would be likely to reject that the mean level of trade had not changed. However, the case of high uncertainty represents a hypothetical regime beyond the range of what researchers have observed in previous studies of the effects of exchange rate variability.

The mean estimated reduced forms are remarkably similar across the different degrees of uncertainty. There is some evidence that a researcher might believe that his coefficient estimates were more precise than they really are. The bracketed standard errors indicate that the coefficients change more across samples than the within-sample standard errors would predict. This discrepancy is especially true for the case of high uncertainty. Nevertheless, even the lower standard errors do not reject constant coefficients across the different degrees of uncertainty.

In order to check the sensitivity of these conclusions to some maintained assumptions about the model, decision rule tables were computed for various alternative assumptions. The first alternative considers the possibility that the exchange rate may follow a random walk. Setting $\rho = 1$ increases the effective exchange rate variability faced by the trader without increasing his uncertainty about next period's exchange rate. While the unconditional variance of the exchange rate is infinite in this case, the model is still well-behaved as long as the cost of trade adjustment is finite and the discount factor is strictly less than one. In Table 5, Case 2 the effect of changing $\rho = .96$ to $\rho = 1$ is to reduce the level of trade in all states of the decision matrix, but the reduction is always much less than 1 percent of steady-state trade. Thus, it does not appear that allowing for a random walk in the exchange rate would significantly alter the conclusions of this section.

As mentioned in the previous section, the model's behavior is not substantially altered by considering the case of a constant-elasticity demand curve. Use of a constant-elasticity demand curve tends to somewhat reduce the effect of uncertainty on the level of trade, but this effect is rather small. Neither is the model sensitive to alternative utility functions, though some care must be taken to calibrate the degree of risk aversion appropriately. The degree of risk aversion employed in this paper appears to be quite high, but even higher degrees of risk aversion would yield larger effects of exchange rate uncertainty on trade.

One assumption that does have interesting implications for the behavior of trade is that of quadratic adjustment costs. Some trial simulations of the model were run under the assumption of linear adjustment costs (in the absolute value of the change in trade) and under the assumption of quartic adjustment costs. The effect of exchange rate uncertainty on the mean level of trade is quite insensitive to these alternative specifications of the adjustment costs. However, as exchange rate uncertainty increases, the speed of adjustment to exchange rate shocks changes notably and the decision matrices become more nonlinear when adjustment costs are not quadratic.[18]

---

18. In the basic model with quadratic adjustment costs, increasing exchange rate uncertainty has little effect on the speed of adjustment of trade as measured by the estimated lag coefficient. In the model with linear adjustment costs, increasing exchange rate uncertainty greatly increases the estimated lag coefficient in trade. In the model with quartic adjustment costs, increasing exchange rate uncertainty somewhat reduces the estimated lag coefficient in trade. These effects are most pronounced in the neighborhood of the deterministic steady state.

These results may not be surprising in light of the recent interest in models of "hysteresis" in trade. Hysteresis is essentially an extreme and

(Footnote continues on next page)

Before concluding this section it is of interest to note that both of two realistic extensions to the model would serve to reduce the effect of uncertainty on trade. First, consider the possibility that the trader might hold inventories in the country where he faces exchange risk. In the case of the trader-exporter, an adverse terms-of-trade shock would present the trader with the choice of selling all his exports at the new lower price (as in the basic model) or holding part of the shipment in a warehouse to await more favorable conditions. If demand is downward-sloping the latter course of action also brings about a higher sales price for the part of the shipment that is sold immediately. Thus, inventories allow the trader to reoptimize his sales decision after he observes the exchange rate. The trader's risk exposure is unambiguously reduced and the effect of exchange rate variability on the level of trade is diminished.

The second extension is the inclusion of futures markets in foreign exchange. Once again, the ability to lock in a future exchange rate provides an opportunity for the trader to reduce the risk he faces. The existence of a currency futures market unambiguously increases the decision-making scope of the trader. Currency futures do not completely eliminate the effect of exchange rate risk, however. In practice, futures markets typically offer contracts for relatively short horizons, so part of the

---

(Footnote continued from previous page)
highly nonlinear form of persistence. Baldwin [1988], Dixit [1989], and Krugman [1989] all generate hysteresis in their models of trade by assuming that any nonzero amount of adjustment is equally costly. Linear adjustment costs are closer to the adjustment costs of hysteresis models than are quadratic adjustment costs, which are nearly zero for small adjustments. Both Dixit and Krugman argue that increases in exchange rate variability tend to increase the sluggishness of trade adjustment. This model supports their claim if one believes that adjustment costs are less convex than the standard quadratic specification.

trader's exchange risk cannot be covered. Also, if there are other sources of uncertainty such as aggregate demand and consumer tastes, then the trader cannot know exactly how much future currency to buy. Finally, there are costs of managing a futures portfolio, including possible risk premia required to induce someone to sell foreign exchange at a guaranteed future price.

III. A Stylized Example

In order to gain further insight into the results of the previous section, this section presents a simplified version of the trade model in which it is easier to isolate the effect of uncertainty on the average level of trade. Consider the case of the trader-exporter with no costs of adjustment. His profit is given by equation (9). The period-by-period utility function is identical to that of the previous section and is repeated in equation (10). Finally, the exchange rate is assumed to follow a simple binomial distribution given by equation (11), in which σ represents the standard deviation of the exchange rate.

(9) $\Pi_t = aX_tR_t - bX_t^2R_t - cX_t - dX_t^2.$

(10) $U[\Pi] = \frac{\exp(-\kappa\Pi)}{-\kappa}.$

(11) $P\left[R_t = 1 + \sigma\right] = 0.5$ and $P\left[R_t = 1 - \sigma\right] = 0.5.$

Once again the trader is assumed to choose X_t before he observes R_t. Since there are no adjustment costs and there is no persistence in the exchange rate, the trader's expected utility maximization problem is particularly simple. Total expected utility is given by equation (12).

$$(12)\quad E_{t-1}U\left[\Pi_t\right] = 0.5U\left[\left(aX_t - bX_t^2\right)(1+\sigma) - cX_t - dX_t^2\right]$$

$$+ 0.5U\left[\left(aX_t - bX_t^2\right)(1-\sigma) - cX_t - dX_t^2\right].$$

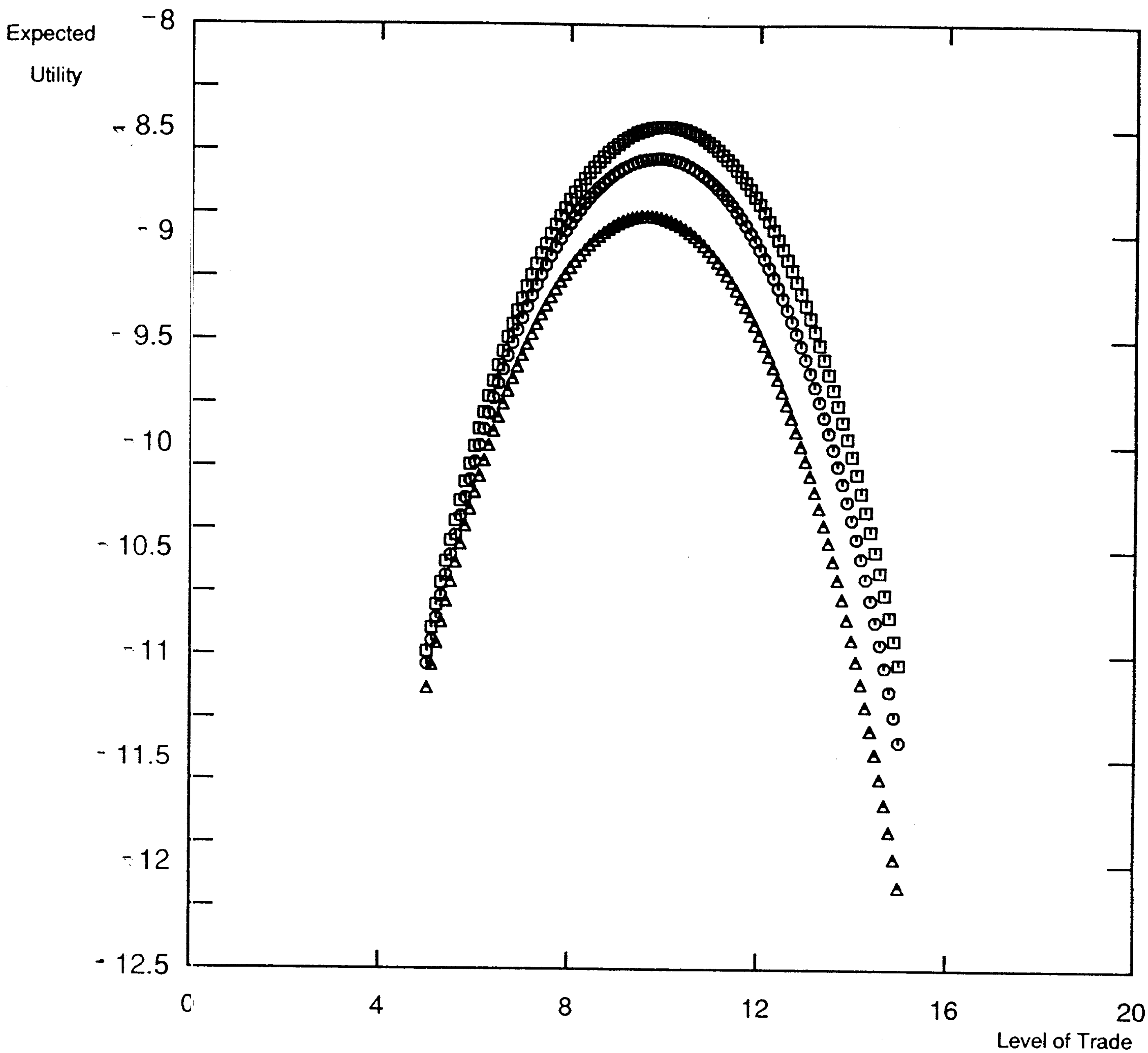
Figure 1 plots the trader's expected utility as a function of his choice of X_t given three different values of the standard deviation of the exchange rate. The values of a, b, c, d, and κ are taken from Table 5. For each value of σ, the trader's optimal decision is to choose the level of trade that yields the highest expected utility. While increasing variability of the exchange rate tends to lower expected utility, it does not change the optimal choice of trade by very much. When $\sigma = .02$ the optimal level of X_t is 9.95. When $\sigma = .05$ the optimal level of X_t is 9.85, and when $\sigma = .08$ the optimal level of X_t is 9.60.

To put the analysis more concretely, when $R_t = 1.08$ and $X_t = 9.60$ the trader's profit is 32.7 on revenues of 104.5. When $R_t = 0.92$ and $X_t = 9.60$ the trader's profit is 17.2 on revenues of 89.0. The trader is clearly facing a lot of risk, and this risk reduces his expected utility. When $\sigma = .08$ and $X_t = 9.60$ the trader's (optimal) expected utility is equal to the (nonoptimal) expected utility associated with a choice of $X_t = 7.80$ when $\sigma = .02$. Although the trader is not happy with an increase in exchange rate uncertainty, it appears that his optimal trade decision is not much affected.

IV. Conclusion

This paper develops a theoretical model of trade under uncertainty that incorporates dynamics explicitly in a framework of intertemporal optimization. This model is used to assess the effect of uncertainty on trade flows when traders are risk averse. As in earlier work, uncertainty

Figure 1



Legend: Standard Deviation of R

- □ = .02
- ⊙ = .05
- △ = .08

about the real exchange rate serves to depress the volume of trade.

Unlike earlier research, this paper attempts to gauge the magnitude of these effects given various plausible parameterizations of the model. Particular attention is devoted to an increase in real exchange rate variability of the magnitude that ocurred after the breakdown of the Bretton Woods system in the early 1970s. The basic version of the model estimates that the switch to floating rates may have reduced the volume of trade by 1 percent. Under a very extreme combination of assumptions, the breakdown of Bretton Woods is estimated to reduce the level of trade by about 3 percent. This effect is shown to be too small to detect statistically. A further increase in exchange rate variability would lower the volume of trade by a statistically significant 9 percent, but this latter scenario requires a degree of exchange rate variability much larger than has been observed historically.

Given the magnitude of global trade, these effects are economically significant, even if they are not statistically significant. However, there are many reasons for believing that these estimates are overstated. Allowing for inventories of traded goods and futures markets in currencies would unambiguously reduce the impact of exchange rate uncertainty on the trader's behavior. Moreover, the degree of risk aversion posited in this model is almost certainly too large, especially if the trading firm has access to capital markets and is owned by investors with diversified portfolios. Realistic extensions of the model in these directions would probably imply a negligible effect of exchange rate variability on trade.

REFERENCES

Baldwin, R., 1988, "Hysteresis in Import Prices: The Beachhead Effect," *American Economic Review* 78, 773-785.

Clark, P.B., 1973, "Uncertainty, Exchange Risk, and the Level of International Trade," *Western Economic Journal* 11, 302-313.

Coleman, W.J., 1989, "An Algorithm to Solve Dynamic Models," *International Finance Discussion Papers* No. 351, Board of Governors of the Federal Reserve System.

Dixit, A., 1989, "Hysteresis, Import Penetration, and Exchange Rate Pass-Through," *The Quarterly Journal of Economics* 104, 205-228.

Edison, H.J., and M. Melvin, 1988, "The Determinants and Implications of the Choice of an Exchange Rate System," unpublished paper.

Farrell, V.S., D.A. DeRosa, and T.A. McCown, 1983, "Effects of Exchange Rate Variability on International Trade and other Economic Variables: A Review of the Literature," *Staff Studies* 130, Board of Governors of the Federal Reserve System.

Gagnon, J.E., 1989, "Adjustment Costs and International Trade Dynamics," *Journal of International Economics* 26, 327-344.

Giovannini, A., 1988, "Exchange Rates and Traded Goods Prices," *Journal of International Economics* 24, 45-68.

Hansen, L.P., and T.J. Sargent, 1980, "Formulating and Estimating Dynamic Linear Rational Expectations Models," *Journal of Economic Dynamics and Control* 2, 7-46.

Hey, J.D., 1979, *Uncertainty in Microeconomics* (New York University Press, New York).

International Monetary Fund, 1984, "Exchange Rate Volatility and World Trade," Occasional Paper No. 28.

Krugman, P., 1989, *Exchange-Rate Instability* (The MIT Press, Cambridge).

Leland, H.E., 1972, "Theory of the Firm Facing Uncertain Demand," *American Economic Review* 62, 278-291.

Lucas, R.E., and E.C. Prescott, 1971, "Investment under Uncertainty," *Econometrica* 39, 659-681.

Magee, S.P., 1974, "U.S. Import Prices in the Currency-Contracts Period," *Brookings Papers on Economic Activity* 1, 117-168.

International Finance Discussion Papers

IFDP NUMBER	TITLES 1989	AUTHOR(s)
369	Exchange Rate Variability and the Level of International Trade	Joseph E. Gagnon
368	A Substitute for the Capital Stock Variable in Investment Functions	Guy V.G. Stevens
367	An Empirical Assessment of Non-Linearities In Models of Exchange Rate Determination	Richard A. Meese Andrew K. Rose
366	Equilibrium in a Production Economy with an Income Tax	Wilbur John Coleman II
365	Tariffs and the Macroeconomy: Evidence from the USA	Andrew K. Rose Jonathan D. Ostry
364	European Integration, Exchange Rate Management, and Monetary Reform: A Review of the Major Issues	Garry J. Schinasi
363	Savings Rates and Output Variability in Industrial Countries	Garry J. Schinasi Joseph E. Gagnon
362	Determinants of Japanese Direct Investment in U.S. Manufacturing Industries	Catherine L. Mann
361	The U.S. and U.K. Activities of Japanese Banks: 1980-1988	Henry S. Terrell Robert S. Dohner Barbara R. Lowrey
360	Policy Rules, Information, and Fiscal Effects in a "Ricardian" Model	Eric M. Leeper
359	A Forward-Looking Multicountry Model: MX3	Joseph E. Gagnon
358	Implications for Future U.S. Net Investment Payments of Growing U.S Net International Indebtedness	Lois E. Stekler William L. Helkie
357	U.S. Policy on the Problems of International Debt	Edwin M. Truman
356	International Economic Policy: The Role of Exchange Rates	Edwin M. Truman
355	An Econometric Analysis of UK Money Demand in *Monetary Trends in the United States and the United Kingdom* by Milton Friedman and Anna J. Schwartz	David F. Hendry Neil R. Ericsson

---

Please address requests for copies to International Finance Discussion Papers, Division of International Finance, Stop 24, Board of Governors of the Federal Reserve System, Washington, D.C. 20551.

International Finance Discussion Papers

IFDP NUMBER	TITLES	AUTHOR(s)
354	Encompassing and Rational Expectations: How Sequential Corroboration Can Imply Refutation	Neil R. Ericsson David F. Hendry
353	The United States as a Heavily Indebted Country	David H. Howard
352	External Debt and Developing Country Growth	Steven B. Kamin Robert B. Kahn Ross Levine
351	An Algorithm to Solve Dynamic Models	Wilbur John Coleman II
350	Implications of the U.S. Current Account Deficit	David H. Howard
349	Financial Integration in the European Community	Sydney J. Key
348	Exact and Approximate Multi-Period Mean-Square Forecast Errors for Dynamic Econometric Models	Neil R. Ericsson Jaime R. Marquez
347	Macroeconomic Policies, Competitiveness, and U.S. External Adjustment	Peter Hooper
346	Exchange Rates and U.S. External Adjustment in the Short Run and the Long Run	Peter Hooper
345	U.S. External Adjustment: Progress and Prospects	William L. Helkie Peter Hooper
344	Domestic and Cross-Border Consequences of U.S. Macroeconomic Policies	Ralph C. Bryant John Helliwell Peter Hooper
343	The Profitability of U.S. Intervention	Michael P. Leahy
342	Approaches to Managing External Equilibria: Where We Are, Where We Might Be Headed, and How We Might Get There	Edwin M. Truman
341	A Note on "Transfers"	David B. Gordon Ross Levine
340	A New Interpretation of the Coordination Problem and its Empirical Significance	Matthew B. Canzoneri Hali J. Edison
339	A Long-Run View of the European Monetary System	Hali J. Edison Eric Fisher