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TERMS OF TRADE, THE TRADE BALANCE, AND STABILITY: THE ROLE OF SAVINGS BEHAVIOR

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Abstract

In conventional models of the open economy, the impact on the trade balance of a change in the terms of trade depends upon whether the Marshall-Lerner condition on demand elasticities is satisfied. This paper shows that, in a model which incorporates rational savings behavior, the link between the Marshall-Lerner condition and stability may survive intact or may be severed, depending upon the precise formulation of savings behavior.

Terms of Trade, The Trade Balance, and Stability: The Role of Savings Behavior

Michael Gavin[1]

"The study of elasticities of supply and demand is, thus, the core of the theory of foreign exchange rates."

Fritz Machlup (1939), p. 381

1. Introduction

Whenever devaluation leads to a deterioration in the terms of trade, the direction of its impact on the trade balance becomes problematical. For this reason, the Marshall-Lerner condition on demand elasticities plays an important role in conventional models of the open macroeconomy. Some variant of that condition generally determines the impact of a change in the terms of trade on the trade balance, and also determines the stability properties of the model.[2] As a practical matter too, some authors and policy makers have taken very seriously the danger that a flexible exchange-rate system would display instability if demand elasticities were sufficiently low, as early econometric evidence suggested they might be.[3] While the possibility of stability has been viewed by

1 This author is a visiting scholar in the Division of International Finance. The paper reflects the views of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff. I would like to acknowledge helpful conversations with Dale Henderson, David Gordon and David Howard. Neither they nor anyone affiliated with the Federal Reserve Board should be implicated in any remaining errors in this paper, for which I take sole responsibility.

2 Bickerdike (1920) appears to have been the first to note the relationship between what has come to be known as the Marshall-Lerner condition and stability of the market for foreign exchange, though Chipman (1987) traces discussion of the condition as far back as Mill (1848). This theme was taken up and discussed at some length by Robinson (1937) and Lerner ((1946). Violation of the Marshall-Lerner condition also implies that the specie-flow mechanism would be unstable, so that not only flexible but also fixed exchange-rate economies would display instability. Jones (1961) contains a very useful discussion of the relationship between the terms-of-trade/trade balance nexus and the stability of equilibrium. See Chipman (1987) and Kemp (1987) for very useful, recent surveys.

3 Bickerdike (1920) and Robinson (1937) took the possibility of instability quite seriously, while Machlup (1939, 1940) and Meade (1951) argued that the stable case is, empirically, the more plausible one.

macroeconomists as a problem, it provides for international trade theorists a solution, for by requiring stability of equilibrium such theorists can often rule out perverse comparative statics results. This is, of course, no more than an application of Samuelson's correspondence principle; here we simply point out that the appropriate stability condition is generally some variant of the Marshall-Lerner condition.

The analyses which assign (static) demand elasticities such a prominent role are based upon essentially static models of the world economy; savings decisions are made in an *ad hoc* fashion, and price dynamics are generally based upon Marshallian or Walrasian tâtonnement processes, rather than being derived from the equilibrium itself. More recently, however, the interplay between the Marshall-Lerner condition and stability has been reexamined within the context of explicitly dynamic models which introduce the assumption of rational intertemporal choice in consumption. Along these lines, Gavin (1990) examines the stability of the trade balance within the context of a two-country, world equilibrium model with rational savings behavior in an overlapping-generations framework. The finding there was that stability cannot be taken for granted, and that instability is in fact associated with low (static) demand elasticities. However, in the special case of infinitely-lived consumers, stability was guaranteed no matter how small are demand elasticities. This last finding is however contradicted in Pitchford (1990), which conducts a small-country analysis with infinitely-lived consumers, in which the interest rate is assumed to rise with the level of foreign indebtedness, and which finds the standard Marshall-Lerner condition to be required for stability. And, within a rather different framework and under the assumption of increasing returns to scale, Ide and Takayama (1991) also investigate related issues.

This paper extends this recent discussion by comparing the interplay between static demand elasticities, the terms of trade and the trade balance that emerges from two popular models of aggregate saving. We show that under the assumption of optimal savings behavior, the link between the Marshall-Lerner condition and stability may survive or it may be severed, depending upon the precise formulation of savings behavior.

The plan of the paper is as follows. In Section 2 we consider a model of savings behavior with infinitely-lived consumers whose rate of time preference is endogenous and depends, in particular, upon the level of utility.[4] In this model, static demand elasticities play no role in determining the stability properties of the model; stability is, in fact, guaranteed regardless of how small are static demand elasticities. In Section 3 of the paper we consider instead the savings behavior generated by the Blanchard-Yaari model of finite-lived, overlapping generations. In this model the link between stability and the Marshall-Lerner condition is restored, except in the limiting, special case of infinite lifetimes. Section 4 concludes.

2. An infinite horizon model

In this section and the next we analyze an open economy which is endowed with a constant flow of a domestic good, X. We focus on one country; the rest of the world enters the analysis only by determining the world interest rate, and by generating a demand for the domestic economy's exports. This focus on a single country has its disadvantages; in particular, it leads one to neglect potentially important income effects on the demand for domestic exports. However, it has the virtue of permitting direct comparison with the results of the traditional literature, which is for the most part similarly structured.

Demand: Consumers obtain utility from consumption of the domestic good, C_X, and of an imported good, C_Y. There is a perfect international market in consumption loans, which are denominated in terms of the foreign good, and we imagine that the economy that we study is too small to influence the world interest rate, $r*$. The problem facing the infinitely-lived representative consumer is:

1)
$$\max \int_0^\infty U(C_X, C_Y) e^{-\Delta(t)} dt$$

$$s.t. \quad \dot{A} = r^* A + \frac{X}{\theta} - \frac{C_X}{\theta} - C_M$$

4 These preferences are due to Uzawa (1968), and were introduced to the international economics literature by Obstfeld (1982).

where A denotes the consumer's financial wealth, comprising claims on the world capital market, and θ is the inverse of the terms of trade, which we shall call the real exchange rate. The discount factor, $\Delta(t)$, is given by:

$$\Delta(t) = \int_0^t \rho(U(C_X(s), C_Y(s)))ds$$

where ρ is the instantaneous rate of time preference, which depends upon utility, $U(C_X, C_Y)$. We follow Obstfeld (1982) in assuming:

$$\rho'(U) > 0, \quad \rho''(U) < 0, \quad \frac{\rho'(U)U}{\rho(U)} < 1$$

We also assume that consumption of the domestic and foreign good can be aggregated into an index of "aggregate consumption", $C \equiv C(C_X, C_Y)$, which is homogenous of degree one in its arguments. Instantaneous utility can then be written:

$$U(C_X, C_Y) = V(C(C_X, C_Y))$$

This assumption is made primarily for expostional purposes; it allows us to speak precisely about aggregate real consumption, which is given by the subutility index $C(\cdot)$. Associated with this subutility index is an exact consumption price index, denoted here by $\pi(\theta)$, which gives the cost, in terms of the domestic good, of raising real consumption, C, by one unit. This cost function depends, of course, upon the real exchange rate, θ. It will be useful later to note that the elasticity of π with respect to θ is equal to the share of imports in domestic consumption expenditure, which we denote by $(1-\alpha)$.

Having imposed this structure on utility, we can write the (present-value) Hamiltonian for problem (1) as follows:

2) $$H = V(C) + \gamma\left\{r^*A + \frac{(X - \pi(\theta)C)}{\theta}\right\}$$

and the first-order conditions for maximization are:

3) $$V'(C) = \gamma\pi/\theta \equiv \lambda$$

4) $$\dot{\gamma} = [\rho(V(C)) - r^*]\gamma$$

The costate variable γ must be set equal to the the shadow value of a unit of foreign assets. The related variable λ is set equal to the shadow value of the amount of foreign assets required to purchase one unit of real consumption; it will prove convenient to work with this latter variable in the following analysis.

At any moment in time, consumers choose the composition of their expenditure to minimize the cost of attaining the above-determined, optimal rate of real consumption. Homogeneity of the subutility function $C(C_X, C_Y)$ implies that the optimal ratio of C_Y to C_X will depend only upon the real exchange rate, θ. Denoting the elasticity of *intratemporal* substitution in consumption by η, we have:

5) $$\frac{C_Y}{C_X} = f(\theta) \qquad\qquad \frac{f'(\theta)\theta}{f(\theta)} \equiv -\eta < 0$$

Equation (3) implies that total consumption depends only upon λ:

3') $$C = C(\lambda) \qquad\qquad \frac{C'(\lambda)\lambda}{C(\lambda)} = \frac{V'(C)}{V''(C)C} \equiv -\sigma < 0$$

where we denote the elasticity of *intertemporal* substitution in consumption by σ. Thus, λ determines real consumption which, in turn, determines utility at any moment of time. It follows that the rate of time preference can be written as a function of λ:

$$\rho(\lambda) = \rho(V(C(\lambda))) \qquad\qquad \rho'(\lambda) < 0$$

The rate of time preference is, in equilibrium, a decreasing function of λ. This is because an increase in λ corresponds to an increase in the marginal utility of consumption, which implies a reduction in consumption and utility. The reduction in utility is associated, by assumption, with a reduction in the rate of time preference, ρ.

The tight link between λ and instantaneous utility allows us to write (compensated) domestic demand for the domestic and the foreign good in terms of λ and the real exchange rate, θ. Combining (3') and (5), we obtain:

6) $$C_X = C_X(\lambda, \theta) \qquad\qquad \frac{\partial C_X}{\partial \lambda} < 0, \quad \frac{\partial C_X}{\partial \theta} > 0$$

$$C_Y = C_Y(\lambda, \theta) \qquad\qquad \frac{\partial C_Y}{\partial \lambda} < 0, \quad \frac{\partial C_Y}{\partial \theta} < 0$$

With respect to foreign demand for domestic exports, we follow the related literature in making export demand an increasing function of the real exchange rate, and denote this demand *Z*:

7) $$Z = Z(\theta) \qquad \frac{\partial Z}{\partial \theta} > 0$$

Goods-Market Equilibrium: Equation (8) gives the condition for domestic goods-market equilibrium:

8) $$X = C_X(\lambda, \theta) + Z(\theta)$$

which implies a positive equilibrium relationship between λ and θ:

8') $$\theta = \theta(\lambda) \qquad \frac{\theta'(\lambda)\lambda}{\theta(\lambda)} = \frac{\alpha\sigma}{(1-\alpha)(\varepsilon^* + \alpha\eta)} > 0$$

where α is the share of domestic expenditure falling upon the domestic good,and ε^* is the elasticity of foreign demand for the domestic good.[5] The interpretation of (8') is straightforward: an increase in λ corresponds to a reduction in domestic consumption, which implies a reduction in demand for the domestic good. Excess supply would emerge unless the relative price of the foreign good, θ, where to rise, thus shifting the composition of demand in favor of the domestic good.

Domestic expenditure and saving: We are now prepared to derive savings behavior. Measured in terms of the domestic good, domestic expendture is the product of real consumption and the price of consumption:

$$E \equiv C_X + \theta C_M = \pi(\theta)C(\lambda)$$

Substituting the equilibrium relationship (8') for the real exchange rate in the above we see that, in equilibrium, expenditure is determined solely by λ:

9) $$E(\lambda) = \pi(\theta(\lambda))C(\lambda)$$

Differentiating (9), and appealing to the comparative statics results discussed above, we obtain:

5 The intermediate results presented here and below are based upon straightforward application of well-known comparative-static results. For the sake of brevity, lengthy derivations are foregone. Derivations are, however, presented in a technical appendix to the paper, which is available from the author upon request.

9') $$\frac{E'(\lambda)\lambda}{E(\lambda)}=\left\{\frac{\pi'(\theta)\theta}{\pi}\right\}\left\{\frac{\theta'(\lambda)\lambda}{\theta}\right\}+\left\{\frac{C'(\lambda)}{C}\right\}$$
$$=(1-\alpha)\left\{\frac{\alpha\sigma}{(1-\alpha)(\varepsilon^*+\alpha\eta)}\right\}-\sigma$$
$$=-\sigma\left\{\frac{\varepsilon^*+\varepsilon-1}{\varepsilon+\alpha\eta}\right\}$$

where $\varepsilon\equiv\alpha\eta+(1-\alpha)>0$ is the uncompensated elasticity of domestic demand for the imported good and ε^* is the price elasticity of export demand. We see from (9') that the sign of the relationship between domestic total expenditure and λ depends upon whether the export and the import demand elasticities sum to a number larger than unity - that is, whether the static Marshall-Lerner condition is satisfied. If it is, then expenditure declines when λ rises. In the perverse but theoretically admissible case in which the condition is not satisfied, expenditure is positively related to λ.

Having determined domestic expenditure, domestic asset-accumulation follows trivially. Substituting (8) and (9) into the representative consumer's budget constraint, we obtain:

10) $$\dot{A}=r^*A+\frac{(X-E(\lambda))}{\theta(\lambda)}$$

Equilibrium dynamics: Equation (10) gives the dynamics for the economy's net foreign assets, A, as a function of λ and A itself. It remains to derive the equilibrium dynamics for λ. Differentiating the definition of λ in equation (3), we obtain:

11) $$\frac{\dot{\lambda}}{\lambda}=\frac{\dot{\gamma}}{\gamma}+\frac{\dot{\pi}}{\pi}-\frac{\dot{\theta}}{\theta}$$

Substituting this relationship into the dynamic efficiency condition, (4), we obtain:

12) $$\frac{\dot{\lambda}}{\lambda}=\left\{\rho(\lambda)-\left(r^*+\frac{\dot{\theta}}{\theta}-\frac{\dot{\pi}}{\pi}\right)\right\}$$
$$=\left\{\rho(\lambda)-\left(r^*+\frac{\dot{\theta}}{\theta}-(1-\alpha)\frac{\dot{\theta}}{\theta}\right)\right\}$$

which sets the growth of λ equal to the difference between the rate of time preference and the consumption rate of interest, where the latter is defined as the rate of interest on the composite of aggregate consumption, $C(C_X,C_Y)$.

Appealing to the equilibrium relationship between the real exchange rate, θ, and λ which is summarized in equation (8') we obtain:

$$\frac{\dot{\theta}}{\theta} = \left\{ \frac{\alpha\sigma}{(1-\alpha)(\varepsilon^* + \alpha\eta)} \right\} \frac{\dot{\lambda}}{\lambda}$$

which, when substituted into (12) finally yields:

13) $$\frac{\dot{\lambda}}{\lambda} = \left\{ 1 + \frac{\alpha^2\sigma}{(1-\alpha)(\varepsilon^* + \alpha\eta)} \right\} (\rho(\lambda) - r^*)$$

Equations (10) and (13) summarize the dynamics of this economy. There are two cases to consider: the one in which the Marshall-Lerner condition is satisfied, and the other in which it is not. Both are summarized in Figure 1, below.

In both panels of Figure 1, the *LL* schedule gives the locus of points along which $\dot{\lambda} = 0$. This occurs for the unique value for λ which sets the rate of time preference equal to the world interest rate. Above the *LL* schedule consumption is low, utility is low, and therefore the rate of time preference is low. Intertemporal optimization then requires that $\dot{\lambda} < 0$. Below the *LL* schedule, λ must be rising.

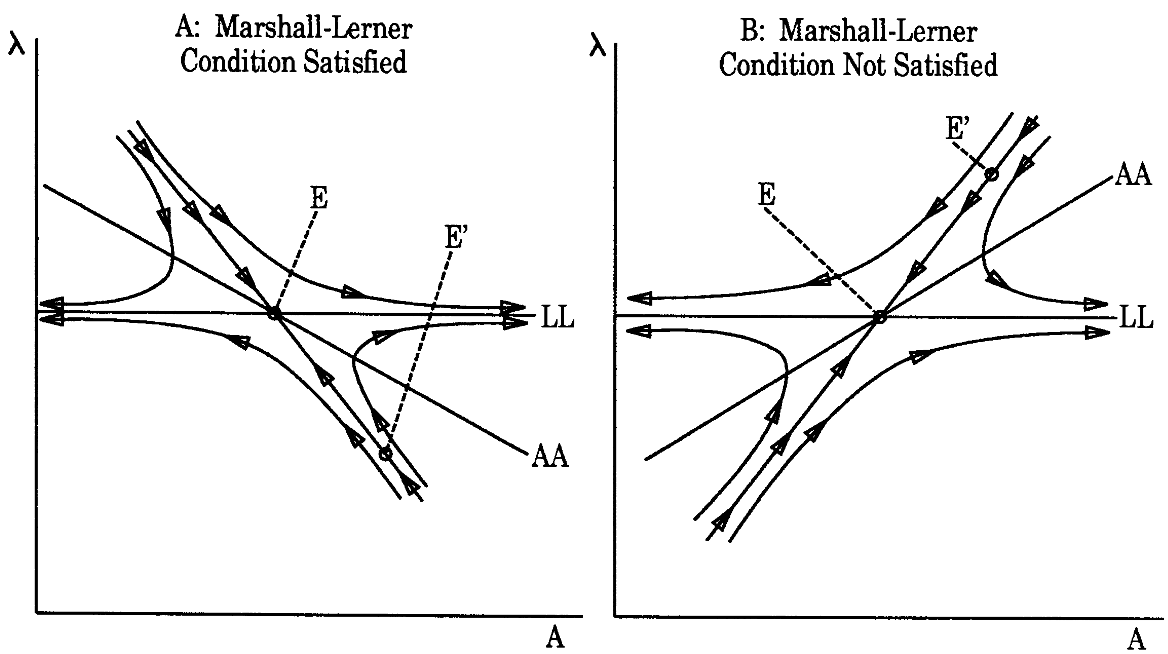
The *AA* schedule gives the locus of points for which $\dot{A} = 0$. Totally differentiating (10), we obtain the slope of this schedule:

$$\frac{d\lambda}{dA} = \frac{r^*\theta}{E'(\lambda) - r^* A \theta'(\lambda)}$$

We follow the traditional literature and simplify the analysis by assuming that the initial steady state is one in which domestic holdings of foreign assets is (approximately) zero; in this case, the trade balance and the current account are both zero. Then the *AA* schedule is downward sloping when the Marshall-Lerner condition is satisfied, (in which case, as noted above, $E'(\lambda) < 0$), and upward sloping when it is not.

Consider first the case in which the Marshall-Lerner condition is satisfied; this case is depicted in panel *A* of Figure 1, in which *AA* schedule is downward sloping. To the right of the *AA* schedule interest income is high, so that total income exceeds expenditure and the representative consumer is accumulating assets and the current account is in surplus. To the left of the *AA* schedule assets are being decumulated, and the current account is in deficit.

Figure 1
Endogenous Time Preference



It is easy to see that the equilibrium described in Figure 1 is saddle-path stable, with a downward-sloping saddle path.[6] Any point off the saddle path is ruled out either by the consumer's transversality condition, or the intertemporal solvency condition, which we now impose:

$$\lim_{T \to \infty} A_T e^{-r^* T} \geq 0$$

Suppose that the economy begins in steady state at the point labeled E in panel A of Figure 1, when it is disturbed by a receipt of foreign assets. The new equilibrium is at point E' in the same figure, at which the economy is running a current-account deficit, thus running down its holdings of foreign assets. The increase in consumption expenditure is accompanied by a decline in λ, which implies that real consumption, as well as the value of

6 We rely here upon graphical arguments here, as they provide somewhat more intuition than do the very simple analytical arguments. The technical appendix provides analytical support for the points discussed in this paper.

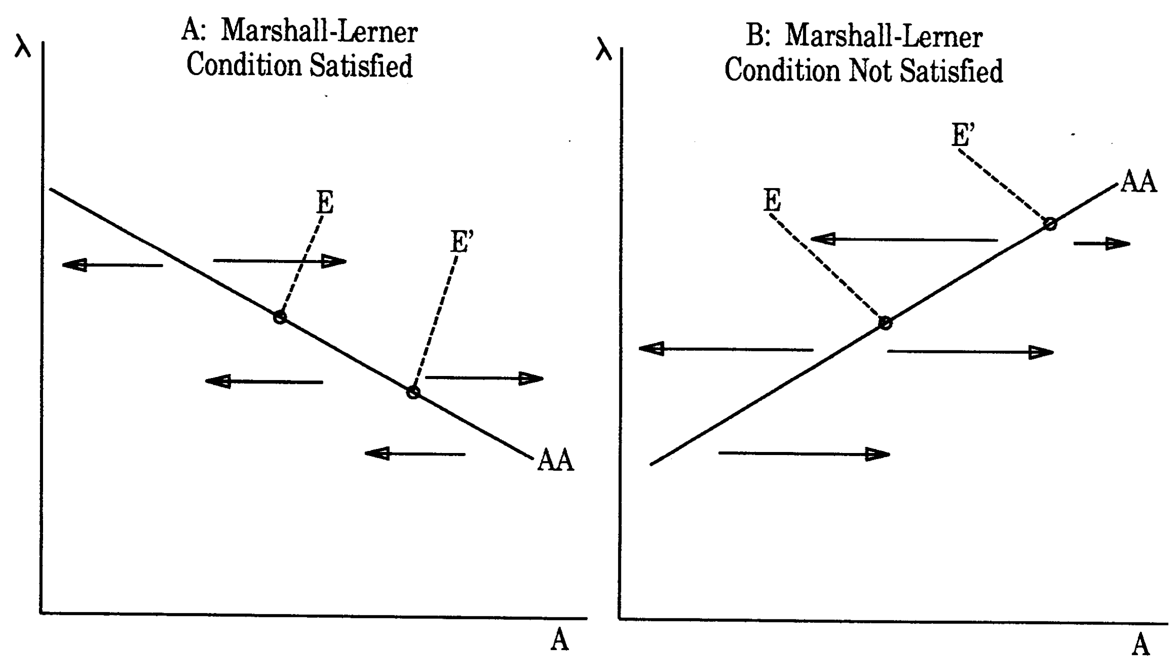
expenditure on consumption, must have risen. We know from (8') that λ and the real exchange rate are inversely related, so it must be the case that at the new equilibrium the domestic economy's terms of trade have improved. In summary, we obtain here the conventional results that the receipt of foreign assets leads to real exchange-rate appreciation, an increase in real consumption, and current-account deficits. These deficits, in turn, are the mechanism by which the economy converges to the original steady state in a stable fashion.

Now consider the case in which the Marshall-Lerner condition is not satisfied, which is depicted in panel *B* of Figure *1*. In this case both the *AA* schedule and the saddle-path are upward, rather than downward sloping. Notice that the equilibrium is still saddle-path stable. As before, a gift of foreign assets will lead to an equilibrium in which consumer expenditure rises and savings declines, as we can see by noting that at the new equilibrium *E'* assets are being decumulated. This ensures stability which is not, therefore, threatened by violation of the Marshall-Lerner condition. In the model described in this section rational savings behavior rules out the destabilizing asset-accumulation that is generally associated with violation of the Marshall-Lerner condition.

The comparative-static implications of the model are, however, highly counterintuitive. In particular, the rise in expenditure is due entirely to the impact on expenditure of a large real exchange-rate *depreciation*, i.e., deterioration in the terms of trade. Real consumption and utility decline, as can be seen by noting that λ rises when moving from the initial equilibrium *E* to the new one, *E'*. The association of transfer receipt with deteriorations in the terms of trade and a decline in real consumption and utility is, of course, perverse. However, the perverse comparative statics are not associated with instability; an implication of this finding is that such perversities cannot, in this model, be ruled out by an appeal to stability of equilibrium.

Constant time preference: An important special case of this model arises when the rate of time preference is constant.[7] In this case the model possesses a stationary state only if this constant rate of time preference equals the world interest rate, which we will assume to be true. In this case the consumer's dynamic optimization requires that λ be constant, as can be seen by setting $\rho = r^*$ in equation (13). The dynamics for asset-accumulation were derived without reference to this dynamic efficiency condition, and are therefore unaffected by the change in assumptions. The system of dynamic equations now has a zero root, which implies path-dependence in wealth, consumption and utility, as can be seen from Figure 2.

Figure 2
Constant Time Preference



7 Actually, this is a special case of all three models of saving discussed in the paper. It emerges in Pitchford's model when $r^{*\prime}(A) = 0$, in the Uzawa model when $\rho'(U) = 0$, and in the Blanchard-Yaari model when $\delta = 0$. Though the assumption of infinite lifetimes is, of course, somewhat unrealistic, the implications of this model have been the subject of intense empirical scrutiny since Hall (1978). Hall's finding that consumption should follow a random walk is the discrete-time, stochastic analogue to the zero-root in the model discussed here.

In Figure 2 the *AA* schedule is, as in Figure 1, the locus of points for which income equals expenditure on consumption and therefore asset-accumulation is zero. The *LL* schedule vanishes, and is replaced by the condition that λ be constant along any equilibrium trajectory. The dynamics are therefore as indicated by the horizontal phase arrows. Any point on the *AA* schedule is a steady-state equilibrium, and at which point on this schedule the economy will be located depends upon initial conditions for financial assets, A. Notice that the comparative statics of a wealth transfer depend, as above, on whether the Marshall-Lerner condition is satisfied, though stability is guaranteed in either case. In the stable case illustrated in panel A of Figure 2, a country which receives a transfer of financial assets would move from point E to E', where real consumption is higher and the terms of trade have improved. In the unstable case, illustrated in panel B, a country which receives a transfer would move from point E to point E', at which the terms of trade have deteriorated, and real consumption has fallen (though expenditure on consumption, in terms of the domestic good, has risen.)

Endogenous rate of interest: These results contrast sharply with those in Pitchford (1990), and it may be useful at this point to provide an explicit comparison.[8] In that paper the representative consumer's rate of time preference is assumed to be constant, but the interest rate which the small economy must pay is taken to be a decreasing function of the economy's foreign assets. In our notation, this leads to the following dynamic efficiency condition:

14) $$\frac{\dot{\lambda}}{\lambda} = k(\rho - r^*(A)) \qquad r^{*\prime}(A) < 0, \quad \frac{r^{*\prime}(A)A}{r^*(A)} > -1$$

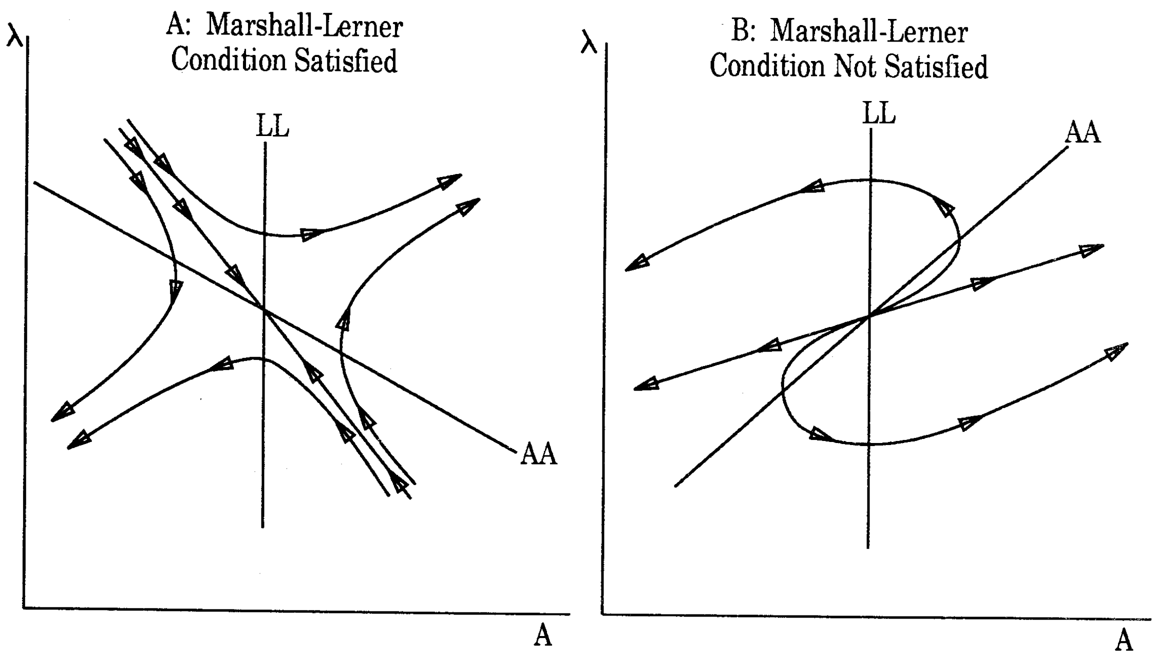
where k is a positive parameter. The partial-equilibrium comparative static results and the economy's asset-accumulation condition are essentially unaffected by the change in assumptions, and equation (10) becomes:

15) $$\dot{A} = r^*(A)A + \frac{(X - E(\lambda))}{\theta(\lambda)}$$

8 See Pitchford (1990) for a more complete exposition. The technical appendix to this paper also contains a more complete derivation of the following relations, in terms of the notation and assumptions used in this paper.

Note that the second assumption on $r^*(A)$ in (14) ensures that total interest income is an increasing function of asset holdings. As in the previous model, the expenditure function $E(\lambda)$ is decreasing in λ when the Marshall-Lerner condition is satisfied, and increasing when it is not. Equations (14) and (15) are summarized in Figure 3:

Figure 3
Endogenous Interest Rate



The LL schedule, along which $\dot{\lambda} = 0$, is now vertical. To the right of LL foreign assets are high (equivalently, debt is low), and the interest rate is therefore low; the consumers intertemporal optimization then requires that λ be increasing. To the left of the LL schedule optimality requires that λ be declining. As in the previous model, foreign assets are increasing to the right of the AA schedule, which is downward sloping when the Marshall-Lerner condition is satisfied, and upward sloping when it is not.

Notice that, as in the previous model, the economy is saddle-path stable when the Marshall-Lerner condition is satisfied, and the response of the economy to a receipt of foreign assets would be qualitatively identical. There is a difference, however, when the

Marshall-Lerner condition is not satisfied. In that case, it is easy to show that (the real parts of) both characteristic roots of (14) and (15) are positive. It follows that the equilibrium is locally unstable. In the extremely unlikely event that the economy had attained some such unstable steady state, any perturbation would send it hurtling off in one direction or another - presumably to some neighboring, locally stable steady state[9] - as is illustrated by the phase arrows in panel *B* of Figure *3*.[10]

This model, then, provides a relationship between the static Marshall-Lerner condition and stability which contrasts with that of the previous model. Unlike in the case with endogenous time preference, the usual Marshall-Lerner condition is precisely the stability condition required for convergence to steady state. We explore now to a popular overlapping-generations model of saving.

3. A finite horizon model

In this section we assume that consumers have finite lives, as in Blanchard (1985) and Yaari (1968). The Blanchard-Yaari model of saving is by now very well known, and has been applied to international economics in a variety of contexts. For this reason, we abbreviate derivations here in order to focus on the results and the underlying intuition; readers who are interested in a more complete derivation are referred to Blanchard (1985) or, for the results specific to this paper, to the appendix.

The basic assumption of the model is that all individuals face a constant probability of death, here denoted δ, which is independent of age, and which gives the proportion of individuals who die at any moment. We assume here that the birth rate is the same as the rate of death, which implies that the size of the population is constant. Individuals work and earn labor income for as long as they are alive, and they have access to a perfect

9 Evaluation of the global dynamics is beyond the scope of this project, which is aimed at deriving some simple lessons concerning the relationship between the Marshall-Lerner condition, comparative statics, and stability in intertemporal optimizing models. Gavin (1990) evaluates global dynamics in a two-country variant of the overlapping-generations model discussed below. Assuming CES utility, it is shown there that any unstable steady state is necessarily surrounded by exactly two saddle-path stable steady states. Bhagwati, Brecher and Hatta discuss global comparative static properties of economies which might be in an unstable equilibrium like the one discussed here.

10 In the unstable case the roots of the dynamics system depicted in Figure 3 could be either real, as drawn, or complex, which would imply cyclical behavior. In either case, the real parts of the roots are positive and the economy would be unstable.

insurance market, in which they pledge the value of their financial assets (or debts) against the event of their death. When death arrives, an individual's assets are turned over to the insurance company, and as long as an individual does not die the insurance company pays a premium equal to δ times the value of the assets which have been pledged.[11] Readers who find the demographic or microeconomic assumptions of this model unconvincing are invited to think of the following as a highly parametrized synthesis of the permanent income with the Metzlerian savings hypothesis for, while the Metzlerian structure of savings generated by this framework is of central importance here, the underlying model used to obtain that structure matters much less.

Supply: As before, we assume that the domestic economy produces a constant stream of the domestic good, which we label X. Because the Blanchard model leads to a distinction between labor and capital income, we assume that some fraction, denoted β, of the economy's output accrues to workers, while the remainder accrues to owners of land, which is the second factor of production. Individuals are born without any claims on land income, and they acquire such claims by saving. They can also purchase foreign assets, as in the previous model.

Demand: Our assumptions on utility are the same as in the previous section, except that we assume that the elasticity of intertemporal substitution, σ, is unity, and that the rate of time preference, ρ, is exogenously given and constant. The central implication of this model, for our purposes, is that the aggregate consumption expenditure resulting from individual optimization is proportional to total wealth, where total wealth includes the value of capital assets owned by domestic consumers, A, and human wealth, H. In equation (14) we measure both components of wealth in terms of the foreign good:

14) $$C_X + \theta C_Y = \theta(\rho + \delta)(A + H)$$

where C_X and C_Y refer to aggregate domestic consumption of the domestically-produced and the imported good, respectively. The marginal and average propensity to consume wealth is the sum of the pure rate of time preference and the instantaneous probability of death, $(\rho + \delta)$.

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11 See Blanchard (1985) for details on the demographic and market structure, and for a demonstration that, in equilibrium, the insurance industry breaks even.

Human wealth is defined as the present value of expected future wage income of currently-living workers, expressed here in terms of the foreign good:

15) $$H_t \equiv \int_t^{\infty} \left(\frac{\beta X}{\theta_s} \right) e^{-(r^* + \delta)(s-t)} ds$$

The expectation discounts future wage income at rate $(r^* + \delta)$ rather than r^* because the probability of a worker being alive $(s-t)$ periods from the present is proportional to $e^{-\delta(s-t)}$, and if a worker is not alive his labor income is, of course, zero. Differentiating (15) with respect to time, we obtain an analytically equivalent expression for human wealth:

15') $$\dot{H}_t = (r^* + \delta)H_t - \beta X / \theta_t$$

As in the previous section, we assume homothetic utility, from which it follows that:

16) $$\frac{C_Y}{C_X} = f(\theta) \qquad\qquad \frac{f'(\theta)\theta}{f(\theta)} \equiv -\eta$$

where η is here, as in the previous section, the elasticity of intratemporal substitution in consumption.

Combining (16) and (14), we obtain domestic demand for the domestic good as a function of the terms of trade, human wealth, and nonhuman wealth:

17) $$C_X = \left[\frac{\theta}{1 + \theta f(\theta)} \right] (\rho + \delta)(A + H)$$

Financial wealth accumulation is the excess of total income over expenditure on consumption:

18) $$\dot{A} = r^* A + \frac{\beta X}{\theta} - (\rho + \delta)(A + H)$$

where A includes both foreign assets and the value of claims on the domestic capital asset, and we have imposed equality of the domestic and the foreign asset returns, expressed in common numeraire, as required by asset-market equilibrium.

The World Interest Rate: A steady state will not exist in this model unless the world interest rate is greater than the domestic consumers' pure rate of time preference, ρ, and less that the sum of the rate of time preference and the index of mortality, δ. If domestic and foreign consumers were identical, this would hold as an implication of the global goods market equilibrium. In any event, for the remainder of this section it will be assumed that $\rho < r^* < (\rho + \delta)$. Notice that as the probability of death, δ, approaches zero, this inequality requires the interest rate to approach the rate of time preference, ρ.

Goods-Market Equilibrium: As in the previous section we assume that export demand is an increasing function of the real exchange rate, θ. The condition for goods-market equilibrium is:

19) $$X = C_X(\theta, A+H) + Z(\theta) \qquad \frac{\partial C_X}{\partial \theta} > 0, \quad \frac{\partial C_X}{\partial (A+H)} > 0$$

which implies an inverse relationship between the real exchange rate and domestic total wealth:

19') $$\theta = \theta(A+H) \qquad \frac{\theta'(A+H)(A+H)}{\theta(A+H)} = \frac{-\alpha}{\alpha + (1-\alpha)(\varepsilon + \varepsilon - 1)} < 0$$

where, as in the previous section, α is the share of domestic expenditure falling upon the domestic good, $\varepsilon \equiv (1-\alpha) + \alpha\eta$ is the uncompensated price elasticity of domestic demand for imports, and ε^* is the price elasticity of demand for exports. Substituting equation (19') for the real exchange rate in (18), we obtain an expression for domestic financial asset accumulation, as a function of financial wealth and human wealth. Linearizing this relationship about some initial steady state, we obtain:

20) $$\dot{A} = r^*\hat{A} + a(\hat{A} + \hat{H}) - (\rho + \delta)(\hat{A} + \hat{H})$$

where carets represent (linear) deviations from some initial steady state, and the parameter a is given by:

$$a \equiv \left\{ \frac{(r^* + \delta)(\rho + \delta - r^*)}{\delta} \right\} \left\{ \frac{\alpha}{\alpha + (1-\alpha)(\varepsilon + \varepsilon^* - 1)} \right\} > 0$$

This parameter measures the strength of relationship between domestic wealth, the terms of trade, and wage income. As we shall see, this positive link between domestic wealth and wage income, combined with the Metzlerian behavior of savings, is what generates the possibility of instability, and therefore a determines the stability properties of the model. Note that the smaller are the demand elasticities ε and ε^*, the larger is a, and therefore the larger is the impact of high domestic wealth on domestic wage income.

Linearizing (15') around some steady state, we obtain:

21) $\dot{H} = (r^* + \delta - a)\hat{H} - a\hat{A}$

Thus, in the vicinity of a steady state, the dynamic behavior of this economy can be summarized by the following pair of equations:

22) $$\begin{bmatrix} \dot{A} \\ \dot{H} \end{bmatrix} = \begin{bmatrix} r^* - \rho - \delta + a & a - \rho - \delta \\ -a & r^* + \delta - a \end{bmatrix} \begin{bmatrix} \hat{A} \\ \hat{H} \end{bmatrix}$$

Local stability depends upon the eigenvalues of the matrix in (22). The sum of the eigenvalues, given by the trace of the matrix, is $r^* + (r^* - \rho) > 0$, from which we conclude that at least one is positive. The product of the eigenvalues is given by the determinant of the matrix in (22) which, after some simplification, can be seen to equal:

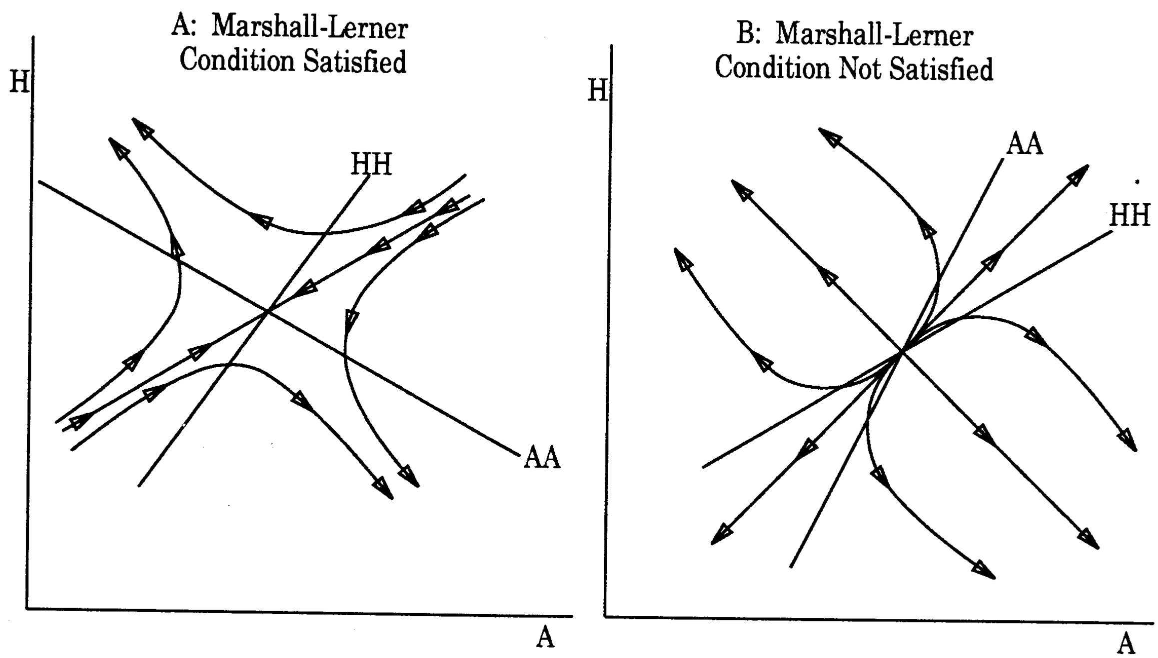
23) $r^*(r^* - \rho) - \delta(\rho + \delta - a)$

Equation (22) describes a saddle-path stable system if this determinant is negative, so the stability condition is, after rearranging for a:

24) $$a < \left(\frac{\delta(\rho + \delta) - r^*(r^* - \rho)}{\delta} \right)$$

Thus, stability requires that the income effects of wealth transfers not be too large. Before investigating the link between (24) and the static Marshall-Lerner condition, we clarify the intuition for the unstable case. It is useful to begin with the stable case, which is depicted in panel *A* of Figure 4. In that figure the *HH* schedule is the locus of points for which equilibrium human wealth is constant; in the stable case it is upward-sloping and can be derived by setting equation (21) equal to zero. Above the schedule human wealth must be increasing, and below it human wealth must be declining. The *AA* schedule is the locus of points along which equilibrium financial wealth is constant. It may be upward- or downward-sloping, depending upon the strength of the relationship between wealth, the equilibrium terms of trade, and wage income which depends, as noted above, on the parameter a. In the stable case the *AA* schedule is downward-sloping, as drawn, or upward-sloping and flatter than the *HH* schedule. Above the schedule total wealth and therefore expenditure are high relative to income, and hence assets must be declining. Below the schedule assets must be rising.

Figure 4
Finite Lifetimes



It is readily seen that, so long as the *AA* schedule is downward-sloping, or upward-sloping and less steep than the *HH* schedule, the equilibrium is saddle-path stable, with an upward-sloping saddle path as drawn in panel *A* of Figure 4. An increase in foreign assets arising from, say, a war indemnity, would by raising total wealth lead to high domestic expenditure. This high domestic demand would generate current-account deficits, and also an appreciation of the domestic real exchange rate. This appreciation would, in turn, increase current and anticipated future wage income, as measured in foreign goods, thus increasing human wealth.

The key aspect of the stable case is that, in equilibrium, human wealth rises less than in proportion to the rise in financial wealth. The importance of this can be seen by adding (15') and (18), to obtain a broad measure of savings:

25) $(\dot{A} + \dot{H}) = (r^* - \rho - \delta)A + (r^* - \rho)H$

Remembering that $\rho < r^* < (\rho + \delta)$, we note that, as in Metzler (1951), aggregate wealth accumulation is negatively related to financial wealth. Also as in Metzler, wealth accumulation is increasing in labor income or, more precisely, human wealth. It follows from (25) that an increase in financial wealth will lead to a decline in wealth accumulation only if it generates, in equilibrium, an increase in human wealth which is less than proportional to the original increase in financial wealth. If so, then the rise in financial wealth leads in equilibrium to dissavings, and wealth returns in stable fashion to its original level. If not, a rise in financial wealth is followed by wealth accumulation, which implies yet higher financial wealth, which generates yet more rapid wealth accumulation. It is easy to see how the possibility of instability arises. The unstable case is drawn in panel *B* of Figure 4.

We turn now to the relationship between the stability condition for this model and the static Marshall-Lerner condition. Substituting the definition of a, given above, into (24), we obtain a relationship between this stability condition and the Marshall-Lerner condition with which we have been concerned:

24') $$\left\{\frac{\alpha}{\alpha + (1-\alpha)(\varepsilon + \varepsilon^* - 1)}\right\} < \left\{\frac{\delta(\rho+\delta) - r^*(r^* - \rho)}{(r^* + \delta)(\rho + \delta - r^*)}\right\} = 1$$

Rearranging slightly, we obtain an expression for the stability condition which makes the relationship between it and the Marshall-Lerner condition transparent:

24") $$(\varepsilon + \varepsilon^* - 1) > 0$$

Thus, in this model the stability condition is exactly equivalent to the static Marshall-Lerner condition. It should be noted that this stability condition was derived under the assumption that the index of mortality, δ, is strictly greater than zero. When $\delta = 0$ this model is identical to the infinite-horizon, constant time-preference model which was discussed as a special case of the Uzawa model, above.[12] Thus, the potential instability identified in this model is closely related to the assumption of finite lives or, to be more precise, to the Metzlerian structure of savings which is, in the Blanchard-Yaari model, associated with that assumption.

12 This can be seen by setting $\delta = 0$ and $r^* = \rho$ in equations (15') and (18) or, equivalently, (22), and noting that one of the eigenvalues of the system must be zero, and the other positive.

4. Conclusion

Some variant of the Marshall-Lerner condition affects the stability and comparative-static properties of most models of the open economy. This paper tried to determine whether this remains true in models where the dynamics are generated by equilibrium asset accumulation, and in which savings decisions are made rationally. The answer is that it depends upon precisely how the savings decision is modeled. In the Uzawa model of endogenous time preference, stability is guaranteed regardless how low are demand elasticities, and the same is true in the infinite-horizon model with constant time preference and interest rates. In the Blanchard-Yaari model, however, the Marshall-Lerner condition is necessary for stability, except in the limiting case of infinite lifetimes.

The intuition for the strong tendency toward stability in the Uzawa model follows from the "target savings" behavior which it implies, in which consumers save as required to achieve a target level of utility in the steady state. This leads them to dissave when financial assets are above the level required to finance the given, steady-state level of utility, which means that savings decisions never lead to unstable patterns of accumulation. The intuition for the possibility of instability in the Blanchard-Yaari model follows from the Metzlerian savings behavior implied by that framework. Aggregate savings can also be described as "target savings" behavior, with the crucial difference that the target for financial wealth is proportional to labor income,[13] and therefore is affected by the terms of trade. Instability arises when an increase in financial assets generates such a large improvment in the terms of trade that it raises target wealth more than actual, current wealth has risen, thus triggering savings, rather than the dissavings required to reverse the increase in financial wealth. This explanation also makes it clear why the instability cannot arise in the limiting case of infinite lifetimes. In that case there is no useful distinction between financial wealth and human wealth; both are claims on assets that live forever. The Metzlerian structure of savings vanishes, and an extreme version of the Friedman permanent-income hypothesis emerges, in which consumers' optimal rule is

13 More precisely, to human wealth which is the present value of anticipated future labor income.

to smooth consumption completely, which implies maintaining constancy of the broad measure of wealth, $(A+H)$. Unstable patterns of wealth accumulation are, then, ruled out by the conditions for intertemporal optimization.

It's not entirely clear how to think about these results. Macroeconomists have for the most part viewed potential violation of the Marshall-Lerner condition as a problem; it means that devaluation would be ineffective in affecting, or even perversely related to employment and the trade balance, and it means that the monetary mechanism for correction of payments imbalances under fixed exchanges rate may display instability. From such a perspective the finding that at least some models of rational savings behavior rule out instability would seem both natural and encouraging. After all, it seems intuitively implausible that rational, forward-looking consumers would choose the savings paths implicit in the "explosive" paths for the current account which are generated by the unstable case. The unexpected and uncomfortable result of this paper would then be that instability can arise even after accounting for rational intertemporal choice.

A trade theorist, accustomed to ruling out perverse comparative statics results on the basis of stability arguments, may react rather differently. Here the finding that both the Uzawa model and the infinite-horizon model of saving robs violation of the Marshall-Lerner condition of its implications for stability, while maintaining the possibility of perverse comparative statics, might seem unsettling. There are at least two responses to this finding. The first would be to argue that these models are psychologically implausible frameworks for studying savings.[14] Second, it would probably be possible to overlay dynamics arising from the Marshallian tâtonnement process on the savings dynamics emphasized here, and show that the dynamic trajectory described in Figure 1 to be unstable off the equilibrium path. The problem with this is that, as soon as intertemporal choice is taken seriously, equilibria are nearly always unstable in this sense. Such equilibria are

14 The model does have some implausible features. First, in order to obtain the well-behaved savings behavior described above, it is assumed that wealthy consumers are more impatient than are poor consumers, and assumption which some find counterintuitive. Lawrance (1991) presents econometric evidence that poor consumers have higher rather than lower rates of time preference, though this begs the question whether the consumers have high rates of time preference because they are poor, or are poor because they have high rates of time preference. The Uzawa model also has the property that an increase in labor income leads to a decline in savings and, in the long run, to lower financial assets. Nevertheless, the model is certainly internally consistent, and some authors find it a compelling framework for the analysis of savings.

nearly always saddle-path stable, and any move off the equilibrium saddle path leads to aberrant dynamics whether the Marshall-Lerner condition is satisfied or not. If we were to rule out *all* equilibria whose out-of-equilibrium behavior displays instability, a whole class of models would become unusable. Whatever the response, this paper makes clear that the link between stability and the Marshall-Lerner condition depends very much upon the precise formulation of saving.

References

Bhagwati, Jagdish, Richard Brecher, and Tatsuo Hatta (1987) "The Global Correspondence Principle: A Generalization", ***American Economic Review***,

Bickerdike, C. (1920) "The Instability of Foreign Exchange", ***Economic Journal***, 30 (March): 118-122.

Blanchard, Olivier (1985) "Debt, Deficits and Finite Horizons", ***Journal of Political Economy***, 93 (April): 223-247.

Chipman, John S. (1987) "International Trade", in John Eatwell, Murray Milgate and Peter Newman (eds.) ***The New Palgrave***, vol. New York: The Stockton Press, 922-955.

Gavin, Michael (1990) "Stability of the Trade Balance with Rational Consumers: The Marshall-Lerner Condition Reconsidered", mimeo, Columbia University.

Hall, Robert (1978) "Stochastic Implications of the Life-Cycle Permanent-Income Hypothesis: Theory and Evidence", ***Journal of Political Economy***, 86 (Oct.): 971-987.

Kemp, Murray (1987) "Marshall-Lerner Condition", in John Eatwell, Murray Milgate and Peter Newman (eds.) ***The New Palgrave***, vol. 3, New York: The Stockton Press, 364-365.

Ide, T. and A. Takayama (1991) "Variable Returns to Scale and Dynamic Adjustments: The Marshall-Lerner Condition Reconsidered", mimeo, Tulane University.

Jones, Ronald (1961) "Stability Conditions in International Trade: A General Equilibrium Approach", ***International Economic Review***, 2, (May): 199-209.

Lerner, Abba (1946) ***The Economics of Control***, New York: Macmillan.

Machlup, Fritz (1939, 1940) "The Theory of the Foreign Exchanges", ***Economica***, (Nov. 1939 and Feb. 1940): 375-397, 23-49.

Meade, James E. (1951) ***The Theory of Economic Policy: The Balance of Payments***, London: Oxford University Press.

Metzler, Lloyd (1951) "Wealth, Saving, and the Rate of Interest", ***Journal of Political Economy***, 59 (April): 93-116.

Obstfeld, Maurice (1982) "Aggregate Spending and the Terms of Trade: Is There a Laursen-Metzler Effect?", ***Quarterly Journal of Economics***, 97 (May): 251-270.

Pitchford, John (1990) "Elasticities and the Trade Balance: The Significance of the Marshall-Lerner Condition", mimeo, Australian National University.

Robinson, Joan (1937) "The Foreign Exchanges", in ***Essays in the Theory of Employment***, Oxford: Basil Blackwell.

Lawrance, Emily C. (1991) "Poverty and the Rate of Time Preference: Evidence from Panel Data", ***Journal of Political Economy***, 99, (February): 30-53.

Uzawa, Hirofumi (1968) "Time Preference, the Consumption Function, and Optimum Asset Holdings", in J.N. Wolfe (ed.) ***Value, Capital and Growth: Papers in Honor of Sir John Hicks***, Chicago: Aldine Publishing Co.

Technical Appendix:
"Terms of Trade, The Trade Balance, and Stability:
The Role of Saving Behavior"

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April, 1991

This appendix fills in the rather cursory derivations of results discussed in the body of the paper, and provides analytical support for those results which are defended graphically in the paper. We begin with the Uzawa model of endogenous time preference.

A. Endogenous Time Preference:

We begin by deriving the relationship between the equilibrium terms of trade and λ, equation (8'). Noting that supply of the domestic good, X, is fixed, we totally differentiate (8), to obtain:

A1) $$0=\left(\frac{\partial C_X}{\partial\lambda}\frac{\lambda}{C_X}\right)C_X\hat{\lambda}+\left(\frac{\partial C_X}{\partial\theta}\frac{\theta}{C_X}\right)C_X\hat{\theta}+\left(\frac{\partial Z}{\partial\theta}\frac{\theta}{Z}\right)Z\hat{\theta}$$

where variables with carets over them denote proportional deviations from the initial steady state.

Homothetic utility implies that the elasticity of C_X with respect to λ equals the elasticity of total consumption with respect to λ. Thus, the first term in parentheses is equal to the term defined in equation (3') as $-\sigma$, and it is equal to the elasticity of intertemporal substitution. The second term in parentheses is the *compensated* demand elasticity for the domestic good, which is equal to $(1-\alpha)\eta$, where α is the share of domestic consumers' spending which falls on the domestic good and η is the elasticity of intratemporal substitution in consumption. The third term in parentheses is the elasticity of foreign demand for domestic goods, denoted by ε^*. Substituting into (A1), we obtain:

$$0=-\sigma C_X\hat{\lambda}+(1-\alpha)\eta C_X\hat{\theta}+\varepsilon^* Z\hat{\theta}$$

A2) $$=-\sigma C_X\hat{\lambda}+(1-\alpha)\eta C_X\hat{\theta}+\varepsilon^*\theta C_Y\hat{\theta}$$

$$=-\sigma\alpha\hat{\lambda}+(1-\alpha)\eta\alpha\hat{\theta}+\varepsilon^*(1-\alpha)\hat{\theta}$$

where the second equality holds under the assumption that trade is initially balanced, and the third equality merely divides the second by total domestic expenditure. Equation (8') then follows by trivial rearrangement of the above.

The other comparative static results should be transparent. We now prove saddle-path stability of the system of equations summarized in (10) and (13). Under the assumption that trade is initially balanced and that initial holdings of foreign assets are negligibly small, the linearization of (10) is as follows:

A3) $$\dot{A}=r^*\hat{A}+\frac{E'(\lambda)}{\theta}\hat{\lambda}$$

where carets now denote linear deviations from the initial steady state. Linearizing (13), we obtain:

A4) $\dot{\lambda} = k\rho'(\lambda)\hat{\lambda}$

where k is the positive constant in equation (13), and it will be remembered that $\rho'(\lambda) < 0$. We summarize the dynamics as follows:

A5)
$$\begin{bmatrix} \dot{A} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} r* & E'(\lambda)/\theta \\ 0 & k\rho'(\lambda) \end{bmatrix} \begin{bmatrix} \hat{A} \\ \hat{\lambda} \end{bmatrix}$$

Where derivatives in the first matrix on the right side of (A5) are of course to be evaluated at the steady state of interest. The determinant of the matrix in (A5) is $r^*k\rho'(\lambda) < 0$ which, being negative, informs us that one eigenvalue is positive and one negative. Notice that the sign of $E'(\lambda)$ does not affect this property of equilibrium; as noted in the paper, saddle-path stability is ensured whether the Marshall-Lerner condition is satisfied or not.

The comparative static properties of the model depend upon the slope of the saddle path, which is in turn given by the eigenvector corresponding to the stable root. This root is equal to $k\rho'(\lambda)$. It is trivial to show that the eigenvector corresponding to this stable root is:

$$e \equiv \frac{(r^* - k\rho'(\lambda))\theta}{E'(\lambda)}$$

The numerator of this expression is positive, so the eigenvector is negative in the case in which the Marshall-Lerner condition is satisfied ($E'(\lambda) < 0$), and conversely.

We have only to derive the Pitchford (1990) model more fully. Using the notation established in the paper, the consumer's problem is:

A6)
$$\max \int_t^\infty V(C_s)e^{-\rho(s-t)}ds$$

$$s.t. \dot{A} = r^*(A)A + (X - \pi(\theta)C)/\theta$$

$$C_s = C(C_X(s), C_Y(s))$$

The associated (present-value) Hamiltonian is:

$$H = V(C) + \gamma(r^*(A)A + (X - \pi(\theta)C)/\theta)$$

and the first order conditions are:

A7) $V'(C) = \gamma\pi(\theta)/\theta \equiv \lambda$

A8) $\dot{\gamma} = \gamma(\rho - r^*(A))$

A9) $\dfrac{C_Y}{C_X} = f(\theta)$

As before, optimal consumption depends upon λ, and the partial-equilibrium comparative statics are identical to those in the Uzawa model, above. This means that equation (10) applies exactly, except for the dependence of r^* on A, as in (15). Equation (14) is derived in exactly the same way as was equation (13). The linearized dynamic system is:

A10)
$$\begin{bmatrix} \dot{A} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} (r^* + r^{*\prime}(A)A) & E'(\lambda)/\theta \\ -r^{*\prime}(A) & 0 \end{bmatrix} \begin{bmatrix} \hat{A} \\ \hat{\lambda} \end{bmatrix}$$

The trace of this matrix is positive, so that at least one eigenvalue is positive. The determinant is $r^{*\prime}(A)E'(\lambda)/\theta$, which is negative when $E'(\lambda) < 0$ and positive otherwise. It follows that when the Marshall-Lerner condition is satisfied ($E'(\lambda) < 0$) the system is saddle-path stable, and when the Marshall-Lerner condition is not satisfied ($E'(\lambda) > 0$) (the real parts of) both roots are positive, signifying instability.

B. The finite horizon model

We provide here a more complete derivation of the partial-equilibrium comparative static results contained in the body of the paper. We begin with domestic demand for the domestic good, given by (17). The elasticity of demand with respect of wealth is clearly unity. The price elasticity is derived as follows:

A11)
$$dC_X = \left\{\frac{1}{(1+\theta f(\theta))} - \frac{\theta}{(1+\theta f(\theta))^2}(f(\theta)+\theta f'(\theta))\right\}(\rho+\delta)(A+H)d\theta$$
$$= \left\{1 - \frac{\theta}{(1+\theta f(\theta))}(f(\theta)+\theta f'(\theta))\right\}\left\{\frac{\theta}{(1+\theta f(\theta))}\right\}(\rho+\delta)(A+H)\frac{d\theta}{\theta}$$
$$= \left\{1 - \frac{\theta f(\theta)}{(1+\theta f(\theta))}\left(1+\frac{\theta f'(\theta))}{f(\theta)}\right)\right\}C_x\hat{\theta}$$
$$= \{1-(1-\alpha)(1-\eta)\}C_x\hat{\theta}$$
$$= \{\alpha+(1-\alpha)\eta\}C_x\hat{\theta}$$

Totally differentiating the goods-market equilibrium condition (19), we obtain:

A12)
$$0 = \{\alpha+(1-\alpha)\eta\}C_x\hat{\theta} + C_X\left(\frac{dA+dH}{A+H}\right) + \varepsilon^* Z\hat{\theta}$$
$$= \{\varepsilon^*(1-\alpha)+\alpha[\alpha+(1-\alpha)\eta]\}\hat{\theta} + \alpha\left(\frac{dA+dH}{A+H}\right)$$

where the second inequality assumes that trade is balanced in the initial, steady-state equilibrium. Remembering that $\varepsilon = \alpha+(1-\alpha)\eta$, minor rearrangement of (A12) leads directly to (19') in the text.

We need now to derive the parameter a which plays such a central role in the text.

A13)
$$a \equiv \frac{d(\beta X/\theta(A+H))}{d(A+H)}$$
$$= -\frac{\beta X}{\theta^2}\theta'(A+H)$$
$$= \frac{\beta X}{\theta^2}\left\{\frac{\alpha}{\alpha+(1-\alpha)(\varepsilon+\varepsilon^*-1)}\right\}\frac{\theta}{A+H}$$
$$= \frac{\beta X}{\theta}\left\{\frac{\alpha}{\alpha+(1-\alpha)(\varepsilon+\varepsilon^*-1)}\right\}\frac{1}{A+H}$$
$$= (r^*+\delta)\left\{\frac{H}{A+H}\right\}\left\{\frac{\alpha}{\alpha+(1-\alpha)(\varepsilon+\varepsilon^*-1)}\right\}$$

where the last equality holds because, in steady state:

$$H = \left(\frac{\beta X/\theta}{r^* + \delta} \right)$$

as can be seen be setting (15') equal to zero. Similarly setting (25) equal to zero and rearranging, we obtain:

A14) $$\left(\frac{H}{A + H} \right) = \left(\frac{\rho + \delta - r^*}{\delta} \right)$$

Substituting (A14) into (A13) we obtain the expression for a which is used in the paper.

We end with a brief digression on the world interest rate. A condition for a steady-state equilibrium to exist (see Blanchard (1985)) is:

$$\rho < r^* < (\rho + \delta)$$

and we imposed this condition in the paper. Note that as the index of mortality, δ, goes to zero the interest rate must approach the pure rate of time preference, ρ for a steady state to exist. Strictly speaking, the stability condition which we derived applies only when the trade balance is zero in the original steady state. This, in turn, holds only when the (endogenously determined) level of net foreign assets is zero. This, in turn, is guaranteed if domestic and foreign consumers share the same rates of time preference and mortality index, and the share of labor in national income is the same in both countries.

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