Uncertainty shocks, monetary policy and long-term interest rates

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Abstract

We study the relationship between monetary policy and long-term rates in a structural, general equilibrium model estimated on both macro and yields data from the United States. Regime shifts in the conditional variance of productivity shocks, or "uncertainty shocks", are an important model ingredient. First, they account for countercyclical movements in risk premia. Second, they induce changes in the demand for precautionary saving, which affects expected future real rates. Through changes in both risk-premia and expected future real rates, uncertainty shocks account for about 1/2 of the variance of long-term nominal yields over long horizons. The remaining driver of long-term yields are changes in inflation expectations induced by conventional, autoregressive shocks. Long-term inflation expectations implied by our model are in line with those based on survey data over the 1980s and 1990s, but less strongly anchored in the 2000s.

1   Introduction

Following Smets and Wouters (2007), estimated general equilibrium models with sticky prices have become popular tools for monetary policy analysis in central banks. Partly as a result of the 2008 financial crisis and the ensuing recession, these models have become much richer in a number of directions, notably including financial frictions (e.g. Christiano, Motto and Rostagno, 2014, or Del Negro *et al.*, 2017). Due to linearization, however, most of these models are not ideally suited to describe bond risk premia and they in fact almost always abstract from bond yields in estimation. Nevertheless, long-term rates and risk premia are often an object of interest in monetary policy analysis—see for example Goodfriend's (1993) discussion of an "inflation scare" in bond yields in the early 1990s, or the famous Greenspan (2005) speech on the "bond markets conundrum".[1] It would be desirable to have models allowing for a joint and coherent explanation of macroeconomic dynamics and bond risk premia.

This paper shows that the estimated, nonlinear version of a simple new-Keynesian model can provide an internally consistent account of the evolution of macroeconomic and yields data in the United States. To achieve this goal, two model modifications are necessary compared to versions used in standard macro-only applications: non-expected utility preferences and stochastic volatility. Non-expected utility—a model feature which is now standard in calibrated analyses of asset prices in production economies with endogenous labor supply—allows one to increase risk-aversion independently of the elasticity of intertemporal substitution.[2] Stochastic

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[1]Goodfriend (1993) defines an inflation scare as a significant increase in long term nominal interest rates in the absence of an increase in policy rates.

[2]See e.g. Tallarini (2000), Rudebusch and Swanson (2012), van Binsbergen *et al.* (2012), Andreasen (2012a), Colacito and Croce (2013), Swanson (2018).

volatility, which in our model takes the form of regime switching, leads to "uncertainty shocks" that produce variations in bond risk premia and, at the same time, to changes in households' demand for precautionary saving. Through the latter mechanism, uncertainty shocks contribute to affect macroeconomic fluctuations.[3]

We show that our simple model specification provides a reasonably good and parsimonious account of U.S. data on aggregate consumption, inflation, short and long-term interest rates. The model also fits well dimensions of the data which were not directly used in estimation, such as long forward rates. The conditional standard deviations implied by our DSGE model for the macro-variables are broadly in line with those generated by a reduced-form Markov-switching VAR including the same variables. Finally, our model does not suffer from the puzzle identified in Backus, Gregory and Zin (1989), as it can simultaneously account for the positive slope of the term structure and a positive autocorrelation of consumption growth. The serial autocorrelation of yields is strikingly consistent with their sample autocorrelation coefficients.

Our results shed new light on the determinants of long-term interest rates over the business cycle. Long-term rates are not roughly constant, as implied by simple, linearized models, but subject to three broad sources of time-variation: risk-premia, expected future real rate and expected future inflation.

Fluctuations in bond risk-premia are induced by uncertainty shocks. Our results suggest that such fluctuations are cyclical. Recessions are persistent periods of heightened uncertainty, which leads to a rise in risk premia. During economic recoveries, uncertainty returns to normal levels and risk premia become again small. If this reduction in risk premia occurs at the turning point of the policy interest rates cycle—as was the case in 2004 according to our estimates—observed long-term yields can remain roughly unchanged masking the expectations of a future policy tightening and producing the apparent "conundrum" noted in Greenspan (2005). If, by contrast, the fall in uncertainty occurs before the monetary policy tightening phase

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[3]See also Gourio (2012, 2013), Andreasen (2012b) for the case of time variations in disaster risk.

begins, as was the case e.g. in 1994 according to our estimates, the response of observed long-term rates is more directly informative of expected changes in future interest rates.

The second source of fluctuations in long-term nominal rates are developments in current and expected future real rates. Once again, uncertainty shocks over future realizations of technology play an important role. The higher level of uncertainty which characterizes recessions leads to an increase in households' demand for precautionary saving. Current and expected future *real* rates, therefore, tend to fall to clear the savings market. For given long-term inflation expectations, this mechanism also leads to a fall in expected future *nominal* interest rates. During recoveries, uncertainty over future realizations of technology switches back to lower levels and "confidence" returns. The demand for precautionary saving falls back to normal levels and expected real yields increase. The ensuing increase in nominal yields is neither a signal of expected inflationary pressure, nor of changes in the anti-inflationary credibility of the central bank.

Such movements in real rates also induce macroeconomic fluctuations. The monetary policy reaction function shapes the strenghts of their effects on consumption and inflation. As described above, uncertainty shocks produce changes in the demand for precautionary saving. If monetary policy is conducted according to the standard Taylor rule, a hypothesis we maintain in our analysis, it does not internalize the ensuing changes in equilibrium real interest rates. For example, after an increase in uncertainty, policy rates are cut due to the ensuing weakness in consumption and inflation, but they do not fall enough to offset the drop in equilibrium real rates induced by the stronger demand for precautionary saving. Consequently, consumption, output and inflation remain weak for a prolonged period of time.

The third driver of developments in bond yields are long-term inflation expectations. In our model these movements mostly arise as a result of linear shocks and conventional, first-order dynamics. More specifically, conventional technology shocks are responsible for the bulk of the secular change in long-term nominal yields. To affect yields at the 10-year horizon, they must be even more persistent, than typ-

ically found in applications solely based on macroeconomic data.

To assess the plausibility of our estimates, we compare model-based long-term inflation expectations with measures of expectations available from the Federal Reserve Bank of Philadelphia's quarterly Survey of Professional Forecasters. There is no reason why the two measures should agree, since they reflect the different information sets of survey participants and of "financial markets," as embodied in bond prices and extracted through the lens of our model. Nevertheless, the broad level and the medium term trends in the two series are quite similar. They both point to a progressive and parallel fall in inflation expectations over the 1980s, then hover around the 2.5% level since the end of the 1990s.

At the higher frequency, however, our model is consistent with a less extreme anchoring of 10-year inflation expectations compared to surveys.[4] After falling towards 2.5 percent over the 1990s, survey measures remain constant at that level thereafter. By contrast, model-implied measures are generally more volatile. For example they temporarily increase again, sharply, during the "inflation scare" of 1993. They hover closely around 2.5 percent at the turn of the millennium, but then fall to levels close to 1 percent during the recession of the early 2000s and even below 1 percent ahead of the Great recession. In sum, bond prices suggest that 10-year inflation expectations over the 2000s are less dogmatically anchored than one would conclude based on survey data. It is noticeable that inflation developments after the Great recession turned out to be more in line with the expectations implied by our model than with survey expectations.

We also derive model-based expectations measures for the late 1960s and early 1970s, a period where the survey measures are unavailable. Unsurprisingly, our results suggest an increase in 10-year inflation expectations over this early part of the sample. The rise in expectations is not abrupt, but very gradual and persistent.

Our paper is related to the literature exploring the term structure implications of macro-models. Many of these papers are theoretical and look at the asset pricing

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[4]We assume a constant inflation target in the monetary policy rule, so inflation expectations are by assumptions always anchored in the very long run.

implications of macro models–see e.g. Piazzesi and Schneider (2006), Rudebusch and Swanson (2012), Swanson (2014). Amongst the empirical papers, De Graeve, Emiris and Wouters (2007) estimate a standard DSGE model using both macroeconomic and term structure data, but rely on the loglinearized version of that model and must therefore introduce additional parameters to allow for non-zero risk-premia. Christoffel, Jaccard and Kilponen (2011) also estimate the linearized version of a new Keynesian model, and then draw bond pricing implications using a higher order approximation. Bekaert, Cho and Moreno (2010) and Campbell, Pflueger and Viceira (2013) follow an intermediate route and study asset prices in a linearized New Keynesian model assuming a stochastic discount factor that is related to the new Keynesian model's equations in a reduced-form manner. The papers most similar to ours are Doh (2011, 2012), van Binsbergen *et al.* (2012) and Andreasen (2012), which estimate nonlinear models with macroeconomic and term structure data. In contrast to all these papers, we allow for regime switches in the variance of shocks and argue that this is an essential model feature to fit bonds and macro data. Moreover, the focus of all these papers is on the model's ability to fit yields, while we highlight the model's implications for the transmission of monetary policy to long-run rates. From this perspective, we are closer to Cochrane (2008, 2017) and Atkeson and Kehoe (2008). All these papers are related to the huge consumption-based asset pricing literature and build on the results of either Campbell and Cochrane (1999) or Bansal and Yaron (2004). In a recent contributions to this literature, Schorfheide, Song and Yaron (2018) highlight the importance of allowing for measurement error in consumption in a long-run risk model. We also allow for measurement errors in our estimation.

Our paper is also related to the literature documenting time variation in macroeconomic volatility in a reduced form setting, including e.g. McDonnell and Perez-Quiros (2000), Sims and Zha (2006), Primiceri (2005). Justiniano and Primiceri (2008) allows for shifts in the volatility of structural shocks in a linearized, medium-scale DSGE model applied to the U.S. economy. Compared to Justiniano and Primiceri's, we rely on a smaller, but non-linear model, which allows us to explore the

effects of uncertainty shocks on households' demand for precautionary saving and on bond risk premia. Our modelling of second moments is however more parsimonious and less flexible in uncovering trend shifts in volatility.

Finally, our paper is related to the literature on uncertainty shocks spawned from Bloom (2009). In Bloom (2009), an increase in uncertainty induces firms to temporarily reduce investment and hiring. In our model, higher uncertainty over future technology shocks induces households to increase their precautionary saving. Consumption demand will tend to fall. Due to monopolistic competition and sticky prices, this will bring down output and inflation. Uncertainty shocks therefore act like demand shocks. This is consistent with the results in Basu and Bundick (2012), which relies on a more comprehensive, calibrated model of the U.S. economy and analyses uncertainty shocks in both technology and preferences. Bianchi, Ilut and Schneider (2014) put forward a model with ambiguity averse investors, where regime shifts generate large low frequency movements in asset prices.

The rest of the paper is organized as follows. Section 2 describes the model, focusing on its distinguishing features: the distribution of the shocks and the utility function, which is of the class proposed by Epstein and Zin (1989) and Weil (1990), but extended to allow for habit persistence in consumption. The methods that we adopt to solve and estimate the model are described next, in section 3. Such methods are non-standard, because we need to solve the model to a second order approximation in order to capture precautionary savings effects. We demonstrate that the reduced form of the model is quadratic in the state variables with continuous support and includes regime-switching intercepts, as well as variances. We then estimate the non-linear reduced form using Bayesian methods. Section 4 described the estimation results and presents a few goodness-of-fit measures and Section 5 illustrates the role of uncertainty shocks through the analysis of impulse responses and the variance decomposition of forecast errors. The implications of our estimates for the relationship between monetary policy and risk premia and for the transmission of monetary policy to long-term rates are discussed in Section 6.

Section 7 concludes. A technical appendix, available on-line[5], describes in details some of the computations carried out in this paper.

2 The model

We start from a simple version of the new-Keynesian model that has been shown to account relatively well for the dynamics of key nominal and real macroeconomic variables—see e.g. Smets and Wouters (2007). We thus assume nominal price rigidities, external habit persistence, inflation indexation, and a monetary policy rule with "interest rate smoothing". Since our interest is on the model's implications for long-term interest rates, we simplify it by abstracting from capital accumulation and real wage rigidities. Our results suggest that even our simple model can go a long way in explaining the data of interest to us.

Compared to the new Keynesian benchmark, we introduce two key modifications. The first is to allow for stochastic regime switching in the variance of structural shocks. The evidence of time variation in the variance of macroeconomic shocks is well-established–see e.g. Justiniano and Primiceri (2008), McDonnell and Perez-Quiros (2000), Primiceri (2005) and Sims and Zha (2006). The novelty in our paper is to explore the implications of time varying variances on bond prices. Our second modification, which is already common in the consumption-based asset pricing literature, is to adopt the non-expected utility specification for preferences proposed by Epstein and Zin (1989) and Weil (1990). Here we extend this specification to a general equilibrium model in which we also allow for habit persistence in consumption and labour-leisure choice.

2.1 Structural shocks

A key distinguishing feature of our model are changes in the demand for precautionary saving induced by variations in the conditional variance of the structural shocks. We therefore start the description of our model from the distribution of structural

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[5]https://sites.google.com/site/gianniamisanowebsite/

shocks.

In macroeconomic applications, exogenous shocks are almost always assumed to be (log-) normal, partly because models are typically log-linearized and researchers are mainly interested in characterizing conditional means. However, Hamilton (2008) argues that a correct modelling of conditional variances is always necessary, for example because inference on conditional means can be inappropriately influenced by outliers and high-variance episodes. The need for an appropriate treatment of heteroskedasticity becomes even more compelling when models are solved nonlinearly, because conditional variances have a direct impact on conditional means.

In this paper, we assume that variances are subject to stochastic regime switches. We will allow for shocks to the level and growth rates of technology, to mark-ups, to the monetary policy rule and to a non-interest-rate-sensitive component of output G_t. G_t will enter GDP like government spending, but we do not model it explicitly since our interest is not on fiscal policy. We only use G_t to allow for a demand-type shock and we therefore refer to it generically as "demand shock". The conditional variance of any of these shocks could in principle be subject to regime switching, but in this paper we adopt a parsimonious specification such that only (level) productivity, monetary policy and demand shocks have regime switching variances.[6]

More specifically, we will assume that the technology shock z_t, the monetary policy shocks η_t and the demand shock G_t have standard deviations that can independently switch between a high and a low regime. Denoting the low variance regime by 1 and the high variance regime by 0, we write

$$\sigma_{z,s_{z,t}} = \sigma_{z,0}s_{z,t} + \sigma_{z,1}\left(1 - s_{z,t}\right)$$

$$\sigma_{G,s_{G,t}} = \sigma_{G,0}s_{G,t} + \sigma_{G,1}\left(1 - s_{G,t}\right)$$

$$\sigma_{\eta,s_{\eta,t}} = \sigma_{\eta,0}s_{\eta,t} + \sigma_{\eta,1}\left(1 - s_{\eta,t}\right)$$

where the variables $s_{z,t}$, $s_{G,t}$ and $s_{\eta,t}$ can assume the discrete values 0 and 1. For

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[6]We have also estimated versions of the model allowing for regime-switching in the variance of mark-up and technology growth shocks. These dimensions of regime switching receive little support from the data.

each variable $s_{j,t}$ $(j = z, G, \eta)$, the probabilities of remaining in states 0 and 1 are constant and equal to $p_{j,0}$ and $p_{j,1}$, while the probabilities of switching to the other state will be $1 - p_{j,0}$ and $1 - p_{j,1}$, respectively.

2.2 Households

We assume that each household i provides $N(i)$ hours of differentiated labor services to firms in exchange for a labour income $w_t(i) N_t(i)$. Each household owns an equal share of all firms j and receives profits $\int_0^1 \Psi_t(j) \mathrm{d}j$. As in Erceg, Henderson and Levin (2000), an employment agency combines households' labor hours in the same proportions as firms would choose. The agency's demand for each household's labour is therefore equal to the sum of firms' demands. The labor index L_t has the Dixit-Stiglitz form $L_t = \left[\int_0^1 N_t(i)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}} \mathrm{d}i \right]^{\frac{\theta_{w,t}}{\theta_{w,t}-1}}$, where $\theta_{w,t} > 1$ is subject to exogenous shocks. At time t, the employment agency minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $w_t(i)$ as given, and then sells units of the labor index to the production sector at the aggregate wage index $w_t = \left[\int_0^1 w(i)^{1-\theta_{w,t}} \mathrm{d}i \right]^{\frac{1}{1-\theta_{w,t}}}$. The employment agency's demand for the labor hours of household i is given by

$$N_t(i) = L_t \left(\frac{w_t(i)}{w_t} \right)^{-\theta_{w,t}}. \tag{1}$$

Each household i maximizes its intertemporal utility with respect to consumption, the wage rate and holdings of a complete portfolio of state-contingent assets, subject to the demand for its labour (1) and the budget constraint

$$P_t C_t(i) + \mathrm{E}_t Q_{t,t+1} W_{t+1}(i) \leq W_t(i) + w_t(i) N_t(i) + \int_0^1 \Psi_t(j) \, \mathrm{d}j \tag{2}$$

where C_t is a consumption index satisfying

$$C_t = \left(\int_0^1 C_t(z)^{\frac{\theta-1}{\theta}} \, \mathrm{d}z \right)^{\frac{\theta}{\theta-1}}.$$

In the budget constraint, $W_t(i)$ denotes the beginning-of-period value of a complete portfolio of state contingent assets held by household i, $Q_{t,t+1}$ is their price

and $\Psi_t(j)$ are the profits received from investment in firm j. The price level P_t is defined as the minimal cost of buying one unit of C_t, hence equal to

$$P_t = \left(\int_0^1 p(z)^{1-\theta} \, \mathrm{d}z \right)^{\frac{1}{1-\theta}}.$$

Equation (2) states that each household can only consume or hold assets for amounts that must be less than or equal to its salary, the profits received from holding equity in all the existing firms and the revenues from holding a portfolio of state-contingent assets.

Households' preferences are described by the Kreps and Porteus (1978) specification proposed by Epstein and Zin (1989). In that paper, utility is defined recursively through the aggregator U such that

$$U\left[C_t, \left(\mathrm{E}_t V_{t+1}^{1-\gamma}\right)\right] = \left\{ (1-\beta) C_t^{1-\psi} + \beta \left(\mathrm{E}_t V_{t+1}^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}} \right\}^{\frac{1}{1-\psi}}, \qquad \psi, \gamma \neq 1, \quad (3)$$

where β, ψ and γ are positive constants. Using a specification equivalent to that in equation (3), Weil (1990) shows that β is, under certainty, the subjective discount factor, but time preference is in general endogenous under uncertainty. The parameter γ is the relative risk aversion coefficient for timeless gambles. The parameter $1/\psi$ measures the elasticity of intertemporal substitution for deterministic consumption paths.

The distinguishing feature of the Epstein-Zin-Weil preferences, compared to the standard expected utility specification, is that the coefficient of relative risk aversion can differ from the reciprocal of the intertemporal elasticity of substitution. In addition, Kreps and Porteus (1978) show that, again contrary to the expected utility specification, the timing of uncertainty is relevant in their class of preferences. The specification in equation (3) displays preferences for an early resolution of uncertainty when the aggregator is convex in its second argument, i.e. when $\gamma > \psi$. Any source of risk will be reflected in asset prices not only if it makes consumption more volatile, but also if it affects the temporal distribution of consumption volatility.

We generalize the utility function in equation (3) by allowing for habit formation and a labour-leisure choice, as in standard, general equilibrium macro-models.

The generalization to allow for the labour-leisure choice has already been used, for example, in Rudebusch and Swanson (2012). We additionally allow for habit formation because it has been shown to be important to match the dynamic behavior of aggregate consumption–see e.g. Fuhrer (2000).

As a result, time-t utility of household i will not only depend on consumption C_t but it will be a more general function of consumption and leisure

$$U_t(i) = u\{C_t(i) - h\Xi_t C_{t-1}, 1 - N_t(i)\}$$

where leisure is written as $1 - N_t$ because total hours are normalized to 1, the h parameter represents the force of external habits and Ξ_t is the rate of growth of technology.[7]

With our more general preferences specification, γ is no-longer related one-to-one to risk aversion. Swanson (2012) discusses the appropriate measures of risk aversion in a dynamic setting with consumption and leisure entering the utility function. However, $1/\psi$ continues to measure the long-run elasticity of intertemporal substitution of consumption.[8]

The first order conditions for household i include (ignoring the i subscript for simplificy)

$$\frac{u_{N,t}}{u_{c,t}} = \mu_{w,t}\frac{w_t}{P_t}$$

and

$$Q_{t,t+1} = \beta\left[\mathrm{E}_t\left(\frac{J_{t+1}}{J_t}\right)^{1-\gamma}\right]^{\frac{\gamma-\psi}{1-\gamma}}\left(\frac{J_{t+1}}{J_t}\right)^{-(\gamma-\psi)}\left(\frac{u_{t+1}}{u_t}\right)^{-\psi}\frac{u_{c,t+1}}{u_{c,t}}\frac{1}{\Pi_{t+1}}, \qquad (4)$$

where $J_t = J(W_t(i))$ is household i's maximum value function [9], Π_t is the inflation rate between t and $t-1$, and the mark-up $\mu_{w,t} \equiv (\theta_{w,t} - 1)/\theta_{w,t}$ follows an exogenous autoregressive process

$$\mu_{w,t+1} = \mu_w^{1-\rho_\mu}(\mu_{w,t})^{\rho_\mu} e^{\varepsilon_{t+1}^\mu}, \qquad \varepsilon_{t+1}^\mu \sim N(0, \sigma_\mu).$$

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[7]Guariglia and Rossi (2002) also use expected utility preferences combined with habit formation to study precautionary savings in UK consumption. Koskievic (1999) studies an intertemporal consumption-leisure model with non-expected utility.

[8]See section A.9 of the online appendix.

[9]For details, see section A.1 of the online appendix.

The gross nominal, one-period interest rate, I_t, equals the conditional expectation of the stochastic discount factor, i.e.

$$I_t^{-1} = P_{1,t}^b = \mathrm{E}_t Q_{t,t+1}, \tag{5}$$

where $P_{1,t}^b$ denotes the price of the one- period bond.

Based on the stochastic discount factor $Q_{t,t+1}$, the price of a n-period zero coupon bond $P_{n,t}^b$ can be written as

$$P_{n,t}^b = \mathrm{E}_t \left[Q_{t,t+1} P_{n-1,t+1}^b \right]$$

and the corresponding nominal yield $R_{n,t}$ as

$$1/I_{n,t} = P_{n,t}^b.$$

Note that we will focus on a symmetric equilibrium in which nominal wage rates are all allowed to change optimally at any point in time, so that individual nominal wages will equal the average w_t.

Equation (4) highlights how our model nests the standard power utility case, in which $\psi = \gamma$ and the maximum value function J_t disappears from the first order conditions. The same equations also demonstrate that the parameter γ only affects the dynamics of higher order approximations. It is straightforward to see that, to first order, the term $\left[\mathrm{E}_t \left(J_{t+1}/J_t \right)^{1-\gamma} \right]^{(\gamma-\psi)/(1-\gamma)} \left(J_{t+1}/J_t \right)^{-(\gamma-\psi)}$ cancels out in the interest rate equation (5).

2.3 Firms

We assume a continuum of monopolistically competitive firms (indexed on the unit interval by j), each of which produces a differentiated good. Demand arises from households' consumption and from the exogenous component G_t, which is an aggregate of differentiated goods of the same form as households' consumption. It follows that total demand for the output of firm j takes the form $Y_t^D(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t^D$. Y_t^D is an index of aggregate demand which satisfies $Y_t^D = C_t + G_t$.

Firms have the production function

$$Y_t(j) = A_t L_t^\alpha(j)$$

where L_t is the labour index L_t defined above and A_t is a mixture of two shocks $A_t = Z_t B_t$ such that, in logs,

$$b_t = b_{t-1} + \xi + \varepsilon_t^\xi, \qquad \varepsilon_{t+1}^z \sim N(0, \sigma_\xi)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \qquad \varepsilon_{t+1}^z \sim N(0, \sigma_{z,s_{z,t}})$$

where ξ is the long run productivity growth rate. This specification allows for both a standard, stationary technology shock and for a stochastic trend, represented by B_t. For the solution and estimation of the model, we will explicitly take the stochastic trend into account.

As in Rotemberg (1982), we assume the firms face quadratic costs in adjusting their prices. This assumption is also adopted, for example, by Schmitt-Grohé and Uribe (2004b) and it is known to yield first-order inflation dynamics around a zero inflation steady state equivalent to those arising from the assumption of Calvo pricing.[10] From our viewpoint, it has the advantage of greater computational simplicity, as it allows us to avoid including an additional state variable in the model, i.e. the cross-sectional dispersion of prices across firms.

The specific assumption we adopt is that firm j faces a quadratic cost when changing its prices in period t, compared to period $t-1$. Consistently with what is typically done in the Calvo pricing literature, we modify the original Rotemberg (1982) formulation for partial indexation of prices to lagged inflation. More specifically, we assume the following specification for the quadratic adjustment cost

$$\frac{\zeta}{2}\left(\frac{P_t^j}{P_{t-1}^j} - (\Pi^*)^{1-\iota}\Pi_{t-1}^\iota\right)^2 Y_t$$

where Π^* is the central bank's inflation target. In a symmetric equilibrium, firms'

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[10]The equivalence does not hold exactly around a positive inflation steady state – see Ascari and Rossi (2010). Moreover two pricing models have in general different welfare implications – see Lombardo and Vestin (2008).

profits maximization problem leads to

$$(\theta - 1) Y_t + \zeta \left(\Pi_t - (\Pi^*)^{1-\iota} \Pi_{t-1}^\iota \right) Y_t \Pi_t \quad =$$

$$\frac{\theta}{\alpha} \frac{w_t}{P_t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{\alpha}} + \mathrm{E}_t Q_{t,t+1} \zeta \left(\Pi_{t+1} - (\Pi^*)^{1-\iota} \Pi_t^\iota \right) Y_{t+1} \Pi_{t+1}$$

2.4 Monetary policy and market clearing

We close the model with the simple Taylor-type policy rule

$$I_t = \left(\frac{\Pi^*}{\beta} \right)^{1-\rho_I} \left(\frac{\Pi_t}{\Pi^*} \right)^{\psi_\Pi} \left(\frac{\widetilde{Y}_t}{\widetilde{Y}} \right)^{\psi_Y} I_{t-1}^{\rho_I} e^{\eta_{t+1}}$$

where $\widetilde{Y}_t \equiv Y_t / B_t$ is detrended aggregate output, $\widetilde{Y}$ its steady state level, Π^* is the constant inflation target and η_{t+1} is a policy shock such that

$$\eta_{t+1} = e^{\varepsilon_{t+1}^\eta}, \qquad \varepsilon_{t+1}^\eta \sim N \left(0, \sigma_{\eta, s_{\eta,t}} \right).$$

Market clearing in the labour market requires labour demand to equal labour supply. In addition, the total demand for hours worked in the economy must equal the sum of the hours worked by all individuals. Taking into account that at any point in time the nominal wage rate is identical across all labor markets because all wages are allowed to change optimally, individual wages will equal the average w_t. As a result, all households will chose to supply the same amount of labour and labour market equilibrium will require that

$$L_t = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{\alpha}}$$

Market clearing in the goods market requires

$$Y_t = C_t + G_t + \frac{\zeta}{2} \left(\Pi_t - (\Pi^*)^{1-\iota} \Pi_{t-1}^\iota \right)^2 Y_t$$

where G_t is an exogenous stochastic process which captures additional non-interest-rate-sensitive components of output and which we specify in deviation from the stochastic growth trend B_t, so that

$$\frac{G_t}{B_t} = \left(\frac{gY}{B} \right)^{1-\rho_g} \left(\frac{G_{t-1}}{B_{t-1}} \right)^{\rho_g} e^{\varepsilon_t^g} \qquad \varepsilon_{t+1}^G \sim N \left(0, \sigma_{G, s_{G,t}} \right)$$

where the long run level g is specified in percent of output, so that $g \equiv G/Y$.

The variable G_t is a common way to introduce demand shocks in the model (see e.g. Rudebusch and Swanson, 2012). It also breaks the theoretical equivalence between GDP (net of price adjustment costs) and consumption. As a result, we will use both these variables in estimation.

3 Solution and estimation methods

3.1 Solution

To solve the model, we first approximate the system around a deterministic steady state in which all real variables are detrended by the technological level B_t. In the solution, we expand variables around their natural logarithms, which are denoted by lower-case letters.

We collect all (detrended) predetermined variables (including both lagged endogenous predetermined variables and exogenous states with continuous support) in a vector x_t and all the non-predetermined variables in a vector y_t.

The reduced form of the model can thus be written in compact form as

$$y_t = g\left(x_t, \widetilde{\sigma}, s_t\right)$$
$$x_{t+1} = h\left(x_t, \widetilde{\sigma}, s_t\right) + \widetilde{\sigma}\Sigma\left(s_t\right)u_{t+1}$$

for matrix functions $g\left(\cdot\right)$, $h\left(\cdot\right)$, and $\Sigma\left(\cdot\right)$ and a vector of i.i.d. innovations u_t. The vector s_t includes the state variables that index the discrete regimes. $\widetilde{\sigma}$ is a perturbation parameter.

Following Hamilton (1994), we can write the law of motion of the discrete processes s_t as

$$s_{t+1} = \kappa_0 + \kappa_1 s_t + \nu_{t+1}$$

for a vector κ_0 and a matrix κ_1. The law of motion of state $s_{z,t}$, for example, is written as $s_{z,t+1} = \left(1 - p_{z,0}\right) + \left(-1 + p_{z,1} + p_{z,0}\right)s_{z,t} + \nu_{z,t+1}$, where $\nu_{z,t+1}$ is an innovation with mean zero and heteroskedastic variance.

For the solution, we follow the approach described in Amisano and Tristani (2011), which exploits the model property that regime switches only affect the shock variances. We can therefore apply standard perturbation methods (as in, for example, Schmitt-Grohé and Uribe, 2004a, or Gomme and Klein, 2011) and approximate the solution as a function of the state vector x_t and perturbation parameter $\widetilde{\sigma}$, but keep it fully nonlinear as a function of the vector s_t. More specifically, we seek a second-order approximation to the functions $g\left(x_t, \widetilde{\sigma}, s_t\right)$ and $h\left(x_t, \widetilde{\sigma}, s_t\right)$ around the non-stochastic steady state, namely the point where $x_t = \overline{x}$ and $\widetilde{\sigma} = 0$.

Due to the presence of the discrete regimes in the system, both the steady state and the coefficients of the second order approximation could potentially depend on s_t in a nonlinear fashion. Since the discrete states only affect the variance of the shocks, however, they disappear when $\widetilde{\sigma} = 0$ so that the non-stochastic steady state is not regime-dependent. Amisano and Tristani (2011) demonstrate that the second order approximation can be written as

$$g\left(x_t, \widetilde{\sigma}, s_t\right) = F\widehat{x}_t + \frac{1}{2}\left(I_{n_y} \otimes \widehat{x}_t'\right) E\widehat{x}_t + k_{y,s_t}\widetilde{\sigma}^2 \tag{6}$$

and

$$h\left(x_t, \widetilde{\sigma}, s_t\right) = P\widehat{x}_t + \frac{1}{2}\left(I_{n_x} \otimes \widehat{x}_t'\right) G\widehat{x}_t + k_{x,s_t}\widetilde{\sigma}^2 \tag{7}$$

where F, E, P and G are constant vectors and matrices and only the vectors k_{y,s_t} and k_{x,s_t} are regime dependent.

The second order approximation proposed by Amisano and Tristani (2011) can be interpreted as a particular case of the more general solution method provided by Foerster *et al.* (2016), namely a situation in which no Markov switching parameter affects the steady state solution of the model. See Foerster et al. *et al.*, p. 639.

Note that regime-switching plays no role to a first order approximation. The quadratic terms in the vector of predetermined variables with continuous support are also regime invariant. Changes in volatility only have an impact on the quadratic terms in the perturbation parameter $\widetilde{\sigma}$. Such terms would be constant in a model with homoskedastic shocks.

3.2 Estimation

Exploiting this feature of the solution, the reduced form system of equations (6) and (7) can be re-written as

$$y_t = k_{y,j} + F\hat{x}_{t+1} + \frac{1}{2}\left(I_{n_y} \otimes \hat{x}'_{t+1}\right)E\hat{x}_{t+1} + Dv_{t+1} \tag{8}$$

$$x_{t+1} = k_{x,i} + P\hat{x}_t + \frac{1}{2}\left(I_{n_x} \otimes \hat{x}'_t\right)G\hat{x}_t + \widetilde{\sigma} \times \Sigma_i \times u_{t+1} \tag{9}$$

$$s_t \curvearrowright \text{Markov switching with } (8 \times 8) \text{ transition probability } P$$

$$P = P_G \otimes P_\eta \otimes P_z$$

$$P_h = \begin{bmatrix} p_{h,1} & 1 - p_{h,1} \\ 1 - p_{h,0} & p_{h,0} \end{bmatrix}, h = G, \eta, z$$

where

$$k_{y,j} = k_{y,s_{t+1}=j}$$

$$k_{x,i} = k_{x,s_t=i}$$

$$\Sigma_i = \Sigma(s_t = i), i = 1, 2, .., 8; j = 1, 2, ..., 8.$$

Note that there are, overall, 8 states, since the volatility of 3 shocks can be either low or high, with independent probabilities across different shocks.

The vector y_t includes all observable variables, and v_{t+1} and u_{t+1} are measurement and structural shocks, respectively. In this representation, the regime switching variables affect the system by changing the intercepts $k_{y,j}$, $k_{x,i}$ and the loadings of the structural innovations Σ_i (we indicate here with i the value of the discrete state variables at t and with j the value of the discrete state variables at $t+1$).

If a linear approximation were used, we would have a linear state space model with Markov switching where the likelihood could be accurately computed by a finite mixture Kalman filter based approximation (see Kim, 1994, Kim and Nelson, 1999, and Schorfheide, 2005).

In the quadratic case, however, such approximation is not available, and one possible approach is to rely on Sequential Monte Carlo (SMC) techniques for the computation of the likelihood. (see Herbst and Schorfheide, 2016). SMC-based likelihood approximations turn out to be very computationally expensive and not

very reliable in the case of our model, given the frequent occurrence of abrupt volatility changes.

Based on the observation that quadratic terms $1/2 \left(I_{n_y} \otimes \hat{x}'_{t+1} \right) E \hat{x}_{t+1}$ and $1/2 \left(I_{n_x} \otimes \hat{x}'_t \right) G \hat{x}_t$ in equations (8) and (9) tend to be small, we therefore proceed as follows.

At any point in time, we first linearize the two quadratic terms around the conditional mean of the continuous state variables. In a homoskedastic setting, this would correspond to applying the extended Kalman filter (Durbin and Koopman, 2012, section 10.2). In our model with regime switching, the linearisation must be conditional on the prevailing regime. As a result, at any point in time we can rewrite equations (8) and (9) as

$$y_t = \widetilde{k}^{(i,j)}_{y,t+1} + \widetilde{F}^{(i,j)}_{t+1} \hat{x}_{t+1} + D v_{t+1}$$
$$\widehat{x}_{t+1} = \widetilde{k}^{(i)}_{x,t} + \widetilde{P}^{(i)}_t \widehat{x}_t + \Sigma_i u_{t+1}$$

for suitably defined coefficients $\widetilde{k}^{(i,j)}_{y,t+1}$, $\widetilde{F}^{(i,j)}_{t+1}$, $\widetilde{k}^{(i)}_{x,t}$ and $\widetilde{P}^{(i)}_t$. Note that, in contrast to the original system (6)-(6), in the above equations both the intercepts $\widetilde{k}^{(i,j)}_{y,t+1}$, $\widetilde{k}^{(i)}_{x,t}$ and the slope coefficients $\widetilde{F}^{(i,j)}_{t+1}$, $\widetilde{P}^{(i)}_t$ become regime-dependent. Nevertheless, we are still in the world of linear state space models with Markov switching. To compute the likelihood, we can therefore apply Kim's (1994) approximate filter— see the on-line appendix for a description of the likelihood compution procedure. We then combine the likelihood with a prior and sample from the posterior using a tuned Metropolis-Hastings algorithm. This approach based on the extended Kalman Filter linearisation is computationally much faster than using sequential Monte Carlo methods.

4 Empirical results

4.1 Functional forms

In our empirical analysis we need to choose a functional form for the utility aggregator $u \left\{ C_t - h \Xi_t C_{t-1}, 1 - N_t \right\}$. As shown by King, Plosser and Rebelo (1988),

consistency with long run growth requires a functional form of the following type

$$u = (C_t - h\Xi_t C_{t-1}) \, v(N_t)$$

where $v(N_t)$ is a decreasing function. Various options are available for $v(N_t)$. We rely on the particular specification proposed by Trabandt and Uhlig (2011), which implies a constant Frisch elasticity of labour supply in the absence of habits and with standard, expected-utility preferences. The specification implies

$$v(N_t) = \left(1 - \eta(1 - \psi) N_t^{1+\frac{1}{\phi}}\right)^{\frac{\psi}{1-\psi}}.$$

4.2 Data description

We estimate the model on quarterly US data over the sample period from 1966Q1 to 2009Q1. Our estimation sample starts in 1966, because this is the date when a Taylor rule begins providing a reasonable characterization of Federal Reserve policy.[11] We end in 2009Q1 when the zero bound constraint, which we do not explicitly include in our model, becomes binding.

Concerning the macro data, we use per capita total real personal consumption, per capita GDP and inflation. We need to use both GDP and consumption because the former enters the Taylor rule and the latter the Euler equation. Given that we abstract from investment, G_t will capture all not-interest-sensitive components of GDP. Inflation is measured as the logarithmic first-difference in the consumption deflator (all macro variables are from the FRED database of the St. Louis Fed). We use continuously compounded yields on 3-month, 3-year and 10-year zero-coupon bonds from Gürkaynak, Sack and Wright (2007).[12]

Prior to the analysis, we take logarithmic first differences for consumption and GDP, which are assumed to follow a stochastic trend. No other data transformations

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[11]According to Fuhrer (1996), "since 1966, understanding the behaviour of the short rate has been equivalent to understanding the behaviour of the Fed, which has since that time essentially set the federal Funds rate at a target level, in response to movements in inflation and real activity". Goodfriend (1991) argues that even under the period of official reserves targeting, the Federal Reserve had in mind an implicit target for the Funds rate.

[12]The data are available at http://www.federalreserve.gov/pubs/feds/2006.

are applied. All variables are expressed as quarterly rates, so that 0.0025 represents an annualized interest rate, inflation rate, or growth rate equal to 1 percent.

4.3 Prior distributions

Prior and posterior distributions for our model are presented in Table 1.

Concerning regime switching processes, we assume beta priors for transition probabilities. We expect the states to be relatively persistent, so we centre all distributions around a value of 0.9, which implies a persistence of 2.5 years for each state. Since it is well know that in mixture models the likelihood is invariant to label permutations of the discrete states, for each of the three discrete volatility states (government spending, technology and monetary policy shocks), we achieve identification by calling "state 1" the low variance state, and we assign a symmetric prior across the two states. Prior and posterior draws not complying with the inequality constraint are therefore suitably permuted (see Geweke 2007).

We use inverse gamma priors for the standard deviations of the shocks. With the exception of the technology growth shock, which has a tighter prior centred around a small value because the process is a random walk, we keep the prior distribution relatively dispersed around a mean value of 0.003. The regime-switching standard deviations also have the same prior distribution in the high and low regimes. To ensure identification, however, all draws from the prior are first ordered and then assigned to the high or low state.

Table 1 reports the resulting empirical distribution for the prior of regime-switching standard deviations. Concerning the persistence of the shocks, we use beta priors centred around the value of 0.85.

For the policy rule, we use relatively loose priors centred around parameter values estimated from quarterly data over a pre-sample period running from 1953 to 1965, namely $\rho_I = 0.85$, $\psi_\Pi = 0.2$ and $\psi_Y = 0.02$.

The priors for all utility parameters are specified broadly in line with the rest of the literature. For the ϕ parameter we rely on a normal prior centred around 1.0, a value in between macro estimates and micro estimates of the Frisch elasticity

of labour supply (see e.g. the evidence reviewed in Chetty *et al.*, 2011). We use a shifted Gamma distribution for ψ and γ, to ensure that $\psi, \gamma > 1$. We centre the distribution of ψ around a value above but close to 1. For the γ parameter, which contributes to shape risk aversion, we use a very large standard deviation whose prior 95 percent confidence set goes from 2 to 30. The habit parameter has a beta prior centred around 0.5. Finally, for β we use a relatively tight prior with a mean of 0.9985. This is consistent with assumptions made in models with growth–see e.g. Christiano, Motto and Rostagno (2014).

For the long run parameters Ξ and Π^* we rely on more dogmatic priors. For Ξ, which determines the growth rate of the economy in the non-stochastic steady state, we use a tight prior centred around 0.005. This implies an annualized growth rate of 2 percent, which is consistent with the average per-capita U.S. GDP/GNP growth from the 1870s to the 1950s–see Maddison (2013). For the inflation target, we choose a prior centred around 1.0063 that gives most mass to annualized values between 2 and 3 percent.

The price adjustment cost ζ is typically calibrated based on the implied frequency of adjustment of prices in linearized models. In our model, however, the relationship is more complex due to both the nonlinearity of the model and the presence of steady state inflation. We therefore centre the prior around 15, which is roughly consistent, for example, with the value used in Schmitt-Grohé and Uribe (2004b), but allow for a relatively large standard deviation. For inflation indexation, we rely on a beta prior centred around 0.5.

The elasticity of intratemporal substitution θ, which is weakly identified, is set dogmatically at 6. Similarly, we set the gross wage mark-up μ_w to 1.2.

4.4 Posterior distributions

The posterior distributions of structural parameters in Table 1 suggest that the data are informative about the estimation of most parameters, as witnessed by the typically smaller standard deviation of the posterior distribution compared to the prior distribution.

Table 1(a): Structural parameter estimates

	post mean	post sd	prior mean	prior sd
$p_{G,11}$	0.8760	0.0556	0.8997	0.0657
$p_{G,00}$	0.9413	0.0351	0.8994	0.0662
$p_{\eta,11}$	0.9595	0.0196	0.8996	0.0657
$p_{\eta,00}$	0.9079	0.0447	0.8998	0.0658
$p_{z,11}$	0.9728	0.0091	0.9013	0.0651
$p_{z,00}$	0.9317	0.0190	0.8993	0.0662
$\sigma_{G,1}$	0.0033	0.0021	0.0021	0.0008
$\sigma_{G,0}$	0.0316	0.0037	0.0039	0.0028
$\sigma_{\eta,1}$	0.0013	0.0001	0.0021	0.0008
$\sigma_{\eta,0}$	0.0039	0.0005	0.0039	0.0027
$\sigma_{z,1}$	0.0109	0.0021	0.0022	0.0008
$\sigma_{z,0}$	0.0271	0.0050	0.0039	0.0029
σ_μ	0.1768	0.0217	0.0031	0.0026
σ_ξ	0.0062	0.0003	0.0012	0.0005
ρ_μ	0.5487	0.0581	0.8552	0.0916
ρ_z	0.9889	0.0018	0.8582	0.0899
ρ_G	0.9091	0.0298	0.8559	0.0906
Π	1.0061	0.0007	1.0063	0.0007
ψ_π	0.2676	0.0241	0.1990	0.1001
ψ_y	0.0497	0.0075	0.0200	0.0100
ρ_i	0.9135	0.0169	0.8494	0.1002
Ξ	1.0045	0.0004	1.0050	0.0010
ι	0.7334	0.1116	0.5003	0.1899
ϕ	0.6156	0.0846	1.0023	0.5049
γ	11.5185	3.6747	10.9537	6.9730
ψ	1.3075	0.0868	1.2035	0.2900
ζ	33.8071	3.1344	14.9744	6.9819
h	0.8619	0.0261	0.4996	0.1886
β	0.9984	0.0006	0.9986	0.0014

Table 1(b): Measurement errors (annualized)

	post mean	post sd	prior mean	prior sd
$\sigma_{me,\pi}$	0.0006	0.0007	0.0006	0.0005
$\sigma_{me,\Delta c}$	0.0005	0.0003	0.0006	0.0004
$\sigma_{me,\Delta y}$	1.4429	0.2470	0.2018	0.1079
$\sigma_{me,i}$	0.0005	0.0003	0.0005	0.0004
$\sigma_{me,i_{12}}$	0.2878	0.0305	0.5512	0.4147
$\sigma_{me,i_{40}}$	0.1748	0.0199	0.5525	0.3994

Legend: "sd" denotes the standard deviation. Priors: Beta distribution for β, h, ι, ζ, ρ_ξ, ρ_g, ρ_π; Gamma distribution for ψ_π, ψ_y and all standard deviations; shifted Gamma distribution (domain from 1 to ∞) for γ, ϕ, Ξ, Π^*; Gaussian distribution for ρ_i. Posterior distributions are based on 200,000 draws.

More specifically, the different regimes in the volatilities of monetary policy, technology and government spending shocks are clearly identified. For monetary policy, the standard deviations in the low and high regimes are equal to 0.13 percent and 0.39 percent respectively. These values straddle the constant standard deviation of 0.24 percent estimated in Smets and Wouters (2007). The standard deviation of technology shocks change between 1.1 percent in the low volatility regime and 2.7 percent in the high volatility regime. The difference between the two volatility regimes is largest for demand shocks: their standard deviation shifts between 0.33 and 3.2 percent.

The posterior mode of the transition probabilities suggests that the low-volatility states are more persistent for monetary policy and technology shocks. For policy shocks, the ergodic probability of being in the low-volatility state is approximately 0.69, which is consistent with the idea that policy shocks were small over most of the sample, except for the Volcker disinflation period. Both the low and the high volatility states are more persistent for technology shocks. These states are countercyclical, being persistently high during recessions and low over expansions. The ergodic probability of the low volatility state for technology shocks is 0.71. By contrast, the volatility of demand shocks is more persistent in the high state, whose ergodic probability is 0.68. Based on these results, we refer to low-volatility regimes for policy and technology shocks as "normal regimes".

As in estimates solely based on macro data, shock processes tend be highly serially correlated. At 0.99, the correlation of the level technology shock process is especially high. Together with the features of the monetary policy rule, this implies that technology shocks have very persistent effects.

The estimated parameter values of the coefficients of the monetary policy rule are of particular interest. *Ceteris paribus* different parameters of the policy rule will be associated with different expectations of the future path of short-term interest rates, thus different configurations of the yield curve. Since we explicitly use yields data when estimating the model, our estimates of the policy rule parameters should be more informative than those obtained without including yields in the

econometrician's information set. Given the well-known problems of general equilibrium models to match the unconditional volatility of long-term yields (see e.g. Den Haan, 1995), one would expect the degree of interest rate smoothing to be higher than in estimates ignoring yields data. A higher smoothing coefficient would impart persistence to any movements in the short-term rate. Its variability would thus be transmitted to longer rates (for a discussion of this point, see Hördahl, Tristani and Vestin, 2008).

To compare our estimates to those in the literature, it is useful to rewrite the rule (2.4) in partial adjustment form. In deviation from the non-stochastic steady state, our parameter estimates imply under a first order approximation[13]:

$$\widehat{i}_t = 0.09 \ \left[3.09 \ (\pi_t - \pi^*) + 0.57 \ \left(\widehat{\widetilde{y}}_t - \widehat{\widetilde{y}} \right) \right] + 0.91 \ \widehat{i}_{t-1} + \eta_{t+1}. \qquad (10)$$

where hats denote deviations from the non-stochastic steady state and tildes denote variables in deviation from the stochastic growth trend.

Equation (10) confirms the above intuition. Compared to the estimates in Smets and Wouters (2007), our parameters imply a somewhat higher, but not exceedingly high, inflation response coefficient.[14] The more striking feature of our estimates, however, is the increase in the degree of interest rate smoothing (0.91 vs. 0.81 in Smets and Wouters). More inertial movements in short-term rates imply that longer-term yields can be systematically affected by monetary policy. This feature is important for the model to be able to generate variation at longer maturities in the term-structure of interest rates.[15]

The estimates of the other structural parameters are roughly consistent with the existing literature.

Concerning long-run means, the mode of the quarterly trend growth rate of

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[13]Second and higher order terms would be zero, because the policy rule is log-linear.

[14]The parameter estimates are not fully comparable, because the policy rule used in Smets and Wouters (2007) includes additional arguments.

[15]De Graeve, Emiris and Wouters (2009) also uses yields data in estimation, but obtains interest rate smoothing estimates similar to Smets and Wouters (2007). In De Graeve, Emiris and Wouters (2009), however, persistent movements in policy interest rates are driven by changes in a stochastic inflation target, which is almost a random walk.

technology is 0.45 and the quarterly inflation target is 0.61, both within the posterior distribution of estimates obtained in Smets and Wouters (2007).

Amongst preference parameters, the posterior mean of ϕ is 0.6. Our estimate of ψ implies a long-run elasticity of intertemporal substitution of consumption of 0.76, which is in line with other available estimates (see e.g. Basu and Kimball, 2002). The γ parameter is equal to 11.5 and the habit parameter $h = 0.86$. Together, these two parameters are suggestive of a high level of risk aversion, which is in line with the results in Piazzesi and Schneider (2006), or in Rudebusch and Swanson (2012).

4.5 Goodness of fit measures

The model fit is good for both macroeconomic and yields data. This claim is supported by various pieces of evidence.

First, measurement errors on all variables are small. This is perhaps not surprising for macro variables and for the short-term interest rate, given the results in Smets and Wouters (2007). For longer-term yields, however, one could expect a worse performance. Nevertheless, both 3-year and 10-year rates are fit rather well. The measurement errors on these two variables are equal to 29 and 18 basis points, respectively. This is a comparable fit to the results in more empirically flexible models such as Ang and Piazzesi (2003).[16]

Second, we check the implications of our model for the dynamic auto-correlations of all observable variables–see Figure (1). The auto-correlations of consumption growth are particularly interesting, because of the difficulty for simple asset pricing models to simultaneously generate positive serial correlation in consumption growth and a positive slope of the term structure–see Backus, Gregory and Zin (1989). We compare model-implied auto-correlations at lags up to 20 quarters to sample auto-correlations. Model-based correlations are computed for all posterior draws. Figure (1) shows the mean and error bands corresponding to a 95 percent confidence set.

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[16]Ang and Piazzesi (2003) is however estimated on more volatile, monthly data.

Figure 1: Model implied and sample dynamic correlations
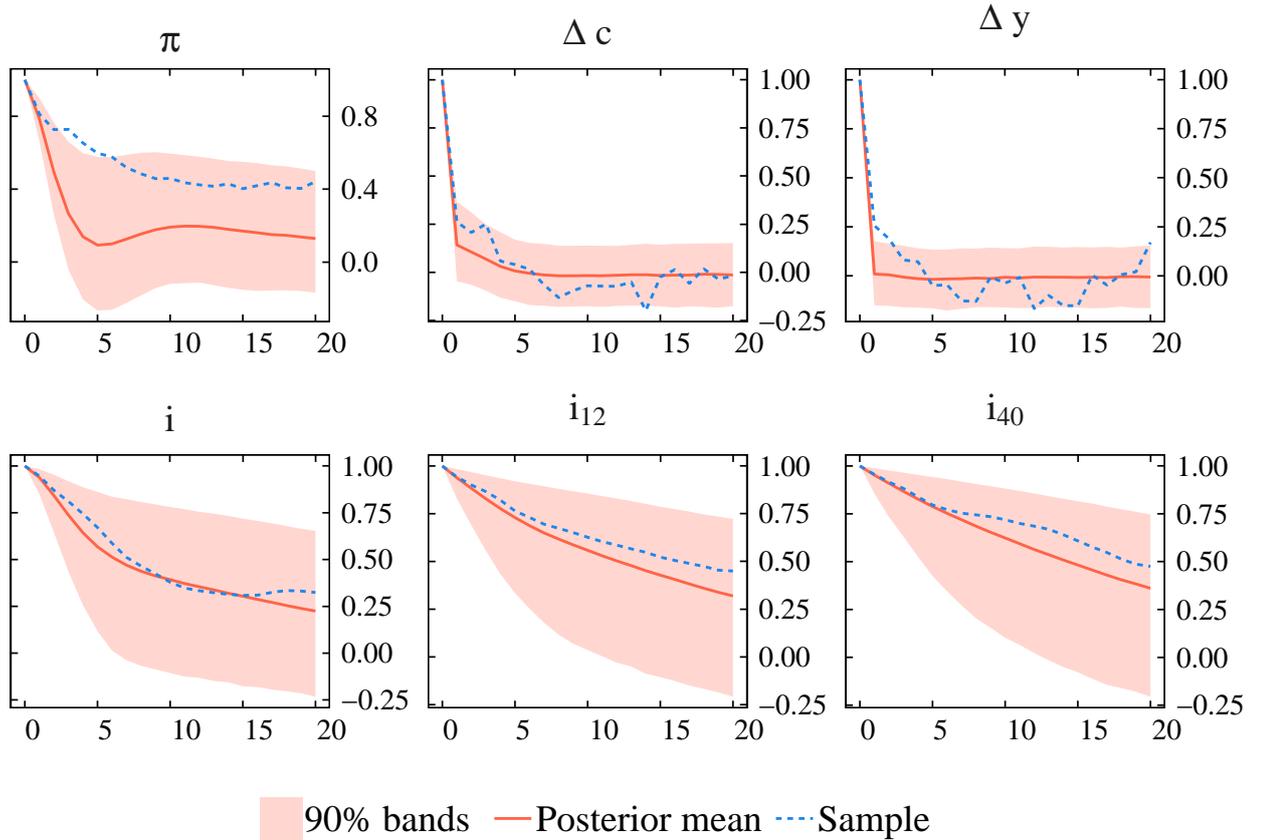
Note: dotted lines display auto-correlation coefficients from the data; solid lines and shaded areas denote the mean and the 5th and 95th percentiles of the distribution across parameter draws of the theoretical correlation coefficients implied by the model.

The figure indicates that the distribution of model-implied auto-correlations always includes its empirical counterpart. The fit is especially good for short and long-term interest rates, whose mean model-implied auto-correlation almost perfectly matches the sample autocorrelation at all lags.

Third, we test the implications of our model for dimensions of the data which were not directly used in estimation, notably for forward rates at various horizons. Model-implied and actual 3-month forward rates in 1, 3 and 10 years are reported in Figure (2). Note that the 1-year rate was not used in estimation. Nevertheless, the model tracks well the evolution of all forward rates.

Figure 2: Model based and actual forward rates at different maturities, % p.a.
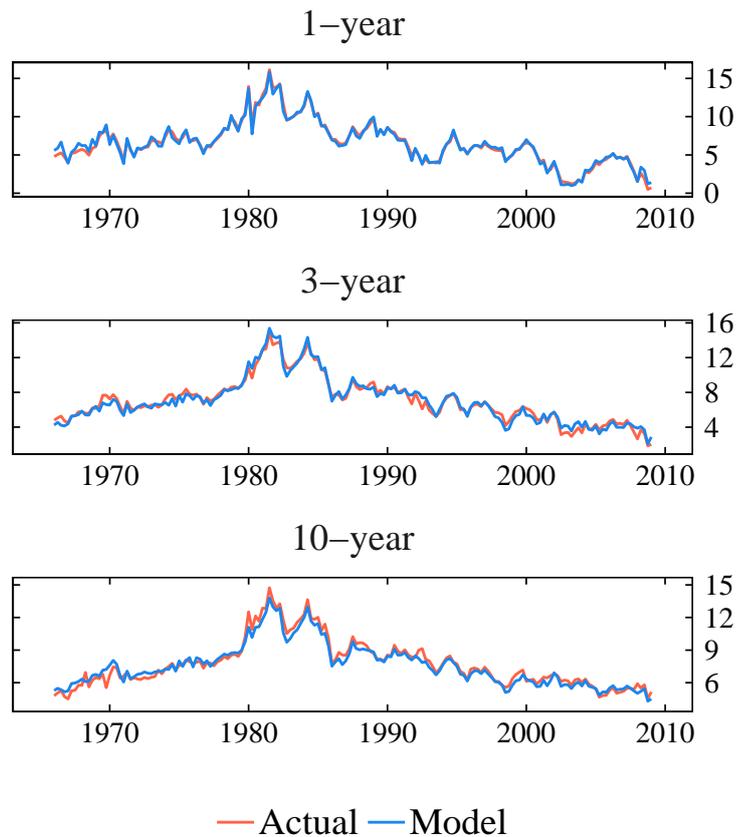
Note: forward rates are taken from Gürkaynak, Sack and Wright (2007).

4.6   Conditional standard deviations and volatility regimes

As a final validation test, we compare model-implied conditional volatilities of the macro variables to a few reduced form counterparts. Since there is no unique way of computing conditional standard deviations, we try two alternative approaches. The first one is a Markov switching VAR including the same observable variables as our DSGE model. It is a natural benchmark for comparison, because it allows for a similar type of parameter variation as the structural model, but without theory-based cross-equation restrictions. To maximize comparability, we allow for 8 parameter regimes for means and variances. Mean and variance parameters are all assumed to

switch simultaneously. In the second approach, we use for each series the simple, univariate unobserved component model proposed by Stock and Watson (2007).

Annualized conditional standard deviations for inflation, the short-term interest rate and the rates of growth of consumption and GDP are reported in Figure (3). For all four variables, conditional standard deviations are computed according to our DSGE model and to the two reduced-form models. The figure also shows conditional standard deviations implied by the Smets and Wouters (2007) model, which are constant over time given that the model is conditionally Gaussian.

Figure 3: Conditional one-step- ahead standard deviations, % p.a.
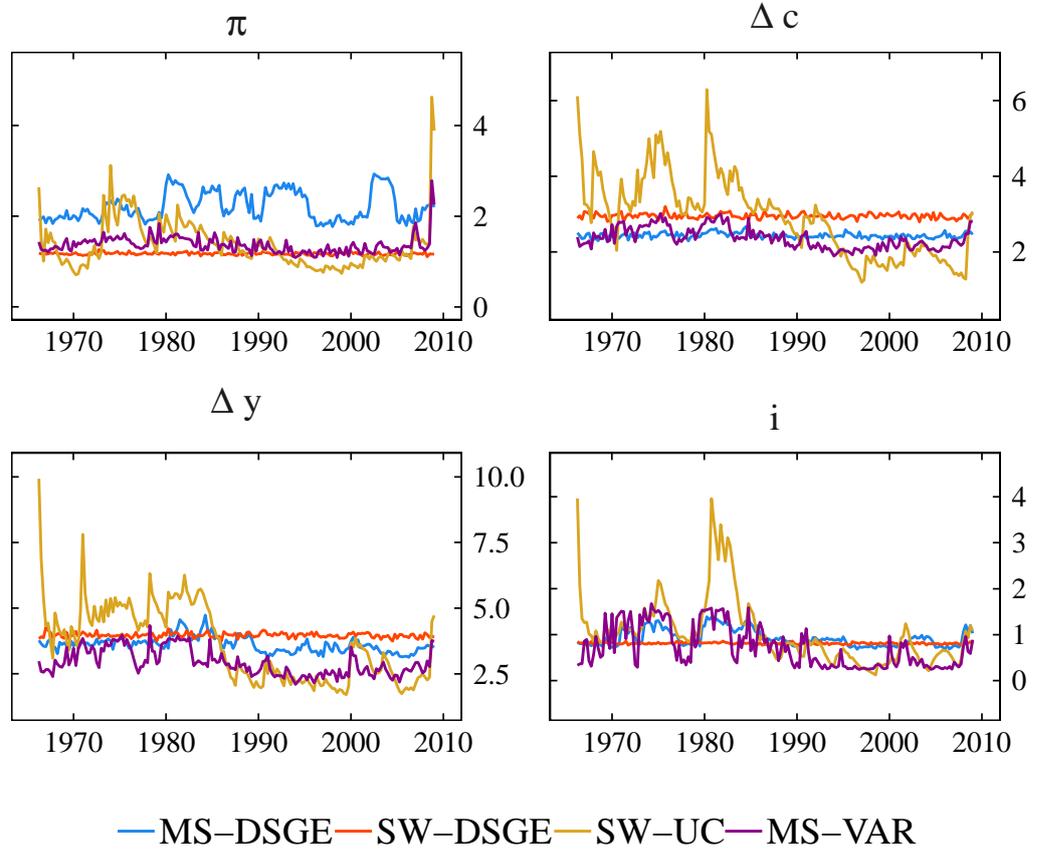

Note: MS-DSGE indicates the model used in this paper, SW-DSGE is the Smets and Wouters (2007) linear DSGE model, SW-UC is the Stock and Watson (2007) univariate unobserved component model with stochastic volatility model, and MS-VAR denotes an 8-state Markov Switching VAR where only the intercepts and the shock covariance matrix is allowed to vary across regimes.

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Estimates from the two reduced form models concur on detecting some degree of time-variation in conditional standard deviations. Compared to the estimates in the Smets and Wouters (2007) model, they tend to find high variances in the seventies and lower variances in the nineties. However, the estimates also show that estimates of conditional standard deviations are somewhat model dependent. The unobserved component model finds very large time-variation over the sample. For inflation, for example, the standard deviation climbs to high peaks in the seventies and then falls to low levels in the nineties. By contrast, the Markov switching VAR finds smaller variations over time.

Taking into account heterogeneity across models, the estimates obtained from our MS-DSGE are broadly in line with the other benchmarks. Our model also tends to find high standard deviations in the seventies and lower in the nineties. The range of variation, however, is smaller than in the reduced-form specifications for consumption and the short-term interest rates. The exception is inflation, which is found to have occasional volatility bursts also in the nineties.

Figure (4) displays filtered and smoothed estimates of the probability of being in a high-variance regime for the three heteroskedastic shocks.

Figure 4: Probabilities of high volatility states
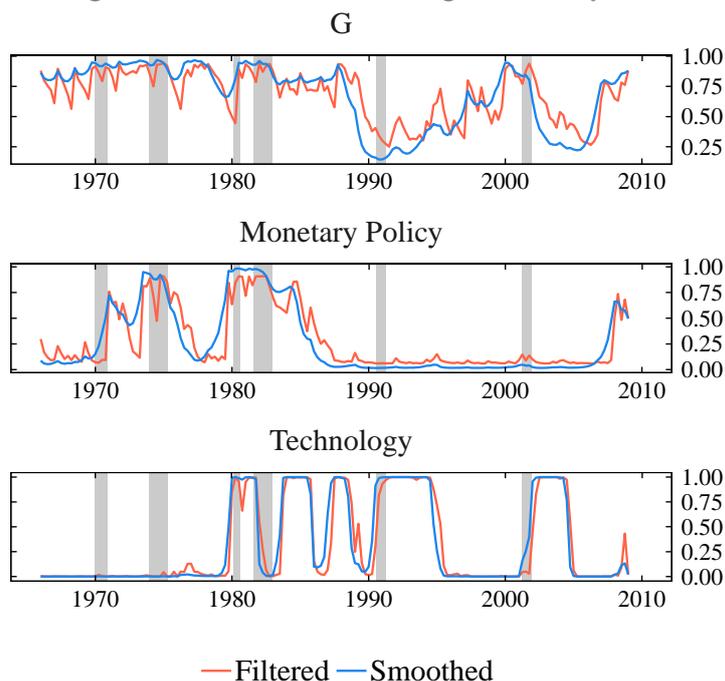
Note: filtered and smoothed probabilities are computed by averaging across draws from the full sample posterior probability of the parameters.

The demand shock has high variance in the first part of the sample and lower variance during the Great moderation period. Concerning the monetary policy shock, our results are consistent with those in Justiniano and Primiceri (2008), where heteroskedasticity takes the form of stochastic volatility, rather than regime switching. The policy shock has a high variance during the mid-1970s and again during the so-called "Volcker disinflation" period in 1979-83. One marginally different feature of our results, is that the increase in volatility in 1979 is estimated to be very rapid in real time. This is arguably consistent with the spikes which can be observed in the short term interest rate over this period. Such sudden increases in volatility can more easily be captured by a regime-switching model than by a stochastic volatility model.

The most striking feature of the regimes for the variance of technology shocks in Figure (4) is that they are strongly cyclical. Starting in 1980, the standard deviation of these shocks tends to increase at the beginning of each recessions and

to fall again after a few quarters. This pattern is quite systematic, especially over the 1990s and the 2000s. The period of the Volcker disinflation is therefore unique in being characterized by high variance of government spending, policy and technology shocks.

5 The role of uncertainty shocks

We next focus on the impulse responses to uncertainty shocks. To understand the impact of these shocks, consider the second-order approximation to the Euler equation

$$
\widehat{\widetilde{c}}_t = \frac{1}{1+h}\mathrm{E}_t(\widehat{\widetilde{c}}_{t+1}) + \frac{h}{1+h}\widehat{\widetilde{c}}_{t-1} - \frac{1}{\overline{\psi}}\left(\widehat{i}_t - \mathrm{E}_t\left[\widehat{\pi}_{t+1}\right]\right) + \frac{1-h}{1+h}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\mathrm{E}_t\Delta\widehat{l}_{t+1}
$$

$$
- \left(\gamma - \psi\right)\frac{1-h}{1+h}\mathrm{Cov}_t\left[\frac{1}{1-h}\left(\Delta\widehat{\widetilde{c}}_{t+1} - h\Delta\widehat{\widetilde{c}}_t\right) + \left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\Delta\widehat{l}_{t+1}, \widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1}\right]
$$

$$
- \frac{\gamma - \psi}{\overline{\psi}}\mathrm{Cov}_t\left[\psi\widehat{\xi}_{t+1} + \widehat{\pi}_{t+1}, \widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1}\right] + \frac{1}{2}\left(\gamma - \psi\right)\frac{\psi - 1}{\overline{\psi}}\mathrm{Var}_t\left[\widehat{\xi}_{t+1} + \widehat{\widetilde{j}}_{t+1}\right]
$$

$$
- \frac{1}{2}\mathrm{Var}_t\Omega_{t+1} \tag{11}
$$

where all variables are expressed in deviation from the non-stochastic steady state. More specifically, $\widehat{\widetilde{c}}_t$ denotes consumption in deviation from the stochastic trend, $\widehat{i}_t$ denotes the nominal interest rate, $\widehat{\pi}_t$ the inflation rate, $\widehat{l}_t$ hours worked, $\widehat{\xi}_t$ the productivity growth rate and $\widehat{\widetilde{j}}_t$ is the conditional present discounted stream of future consumption and hours

$$
\widehat{\widetilde{j}}_{t+1} + \widehat{\xi}_{t+1}
$$

$$
= \sum_{i=0}^{\infty}\left(\beta\Xi^{1-\psi}\right)^i\mathrm{E}_{t+1}\left[\widehat{\xi}_{t+1+i} + \left(1-\beta\Xi^{1-\psi}\right)\left(\frac{\widehat{\widetilde{c}}_{t+1+i} - h\widehat{\widetilde{c}}_{t+1+i-1}}{1-h} - \frac{\psi}{1-\psi}\left(1+\frac{1}{\phi}\right)\frac{\overline{n}}{1-\overline{n}}\widehat{l}_{t+1+i}\right)\right]
$$

Finally $\overline{\psi} \equiv \psi\frac{1+h}{1-h}$ is the inverse of the short-run elasticity of intertemporal substitution, $\overline{n} \equiv \eta\left(1-\psi\right)N^{1+1/\phi}$ and $\mathrm{Var}_t\Omega_{t+1}$ denotes other second order terms such as $\mathrm{Var}_t\widehat{\widetilde{c}}_{t+1}$, or $\mathrm{Var}_t\widehat{\xi}_{t+1}$ which tend to be quantitatively small and which are are made explicitly in the appendix.

The first row on the right-hand-side of equation (11) includes first-order terms. As in the standard new-Keynesian model with habits (see e.g. Woodford, 2003),

consumption is partly backward-looking, partly forward looking, and it is negatively related to the real interest rate. The following terms in the equation arise under Epstein-Zin-Weil utility and involve conditional variance and covariances of expected future utility news, $\widehat{\widetilde{j}}_{t+1} + \widehat{\xi}_{t+1}$. The covariance term in the second row is positive when news about expected future utility growth tend to be associated with high expected growth in consumption and hours worked at $t+1$. When this covariance increases, i.e. expected future utility growth becomes riskier, households increase their precautionary saving and reduce their consumption. This trasmission channel is stronger, the more agents are unwilling to adjust their level of utility across states (i.e. the higher γ). Note, however, that this channel is dampenend by the adjustment in the real interest rate, which falls, thus stimulating the demand for consumption goods, to restore the equilibium in the savings market. A similar effect is produced by increases in the first covariance on the last row of the equation, which is positive when news about expected future utility growth are associated with high inflation and/or high productivity growth at $t+1$. When $\psi \neq 1$ increases in the variance of revisions of expected future utility also play a role and tend to boost consumption, according to the following term in the last row of the equation. This effect is, however, small for our estimated parameter values.

To summarize, increases in the volatility of structural shocks which bring about an increase in the conditional covariance terms in equation (11) tend to produce an increase in precautionary saving and a fall in consumption. For our estimated parameter values, these effects are small in reaction to changes in the conditional variance of demand and policy shocks, but non-negligible after switches in the standard deviation of (level) technology shocks.

We report in Figure 5 the impulse responses to an increase in the variance of technology from the low to the high regime. For illustrative purposes, in this figures we assume that no further changes in the variance regime occur after the shock.

Figure 5: Impulse responses to a one-off volatility shift for the technology shock
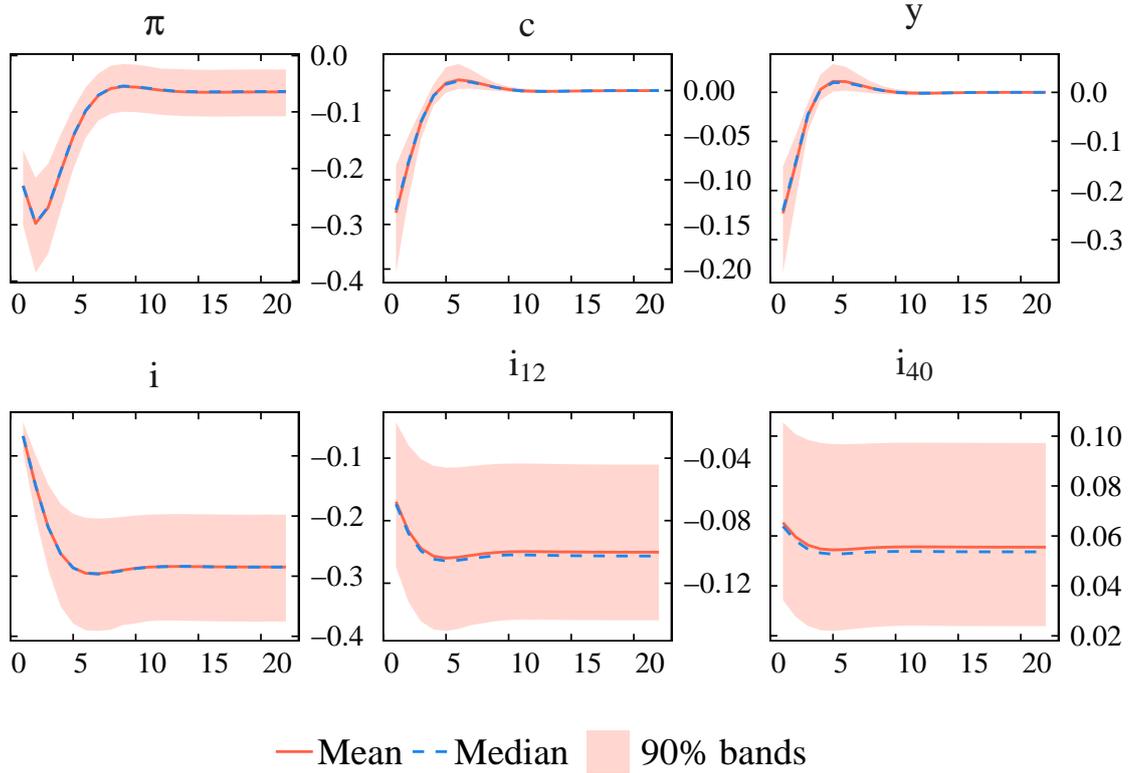
Note: the impulse response functions describe the reaction to a one-off regime shift in volatility, from low to high state. The responses reported here are scaled by the difference between the standard deviations of the technology shocks in the two regimes.

The increase in the variance of technology shocks generates an increase in the demand for precautionary saving. As a result, the demand for consumption goods falls. Given that prices are sticky and output is demand determined, lower demand for consumption goods generates a fall in output and inflation. The policy rate also falls but, due to the high smoothing coefficient in the Taylor rule, very slowly over time. After one to two years, consumption, output and inflation return closer to their initial level, but the policy rate remains low for much longer, because the persistent nature of the uncertainty shock keeps the demand for precautionary saving high and it drives down real rates.

The impulse response of 3-year interest rates also falls, but more mutedly, while 10-year rates increase slighly. These impuse responses compound the aforementioned expectations of lower future policy rates with a generalized increase in risk premia

due to the hightened technological uncertainty. We describe variations in risk-premia in more detail in the next section.

All in all, an uncertainty shock in technology looks like a demand shock, in the sense of being associated with a fall in output, consumption and prices at the same time. Our results corroborate, in the context of an estimated model, Basu and Bundick's (2012) finding that a persistent fall in nominal interest rates is an important part of the macroeconomic adjustment mechanism, following an uncertainty shock. If the fall in the nominal interest rate were prevented by the zero lower bound, the macro-economic effects of the shock would be even larger.

To gauge the role of uncertainty shocks in driving economic dynamics, Table 2 reports the forecast error variance decompositions of all endogenous variables over horizons of 1, 12 and 40 quarters ahead. Consistently with standard results based on linearized models, uncertainty shocks play a negligible role for most macro variables both at short (1 quarter) and business cycle (3-year) horizons. The only exception are changes in the standard deviation of monetary policy shocks, which account for 50% of the 1-quarter ahead variance of short-term rates. This result is in line with the high volatility of policy rates during the high-inflation years at the beginning of the sample and during the Volcker disinflation.

Uncertainty shocks to future productivity play no role in the short-term, but they start playing some role at business-cycle frequency and they become the main driver of the variance of yields over long-term horizons. More specifically, over a forecast horizon of 40 quarters, technological uncertainty shocks account for 53% of the variance of 10-year yields, 52% of the variance of 3-year yields, 45% of the variance of the policy rate, and 33% of the variance of inflation.

Table 2a: Variance decomposition, one step ahead

	π	Δc	Δy	i	i_{12}	i_{40}
σ_G	2[0-4]	0[0-0]	39[24-51]	3[2-6]	9[5-14]	2[1-4]
σ_η	16[7-26]	2[1-5]	1[0-1]	54[31-70]	1[0-2]	0[0-0]
σ_z	2[0-7]	0[0-0]	0[0-0]	0[0-2]	5[0-21]	6[0-26]
ε^G	0[0-0]	0[0-0]	1[0-2]	0[0-0]	0[0-1]	0[0-0]
ε^η	4[3-6]	1[0-1]	0[0-0]	16[8-27]	0[0-0]	0[0-0]
ε^z	15[10-20]	0[0-0]	0[0-0]	4[2-7]	56[41-68]	70[50-81]
ε^ξ	1[1-2]	96[93-98]	43[36-49]	5[2-8]	0[0-0]	0[0-0]
ε^μ	59[48-69]	1[0-1]	0[0-0]	16[11-24]	7[4-11]	1[0-2]
$meas.$	0[0-0]	0[0-0]	16[9-27]	0[0-0]	16[11-22]	13[8-18]

Table 2b: Variance decomposition, 12-steps-ahead

	π	Δc	Δy	i	i_{12}	i_{40}
σ_G	3[2-5]	1[1-2]	35[23-45]	6[3-10]	3[1-5]	1[0-1]
σ_η	14[7-24]	2[1-5]	1[0-2]	7[3-12]	0[0-0]	0[0-0]
σ_z	19[9-30]	3[1-7]	1[0-2]	25[13-40]	37[20-55]	38[21-56]
ε^G	0[0-0]	0[0-0]	1[0-3]	0[0-1]	0[0-0]	0[0-0]
ε^η	5[3-7]	1[0-1]	0[0-0]	2[2-4]	0[0-0]	0[0-0]
ε^z	22[16-29]	4[2-6]	1[1-2]	40[29-50]	56[39-71]	59[42-76]
ε^ξ	2[1-3]	83[75-90]	43[37-50]	1[0-1]	0[0-0]	0[0-0]
ε^μ	34[27-43]	5[3-9]	2[1-3]	18[13-24]	2[1-3]	0[0-0]
$meas.$	0[0-0]	0[0-0]	16[9-26]	0[0-0]	2[1-2]	1[1-2]

Table 2c: Variance decomposition, 40-steps-ahead

	π	Δc	Δy	i	i_{12}	i_{40}
σ_G	2[1-4]	1[1-2]	33[21-44]	3[1-5]	1[0-2]	0[0-0]
σ_η	9[4-17]	2[1-5]	1[0-2]	3[1-6]	0[0-0]	0[0-0]
σ_z	33[19-47]	5[2-10]	2[1-4]	45[28-60]	52[34-68]	53[34-68]
ε^G	0[0-0]	0[0-0]	1[0-3]	0[0-0]	0[0-0]	0[0-0]
ε^η	4[2-5]	1[0-1]	0[0-0]	1[1-2]	0[0-0]	0[0-0]
ε^z	27[20-35]	4[2-6]	1[1-2]	38[26-52]	45[30-62]	46[31-64]
ε^ξ	1[1-2]	81[72-89]	44[37-51]	0[0-1]	0[0-0]	0[0-0]
ε^μ	24[18-31]	5[3-9]	2[1-3]	9[6-13]	1[0-1]	0[0-0]
$meas.$	0[0-0]	0[0-0]	16[9-26]	0[0-0]	1[0-1]	0[0-1]

Legend: i_{40} denotes the 10-year rate; i_{12} is the 3-year rate; i is the short-term rate; π is the inflation rate; Δc denotes the rate of growth of consumption; Δy is the rate of growth of GDP. The shocks indicated in the first column are the three variance shifts and the five continuous shocks: on government spending, monetary policy shock, temporary and permanent technology components, and mark-up. $meas.$ is shorthand for measurement error. The variance decomposition is reported 1, 12 and 40 quarters ahead.

To summarize, uncertainty shocks over the evolution of future productivity play a central role in driving nominal yields over long-term horizons. In the next section, we will demonstrate that this result is related to variations in risk-premia and in expected future real interest rates. The remaining half of the variance of yields over long horizons is explained by standard technology shocks, which produce effects on yields partly because they are extremely persistent, partly due to the endogenous persistence induce by the high interest rate smoothing coefficient.

6 Monetary policy and long term rates

We have shown that uncertainty shocks have macroeconomic effects. We now investigate their effects on bond prices.

6.1 Monetary policy and risk premia

Nominal bonds reflect risk premia associated with both consumption risk and with inflation risk. Hördahl, Tristani and Vestin (2008) demonstrate that models with homoskedastic shocks solved to a second order approximation can only generate constant risk premia. Consistently with this result, our model can produce changes in risk premia only when there is a change in the standard deviation of the structural shocks. In other words, time variation in risk premia is associated with switches in the variance regimes.

A typically used measure of risk premia which is independent of expected changes in the future path of short term interest rate is the expected excess holding period return on a bond of maturity n. This corresponds to the expected return that can be earned by holding an n-maturity bond for one quarter in excess of the one quarter interest rate.

To second order, the expected excess holding period return can be written as

$$
\begin{aligned}
\widehat{h}_{n,t} - \widehat{i}_t \;=\; & F_{B_{n-1}} \mathrm{E}_t \left[u_{t+1} u'_{t+1} \right] \times \\
& \left[\psi \frac{1}{1-h} F'_c + \psi \left(1 + \frac{1}{\phi} \right) \frac{\overline{n}}{1-\overline{n}} F'_l + \psi F'_\xi + F'_\pi + (\gamma - \psi) F'_j \right] \; (12)
\end{aligned}
$$

where F_z denotes the vector of parameters of the first order approximation to the law of motion of any variable z. This equation highlights that all terms in the excess holding period return are constant, except for the variace-covariance matrix of the structural shocks $E_t \left[u_{t+1} u'_{t+1} \right]$. Hence, changes in expected excess holding period returns occur as a result of regime switches in conditional variances. Our model approximation generates differences in the market prices of risk across variance regimes, but regime-switching risk is not priced.

The first term on the right-hand-side of equation (12), $F_{B_{n-1}}$, represents the first-order impact of each of the structural shocks in vector u_{t+1} on the price of a bond with maturity $n-1$. This term captures the degree of riskiness of the bond, namely the typical fluctuation in its price which the representative household can expect. The expression in brackets on the right-hand-side of equation (12) has to do with the household's willingness to bear a risky bond, or price of risk. It suggests that the price of risk is a function of all dimensions which affect household utility, not just the curvature of the utility function with respect to consumption.[17] Swanson (2012, 2018) derive general relationships between the excess holding period return (simply referred to as "risk premium" in those papers) and an appropriate measure of relative risk aversion in models with endogenous labor supply.

The (filtered) expected excess holding period return generated by our model for the 3-year and 10-year bonds is displayed in Figure (6).

---

[17]The parameters h, ψ, ϕ and γ may enter equation (12) also through the endogenous vectors F_c, F_l, F_π and F_j.

Figure 6: Model implied premia (expected excess holding period returns) at different maturities, % p.a.
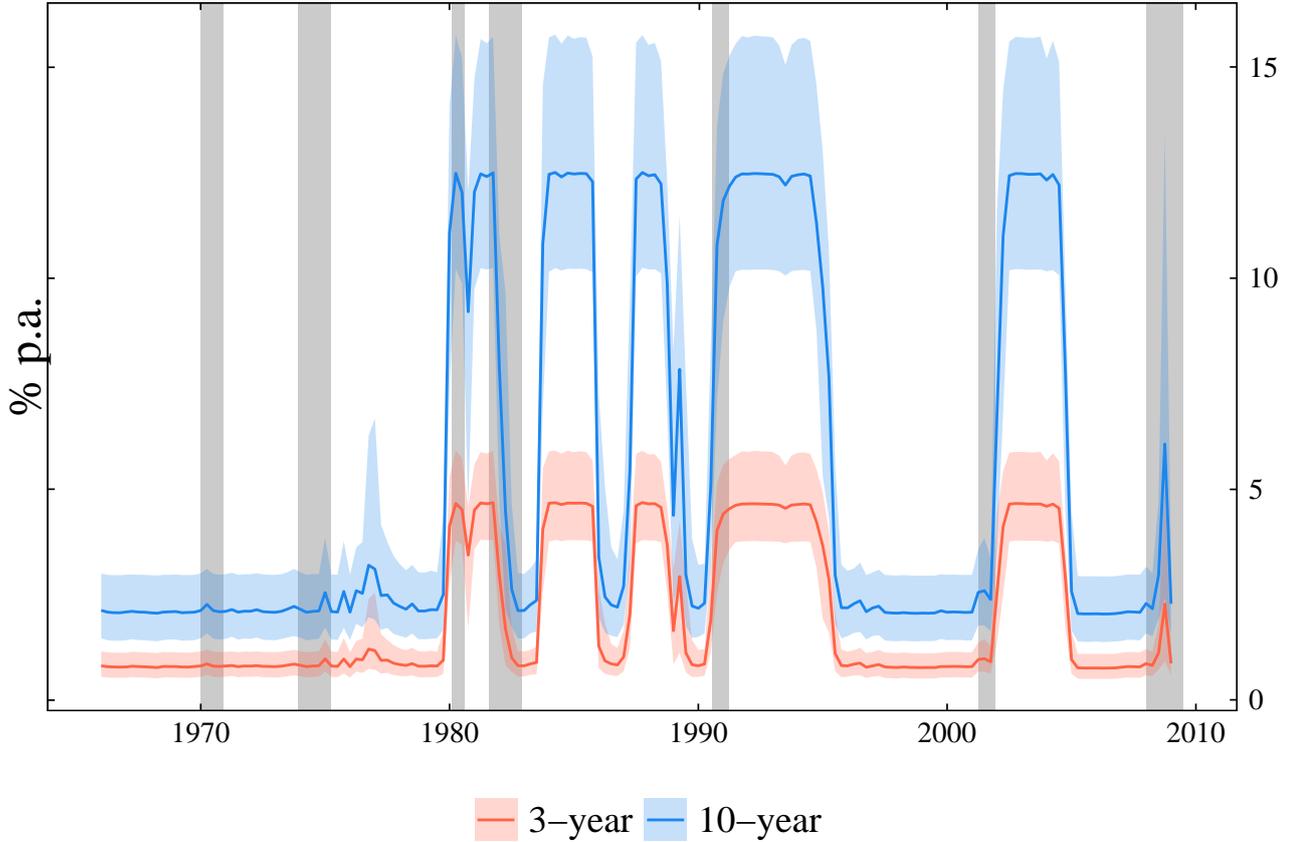
Note: filtered and smoothed premia are computed by averaging across draws from the full sample posterior probability of the parameters.

Excess holding period returns are relatively large. At the 10-year maturity, they average about 8 percentage points in annualized terms over our estimation sample. This value is in the same order of magnitude as some estimates from the finance literature–see e.g. Figure 1 in Duffee (2002). Also consistently with the finding in that literature (see e.g. Fama and French, 1989), risk premia tend to be countercyclical. After remaining around 3 percentage points over the first decade of our sample, they spike to 12 percentage points in annualized terms around most recessions from 1980 to 2008.

In contrast to the finance literature, however, we find variations in risk premia to be much more infrequent. Our results suggest that periods of either high or

low risk premia are very persistent, since they go along with stochastic switches in conditional variance regimes. We therefore miss the higher frequency fluctuations which are typically uncovered by finance estimates.

One would expect that regime switches in the volatility of all shocks lead to variation in risk premia. In the model, however, variations in risk premia must be associated with uncertainty about revisions in the rate of growth of future utility and with their correlations with inflation and with the marginal utility of consumption–see also Restoy and Weil (2011) and Piazzesi and Schneider (2006). From a quantitative perspective, monetary policy and demand shocks have a small impact on the rate of growth of utility over long future horizons. Changes in their variance have therefore a small impact on the size of risk premia. This can be observed through a comparison of figures 4 and 6.

The key source of quantitatively sizable time-variation in risk premia are switches in the variance of technology shocks. Since these variance regimes are estimated very precisely, also in real time, risk premia oscillate mostly between a high and a low value. Consistently with the cyclicality of technological uncertainty shocks, risk premia increase during every NBER-dated recession, then fall again after a few years.

It goes without saying that considerable uncertainty characterizes any estimates of risk premia, because of estimation and model uncertainty. Figure 6 shows that filtering uncertainty is around 5 percentage points at the 10-year maturity. In a classical econometric setting, the small sample bias in maximum likelihood estimates also plays a role–see e.g. Bauer, Rudebusch and Wu (2014), and Wright (2014).

6.2 Yields and the monetary policy transmission mechanism

We have shown that changes in the conditional variances of technology shocks play an important cyclical role. At the beginning of recessions, the increase in volatility is tantamount to a persistent fall in confidence: precautionary saving increases, con-

sumption, output and inflation fall, and so do current and expected future nominal interest rates. After the recovery sets in, the conditional variance of technology switches back to lower levels and confidence returns. The demand for precautionary saving becomes low again, expected future policy interest rates increase.

The dynamics induced by uncertainty shocks also have implications for long-term yields. Their movements are the results of three different forces.

First, as we have already discussed in the previous section, uncertainty shocks generate fluctuations in risk premia. Cyclical increases in uncertainty lead to larger risk premia, thus higher yields levels.

Second, uncertainty shocks produce an increase in precautionary saving, which exerts downward pressure on current and expected future real rates. As a result, nominal yields also tend to fall.

These two effects are not present in a model with constant premia, which therefore does not provide a good description of long-term yields during cyclical turning points. A famous example of this shortcoming occurred in 2004, when long-term yields did not increase in the face of an increase in expected future policy rates. Such behavior of long term yields appeared to be an anomaly compared to previous cyclical developments. In his semiannual Monetary Policy Report to the Congress, Chairman Greenspan stated that "the broadly unanticipated behavior of world bond markets remains a conundrum"–see Greenspan (2005). From the perspective of our model, the conundrum is accounted for by the timing of the fall of uncertainty back to normal levels after the recession. In 2004 it overlapped with the beginning of the monetary policy tightening cycle. The combination of lower risk premia and higher expected future policy rates left observed yields essentially unchanged.

The third determinant of long-term yields is more conventional. Over periods in which volatility stays constant, long-term rates react to changes in monetary policy rates according to the expectations hypothesis. Changes in long term rates reflect variations in *long-term inflation expectations*, which are due to standard, Gaussian shocks.

This result may explain the acceptable forecasting performance of linearized

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models over specific periods of time. For example, De Graeve, Emiris and Wouters (2009) finds that a linearized model is competitive with the random walk in forecasting 1-year yields up to 3-year ahead over the 1990:Q1-2007Q1 period, but less successful in forecasting longer maturity yields. This is not so surprising given that, according to our estimates, risk premia tend to be smaller at short horizons and they only increased and fell four times over the 1990:Q1-2007Q1 period.

To produce changes in long-term rates, Gaussian shocks must be extremely persistent and they need to be coupled with a high degree of inertia in the monetary policy rule. A single shock plays this role in our model: the level technology shock z_t.

Figure (7) shows an impulse response to the technology shock. The shock has the usual opposite effects on output and inflation: real variables increase, while inflation falls. The shock also generates extremely drawn out responses of macroeconomic variables. This is due, first, to the high persistence of the autoregressive process for z_t, whose half-life is of about 15 years.[18] Second, it is due to the high interest rate smoothing coefficient of the Taylor rule, which keeps the short-term real interest rate positive over many quarters after the shock. The increase in the real interest rates reinforces the initial fall in inflation and requires an increasingly loose monetary policy stance over the first year after the shock. It is only after two years that all variables slowly start returning towards their long-run value in a monotonic fashion.

---

[18]The half-life is defined as the number of periods over which the effect of a unit shock remains above 0.5. For an autoregressive process with serial correlation coefficient ρ, the half-life is $hl = \ln(0.5) / \ln(\rho)$.

Figure 7: Generalized impulse responses to a continuous technology shock

90% bands ——— mean - - median
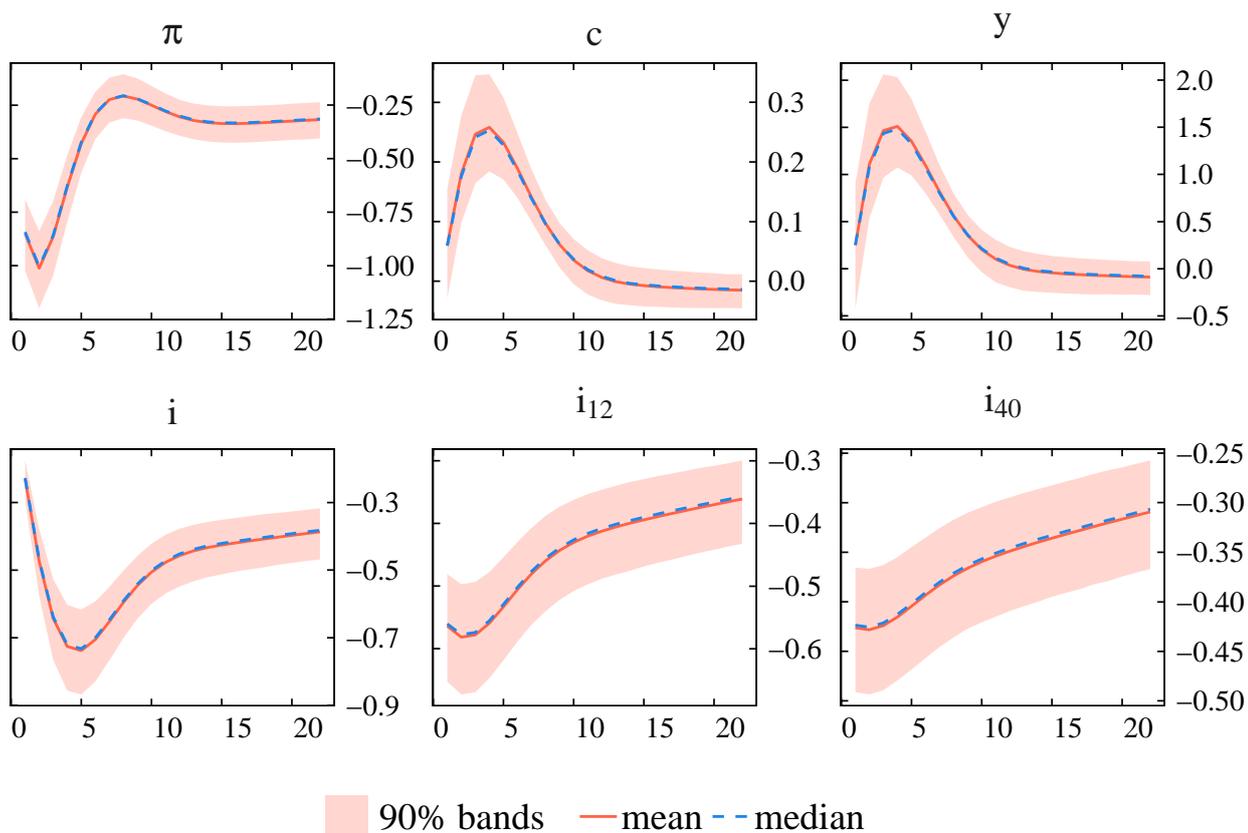
Note: responses to a unit shock on the transitory technology component, with variables expressed in % p.a., and the responses to consumption and output are obtained by cumulating responses to the respective changes.

6.3 Implications for 10-year inflation expectations

Both uncertainty shocks and level technology shocks affect inflation over a prolonged period. It is therefore instructive to analyze the overall implications of our estimates for long-term inflation expectations–i.e. expected inflation over the next 10-year. These expectations are important as their stability, or "anchoring", is often interpreted as a measure of central banks' anti-inflationary credibility. As a benchmark for comparison, we use expectations by the Federal Reserve Bank of Philadelphia's quarterly Survey of Professional Forecasters combined with the Blue Chip Economic Indicators, which is available since 1979:Q4.[19] These expectations relate to inflation as measured by the consumer price index, which tends to be a bit higher than inflation measured by the index for personal consumption expenditures, the inflation rate that we use in our estimation and that since January 2012 represents the Feds primary measure of inflation.

From a secular perspective, a downward trend can clearly be identified in long-term inflation expectations over the 1980s. Over this period, model-implied 10-year inflation expectations are roughly consistent with the survey data–see Figure (8).

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[19]Both surveys report forecasts for the average rate of CPI inflation over the next 10 years. The Blue Chip survey reports long-term inflation forecasts taken twice a year (March and October). Prior to 1983, and in 1983:4, the variable was the GNP deflator rather than the CPI. As of 1991:Q4, we rely on the Philadelphia Survey of Professional Forecasters.

Figure 8: Model based and survey long-run inflation expectations, % p.a.
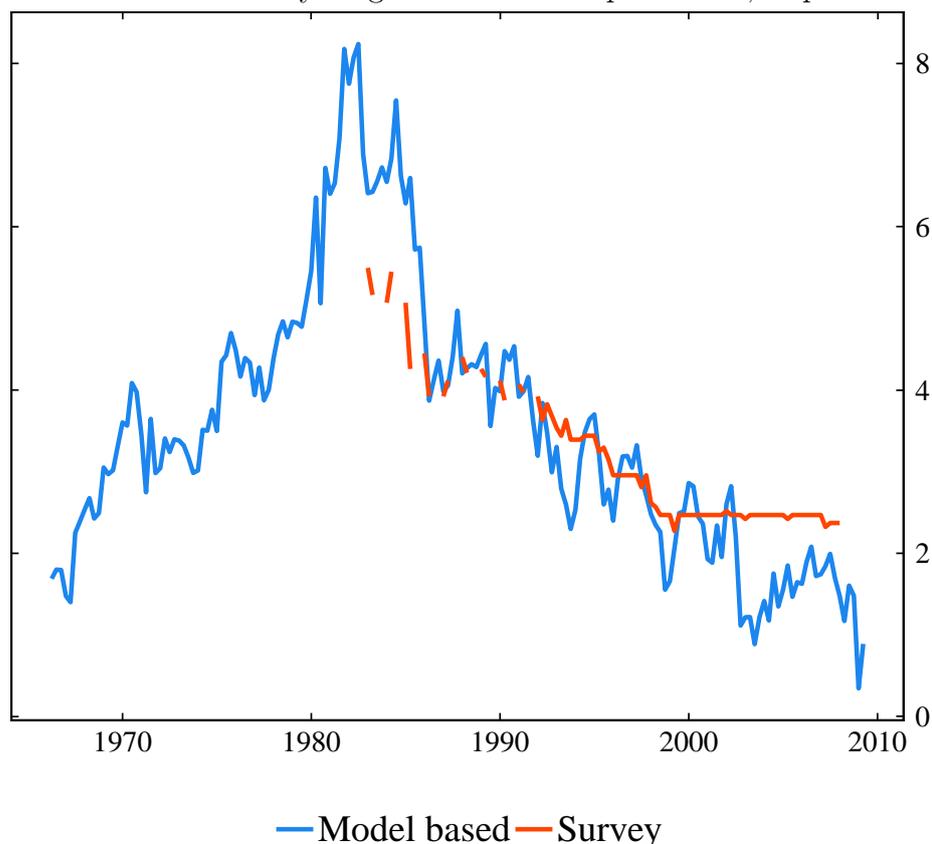
Note: survey inflation expectations reported here taken from the Federal Reserve Bank of Philadelphia's quarterly Survey of Professional Forecasters combined with the Blue Chip Economic Indicators, available since 1979:Q4.

The high volatility of the early 1980s kept risk premia and yields high, even as expected future inflation and expected future policy rates were coming down. From this long-term perspective, the improved anchoring of inflation expectations in the U.S. is undoubtable.

From a more cyclical perspective, however, survey and model-implied results differ. Survey expectations fall steadily towards 2.5 percent over the 1990s and then remain constant at that level through the 2000s. In contrast, yields dynamics interpreted through the lens of our model suggest a much less tight anchoring of inflation expectations.

Model-implied measures fall faster than surveys during the policy tightening phase which started in spring 1988 and was followed by the 1990 Gulf War and the ensuing recession. The fall in long-term inflation expectations is smaller than the fall in 10-year yields, because it is accompanied by a surge in volatility and a fall in real rates.

Model-implied inflation expectations increase again sharply in 1993. An increase in long-term inflation expectations is in line with the idea of an "inflation scare", which was put forward by some commentators in this period. For example, Goodfriend (2002) states: "Starting from a level of 5.9 percent [in October 1993], the 30-year bond rate rose through 1994 to peak at 8.2 percent just before election day in November. The nearly 2 1/2 percentage point increase in the bond rate indicated that the Fed's credibility for low inflation was far from secure in 1994."

Following this period, model-implied inflation expectations remain roughly close to the survey measures. However, model-implied expectations diverge again during the recession of the early 2000s, when they fall sharply to levels around 1 percent. These dynamics are arguably consistent with the views of Federal Reserve officials, who expressed concerns about the, albeit remote, possibility of deflation from late 2002 through 2003. In November 2002, the then Governor Bernanke (2002) judged that "the chance of significant deflation in the United States in the foreseeable future is extremely small", but added that "having said that deflation in the United States is highly unlikely, I would be imprudent to rule out the possibility altogether."

After a return towards 2.5 percent, model-implied long-term inflation expectations fall again ahead of the Great recession, i.e. a period when the possibility of a protracted, low-inflation period was difficult to rule out.

To summarize, our model-implied estimate of long-term inflation expectations implicit in bond prices complements comparable information available from survey data. It suggests that long-term inflation expectations are less rigidly anchored than one would conclude, based on survey data.

Since our estimates are model-based, they may of course be affected by model misspecification. For example, it is possible that either the transition probability

of technology shocks, or the value of their standard deviation, changed in the mid-2000. As a result, the low-volatility shock could have become an absorbing state. Alternatively, the standard deviation of the high-volatility state could have become much closer to that of the low-volatility state. In both cases, estimated risk premia would have been smaller and, for given level of the nominal yield, inflation expectations would have been higher. We cannot exclude this possibility, but it would only reveal itself slowly over time, as more and more data are accumulated.

It is nevertheless noticeable that inflation developments after the Great recession turned out to be more in line with the expectations implied by our model than with survey expectations. With the exception of 2012Q1, the personal consumption expenditure deflator remained persistently below 2 percent after 2008. Over the 9 years between 2009Q1 and 2017Q4 it only averaged 1.5%. By contrast, average inflation over the 10 years starting in 2019Q1 is equal to 1 percent according to model-implied expectations and to 2.5 percent according to the survey.

Our model also provides us with measures of long-term inflation expectations over the late 1960s and 1970s, a period for which survey measures are not available. Levin and Taylor (2013) compute far forward inflation expectations over this period based on a simple methodology and suggest that they started drifting up steadily as of 1965 reaching an estimated peak of about 4.5 percent in 1970 and then remained between 3.5 and 4.5 percent over the next several years. Our estimates are broadly consistent with the results in Levin and Taylor (2013). Average 10-year inflation expectations increase to 4 percent in 1971 and then hover between 3 and 3.5 percent in the early 1970s. In the second half of the 1970s they increase again to levels around 4.5 percent and remain at levels above 4 percent until 1980. These estimates are also consistent with the results in Ang, Bekaert and Wei (2008), which are based on a no-arbitrage factor model of the term structure. Compared to the aforementioned two papers, ours has the additional feature of providing consistent estimates of the structural shocks which led to the model-implied developments in inflation expectations.

7   Conclusions

This paper presents the results of the estimation of a nonlinear macro-yield curve model with Epstein-Zin-Weil preferences, in which the variance of structural shocks is subject to changes of regime. It shows that the model fits the data well: measurement errors are small; the dynamic cross-correlation matrix of the data is closely replicated; long-term forward rates are matched.

An important role to account for this performance is played by changes in variance regimes, or uncertainty shocks, which tends to occur during recessions. On the one hand, uncertainty shocks induce changes in the demand for precautionary savings. Expected real and nominal yields also fall, which is consistent with the empirical evidence. On the other hand, the increase in volatility during recessions also boosts uncertainty over future consumption growth. Risk premia increase in a countercyclical fashion, which is consistent with results from the finance literature.

Compared to survey evidence, model-based measures of long-term inflation expectations are more variable over the economic cycle. They fall to low levels over the 1980s, but are subject to cyclical "scares"–either upwards or downwards. They suggest that the Federal Reserve's credibility for low inflation is less dogmatically established than one would conclude, based on survey data.

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