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THE ROLE OF FISCAL POLICY IN AN INCOMPLETE MARKETS FRAMEWORK

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Abstract

A general-equilibrium model is developed to highlight the link between neo-Keynesian models of unemployment and recent results on the constrained suboptimality of competitive economies with incomplete asset markets. Although the model deviates from the Arrow-Debreu paradigm only by the absence of some contingent claims, the competitive equilibrium exhibits under-employment and balanced-budget fiscal policies have Keynesian effects which are Pareto improving.

The Role of Fiscal Policy in an Incomplete Markets Framework

Charles P. Thomas[1]

I. Introduction

Will a market economy, left to itself, fully employ its resources? In The General Theory, Keynes exploited several mechanisms through which real economies differ from the classical paradigm where unemployment is either voluntary or non-existent. A common characteristic of these mechanisms is the difficulty of coordinating production and consumption plans in a decentralized economy. In recent years neo-Keynesians have sought to develop general-equilibrium models from micro foundations where this coordination problem can be analyzed. These efforts are of interest for two reasons. First, as an intellectual exercise it is of interest to explore which deviations from the classical paradigm are necessary and/or sufficient to generate Keynesian effects. Second, from a policy perspective, it is important to start from micro foundations to verify the welfare implications of policy.

The neo-Keynesian literature has gone in several directions. We will not review them here.[2] At this point it is sufficient to note that the argument developed below is based on incomplete financial markets and thus is related to the asymmetric information models of Greenwald and Stiglitz.[3]

The main contribution of this paper is the development of a model where we can make the following three points: 1) The Keynesian coordination problem can arise naturally in a competitive system with perfectly flexible prices and rational expectations. 2) The mechanism underlying the coordination problem is related to recent work on the constrained suboptimality of competitive equilibria with incomplete markets. As such, the coordination problem is generic to this and other models with incomplete financial markets. 3) Familiar, and feasible, policies can be welfare improving even though they do not directly address the fundamental microeconomic causes of the coordination failure. We study in detail the welfare effects of two fiscal policies and discuss briefly how other policies can be studied within this model.

The paper is organized as follows: The rest of the introduction contains a review of recent results on the constrained suboptimality of economies with incomplete markets along with an overview of the model and argument. Section II lays out the formal model and computes its equilibrium. Section III discusses why the constrained suboptimality takes the form of under-employment and why agents do not act to alleviate the underemployment. Section IV looks at the welfare effects of two balanced-budget fiscal policies and shows they can be Pareto improving. Section V outlines how other policies can be studied within this framework and concludes.

Constrained Suboptimality

In 1986 Geanakoplos and Polemarchakis (GP) established that with incomplete financial markets the competitive allocation is constrained suboptimal (CS) in the following sense: If we consider the set of allocations that can be obtained by arbitrary trades in the existing (incomplete) assets and competitive trades in the goods markets, the competitive allocation is Pareto dominated by some other allocation in this set. It is important to note that we are not ranking the competitive allocation against those that could be obtained in a complete-market setting. Instead we are ranking it against those that can be obtained with the existing markets. That is, competitive agents do not efficiently use the markets they have. GP (1986) showed that this inefficiency is generic for exchange economies with two or more agents and two or more goods in each state. A similar result for production economies with stock markets was obtained by Geanakoplos, Magill, Quinzii and Dreze (1990).[4]

The intuition behind this suboptimality is straightforward, but focuses on a channel often ignored by macro economists.

> "...[F]or generic utilities (for which different agents have different marginal propensities to consume), a redistribution of assets redistributes wealth in any given state, leading to a change in relative prices. This relative price change yields yet another redistribution of wealth which is not necessarily achievable through the [asset] market. By judiciously choosing the right portfolio adjustments, the government can use the pecuniary externality to make everyone better off."[5]

Since this suboptimality is generic, it would appear to provide a wide avenue for policy. Its policy relevance can be challenged on two, so far unanswered, grounds, however. First, there is the information-based question of how the policy maker could learn enough about preferences to devise a useful policy. GP (1990) show that existing asset and goods prices do not reveal enough information about preferences to construct the Pareto improving trades. Second, even if the preferable trades could be discerned, there is a "feasability" question, since the direct application of the suboptimality result would suggest that a market be closed and agents be forced to make trades that they choose not to make on their own.

The model developed below addresses both of these questions. The first is addressed by showing that, for standard preferences, the suboptimality can manifest itself in terms of underemployment and the mechanism behind this underemployment can be understood with the more familiar intuition from neo-Keynesian models. As such, policy makers are able to consider policies that use this mechanism even though they cannot know everyone's preferences. The second issue, that it is infeasible to close some markets and force trades on people, is addressed more directly. We show that without closing any markets or imposing any trades two standard fiscal policies can alleviate some of the inefficiency introduced by incomplete asset markets.

The strategy of what follows is to develop a static two-period economy with incomplete asset markets where the constrained suboptimality has a clean, macro interpretation in terms of equilibrium underemployment. We then show how balanced-budget fiscal policies, within the given market structure, have Keynesian effects and can be welfare improving.

Overview of the Argument

Before presenting the formal model, it is useful to have an overview of the model structure and the argument. The setting is a two-period production economy with demand uncertainty arising from a random element in preferences. Production takes time which forces producers to hire labor, and thus commit themselves to a wage bill, before they learn what the demand for their product is. Bankruptcy is prohibited, so we require that the revenue from sales in each state be sufficient to meet the pre-committed wage bill. The producer does not consume the good he produces nor does he derive any direct (dis)utility from the production process. Consumers provide the labor in the production process.

The only financial asset is a riskless bond that producers issue in exchange for labor services. As such, there are no assets for trading demand risk. From the constrained sub-optimality results we know that the competitive trade in the existing asset will not be efficient. That is, if the competitive trade of bonds for labor were replaced with another, using the same bonds, the allocation from subsequent clearing of competitive markets would Pareto dominate the allocation from competitive trades in both the labor and goods markets. In this model, the superior trade includes a higher level of employment. Thus we can interpret the inefficiency as a form of equilibrium underemployment.

The underemployment arises in a straightforward way. The asset structure does not allow producers to insure against the demand shocks, which leads to uncertainty in revenue and profits. When producers are risk averse, this revenue uncertainty induces them to limit production to a level below the benchmark level where marginal cost equals expected price. Even if they are risk neutral, the possibility that revenues can fall short of the wage bill can limit production to a level below this benchmark level.

This benchmark level of production--where marginal cost equals expected price--is of interest because it represents the point at which the incremental disutility to the consumer from the labor required to produce a marginal unit of production (the incremental wage bill) equals the expected utility from having a marginal unit of the produced good available for consumption (expected price). Since the consumer is the only agent who derives direct disutility from the production process and he is also the only agent who derives direct utility from consuming the produced good, it is clear that a higher level of production would make him better off whenever expected marginal profit is positive.

Without the constrained suboptimality results we might be inclined to attribute this to the general inefficiencies arising from incomplete asset markets: Agents are unable to transfer risk fully which has general equilibrium effects and the lower level of employment is one manifestation of them. But the CS results indicate that agents could do better with the assets they have. In this setting better includes a higher level of employment. In particular, the consumer would be better off if the competitive bond-for-labor trade were replaced with one where more labor is traded for the same number of, or slightly fewer, bonds.

From a policy perspective this may seem to be a useless result since there is no apparatus for closing the labor market and forcing workers to supply more labor than they want at an "official" wage rate. There are, however, feasible policies which have much the same effect. In particular an ex-ante transfer from the consumer to the producer or a balanced-budget fiscal-spending policy which is financed by a lump-sum tax on the consumer can be Pareto improving by exploiting these same inefficiencies. Two such policies are explored below.

II. Formal Model

Basic Structure

Consider a two-period, two-agent production economy with an aggregate producer and an aggregate consumer. In the first period, labor/leisure is the only factor or good. In the second period, there are two goods -- a produced good (G) and a non-produced good (N).[6] There are S second-period states of nature which correspond to a random element of the consumer's preferences. This is the only exogenous random element in the model.

The Consumer

The consumer is endowed with one unit of leisure and e_c units of N. His preferences, which are intertemporally separable and stochastic, are described by the following two-period utility function:

$$u_{cs}[L, c_{cs}, x_{cs}] = \ln[1-L] + \rho \cdot ((1-\alpha_s) \cdot \ln[c_{cs}] + \alpha_s \cdot \ln[x_{cs}])$$

where $1\text{-}L$ = First-period leisure;

α_s = A random variable where $s \in \mathbf{S} = \{1,2,\ldots,S\}$ and

$\alpha_s \in [\alpha_-, \alpha_+], 0 < \alpha_- < \alpha_+ < 1$;

c_{cs} = Consumption of N in state s;

x_{cs} = Consumption of G in state s;

ρ = A positive time discount factor.

State s occurs with probability π_s.

The realization of α is learned at the beginning of the second period. The consumer knows the true state probabilities and satisfies the expected utility hypothesis.

The Producer

The producer is endowed with e_p units of N and no leisure. His preferences are described by the one-period utility function $u_p[c_{ps}]$ where c_{ps} is his consumption of N. We assume that $u_p' > 0$, $u_p'' \leq 0$, and that u_p' and u_p'' are continuous. He derives no utility from the consumption of G. He is the sole owner and manager of a firm which produces G; there is no stock market. He hires labor in the first period to produce G which he sells in the second period. The firm is useful to him only to the extent that it generates income in terms of the non-produced good.

The Technology

G is produced by the following technology: $q_s = q[L] = L^{1/a}$, $a > 1$. That is, production is non-stochastic, labor is the only factor, and there are diminishing marginal returns.

The Market Structure

In the first period the sole financial asset is a risk-free bond that pays one unit of N in the second period regardless of the state. It is traded against the other first-period good, labor. The bond's price is normalized to one and the wage rate, in terms of the bond, is w. There is no exogenous supply of the bonds and in equilibrium the producer issues them to the consumer in exchange for labor services.

In the second period there is a spot market where N is traded for G. The price of N is normalized to one in each state and the price of G in state s is denoted by p_s.

Some additional notation is helpful. Let

$\mathbf{p} \equiv [p_1,\ldots,p_s,\ldots,p_S]$ Vector of state-specific G prices;

$\bar{\alpha} \equiv E[\alpha] = \Sigma_s \pi_s \alpha_s$ Expectation of α over states;

$\omega \equiv w \cdot L$ Wage bill in terms of N;

$I \equiv \omega + e_c$ Consumer second-period income in terms of N;

$Y_s \equiv p_s \cdot q - \omega + e_p$ Producer income in terms of N.

Consumer Behavior

The consumer acts to maximize expected utility of the form:

$$V_c = E[u_{cs}[L, c_{cs}, x_{cs}]] = E\{\ln[1\text{-}L] + \rho \cdot ((1\text{-}\alpha_s) \cdot \ln[c_{cs}] + \alpha_s \cdot \ln[x_{cs}])\}$$

subject to:

$$0 \leq L \leq 1;$$

$$I = w \cdot L + e_c;$$

$$0 \leq I - p_s \cdot x_{cs} - c_{cs} \ \forall s \in \mathbf{S};$$

$$0 \leq c_{cs},\ 0 \leq x_{cs} \quad \forall s \in \mathbf{S}.$$

When the budget constraints are inserted into the objective function, it becomes:

(C1) $$V_c = E\{\ln[1\text{-}L] + \rho \cdot ((1\text{-}\alpha_s) \cdot \ln[I - p_s \cdot c_{cs}] + \alpha_s \cdot \ln[x_{cs}])\}.$$

This is a two-period dynamic-programming problem. The FOCs yield

(C2) $$x_{cs} = I \cdot \alpha_s / p_s$$

as the consumer's demand for G. Substituting this x_{cs} into the first-period problem and maximizing with respect to L yields the following labor-supply schedule:

(C3) $$L^S = (w \cdot \rho - e_c)/[w \cdot (1+\rho)].$$

This can be expressed as a function of the wage bill alone:

(C4) $$L^S[\omega] = \rho \cdot \omega/[\omega \cdot (1 + \rho) + e_c].$$

Producer Behavior

In the second period the producer sells all of the produced good at price p_s and spends all of his income on N. In the first period, he chooses the level of production, q -- a choice which depends on the the wage rate, his preferences, and the distribution of prices. His first-period problem is to choose q to maximize

$$V_p = E\{u_p[c_{ps}]\} \text{ subject to}$$

(PBC) $$0 \leq c_{ps} \leq p_s \cdot q - w \cdot L + e_p \;\; \forall\, s \in \mathbf{S},\, 0 \leq q \leq L^{1/a}.$$

(PBC) is a sequence of budget constraints. The first inequality says that consumption cannot be negative. The second is the usual budget constraint with uncertain prices. Taken together, they form a bankruptcy constraint (BC) which insures that the producer will be able to meet his wage obligations in all states.[7] Later it is shown that in equilibrium $\min_s[p_s]$ is well defined, so there is a production/consumption plan that satisfies (PBC). The third and fourth inequalities represent the technology.

When BC is not binding, the producer's FOCs imply

(P1) $$\tilde{\Phi}[q;\mathbf{p},w] \equiv \Sigma_s \pi_s u'_p[Y_s] \cdot (p_s - w \cdot a \cdot q^{(a-1)}) = 0.$$

The expression in parentheses is the difference between the price of G in state s and its marginal cost, i.e., perceived, state-specific marginal profit. $\tilde{\Phi}[q;\mathbf{p},w]$ is the expected marginal utility from an incremental unit of production for given wages and state-specific prices. Although $\tilde{\Phi}$ is used to characterize interior solutions (BC not binding), it is convenient for it to be defined when income is negative for some states; let $u[Y_s] = u[0] + u'[0] \cdot Y_s$ when $Y_s < 0$.

The solution to (P1) is unique, so we can define $\tilde{q}[\mathbf{p},w]$ to be the zero of $\tilde{\Phi}[q;\mathbf{p},w]$.[8]

When BC is binding, the producer's net income is zero in some state, which means the level of production is determined by his ability to meet the wage bill in that state. This choice is characterized by

(P2) $$\bar{\Phi}[q;\mathbf{p},w] \equiv \min_s[p_s] \cdot q - w \cdot q^a + e_p = 0.$$

In the minimum-price state, the incremental revenue from production is less than the incremental wage bill, so any further production would violate the bankruptcy constraint.

Since the solution to (P2) is also unique, we can define $\bar{q}[\mathbf{p},w]$ to be the zero of $\bar{\Phi}[q;\mathbf{p},w]$ and characterize the solution to the producer's problem as

(P3) $$\overset{*}{q}[\mathbf{p},w] = \min\{\tilde{q}[\mathbf{p},w], \bar{q}[\mathbf{p},w]\}.$$

<u>Computation of the Equilibrium</u>

A perfect stochastic foresight (PSF) equilibrium is an S+1 price vector $(w^*,\mathbf{p}^*)$ such that the wage rate w^* clears the first-period labor market when agents condition on second-period prices $\mathbf{p}^*$, $\mathbf{p}^*$ clears the second-period goods markets with the income and level of production determined in the first period, and agents know the true state probabilities.

Note that w, L, q, and I are determined in the first period, so they are independent of the realization of α. For the G market to clear $x_{cs} = q$ for each s. From (C2),

(E1) $$p_s = (I/q) \cdot \alpha_s$$

which is unique for each s, given the levels of income and production. Substituting for p_s in $\tilde{\Phi}[q;\mathbf{p},w]$ and $\bar{\Phi}[q;\mathbf{p},w]$ yields

(E2) $$\tilde{\Phi}[q;I,w] = \Sigma_s \pi_s u'\{I \cdot \alpha_s - w \cdot L[q] + e_p\} \cdot (I \cdot \alpha_s - a \cdot w \cdot L[q])/q \;;$$

(E3) $$\bar{\Phi}[q;I,w] = \min_s[I \cdot \alpha_s] - w \cdot L[q] + e_p.$$

Since L is monotonic in the wage bill, ω, (see (C4)) and q is monotonic in L, q can be written as a function of ω alone. The wage rate enters (E2) and (E3) through the wage bill alone and consumer income is the wage bill plus endowment. Therefore, $\tilde{\Phi}[q;I,w]$ and $\bar{\Phi}[q;I,w]$ can be written as functions of ω alone.

(E4) $$\tilde{\Phi}[\omega] = \Sigma_s \pi_s u'\{\omega \cdot (\alpha_s - 1) + \alpha_s \cdot e_c + e_p\} \cdot (\omega \cdot (\alpha_s - a) + \alpha_s \cdot e_c)/q[\omega].$$

(E5) $$\begin{aligned}\bar{\Phi}[\omega] &= \omega \cdot (\min_s[\alpha_s] - 1) + \min_s[\alpha_s] \cdot e_c + e_p. \\ &= \omega \cdot (\alpha_- - 1) + \alpha_- \cdot e_c + e_p.\end{aligned}$$

The zeros of (E4) and (E5) are used to characterize the equilibria of the system. Although the producer's choice is unique -- (P1) has a single zero in q -- (E4) can have several zeros. (This is the familiar "third derivative" problem; see Rothschild and Stiglitz (1971).)

Let $$\begin{aligned}\Omega^{\sim} &\equiv \{\omega : \tilde{\Phi}[\omega] = 0\} \text{ and} \\ \omega^{-} &\equiv (\omega : \bar{\Phi}[\omega] = 0) = (\alpha_- \cdot e_c + e_p)/(1-\alpha_-).\end{aligned}$$

$\Omega^{\sim}$ is the set of of wage bills that satisfy the equilibrium conditions without regard for the bankruptcy constraint. ω^{-} is the wage bill at which the bankruptcy constraint is just binding. Finally, let

$$\begin{aligned}\Omega^{*} &\equiv \{\omega \in \Omega^{\sim} : \omega < \omega^{-}\} \qquad \text{if } \tilde{\Phi}[\omega^{-}] < 0 \\ &\equiv \{\omega \in \Omega^{\sim} : \omega < \omega^{-}\} \cup \{\omega^{-}\} \text{ otherwise.}\end{aligned}$$

This is the set of wage bills that satisfy the bankruptcy constraint as well as the FOC for both consumer and producer maximization. The existence of the equilibrium is given by the following claim:

Claim 1: (Existence of PSF Equilibrium)

Ω^{*} is non-empty; each element of Ω^{*} fully characterizes an equilibrium of the system and every equilibrium is characterized by an element of Ω^{*}.

Proof: (See Appendix for details.)

That Ω^{*} is non-empty is proved by showing that $\tilde{\Phi}[\omega]$ is a continuous function that is positive for low values of ω and is negative for high values of ω. That this is an equilibrium drops out directly from the construction of Ω^{*}.

The argument that follows does not depend on the multiplicity of equilibria, so we will deal with the equilibrium that corresponds to the highest level of employment. Let $\omega^{*} \equiv \max[\omega \in \Omega^{*}]$.

Benchmark Case: Risk-Neutral Producers with BC not Binding

Before looking at the properties of this equilibrium, it is useful to have a benchmark case where the producer is risk neutral and the bankruptcy constraint is not binding. It is against the level of employment for this benchmark economy that we measure the equilibrium underemployment. In this special case (E4) becomes

$$(\hat{E4}) \qquad \hat{\Phi}[\omega] = u'_p[] \cdot \Sigma_s \pi_s \cdot (\omega \cdot (\alpha_s - a) + \alpha_s \cdot e_c)/q[\omega].$$

Solving for $\hat{\omega} = (\omega : \hat{\Phi}[\omega] = 0)$ yields

$$(\hat{E5}) \qquad \hat{\omega} = \bar{\alpha} \cdot e_c/(a-\bar{\alpha})$$

Using (C4), the level of employment is given by

$$(\hat{E6}) \qquad \hat{L} = \rho \cdot \bar{\alpha}/(a + \rho \cdot \bar{\alpha}).$$

Note that this level of employment is independent of the producer's preferences and both agents' endowments. It can be shown that this is also the level of employment when there is a complete set of financial markets, which implies it is the level of employment at a fully Pareto efficient allocation.[9] It is understandable that only the consumer's preferences enter into the determination of the fully Pareto efficient level of production, since the consumer is the only agent who is made worse off by not consuming more of the factor of production and he is also the only agent who derives utility from the consumption of the produced good.

It is a straight-forward calculation to show that at $\hat{L}$,$E[p] = a \cdot w \cdot q^{a-1}$. That is, expected price equals marginal cost when the producer is risk neutral (RN) and the BC is not binding.

Level of Employment Relative to Benchmark

The significance of the benchmark level of employment is discussed in the next subsection. At this point it is useful to establish that in general the equilibrium level of employment will be less than that of the benchmark case. This is done in claim 2. Part (i) of Claim 2 is that producer risk-aversion is sufficient to yield employment below $\hat{L}$; part (ii) is that risk-aversion is not necessary, because of the bankruptcy constraint.

Claim 2: (Employment is Below the Benchmark Level)

i) When the producer is risk-averse, ω^* is less than $\hat{\omega}$ and $L[\omega^*] < \hat{L}$. ii) When the producer is risk neutral, there are endowments and consumer α_s's such that the bankruptcy constraint is binding and $L[\omega^*] < \hat{L}$.

Proof: (See Appendix details)

Part i: At $\hat{\omega}$, expected price equals marginal cost. For a risk-averse producer this implies that $\tilde{\Phi}[\omega]$ is negative at $\hat{\omega}$, since a risk-averse agent will avoid what he perceives as a fair bet when it increases the volatility of his income. Since $\tilde{\Phi}[\omega]$ is negative at $\hat{\omega}$, $\tilde{\Phi}[\omega]$ crosses through zero to the left of $\hat{\omega}$, and $\max[\omega \in \Omega^*]$ is less than $\hat{\omega}$. Since labor-supply is monotonic in the wage bill, the level of employment at this wage bill is below the level of employment at $\hat{\omega}$.

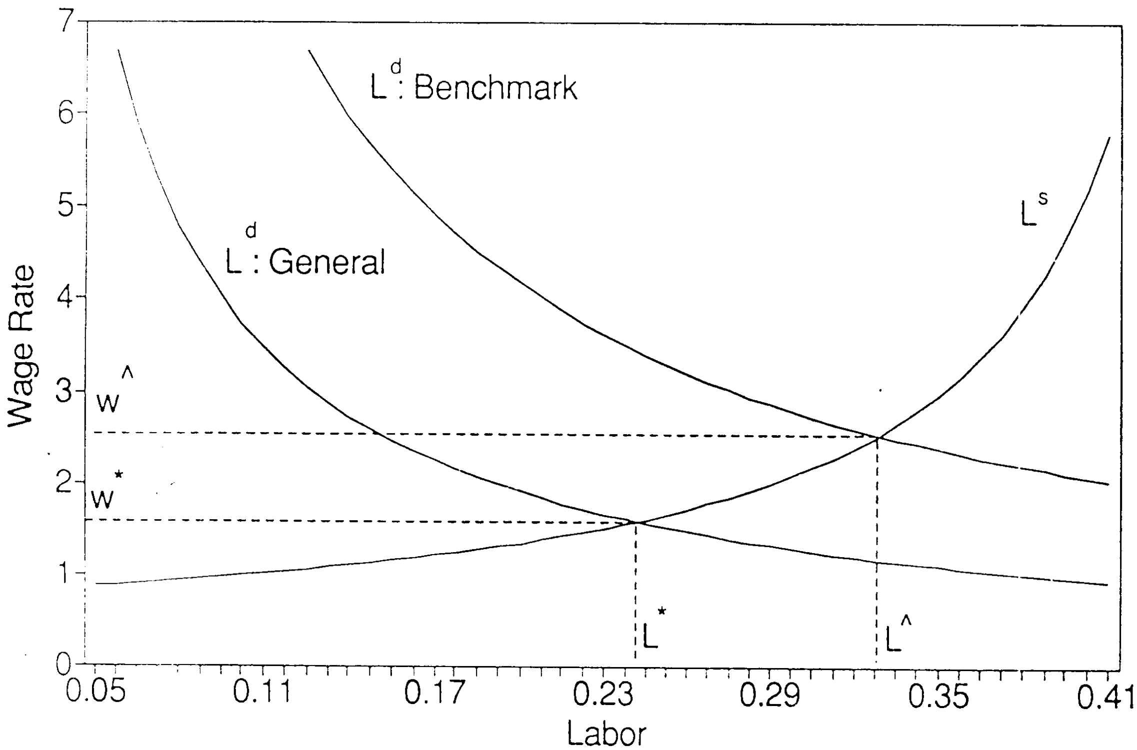
Part ii: The wage bill where the bankruptcy constraint binds, $(\alpha_- \cdot e_c + e_p)/(1 - \alpha_-)$, is independent of producer preferences. Even if the producer is risk neutral, if $(\alpha_- \cdot e_c + e_p)/(1-\alpha_-) < \bar{\alpha} \cdot e_c/(a-\bar{\alpha})$, the bankruptcy constraint will bind and employment will be below $\hat{L}$. By choosing endowments and consumer preferences appropriately we can clearly make this hold.[10]

Labor Market

The wage bill ω^* represents the employment/wage combinations consistent with the producer's maximization. It is, then, a hyperbolic labor-demand schedule. Equation (C3) is a labor-supply schedule. Figure 1 depicts the labor market for the benchmark and general cases. Since ω^* is less than $\hat{\omega}$, L^* is to the left of $\hat{L}$.

Figure 1

Labor Market
Benchmark and General Cases



III: Underemployment and Constrained Suboptimality in the Model

The fact that the level of employment is higher when the producer is risk neutral and BC is not binding than it is when the producer is risk averse or the BC is binding is not, in itself, any indication of inefficiency. It could reflect the best agents can do with the markets they have. In this section we show why a level of employment below $\hat{L}$ is indeed an indication of inefficiency. In particular, we show that there are trades other than the competitive ones that the agents could make with the existing assets that yield allocations that Pareto dominate those of the competitive equilibrium. Claim 3 establishes the constrained suboptimality of the equilibrium.

Claim 3: (The Equilibrium is Constrained Suboptimal)

Let L^* and ω^* be the level of employment and the wage bill in the above economy.

When the producer is risk averse or BC is binding, there is a first-period trade $(L^{\#},\omega^{\#})$ such that the allocation which results from the competitive clearing of subsequent markets Pareto dominates the allocation arising from (L^{*},ω^{*}) and subsequent market clearing.

Proof:

To prove this, we compute the ex-ante welfare of the agents in terms of employment and income -- claims to N -- when period-two markets clear competitively. We then show that for unchanged claims to N, an increase in the level of production makes the consumer better off whenever $L^{*} < \hat{L}$. It is then shown that for the risk-averse or BC cases, there is a first period trade $(L^{\#}, \omega^{*})$ with $L^{*} < L^{\#} < \hat{L}$ which makes the consumer better off without changing the producer's welfare. Finally, it is argued that a small decrease in ω allows the producer to share in this increase in welfare while keeping the consumer's welfare above the competitive level.

Consumer Welfare

Consumer ex-ante welfare is given by

(W1) $$W_c = \ln[1-L] + \rho \cdot E\{(1-\alpha_s) \cdot \ln[c_{cs}] + \alpha_s \cdot \ln[x_{cs}]\}$$

When the second-period allocations are determined by market clearing, the following obtain: $c_{cs} = I \cdot (1-\alpha_s)$ and $x_{cs} = L^{1/a}$. Substituting these into (W1) yields

(W2) $$W_c = \ln[1-L] + \rho \cdot E\{(1-\alpha_s) \cdot \ln[I \cdot (1-\alpha_s)] + \alpha_s \cdot \ln[L^{1/a}]\}.$$

This is an expression for the consumer's welfare in terms of his claims to N and the amount of labor supplied. Taking the total differential of W2 yields

(W3) $$dW_c = \partial W_c/\partial L \cdot dL + \partial W_c/\partial I \cdot dI \quad \text{where}$$

$$\partial W_c/\partial L = -1/(1-L) + \rho \cdot \bar{\alpha} /(a \cdot L) \lesseqgtr 0 ;$$

$$\partial W_c/\partial I = \rho \cdot (1-\bar{\alpha}) /I > 0 .$$

With a little algebra it is easily seen that

(W4) $\quad (L < \rho \cdot \bar{\alpha}/(a + \rho \cdot \bar{\alpha})) \Rightarrow (\partial W_c/\partial L > 0).$

The key to the suboptimality result is to note that the critical value of L in (W4) is the $L^{\wedge}$ from the benchmark case where the producer is risk neutral and the BC is not binding.

Producer Welfare

Ex-ante producer welfare, when the period-two markets clear, is given by

(W5) $\quad W_p = E[u_p[Y_s]] = E\{u_p[\omega \cdot (\alpha_s - 1) + \alpha_s \cdot e_c + e_p]\}$

Taking the total differential yields

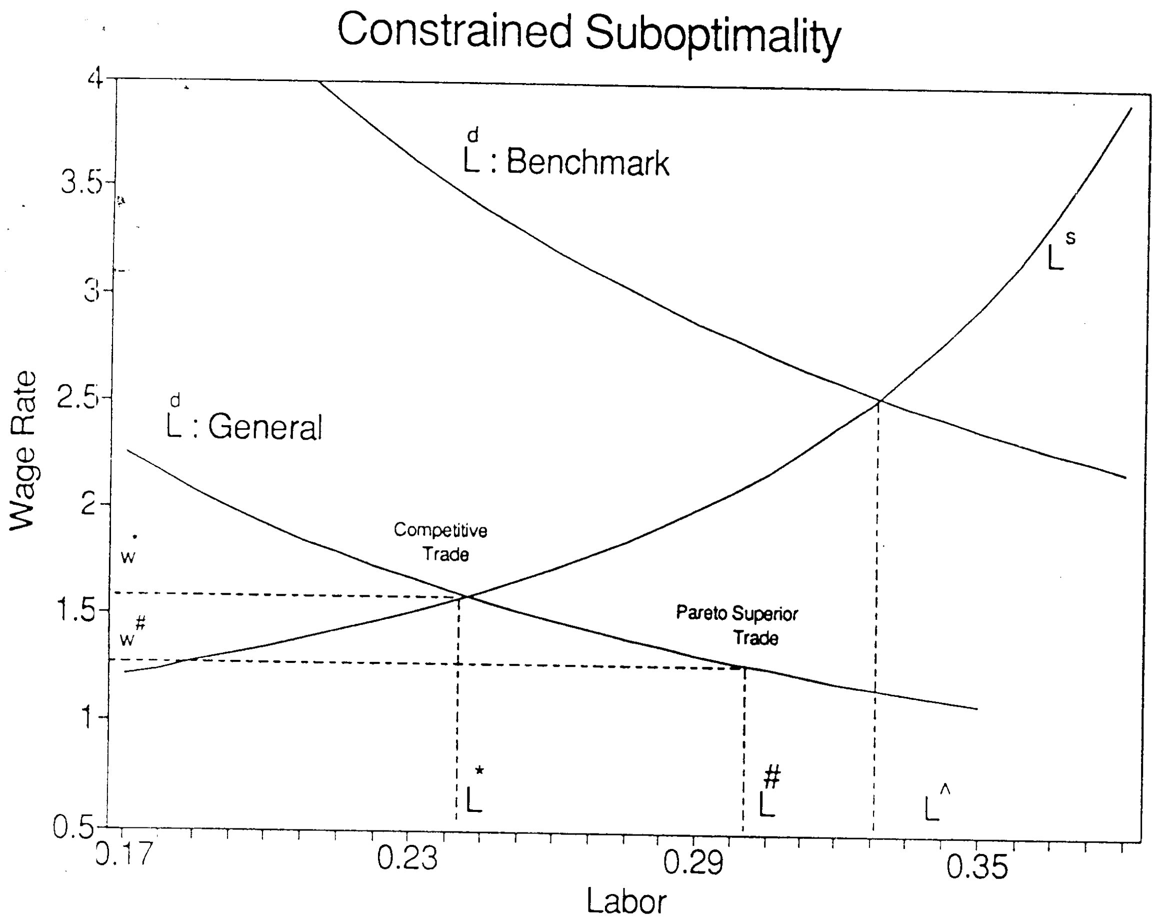
$$\partial W_p/\partial \omega = E\{u'[Y_s] \cdot (\alpha_s - 1)\} < 0$$
$$\partial W_p/\partial e_c = E\{u'[Y_s] \cdot \alpha_s\} > 0$$
$$\partial W_p/\partial e_p = E\{u'[Y_s]\} > 0.$$

Suboptimality

From claim 2, we know that if the producer is risk averse or the BC is binding, then L^* is less than $L^{\wedge}$. Let $L^{\#}$ be any value between L^* and $L^{\wedge}$. Since $L < L^{\wedge}$ implies that $\partial W_c/\partial L$ is positive, the consumer is made better off if the competitive trade in period one is replaced by ($L^{\#}$, ω^*). Since L does not enter directly into W_p and ω, e_c, and e_p are unchanged, the producer's utility is unchanged. From the continuity of the utility functions, there is some positive ε such that ($L^{\#}$, $\omega^* - \varepsilon$) yields an allocation which both the consumer and producer prefer to the competitive allocation. This proves the constrained suboptimality of the competitive trades.

Figure 2 depicts the weakly-Pareto superior trade.

Figure 2



Interpretation of Underemployment

One interpretation of this suboptimality is that the consumer's wage schedule is too high -- he demands too high a wage for any given amount of labor supplied. For the weakly Pareto improving trade of $(L^{\#}, \omega^{*})$, the "wage rate" is $w^{\#} \equiv \omega^{*}/L^{\#}$, which is less than the wage under competitive market clearing. Even so, consumer welfare is higher under the alternative labor agreement.

The microeconomic intuition for this is clear. When the worker supplies more labor for an unchanged wage bill, the level of production rises and consumer income, in terms of N, is unchanged. The Cobb-Douglas preferences imply that the shares of income spent on G and N will remain unchanged, so the price of G falls in proportion to the increase in production. The net effect for the consumer is less leisure, more consumption of G, and unchanged consumption of N. So long as L is less than $\hat{L}$, consumer welfare is improved by giving up more labor in exchange for its marginal physical product in G. For the producer, the wage bill and all state-specific revenues are unchanged, so producer profit and welfare are unchanged in each state.

The macroeconomic intuition is also clear and echoes many of the neo-Keynesian arguments. In a decentralized economy, production and consumption decisions are made by different people at different times. Production decisions require a commitment of resources and are made in anticipation of the future consumption decisions to be made by others. Without a mechanism by which employees can, at the production stage, commit to later consumption, the direct link between productive activity and future consumption opportunities is broken. Once this link is broken, financial market imperfections, either in the form of financing constraints or inadequate vehicles for risk sharing, drive a wedge between the wage rate and the value of productive activity. The result is a level of activity below that which agents would prefer.

Persistence of Underemployment

Why do agents not make the Pareto-superior trade themselves and eliminate the underemployment? There are at least two reasons for this. The first is the usual competitive assumption that agents do not know how their actions affect prices. The consumer will voluntarily supply the requisite labor at the lower wage only if he believes that the amount he works affects the second-period price.

The second reason is that even if the consumer knew how his labor supply affected second-period prices, the Pareto-superior trades will not be stable when the economy is replicated to include many identical consumers and producers. Bester (1984) and Repullo (1988) have shown that in exchange economies with incomplete securities markets the competitive equilibrium will coincide with the core after sufficient replication. If this economy were replicated many times, the first-period trade of $(L^{\#}, \omega^{*})$ would not be stable. A coalition of workers and (a smaller number of) producers could defect from the rest and make a trade closer to (L^{*}, ω^{*}). By choosing the proportion of consumers and producers in the coalition appropriately we can leave each producer's level of production and wage bill the same as it would have been at $(L^{\#}, \omega^{*})$, but at the same time give the workers the w^{*} wage rate with less than the $L^{\#}$ labor supply. From the labor supply curve, we know that the workers would prefer this trade to $(L^{\#}, \omega^{*})$. Thus the $(L^{\#}, \omega^{*})$ trade is not stable to free coalition formation.

This argument is similar to the one Keynes (1936, Chapt. 19) gives as to why workers who negotiate their contracts at different times cannot change the real wage when it is an effective way to increase employment.

Policy Relevance

Since constrained suboptimality is a generic result, we can expect that the kind of inefficiency described above is pervasive. It is a separate question as to whether this insight is useful for policy. It would seem not for two reasons. First, it is not clear how the planner could learn of this superior trade. Second, the direct implementation of the superior trade would require that the labor market be closed and that people be forced to work beyond their will.

To show that the competitive trades were suboptimal we used the individual utility functions. In practice the planner cannot know these functions and, as discussed earlier, the planner cannot learn enough about preferences from the observed asset demands to discern what the superior trades in the asset market are. But this same objection can be raised about any macroeconomic policy analysis that starts with individual utility. Economists presume to have some understanding of the structure of the economy and develop models to highlight certain aspects of it. The individual utility is used both to motivate individual actions and to verify that welfare gains from policy are possible. If the analysis is relevant, in that it highlights important problems with the economy, then the usefulness of policy will be robust to errors in the specification of individual utility. As is shown below, a general understanding of the source of inefficiency can be sufficient to find an improving policy even if it is not sufficient to find an optimal one.

The second objection, that to implement the policy requires forcing trades on some agents, may seem more daunting. In the next section we show how policies which leave the labor market to clear competitively can be useful. In particular, feasible government redistribution and spending policies have Pareto improving effects because they act to mitigate the inefficiency described above.

IV. Effectiveness of Fiscal Policy

We consider two types of fiscal policies, both of which are state independent. The first is a transfer policy implemented by an ex-ante lump-sum transfer of endowments from the consumer to the producer. The second is a balanced-budget spending policy where the government imposes a fixed lump-sum tax on the consumer, spends all of the tax revenue on the produced good, and throws its purchases away. That is, although we attach no social utility to government consumption, the government spending is still useful.

Transfer Policy

The transfer is fully anticipated before the first-period labor market opens and agents condition on the post-transfer equilibrium prices. The first effect of the transfer is purely accounting -- it lowers e_c and raises e_p by the same amount. The decrease in e_c lowers the labor-supply schedule which, *ceteris paribus*, increases the level of production.

In general, the transfer will also shift the labor-demand schedule. The direction and size of this shift is indeterminate, which makes the net effect on employment ambiguous.[11]

To sort out these effects and show how the tax can improve the welfare of both agents, we look at two cases. In the first case the BC is binding and the change in ω resulting from the transfer can be computed directly. We show that whenever the BC is binding the transfer makes the consumer better off and leaves producer welfare unchanged. For the second case, where the BC is not binding, we compute an example to show that when the producer's preferences are logarithmic, a small transfer can increase both agents' welfare while a large transfer makes the consumer worse off.

Welfare

The change in consumer welfare from a change in the endowments of N can be written as

(F1)
$$dW_c = \partial W_c/\partial L \cdot (\partial L^s/\partial\omega) \cdot (\partial\omega/\partial e_c \cdot de_c + \partial\omega/\partial e_p \cdot de_p) +$$
$$\partial W_c/\partial L \cdot (\partial L^s/\partial e_c) \cdot de_c +$$
$$\partial W_c/\partial I \cdot (\partial\omega/\partial e_c + 1) \cdot de_c +$$
$$\partial W_c/\partial I \cdot (\partial\omega/\partial e_p) \cdot de_c$$

where $\partial W_c/\partial L = -1/(1-L) + \rho \cdot \bar{\alpha}/(a \cdot L)$ and $\partial W_c/I = \rho \cdot (1-\bar{\alpha})/I$.

The analogous expression for producer welfare is

(F2)
$$dW_p = \partial W_p/\partial\omega \cdot (\partial\omega/\partial e_c \cdot de_c + \partial\omega/\partial e_p \cdot de_p) +$$
$$\partial W_p/\partial e_c \cdot e_c +$$
$$\partial W_p/\partial e_p \cdot e_p$$

where $\partial W_p/\partial\omega = E[u'[Y_s] \cdot (\alpha_s - 1)]$, $\partial W_p/\partial e_c = E[u'[Y_s] \cdot \alpha_s]$ and $\partial W_p/\partial e_p = E[u'[Y_s]]$.

The policy we are considering is an ex-ante redistribution of second-period endowments, i.e., $de_c = -de_p$. Inserting this equality into (F1) and (F2) and simplifying yields

(F3)
$$dW_c = \partial W_c/\partial L \cdot (\partial L^s/\partial\omega \cdot (\partial\omega/e_p - \partial\omega/\partial e_c) - \partial L^s/\partial e_c) \cdot de_p +$$
$$\partial W_c/\partial I \cdot (\partial\omega/\partial e_p - \partial\omega/\partial e_c - 1) \cdot de_p ;$$

(F4)
$$dW_p = E[u'[Y_s] \cdot (1 - \alpha_s)] \cdot (1 - \partial\omega/e_p - \partial\omega/\partial e_c) \cdot de_p .$$

Case One: Bankruptcy Constraint Binding

Claim 4: (With BC Binding, Transfer is Pareto Improving)

When the BC is binding, an ex-ante transfer of endowments from the consumer to the producer increases consumer welfare and leaves producer welfare unchanged.

Proof:

We look at marginal changes in endowments which do not move the economy off of the BC. At a BC equilibrium $\omega^* = \omega^- = (\alpha_- \cdot e_c + e_p)/(1 - \alpha_-)$. Therefore,

$$\text{(F5)} \qquad \partial\omega/\partial e_p - \partial\omega/\partial e_c = 1/(1-\alpha_-) - \alpha_-/(1-\alpha_-) = 1.$$

Substituting (F5) into (F1) yields

$$\text{(F6)} \qquad dW_c = \partial W_c/\partial L \cdot (\partial L^S/\partial\omega - \partial L/\partial e_c) \cdot de_p.$$

Since $\partial L^S/\partial\omega$ is positive and $\partial L/\partial e_c$ is negative, the expression in parentheses is positive. At a BC equilibrium we know $\partial W_c/\partial L > 0$, therefore, $dW_c/de_p > 0$. That is, the initial shift of endowments from the consumer to the producer makes the consumer better off.

Substituting (F5) into (F4) yields $dW_p/de_p = 0$. Therefore, when the BC constraint is binding an ex-ante increase in producer endowment accompanied by an equal decrease in consumer endowment raises consumer welfare and leaves producer welfare unchanged. This establishes Claim 4.

By looking at the two components of the transfer, we can see what is happening to the system. The decrease in e_c shifts the labor supply curve down as the consumer tries to transfer wealth into the second period to offset the tax. Lower income, for a given wage bill, means less spending on the produced good in each state which tends to decrease the demand for labor.

The increase in e_p does not affect the supply of labor, but it does increase its demand since the producer has a larger reserve out of which to pay the wage bill in the low-demand states. The net effect on the demand for labor is to increase the wage bill by the amount of the transfer.

The increase in the labor demand alone, for an unchanged labor supply schedule, would be sufficient to increase consumer welfare. There would be an increase in production with no change in consumer income which is sufficient to make the consumer better off. As it is, however, the labor supply schedule moves down with the transfer and there is a further increase in the level of production which increases consumer welfare further. The final allocation is identical to that with the unfeasible policy of closing the labor market and requiring trades below the labor supply schedule.

Case 2: BC not binding

Claim 5: (With BC Not Binding, Transfer Can be Pareto Improving)

At an interior equilibrium for some preferences and endowments, an ex-ante transfer of N endowments from the consumer to the producer makes both agents better off.

Proof:

An example is sufficient to prove the claim. Let technology, preferences, and initial endowments be given as follows:

$a = 1.25$; $u_p[c_{ps}] = \ln[c_{ps}]$;
$S = \{1,2\}$; $\alpha = \{.2, .8\}$; $\pi = \{.2, .8\}$; $\rho = .9$;
$e_p = 0$, $e_c = 1$.

Figure 3 plots an index of welfare for each agent for various tax rates. The example is constructed so that a transfer from the consumer to the producer makes both agents better off for small transfers. After a point, however, further increases in the transfer make the consumer worse off while continuing to make the producer better off. Figure 4 depicts the labor market without any tax and with the tax corresponding to that which maximizes consumer welfare.

Figure 3

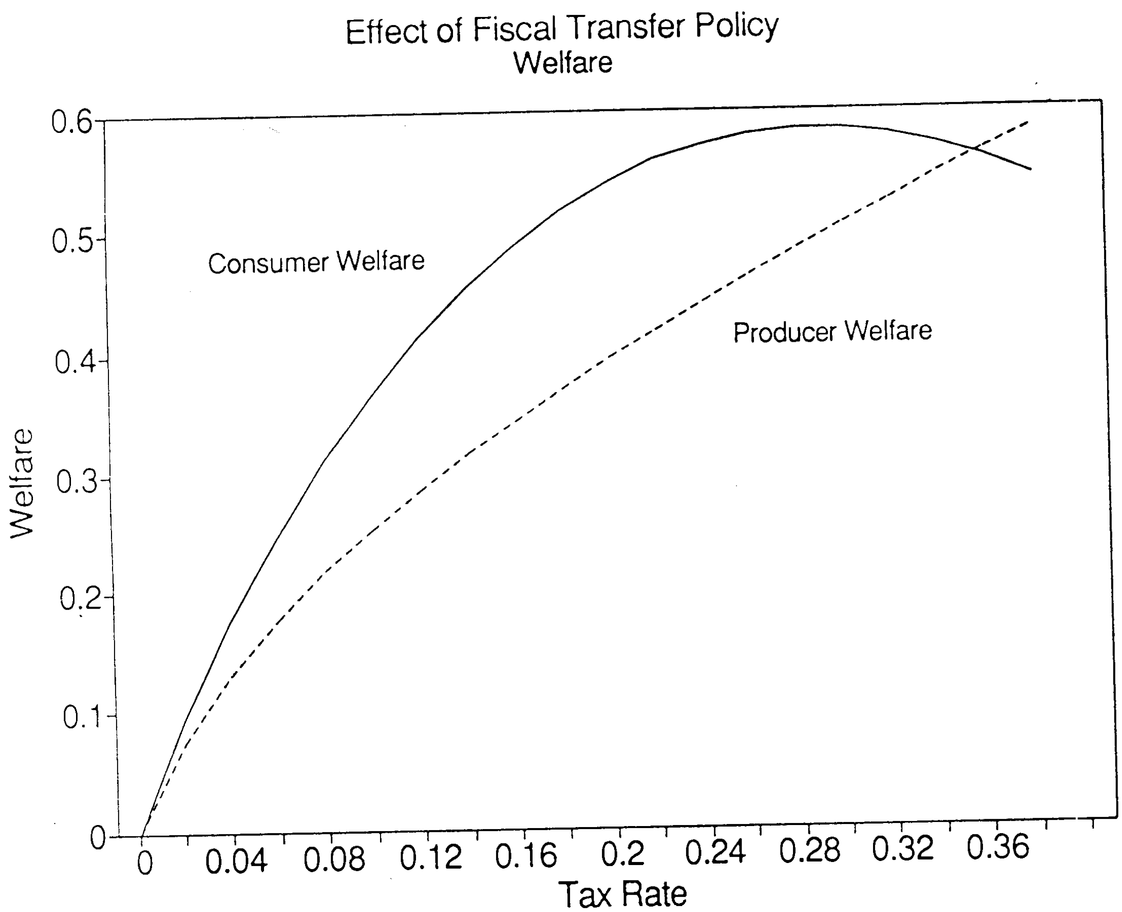
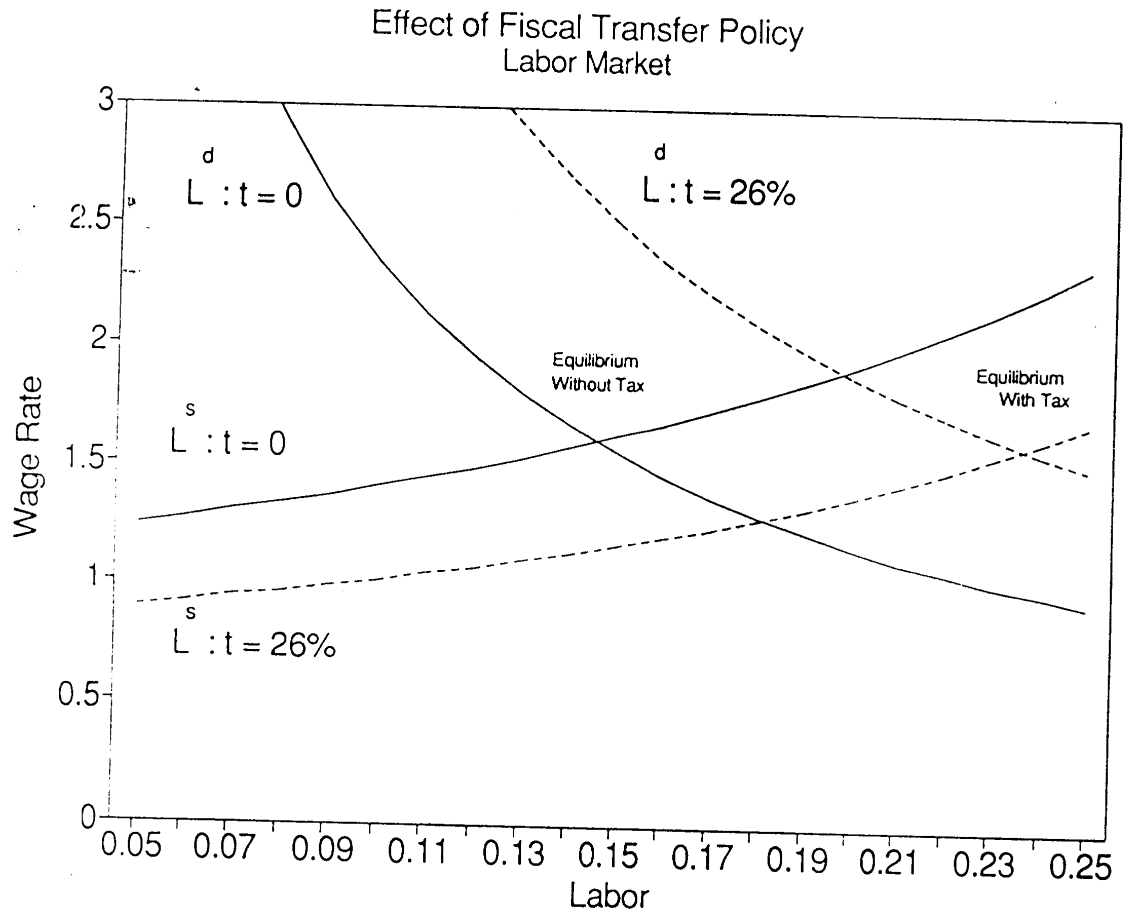

Effect of Fiscal Transfer Policy
Welfare
Consumer Welfare
Producer Welfare
Welfare
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.04
0.08
0.12
0.16
0.2
0.24
0.28
0.32
0.36
Tax Rate

Figure 4



The mechanism is essentially the same as in the BC binding case. The tax lowers the labor supply curve, which *ceteris paribus* increases employment and tends to make the consumer better off. There are, however, other effects of the tax which make the general characterization of the comparative statics impossible. The fall in consumer endowment can decrease the demand for labor. Similarly, the increase in producer endowment can increase the demand for labor.

Fiscal Spending Policy:

The second policy we consider is a balanced-budget spending policy where the government levies a constant lump-sum tax on the consumer, purchases the produced good and throws the purchases away. This policy tends to stabilize the price of the produced good and increase production. For some preferences and endowments, the consumer's consumption of G is higher for some states and lower for others which can lead to a welfare gain.

Model with Government Spending

Let I^d and t represent the consumer's disposable income and a lump-sum tax on consumers. When bankruptcy is prohibited, I^d is a constant equal to $\omega + e_c$ -t. Consumer maximization again yields $x_{cs} = I^d\alpha_s/p_s$ and $L^S = (\rho \cdot w - (e_c - t))/(w \cdot (1 + \rho))$.

The producer's FOC are again $\tilde{\Phi}[q;\mathbf{p},w] \geq 0$, $Y_s \geq 0$ $\forall s \in \mathbf{S}$ where Y_s and $\tilde{\Phi}[q;\mathbf{p},w] = 0$ if $Y_s > 0$ $\forall s \in \mathbf{S}$, where Y_s and $\tilde{\Phi}[q;\mathbf{p},w]$ are the same as above.

The government spends all of its tax revenue on the produced good, so its demand is $x_{gs} = t/p_s$.

Equilibrium

Total demand for G is $x_{cs} + x_{gs} = (I^d\alpha_s + t)/p_s$. For market clearing $x_{cs} + x_{gs} = q$, so market clearing prices are

(F7) $$p_s = (I^d\alpha_s + t)/q = [(\omega + e_c) \cdot \alpha_s + (1-\alpha_s) \cdot t]/q.$$

To compute the equilibria, we substitute for p_s in $\tilde{\Phi}[q;\mathbf{p},w]$ to obtain

(F8) $$\tilde{\Phi}[\omega;t] = \Sigma_s \pi_s u'\{Y_s[\omega,t]\} \cdot [e_c \cdot \alpha_s + (1-\alpha_s) \cdot t + (\alpha_s\text{-a}) \cdot \omega] \text{ where}$$

$$Y_s[\omega;t] = e_c \cdot \alpha_s + (1-\alpha_s) \cdot t + (\alpha_s-1) \cdot \omega + e_p.$$

The zero of $\tilde{\Phi}[\omega;t]$ is the wage bill consistent with market clearing and producer maximization. Let $\omega^{\sim}[t] \equiv (\omega\text{: } \tilde{\Phi}[\omega;t] = 0)$. The maximal ω consistent with the BC is

(F9) $$\omega^{-}[t] = (\omega\text{: } \min_s[p_s] \cdot q - \omega + e_p = 0) = (e_c \cdot \alpha_{-} + e_p)/(1-\alpha_{-}) + t.$$

The equilibrium is characterized by $\omega^* \equiv \min[\omega^{\sim}[t], \omega^{-}[t]]$.

Comparative Statics

From the price equation, we see that for given q and ω, an increase in t increases the price of G in all states. From the labor-supply equation, we see an increase in t shifts the labor-supply schedule down. The effect of the tax on the wage bill is ambiguous. When the BC is binding, $d\omega/dt$ equals one. For the interior case, over the range of policy interest, $d\omega/dt$ switches from being positive for small tax rates to being negative for large tax rates. As a result, employment is increasing as the policy begins, but later falls as the policy is expanded. The rise in q induced by the increase in t tends to lower prices and we cannot, in general, put a sign on the net change to prices.

The difficulties in signing the effects of this policy are similar to those encountered with the redistribution policy. As with the redistribution policy, we study the effects analytically for the case where the BC is binding and look at simulations for the interior case.

Consumer Welfare

The consumer's ex-ante welfare is

$$\text{(F10)} \qquad W_c = E\{\ln[1-L] + \rho \cdot \alpha_s \cdot \ln[x_{cs}] + \rho \cdot (1-\alpha_s) \cdot \ln[c_{cs}]\}.$$

Substituting the maximizing x_{cs} and c_{cs} yields

$$\text{(F11)} \qquad W_c = \ln[1-L] + \rho \cdot \ln[I^d] - \rho \cdot E\{\alpha_s \cdot \ln[p_s]\} + \rho \cdot E\{\alpha_s \cdot \ln[\alpha_s] + (1-\alpha_s) \cdot \ln[1-\alpha_s]\}$$

The policy is useful to the extent that the induced increase in q lowers the weighted geometric average of prices in the third term. Substituting for p yields

$$\text{(F12)} \qquad W_c = \ln[1-L] + \rho \cdot \ln[I^d] - \rho \cdot E\{\alpha_s \cdot \ln[I^d \cdot \alpha_s + t]\} + \rho \cdot E\{\alpha_s \cdot \ln[q]\} + \rho \cdot E\{\alpha_s \cdot \ln[\alpha_s] + (1-\alpha_s) \cdot \ln[1-\alpha_s]\}.$$

Taking the total derivative w.r.t. the tax yields

(F13) $$dW_c/dt = (-1/(1-L) + \rho \cdot \bar{\alpha}/(a \cdot L)) \cdot dL/dt + \rho/I^d \cdot dI^d/dt - \rho \cdot E\{\alpha_s \cdot (\alpha_s \cdot dI^d/dt + 1)/(I^d \cdot \alpha_s + t)\}.$$

At t = 0 this becomes

(F14) $$dW_c/dt = (-1/(1-L) + \rho \cdot \bar{\alpha}/(a \cdot L)) \cdot dL/dt + \rho \cdot (1-\bar{\alpha})/I^d \cdot dI^d/dt - \rho \cdot \bar{\alpha}/I^d.$$

Producer Welfare

Producer welfare is given by

(F15) $$W_p = E\{u_p[Y_s]\} = E\{u_p[\omega \cdot (\alpha_s - 1) + \alpha_s \cdot e_c + (1-\alpha_s) \cdot t + e_p]\}, \text{ hence}$$

(F16) $$dW_p = \partial W_p/\partial\omega \cdot d\omega + \partial W_p/\partial e_c \cdot de_c + \partial W_p/\partial e_p \cdot de_p + \partial W_p/\partial t \cdot dt \text{ where}$$

$$\partial W_p/\partial\omega = E\{u'[Y_s] \cdot (\alpha_s - 1)\} < 0;$$
$$\partial W_p/\partial e_c = E\{u'[Y_s] \cdot \alpha_s\} > 0;$$
$$\partial W_p/\partial e_p = E\{u'[Y_s]\} > 0;$$
$$\partial W_p/\partial t = E\{u'[Y_s] \cdot (1-\alpha_s)\} > 0.$$

Case 1: BC Binding

When the BC is binding, $d\omega^-/dt = 1$, so the spending policy does not change the consumer's disposable income. It does, however increase the level of production as the labor supply schedule shifts down and the labor-demand schedule, ω^-, shifts out. The policy's effect on consumer welfare is given by

(F17) $$dW_c/dt = [-1/(1-L) + \rho \cdot \bar{\alpha}/(a \cdot L)] \cdot dL/dt - \rho \cdot \bar{\alpha}/I^d.$$

The policy has no effect on producer welfare since $\partial W_p/\partial\omega = -\partial W_p/\partial t$ and $d\omega^-/dt = 1$.

Claim 6:

There are endowments and preferences such that the BC will bind and a balanced-budget spending policy financed by a tax on the consumer will increase consumer welfare without changing producer welfare.

Proof:

For $e_p = 0$, there is an α_- such that the BC binds. From this point, as $\alpha_- \to 0$, $\omega \to 0$ and $L \to 0$. Provided $\bar{\alpha}$ does not go to zero, the first term in dW_c/dt goes to positive infinity. The second term is bounded, so the derivative on consumer welfare becomes positive for some α_-. As shown above, the policy has no effect on producer welfare. Which completes the proof.

The economic interpretation of this is clear. When the BC is binding, the level of production is determined by the level of demand in the state when the consumer desires the produced good the least. Demand in this state does not reflect his average preference for it. The fiscal policy raises the floor on prices which enables the producer to honor a larger wage bill. The larger wage bill, coupled with the shifted labor-supply schedule, means that production is increased. The shift in the wage bill just offsets the tax to leave disposable income unchanged.

If the consumer were to enjoy the full product of his increased labor, the argument would be the same as with the transfer policy. However, with the spending policy, the government buys some of the produced good and the amount that the consumer enjoys depends on the state. In the low-demand states, x_{cs} is lower with the policy, but in the high demand states, it is higher. Since it is in the high-demand states that the consumer most enjoys the produced good, the greater consumption of it in these states can outweigh the disutility from less leisure and the lower consumption in the low-demand states to make him better off.

Case 2: Interior equilibrium

Claim 7:

At an interior equilibrium, for some preferences and endowments, a balanced budget fiscal spending policy is Pareto improving.

Proof:

An example is sufficient to prove the claim. Let technology, preferences and endowments be the same as in the example used to prove claim 5. Figure 5 depicts the indexes of welfare. Again, producer welfare is increasing monotonically in the tax rate while consumer welfare improves for small tax rates and decreases for large tax rates. Figure 6 depicts the labor market without any tax and with the tax that maximizes consumer welfare.

Figure 5

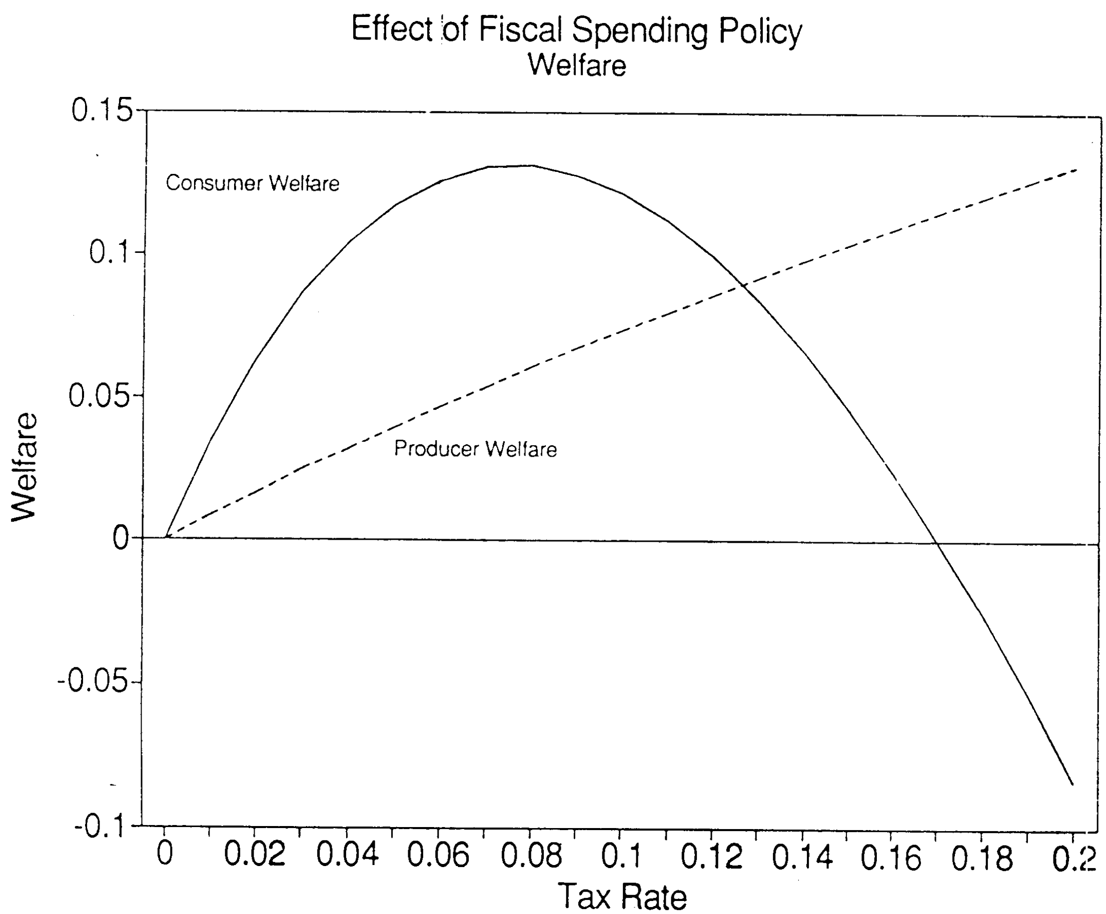
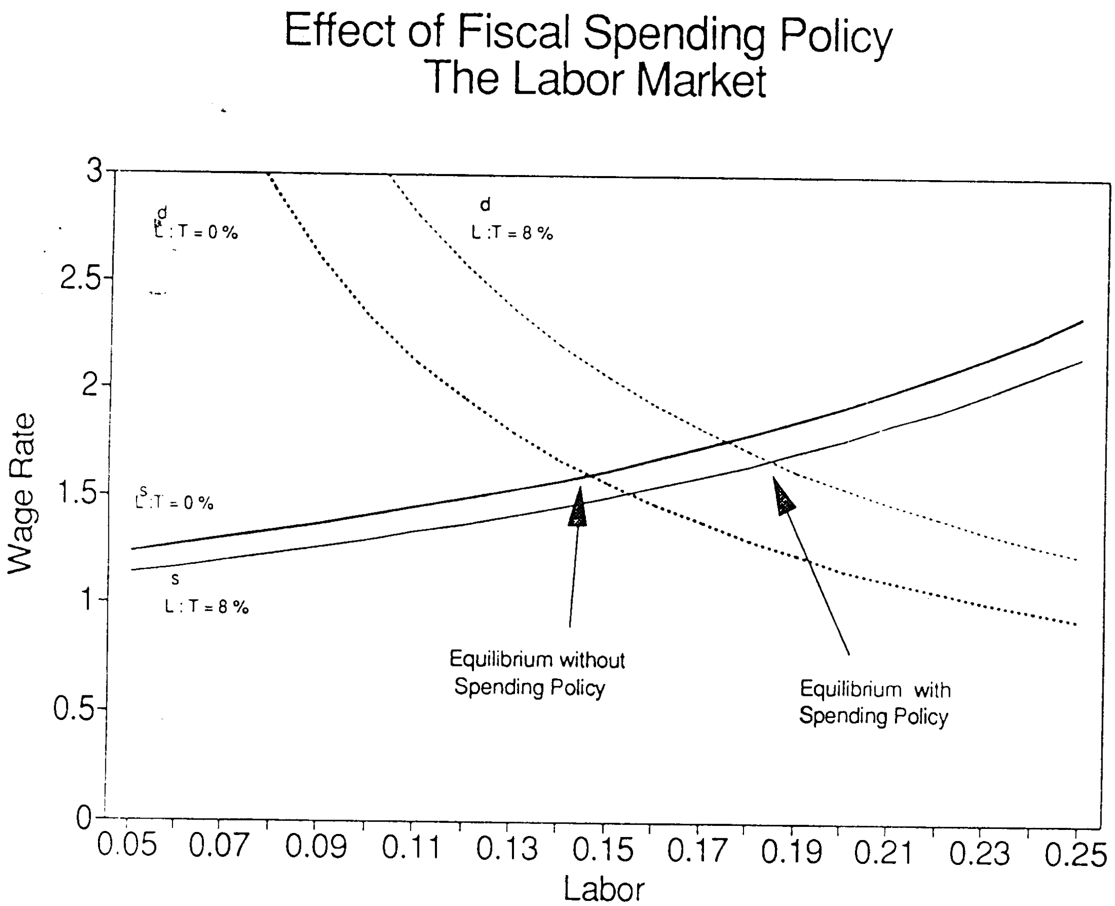


Figure 6



V. Conclusion

This paper presents a small general-equilibrium model to highlight the link between neo-Keynesian models of unemployment and recent work on incomplete financial markets. We show how the constrained suboptimality of the competitive equilibrium, which is generic with two or more agents and two or more goods, can take the form of underemployment.

The direct application of the suboptimality results would imply that policies to improve on the competitive equilibrium would likely be infeasible as they would involve closing a market and forcing agents to make trades which they choose not to make on their own.

The economic intuition behind the underemployment is clear in this small model, however, and allows us to construct feasible fiscal policies that are welfare improving.

The model also provides a framework for analyzing other types of policies, two of which we briefly discuss. Since the inefficiency arises because of a lack of securities markets, it is natural to ask what effect the introduction of another asset would have and if there is a role for policy in introducing or supporting this new market. In Thomas (1986b) use this model to show that the introduction of a financial futures market--linked to the produced good's price--can be Pareto improving even if the worker/consumer does not trade in (or even know of) the new market. In that model, a lump-sum tax on the consumer is used to finance a government "speculator" who trades futures with the producer. This example demonstrates why there can be externalities to financial assets that drive a wedge between the private and public incentives for their introduction. It also provides a rationale for some price stabilization policies.

In discussing the persistence of underemployment above, we argued that the Pareto-superior trade will not be robust to the free formation of coalitions. It has been suggested that the model developed here could be used to explain the national wage-bargaining schemes used in some countries.

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Appendix

Proof of Claim 1:

That ω^* exists is proved by showing that $\tilde{\Phi}[\omega]$ has a zero. $\tilde{\Phi}[\omega]$ is a continuous function that is positive for low values of ω and is negative for high values of ω. When q is low (high) enough, the price of G in each state is above (below) marginal cost which makes each term of $\tilde{\Phi}[]$ positive (negative). We call the largest (smallest) ω where this is true ω_- (ω_+). For ω less (greater) than this critical value, $\tilde{\Phi}[\omega]$ is positive (negative) and the producer will want to increase (decrease) production. $\tilde{\Phi}[]$ is continuous in ω (for $Y_s \geq 0$), so there is at least one point between ω_- and ω_+ where it equals zero. At this zero, the producer has no incentive to change the level of production.

Since producer income is equal to $\omega \cdot (\alpha_s - 1) + \alpha_s \cdot e_c + e_p$, and α_s is always less than one, producer income is falling in ω for all states. ω^-, then, represents the maximal ω consistent with non-negative income in all states, i.e., the no-bankruptcy constraint.

When ω^- is greater than or equal to $\omega^\sim$, the no-bankruptcy constraint does not affect the equilibrium. When ω^- is less than $\omega^\sim$, the producer would like to increase production, but cannot because of the constraint. He has no incentive to decrease production, so ω^- becomes an equilibrium.

That ω^* fully characterizes an equilibrium follows directly from (C2), (C4) and the consumer's income identity. That there is no equilibrium other than ω^* follows from the construction of ω^* and the concavity of the agents' objective functions.

Proof of Claim 2:

The proof has four steps. The first step is to show that at $\hat{\omega}$ expected price equals marginal cost. The second step is to show that if marginal cost equals expected price, a risk averse producer is not maximizing and $\tilde{\Phi}[\hat{\omega}] < 0$. The third step is to show that if $\tilde{\Phi}[\hat{\omega}]$ is negative there is an $\omega^\sim$ less than $\hat{\omega}$ such that $\tilde{\Phi}[\omega^\sim]=0$. The fourth step is to note that $L[\omega]$ is a monotonically increasing function, so $L[\omega^\sim]$ is less than $L[\hat{\omega}]$.

Step One: At $\hat{\omega}$, marginal cost equals expected price.

From equation $\hat{E5}$, $\hat{\omega} = \bar{\alpha} \cdot e_c/(a-\bar{\alpha})$, which implies

(A1) $$p_s[\hat{\omega}] = \alpha_s \cdot I/q[L[\hat{\omega}]] = \alpha_s \cdot (\hat{\omega} + e_c)/q[L[\hat{\omega}]].$$

Substituting $\bar{\alpha} \cdot e_c/(a-\bar{\alpha})$ for $\hat{\omega}$ in A1 yields

(A2) $$p_s[\hat{\omega}] = \alpha_s \cdot (\bar{\alpha} \cdot e_c + (a-\bar{\alpha}) \cdot e_c)/((a-\bar{\alpha}) \cdot q[\hat{\omega}] = \alpha_s \cdot a \cdot e_c/((a-\bar{\alpha}) \cdot q[\hat{\omega}]).$$

Taking expectations over states yields

(A3) $E[p_s[\hat{\omega}]] = \bar{\alpha} \cdot a \cdot e_c/((a-\bar{\alpha}) \cdot q[\hat{\omega}]) = a \cdot \hat{\omega}/q[\hat{\omega}]$.

Since labor is the only productive factor, total cost is the wage bill: $w \cdot L = w \cdot q^a$ and marginal cost is given by $MC = a \cdot w \cdot q^{a-1}$. Since $L = q^a$, we have $MC = a \cdot w \cdot L/q = a \cdot \omega/q$ for all ω. In particular, at $\hat{\omega}$, this is the same expression as that for expected price in (A3). This completes step One.

<u>Step Two:</u> If the wage bill equals $\hat{\omega}$ a risk averse producer is not optimizing and $\tilde{\Phi}[\hat{\omega}] < 0$.

(A4) $\tilde{\Phi}[\hat{\omega}] = E[u'[Y_s[\hat{\omega}]] \cdot (p_s[\hat{\omega}] - a \cdot w[\hat{\omega}] \cdot q[\hat{\omega}]^{a-1})]$

Since at $\hat{\omega}$, $E[p_s] = MC$, we can write (A4) as

(A5) $\tilde{\Phi}[\hat{\omega}] = E[u'[Y_s[\hat{\omega}]] \cdot (p_s[\hat{\omega}] - E[p_s[\hat{\omega}]])]$.

Since $p_s = \alpha_s \cdot I/q$, $E[p_s] = \bar{\alpha} \cdot I/q$ and (A5) can be written as

(A6) $\tilde{\Phi}[\hat{\omega}] = E[u'[Y_s[\hat{\omega}]] \cdot (\alpha_s - \bar{\alpha}) \cdot (\hat{\omega} + e_c)/q]$.

Substituting for $Y_s[\hat{\omega}]$ in (A6) yields

(A7) $\tilde{\Phi}[\hat{\omega}] = E[u'[(\hat{\omega} + e_c) \cdot \alpha_s - \hat{\omega} + e_p] \cdot (\alpha_s - \bar{\alpha}) \cdot (\hat{\omega} + e_c)/q]$.

Since $E[u'[(\hat{\omega} + e_c) \cdot \bar{\alpha}_s - \hat{\omega} + e_p] \cdot (\alpha_s - \bar{\alpha})]$ equals zero, we can subtract it from each term on the RHS of (A7) to yield

(A8) $\tilde{\Phi}[\hat{\omega}] = E\{(u'[(\hat{\omega} + e_c) \cdot \alpha_s - \hat{\omega} + e_p] - u'[(\hat{\omega} + e_c) \cdot \bar{\alpha} - \hat{\omega} + e_p]) \cdot$
$(\alpha_s - \bar{\alpha})\} \cdot (\hat{\omega} + e_c)/q]$.

Under the assumption that $u'' < 0$, $u'[\alpha_s; \cdot\,] < u'[\bar{\alpha}; \cdot\,]$ iff $\alpha_s > \bar{\alpha}$. Therefore, each term in the {} is non positive and must be negative for all s such that $\alpha_s - \bar{\alpha}$. Thus $\tilde{\Phi}[\hat{\omega}] < 0$.

<u>Step Three:</u>

In the proof to claim 1, we showed there is an ω_- such that $\tilde{\Phi}[\omega] > 0$ for all $\omega \leq \omega_-$. Since $\tilde{\Phi}[]$ is a continuous function and $\tilde{\Phi}[\hat{\omega}] < 0$ there must be an $\omega^{\sim}$ between ω_- and $\hat{\omega}$ such that $\tilde{\Phi}[\omega^{\sim}] = 0$. Finally, since $L[\omega]$ is monotonically increasing, $L[\omega^{\sim}] < L[\hat{\omega}]$. QED

Endnotes

[1] The author is a staff economist in the Division of International Finance. This paper represents the views of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff. I would like to thank Herakles Polemarchakis, Ed Green, Guy Stevens, and Jon Faust for their helpful comments.

[2] For an overview see Romer(1993) and the related articles in that volume.

[3] In a series of papers, Greenwald and Stiglitz have shown how asymmetric information can give rise to capital market imperfections. The imperfections generally take the form of missing contingent claims markets. In the presence of these imperfections, firms will behave as though they were risk averse or facing financing constraints. They further show how the competitive equilibrium will not use the existing markets efficiently, which leads to a role for policy. See Greenwald and Stiglitz (1993) for a review of these arguments.

[4] See Geanakoplos (1990) for a review of the literature.

[5] J. Geanakoplos (1990) p. 26.

[6] N can be considered manna, a Hicksian composite good or money claims on production from the rest of the economy.

[7] If we allow bankruptcy, the welfare arguments that follow go through (although existence of an equilibrium is not assured), provided the contract takes the following form: If possible, the wage bill is met; if not, the worker receives all of the firm's revenue plus the producer's endowment. Gale and Hellwig (1984) derive this as an optimal contract when it is costly for the bond holder to observe the state. See Thomas (1986a) Chapter II Section Four.

[8] It is straight-forward to show that $\tilde{\Phi}$ is always falling in q. The difficulty comes from making sure u'(Y) is defined over the relevant range. This is where the definition that for $Y_s < 0$, $u(Y_s) = u[0]+u'[0] \cdot Y_s$ is used.

[9] See Thomas (1986) Chapter I.

[10] As a trivial case, if $\alpha_- = 0$, then BC will be binding for RN producers if $e_p/e_c < \bar{\alpha}/(a-\bar{\alpha})$. For $e_p = 0$, this is clearly true.

[11] This is another example of the "third derivative" problem discussed in Rothschild-Stiglitz (1971).

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