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Equilibrium Yield Curves with Imperfect Information[*]

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Federal Reserve Board of Governors

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Abstract

I study the dynamics of default-free bond yields and term premia using a novel equilibrium term structure model with a New-Keynesian core and imperfect information about productivity. Imperfect information can justify a shock to signals about productivity that does not lead to actual changes in productivity, which can be interpreted as a demand shock. When incorporated in a DSGE term structure model with a standard productivity shock, this demand shock generates term premia that are on average higher, with sizable countercyclical variation that arises endogenously. The model helps reconcile the empirical evidence that term premia have been on average positive and countercyclical, with numerous studies pointing to demand shocks as a key driver of business cycles over the last few decades.

JEL: D83, E12, E43, E44, E52, G12

Keywords: Yield Curve, Term Premium, Term Structure of Interest Rates, DSGE Model, Imperfect Information, Learning.

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1 Introduction

Among various risk premia of financial assets, the term premium—a function purely of the term structure of pricing kernels—has implications that span beyond default-free bonds, and especially for monetary policy in terms of extracting market expectations accurately from bond prices. In this paper, I propose a structural explanation of the term structure of interest rates, with a primary focus on the economic forces behind the (counter)cyclicality of longer-maturity term premia. A number of studies have shown empirical evidence of countercyclical term premia, consistent with the ample evidence of countercyclical risk premia across multiple asset classes. Meanwhile, a large macroeconomic literature based on dynamic stochastic general equilibrium (DSGE) models or structural vector autoregressions has found that "demand" shocks—shocks that move inflation and aggregate quantities such as output in the same direction—have been an important source of business cycle fluctuations in recent decades, after the great inflation period.[1] Taken together, these findings seem to suggest that demand shocks should play an important role in making term premia countercyclical.

Nonetheless, studies that use DSGE models to analyze yield dynamics, such as Rudebusch and Swanson (2012), have typically considered supply shocks to play a dominant role in generating the empirical pattern of yields and term premia. This is because supply shocks can generate inflation risk premia that are on average positive, which helps explain the significantly positive nominal term premia observed over the last several decades. While term premia can be countercyclical in that setting, the mechanism seems at odds with the aforementioned empirical support for demand shocks in explaining business cycles.

To show how the countercyclicality of term premia can endogenously arise from demand shocks, I explore a novel channel in which imperfect information about productivity plays a crucial role. A number of studies, such as Fajgelbaum et al. (2017) have shown that a form of rational learning about the unobservable states where the precision of the signal is increasing in economic activity can explain key aspects of the business cycle, such as countercyclical uncertainty. My contribution is to show that this framework can be embedded tractably in a nonlinear DSGE term structure model, and generate meaningful demand-side effects that can help explain the dynamics of the term structure of interest rates.

There are, in fact, relatively few studies that systematically analyze the cyclicality of longer-maturity term premia including the post-financial crisis sample. Hence, before I present the model, I offer regression-based evidence on the countercyclicality of longer-maturity term premia using a number of business cycle indicators. I find that there is,

[1] Similarly, I refer to "supply" shocks as shocks that move inflation and quantities in the opposite direction. This is consistent with the "traditional interpretation" (Blanchard (1989)) used in many studies.

overall, statistically significant countercyclicality over the sample period from the beginning of 1990 to the end of 2019. I further confirm that term premia are also positively correlated with measures of aggregate uncertainty.

Motivated by the empirical findings, I first build a simple equilibrium term structure model with imperfect information to clarify the intuition for why a shock to productivity and a shock to signals about productivity that does not lead to actual changes in productivity ("noise" shock) can *both* generate countercyclical term premia. The model consists of a state space model of productivity, a consumption rule, and an Euler equation. Productivity is the sum of a persistent and a transitory component, both unobservable. The agent infers levels of the unobservables from productivity itself and a noisy public signal about the persistent component. Importantly, signal precision is increasing in economic activity, which, in turn, is increasing in productivity or the signal. Intuitively, more economic activity leads to further information about productivity via social learning, and hence the collective signal becomes more precise, i.e., uncertainty about productivity is countercyclical. Since term premia are on average positive in the model, this endogenous countercyclicality of uncertainty leads to the countercyclicality of term premia. In contrast, under perfect information, term premia are still positive but constant. The model is nonlinear, but simple enough to be solved without approximation, and allows for some analytical characterizations of the term premium. However, it remains illustrative, and without featuring inflation, it is difficult to interpret the shocks as "demand" or "supply" shocks.

To address these issues, I build a DSGE term structure model with imperfect information by embedding the productivity structure and the learning process of the simple model into an otherwise standard New-Keynesian setup. The New-Keynesian model allows me to analyze the economic determinants of nominal and real yields in a more realistic setting where inflation is determined endogenously through nominal price rigidities and monetary policy. In this model, intermediate goods firms infer the states of productivity from a public signal that becomes more informative as the output gap increases. While models with information frictions can face computational challenges and be hard to solve without linearization, my specification remains relatively tractable. I solve the model using a high-order perturbation method to account for time-varying uncertainty and term premia.

I calibrate the model to US data and show that the mechanism elucidated in the simple model carries over to the DSGE model in a quantitatively meaningful way. In addition, the noise shock can be clearly interpreted as a demand shock. This is because a noise shock, by raising the *belief* about supply capacity, motivates consumption without the actual increase in supply, causing upward pressure on prices. I also find that imperfect information amplifies the countercyclicality of term premia due to *supply* shocks through an intuitive mechanism of endogenous countercyclical uncertainty, without resorting to exogenous volatility shocks. Thus, I show that incorporating imperfect information significantly increases average nominal term premia and amplifies the countercyclicality of term premia through *both* demand and supply shocks, bringing the model-implied term premium dynamics more in line with empirical estimates.[2] I conclude the analysis by clarifying the role of the key parameters that govern the information friction in my model, and show that real term premia have an important role in the amplification of the volatility and countercyclicality of nominal term premia. The result is consistent with recent papers by Duffee (2018) and Chernov et al. (2021) in emphasizing the real term structure to explain the variation of the nominal term structure of interest rates. I further offer a new explanation based on imperfect information.

The structure of the paper is as follows. Following a literature review, Section 2 presents motivating empirical evidence on the countercyclicality of term premia. Section 3 presents a simple term structure model with imperfect information that fleshes out the key mechanism of the DSGE term structure model, which is analyzed in Section 4. Section 5 concludes.

Literature Review This work is related to a few strands of the literature. First, it builds on macroeconomic models with imperfect information that dates back as early as Kydland and Prescott (1982). It is particularly related to New-Keynesian models that embed imperfect information featuring Bayesian learners about unobservable states with homogeneous expectations, such as Blanchard et al. (2013) and Faccini and Melosi (2022). However, a key difference in the information friction is the feature of procyclical signal precision, bringing my work closer to a smaller set of papers such as Van Nieuwerburgh and Veldkamp (2006), Fajgelbaum et al. (2017), and Ilut and Saijo (2021). While these papers focus on how the information friction helps explain the business cycle, my paper argues that the friction has implications beyond that, and is useful to understand term structure dynamics as well.

Second, it complements studies on production-based asset pricing models with imperfect information, such as Cagetti et al. (2002), Ai (2010), Ai et al. (2018), Winkler (2020), and Bianchi et al. (2022). These papers, however, do not focus on Treasury yields and term premia, as I do.

Third, it follows the large literature on equilibrium term structure models. Compared to endowment economy models (e.g., Campbell (1986) or Piazzesi and Schneider (2007), among many others), the DSGE-based models which this paper directly builds on have been developed relatively recently; a selective list includes: Rudebusch and Swanson (2008, 2012),

[2]While not the scope of this paper, term premia can also be procyclical in important ways. For example, procyclicality can arise from preference shocks (Andreasen et al. (2018)), or safety and liquidity of Treasury bonds (Krishnamurthy and Vissing-Jorgensen (2012)) can become more valuable during recessions.

Doh (2011), Andreasen (2012a), Van Binsbergen et al. (2012), Chen et al. (2012), Dew-Becker (2014), Kung (2015), Lopez et al. (2015), Carlstrom et al. (2017), Andreasen et al. (2018), Swanson (2019), Andreasen and Jørgensen (2020), Gourio and Ngo (2020), and Hsu et al. (2021). A common assumption across these papers is perfect information. By contrast, the key feature of my model is imperfect information. This feature leads to endogenous heteroskedasticity in the pricing kernel, and provides a deeper microfoundation to studies that incorporate exogenous stochastic volatility, such as Andreasen (2012b), Nakata and Tanaka (2016), Bianchi et al. (2023), and Bretscher et al. (2020).

2 Empirical Motivation

I start with a discussion on the empirical pattern of term premia that motivates the paper. In line with risk premia across a broad set of asset classes, the countercyclicality of nominal term premia has been documented in a number of empirical studies. However, analyses that include the post-financial crisis period are somewhat limited in the scope of the term premium measures and the business cycle indicators they consider (see Appendix B.1 for references). Thus, I conduct a regression analysis using a different set of term premium estimates and a range of business cycle indicators. The sample period ends before 2020, since the unprecedented economic impact of the pandemic questions the suitability of the standard statistical methods I use for the analysis. Details of the data are relegated to Appendix A.

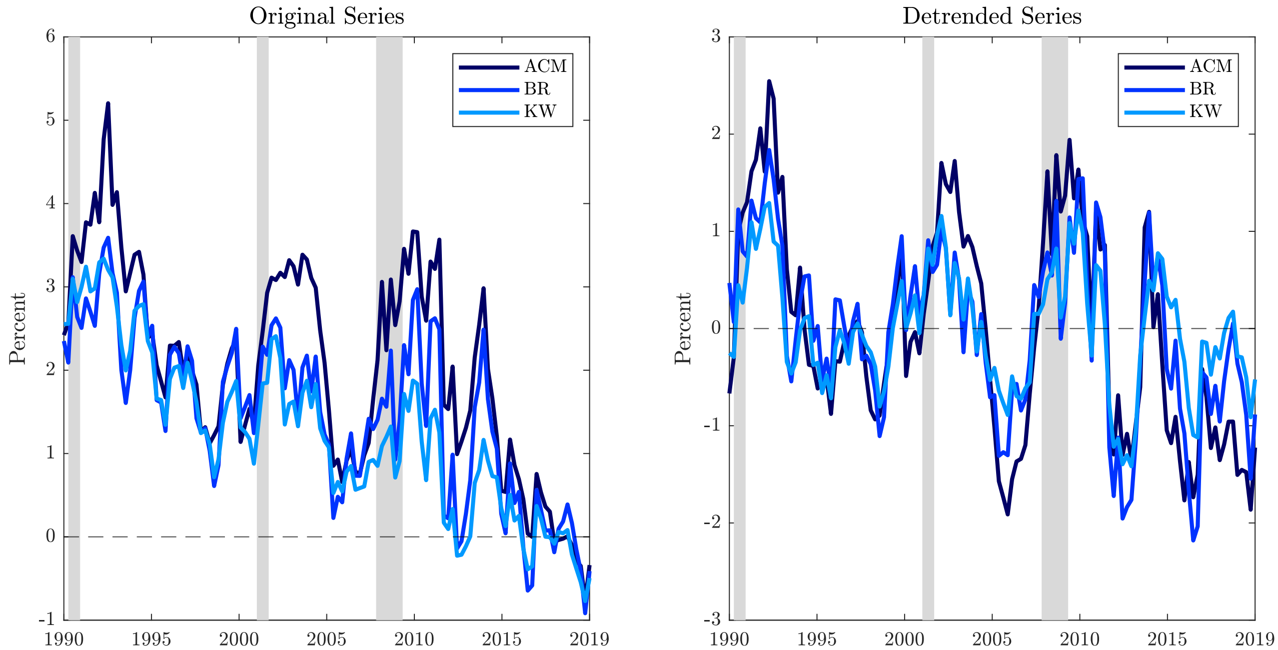

Figure 1: **5-to-10-year Forward Term Premium**

Notes: Quarterly time series from 1990.Q1 to 2019.Q4. ACM is the term premium of Adrian et al. (2013). BR is the term premium estimated from a version of the model by Bauer and Rudebusch (2020). KW is the term premium of Kim and Wright (2005). Shaded grey areas correspond to NBER recession periods. The left panel plots the original term premium series, while the right panel plots the same series detrended using the method of Hamilton (2018).

Figure 1 shows three well-known estimates of the nominal term premium—first, from Adrian et al. (2013) (ACM), second, from a version of Bauer and Rudebusch (2020) (BR), and third, from Kim and Wright (2005) (KW). I focus on longer-maturity measures; in particular, the 5-to-10-year forward measure to mitigate potential measurement issues since the models do not account for the effective lower bound (ELB).[3] Term premia have generally been positive but trending down and were near zero by the end of 2019 (left panel). Meanwhile, the rise in term premia in the last three recessions before 2020 is visually evident, particularly from the detrended series using the method of Hamilton (2018) (right panel).

For the dependent variables, I use these three estimates of nominal term premia. Also, following many other studies, I consider the 1-year expected excess holding return of a 10-year nominal bond.[4] For a sharper focus on the business cycle component of these variables, I use

[3]All three models assume yields and term premia are driven by multiple latent factors, and use no-arbitrage restrictions to help identification, but the three models differ significantly in other aspects. For instance, only KW uses survey forecasts to assist identification, while only BR allows for a non-stationary factor, which may better account for the downward trend in interest rates than strictly stationary models.

[4]Since the excess return per se is not the term premium, I use the expected component of the excess

the detrended series except for the expected excess return, which does not show a significant trend. The independent variables are nonfarm payroll, industrial production, real GDP (all year-on-year changes), GDP gap, unemployment gap, and capacity utilization. I run a series of univariate regressions where each term premium series and the expected excess return are regressed separately on to one of the independent variables with a constant. I take this approach since the goal of the analysis is to gauge the robustness of the countercyclicality of term premia across different indicators rather than to single out the relative importance of a particular one. The results are summarized in Table 1, Panel A, which reports the slope coefficient associated with the macroeconomic indicator for each individual regression.

Results show that the coefficients are negative regardless of the specification, consistent with notable evidence of countercyclicality of the term premium. While the statistical significance varies, it is strong for many specifications, especially for the ACM and the expected excess return. As a robustness check, I provide additional regression results in Appendix B, using the non-detrended series and year-on-year differences of term premia, and find similar countercyclicality. I also show that the evidence of countercyclicality is similarly strong since 2000. This period is interesting since it largely coincides with a period in which the correlation between inflation and consumption growth turned from negative to positive (e.g., Song (2017)) while the correlation between returns on Treasury bonds and equity turned from positive to negative (e.g., David and Veronesi (2013), Campbell et al. (2020)), which could be interpreted as increased relevance of demand shocks as a source of the business cycle.[5]

return as the dependent variable based on Fama and Bliss (1987). Using alternative canonical specifications of Campbell and Shiller (1991) and Cochrane and Piazzesi (2005) lead to broadly similar results.

[5]The subsample from 2000 puts more emphasis on the post-financial crisis period in which unconventional monetary policy such as large scale asset purchases was implemented. Countercyclical term premia can still arise in such an environment if asset purchases contain a shock that lowers term premia and increase economic activity at the same time, as shown by Chen et al. (2012) and Carlstrom et al. (2017).

Table 1: **Regression of Nominal Term Premia on Business Cycle Indicators**

	ACM	BR	KW	FB
Panel A: Macroeconomic Indicators				
ΔNFP	−44.00***	−25.30***	−22.76***	−77.18***
	(8.39)	(5.58)	(4.16)	(13.33)
ΔIP	−7.18**	−2.74	−4.63***	−17.27***
	(3.26)	(2.63)	(1.52)	(4.67)
ΔGDP	−24.64***	−10.05*	−10.67***	−45.54***
	(6.63)	(5.61)	(3.99)	(12.40)
GDP gap	−29.60***	−12.53	−10.28*	−89.14***
	(9.33)	(8.06)	(6.17)	(7.32)
UE gap (negative of)	−25.58**	−10.36	−6.87	−93.94***
	(11.15)	(10.41)	(8.16)	(12.11)
CU	−22.11***	−10.65***	−9.71***	−48.08***
	(3.46)	(2.33)	(1.80)	(7.34)
Panel B: Uncertainty Indicators				
VIX	8.90***	4.33***	3.68***	12.58**
	(2.34)	(1.61)	(1.27)	(5.64)
TFP Vol	1.25***	0.66***	0.44**	2.99***
	(0.30)	(0.24)	(0.20)	(0.51)
JLN (Macro)	4.61***	2.64***	2.56***	6.79**
	(1.35)	(0.96)	(0.75)	(3.24)

Notes: The sample period is 1990.Q1 through 2019.Q4 with quarterly frequency. Each value represents the slope coefficient from a univariate regression of a measure of term premium on to either a macroeconomic or uncertainty indicator plus a constant. The ACM, BR, and KW measures are detrended using the method of Hamilton (2018). The FB measure is the expected excess return based on Fama and Bliss (1987). The first three regressors are year-on-year changes. The regression coefficients are in basis point units per one percentage point change in the regressor. ***, **, and * indicate 1%, 5%, and 10% significance based on Newey-West standard errors with 6 lags (in brackets), respectively.

Given these results, a natural question would be to ask what are the potential macroeconomic drivers behind the countercyclicality. To this end, it is useful to note that DSGE models designed to analyze the source(s) of macroeconomic dynamics have generally implied that, particularly since the mid-1980s, demand shocks have been an important driver of the business cycle. These demand shocks are often shocks to the *level* of a variable (or "first-moment" shocks), but can also be shocks to volatility, as recent studies have emphasized.[6]

[6]Examples from a vast DSGE literature are: Smets and Wouters (2007), Justiniano et al. (2010), Christiano et al. (2014), Blanchard et al. (2013), Gust et al. (2017), Chahrour and Jurado (2018). For a less

Most DSGE models abstract from Treasury yields and term premia, but taken at face value, the findings imply that demand shocks should be a predominant driver of countercyclical term premia. Nevertheless, canonical studies that use DSGE models to analyze the term structure, such as Rudebusch and Swanson (2008, 2012) or Andreasen et al. (2018), have typically considered supply shocks to play a dominant role. This is because supply shocks can generate positive inflation risk premia on average, which helps explain the positive nominal term premia over the last several decades. Recent exceptions to this approach rely on exogenous volatility shocks (e.g., Bretscher et al. (2020), Bianchi et al. (2023)).

To understand the dynamics of yields and term premia in a world of significant demand shocks, I propose an explanation based on imperfect information. As discussed in the rest of the paper, the key mechanism is the link between countercyclical term premia and countercyclical macroeconomic uncertainty, which in turn is caused endogenously through procyclical information production about productivity. Indeed, numerous papers have documented evidence of countercyclical uncertainty. For completeness, I plot several relevant measures of aggregate uncertainty in Figure 2 (left panel): (1) the VIX, (2) the conditional volatility of TFP growth from a GARCH(1,1) model similar to Bloom et al. (2018), (3) the macroeconomic uncertainty index by Jurado et al. (2015), and (4) the standard deviation of the 1-quarter-ahead aggregate forecast distribution of GDP growth from the Survey of Professional Forecasters (SPF). Countercyclical uncertainty can be visually confirmed.

I further run regressions of term premia on measures of uncertainty to provide formal evidence on the link between term premia and uncertainty. Analogous to the previous exercise, I run a series of univariate regressions where each term premium series and the expected excess return are regressed separately on to one of the uncertainty measures with a constant. In particular, I use the first three measures of uncertainty listed above as the independent variables, and report the results in Table 1, Panel B. The results confirm that a rise in uncertainty leads to an increase in term premia, as well as to an increase in expected excess returns. These results complement similar analyses using different measures of uncertainty and sample periods, such as Wright (2011) and Bansal and Shaliastovich (2013). Additional details and robustness checks can be found in Appendix B.[7]

structural approach, see, e.g., Bekaert et al. (2021). Leduc and Liu (2016) and Basu and Bundick (2017) are examples which identify uncertainty shocks as demand shocks.

[7]While measures of aggregate uncertainty are most relevant for the model I develop, I provide some discussion on using a related measure—"disagreement"—in the appendix.

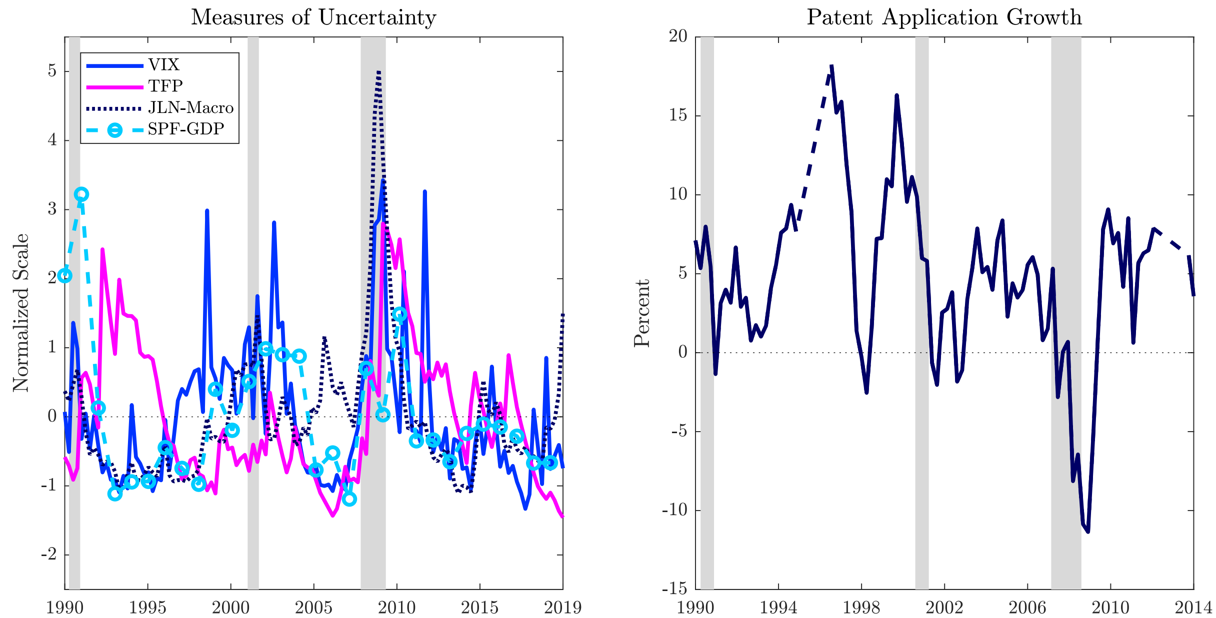

Figure 2: **Measures of Uncertainty and Information**

Notes: "VIX" is the option-based measure of the 30-day expected volatility of the S&P 500 Index. "TFP" is the conditional standard deviation of TFP growth estimated from a GARCH(1,1) model, using the unadjusted TFP growth data by Fernald (2014). "JLN-Macro" is the macro uncertainty index by Jurado et al. (2015). "SPF-GDP" is the standard deviation of the 1-quarter-ahead average forecast distribution of real GDP growth from the Survey of Professional Forecasters. The uncertainty series are quarterly from 1990.Q1 to 2019.Q4 except for "SPF-GDP", which is measured for the last quarter of each year. All uncertainty series are normalized such that they have zero mean and one standard deviation. Patent application is from the dataset compiled by Marco et al. (2015), and the annual growth rate is computed at the quarterly frequency from 1990.Q1 to 2014.Q4. The dashed line portions indicate omitted data corresponding to periods of idiosyncratic volatility due to regulatory changes in 1995.Q2 and 2013.Q1. Shaded grey areas correspond to NBER recession periods.

Meanwhile, the relation between countercyclical uncertainty and procyclical information production has been analyzed by a strand of macroeconomic studies such as Van Nieuwerburgh and Veldkamp (2006) and Fajgelbaum et al. (2017), among others. These studies provide a compelling framework of rational learning with imperfect information which I build on, but they do not necessarily show direct evidence of how information on productivity evolves over time. While such evidence is generally difficult to obtain, I offer one suggestive evidence based on the growth rate of total patent applications in the U.S. (Figure 2, right panel). Intuitively, patent *applications* provide only informative *signals* about potential technological advances, and do not necessarily measure productivity itself.[8] The

[8]Based on the data, the ratio of patent issuance to application was about 60 percent (assuming a 2-year lag between application and issuance), suggesting that a notable share did not result in issuance. That said,

panel shows that the growth rate of applications tends to fall sharply around recessions and recover afterwards, implying that information about new technology may be procyclical.

3 Simple Term Structure Model with Imperfect Information

To build intuition, I first analyze a simple equilibrium term structure model of default-free interest rates with imperfect information.

3.1 Model

The model consists of three parts: (1) a state space system of productivity with unobservable components, (2) a consumption rule that is linear in productivity, and (3) a consumption Euler equation that prices the term structure of interest rates. Apart from transparency, an advantage of its simplicity is that, despite being a nonlinear model, it can be solved easily in a sequential fashion, without approximation methods. In Appendix C, I show that this model is consistent with a stylized real business cycle model without capital.

State space system of productivity Productivity z_t (in logs) consists of a "persistent" component a_t and a "transitory" component $\varepsilon_{z,t}$:

$$z_t = a_t + \sigma_z \varepsilon_{z,t}. \tag{1}$$

z_t is observable to the agent (the representative consumer), whereas its components a_t and $\varepsilon_{z,t}$ are *unobservable*. a_t follows an AR(1) process: $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}$, where $\rho_a \in (0, 1)$. Both $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ are i.i.d. standard normal. All parameters are observable.

In addition to z_t, the agent observes a large number J_t/Δ_j of noisy signals $s_{j,t}$ about a_t, where the indices j are evenly spaced in the interval $[0, J_t]$. Each signal is characterized as:

$$s_{j,t} = a_t + \sigma_s \varepsilon_{s,j,t}, \tag{2}$$

where $\varepsilon_{s,j,t}$ is i.i.d normal with respect to j and t, and has a mean of zero and a variance of $1/\Delta_j$. $\varepsilon_{s,j,t}$ and a_t are unobservable, but each signal is observable. These signals can be interpreted intuitively as "data" (following e.g., Farboodi et al. (2019)) or "news". By

patent applications are found to be informative about future technological advances, and used in recent studies such as Bluwstein et al. (2020) to identify news shocks.

aggregating the signals and taking the limit as Δ_j approaches zero, a sufficient statistic for the collection of signals s_t can be constructed:

$$s_t \equiv \frac{1}{J_t}\int_0^{J_t} s_{j,t}dj = a_t + \sigma_{s,t}\varepsilon_{s,t}, \tag{3}$$

where $\varepsilon_{s,t}$ is i.i.d standard normal, and $\sigma_{s,t}^2 = \sigma_s^2(J_t^{-1})$.[9] $\varepsilon_{s,t}$ can be considered a "noise" shock in line with the literature, since it is the component of s_t which is orthogonal to productivity. This formulation is similar to Fajgelbaum et al. (2017), but while they link J_t specifically to the number of firms entering into production, I interpret the source of J_t more broadly and assume it is an increasing function of an (observable) measure of economic activity from the previous period, defined as h_{t-1}, i.e., $J_t = \phi(h_{t-1})$, where $\phi' > 0$. For simplicity, I assume $\phi(\cdot)$ is exponential, hence $\ln J$ is linear in h:

$$\ln J_t = \xi h_{t-1}.$$

$\xi > 0$ controls the rate of signals produced by h_{t-1}. Since $\sigma_{s,t}$ is inversely related to J_t, h_{t-1} increases the amount of information about a_t, and makes the aggregate signal s_t more precise. This mechanism may be understood intuitively as a "social learning" process where firms, by producing more goods, disseminate noisy information about aggregate productivity in the form of data or news from various media outlets, and in turn, learn about productivity more precisely among themselves as economic activity of others increase.[10]

h_t is specified as an AR(1) process: $h_t = \rho_h h_{t-1} + \sigma_{h,a}\varepsilon_{a,t} + \sigma_{h,s}\varepsilon_{s,t}$, where $\sigma_{h,a}, \sigma_{h,s} \geq 0$, i.e., h_t loads positively on $\varepsilon_{a,t}$ and/or $\varepsilon_{s,t}$. I also assume the agent only observes h_{t-1} at t, and cannot infer $\varepsilon_{a,t}$ or $\varepsilon_{s,t}$ at t to avoid the model becoming trivial. While h_t should, in principle, be explicitly linked to equilibrium variables such as consumption or output, here I simply interpret h_t as representing various activities that produce information about productivity. This specification simplifies the computation considerably, while capturing the "cyclicality" of h_t, and allowing for some endogeneity of the process (in the sense that h_t is not generated from shocks other than what are already in the model). In the DSGE model developed in Section 4, J_t will be fully endogenized as a function of equilibrium output.

[9] s_t can be understood as the limiting distribution of $J^{-1}\sum_{n=1}^{N} s(m_n)\Delta_j$ as $\Delta_j \to 0$, where m_n is the midpoint of interval $[j_{n-1}, j_n] \subseteq [0, J]$ with length Δ_j, and $N = J/\Delta_j$. As mentioned in Fajgelbaum et al. (2017), adjusting the variance of $\varepsilon_{s,j,t}$ by Δ_j is necessary to prevent the signals from perfectly revealing a_t when the number of signals is large and the mass of signals disseminated as $s_{j,t}$ becomes infinitesimal.

[10] My formulation is a simple way to generate procyclical signal precision, which can also be done by alternative mechanisms, such as Van Nieuwerburgh and Veldkamp (2006) and Ilut and Saijo (2021).

The agent updates her belief about a_t via a Kalman filter:

$$a_{t|t} \equiv \mathbb{E}_t[a_t] = \rho_a a_{t-1|t-1} + \boldsymbol{K}_{t-1}(\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}), \tag{4}$$

where $\boldsymbol{s}_t$ is the vector of signals ($\boldsymbol{s}_t \equiv [z_t, s_t]'$), and $\boldsymbol{K}_t$ is the time-varying Kalman gain matrix. $\sigma^2_{a,t}$ is the conditional forecast variance of a_{t+1} ($\sigma^2_{a,t} \equiv \mathbb{V}ar_t(a_{t+1})$), and is updated according to the standard Ricatti equation:

$$\sigma^2_{a,t} = \rho^2_a \left(\frac{\phi(h_{t-1})}{\sigma^2_s} + \frac{1}{\sigma^2_z} + \frac{1}{\sigma^2_{a,t-1}} \right)^{-1} + \sigma^2_a. \tag{5}$$

Consumption Rule and Euler Equation Consumption (in logs) is linear in productivity z_t:

$$c_t = \theta_c z_t. \tag{6}$$

The yield of a n-period default-free (real) bond, $r^{(n)}_t$, is priced by the Euler equation:

$$r^{(n)}_t = \bar{r} - \frac{1}{n} \ln \mathbb{E}_t \left[\exp \left(-\chi_c \sum_{i=1}^{n} \Delta c_{t+i} \right) \right]. \tag{7}$$

θ_c, $\bar{r}$, and χ_c are exogenous parameters. If derived from a fully-specified equilibrium model, the Euler equation is consistent with power utility that has risk aversion of χ_c. It also must be consistent with (6), which imposes a cross-restriction on θ_c and χ_c (see Appendix C). However, such a restriction is largely irrelevant for the discussion in this section.

The term premium is constructed in a standard way. First, define a hypothetical price of an n-period bond $P^{\mathbb{Q}(n)}_t$ formed by discounting cashflows by the risk-free bond price: $P^{\mathbb{Q}(n)}_t = P^{(1)}_t \mathbb{E}_t[P^{\mathbb{Q}(n-1)}_{t+1}]$. Then the n-period hypothetical yield priced under risk-neutrality $r^{\mathbb{Q}(n)}_t = -\frac{1}{n} \ln P^{\mathbb{Q}(n)}_t = -\frac{1}{n} \ln \mathbb{E}_t \left[\exp \left(-\sum_{i=0}^{n-1} r^{(1)}_{t+i} \right) \right]$. The n-period term premium is the difference between the n-period yield $r^{(n)}_t$ and $r^{\mathbb{Q}(n)}_t$:

$$tp^{(n)}_t \equiv r^{(n)}_t - r^{\mathbb{Q}(n)}_t. \tag{8}$$

3.2 Results

I analyze the model dynamics using impulse responses. Since h_t acts as a "time-varying coefficient" in the state space system, the model is nonlinear. Nevertheless, I present standard impulse responses from the ergodic mean for ease of exposition.[11] Parameter values for the

[11] I assume no shocks after the initial one. See Fernández-Villaverde et al. (2011) for a similar approach.

model are chosen for illustrative purposes, and set as follows: $\rho_a = 0.98$, $\rho_h = 0.5$, $\sigma_a = 0.03$, $\sigma_z = 0.08$, $\sigma_s = 0.12$, $\sigma_h = 0.1$, $\xi = 30$, $\theta_c = 1$, $\chi_c = 10$, $\bar{r} = 0$.[12]

3.2.1 Impulse Responses to a Persistent Productivity shock

The dark blue lines in Figure 3 show how the baseline model with imperfect information ("model-BL") responds to a positive one standard deviation (unobserved) shock to a_t. For reference, I also plot impulse responses for a version of the model with perfect information ("model-PI", light blue lines), in which $\sigma_s = 0$, and a version with imperfect information, but when the mass of signals does not vary with respect to economic activity and thus has constant precision ("model-CP", dashed dark blue lines), in which $\xi = 0$. This last version is a popular specification of imperfect information adopted widely in the literature.

[12]For each impulse response, only one of $\sigma_{h,a}$ and $\sigma_{h,s}$ is assigned the value σ_h while the other is set to zero. σ_z is set relatively larger than σ_a so that learning about a_t is gradual, and the additional signal s_t plays a meaningful role in the learning process. The signal production rate ξ needs to be sufficiently high to generate countercyclical uncertainty of consumption and term premia. χ_c is set to generate meaningful variation in yields and term premia. The values for θ_c and $\bar{r}$ are chosen for simplicity.

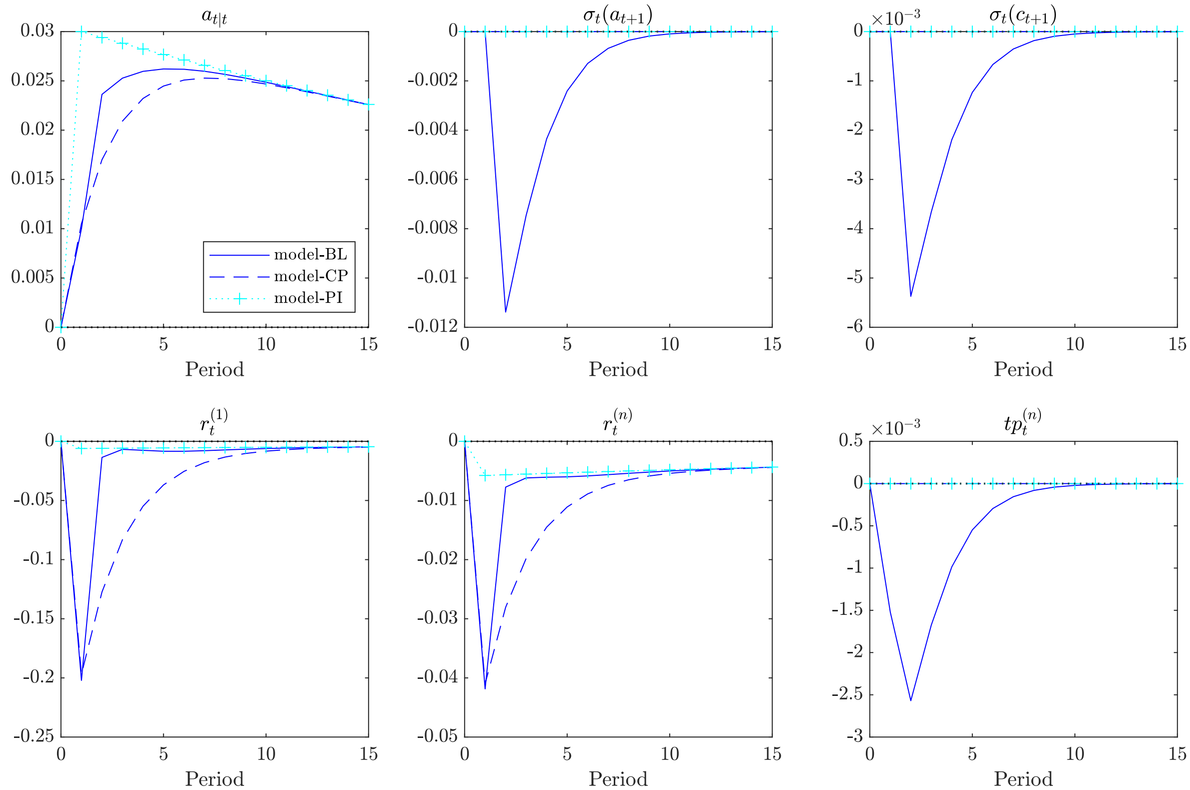

Figure 3: **Impulse Responses to** $\varepsilon_{a,t}$

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI). The yield maturity is $n = 5$.

The top left panel shows the hump-shaped response of the contemporaneous belief about a_t ($a_{t|t}$) under model-BL and CP, which reflects learning about (and hence asymptotes to) the true a_t over time. By contrast, the response of $a_{t|t}$ completely tracks the response of a_t under model-PI. The difference between model-CP and PI is well known (e.g., Edge et al. (2007)). However, compared to model-CP, the agent in model-BL learns faster as economic activity increases and more information about a_t improves signal precision. Indeed, $\sigma_{a,t}$ decreases in model-BL (top middle), but is unchanged in the other models. This is also evident from the characterization of $\sigma_{a,t}$, (5), which implies $\frac{\partial \sigma_{a,t}^2}{\partial h_{t-1}} < 0$.

Due to the underlying increase in a_t, productivity z_t and consumption c_t increases ($c_t = z_t$), but this does not affect the conditional volatility of c_t (z_t) in model-PI and CP (top right). However, in model-BL, we see a significant decrease (dark blue line). In other words, model-BL can generate the well-documented empirical pattern of countercyclical uncertainty.

The bottom three panels of Figure 3 show corresponding impulse responses of the term structure of interest rates. The bottom left and middle panel plot the responses of the 1-period risk free rate ($r_t^{(1)}$) and the n-period yield ($r_t^{(n)}$), respectively. I set $n = 5$ as an example. The declines in $r_t^{(1)}$ and $r_t^{(5)}$ across models are consistent with the Euler equation (7). In the models, an increase in productivity raises consumption, but since consumption is (trend) stationary, consumption growth is expected to decline, which can only be supported with a lower equilibrium rate to induce borrowing. The decline in rates of model-BL and CP are sharper compared to model-PI because when information is imperfect, the agent suspects the shock is transitory, leading to a larger contraction in expected consumption growth.[13]

The key result is the countercyclical drop in the term premium under model-BL (bottom right panel, in dark blue). The reason is twofold. First, with perfect information, the model is basically homoskedastic, with a constant positive term premium. This is because stationarity in consumption leads to a negative autocorrelation in consumption growth, and hence, the pricing kernel.[14] Second, since the term premium is positive for a given level of consumption volatility, countercyclical consumption volatility generated by imperfect information results in countercyclical term premia. The results of model-BL is in stark contrast with model-PI and CP which generate little variation in the term premium. In Section 3.2.3, I make this point more formally through an analytical characterization of the 2-period term premium.

3.2.2 Impulse Responses to a Noise Shock

I now discuss the impulse responses to a positive noise shock $\varepsilon_{s,t}$. For ease of comparison with the impulse responses to $\varepsilon_{a,t}$, the shock is of the same size as $\varepsilon_{a,t}$, which results in the same positive response of h_t. The top left panel of Figure 4 shows distinct responses of $a_{t|t}$ compared to the responses to $\varepsilon_{a,t}$. Since the shock does not impact a_t itself, there is no response to $a_{t|t}$ under model-PI, while under imperfect information, the agent attributes part of $\varepsilon_{s,t}$ to $\varepsilon_{a,t}$ and learns that the signal was actually false only gradually. Similar to the impulse responses to $\varepsilon_{a,t}$, model-BL displays faster learning about a_t compared to model-CP. Since the shock has no impact on productivity, there is no impact on consumption.

[13] The responses of $r_t^{(5)}$ are smaller in magnitude compared to the response of $r_t^{(1)}$ due to the stationarity of interest rates, which is consistent with the empirical evidence of a downward sloping term structure of yield volatility. See Section 4, and studies such as Cieslak and Povala (2016).

[14] This implies that "bad (good)" times are likely to be followed by "good (bad)" times, i.e., there is a stronger demand to hedge for the near-term than for the longer-term, generating a positive term premium. This mechanism has been documented in studies as early as Campbell (1986).

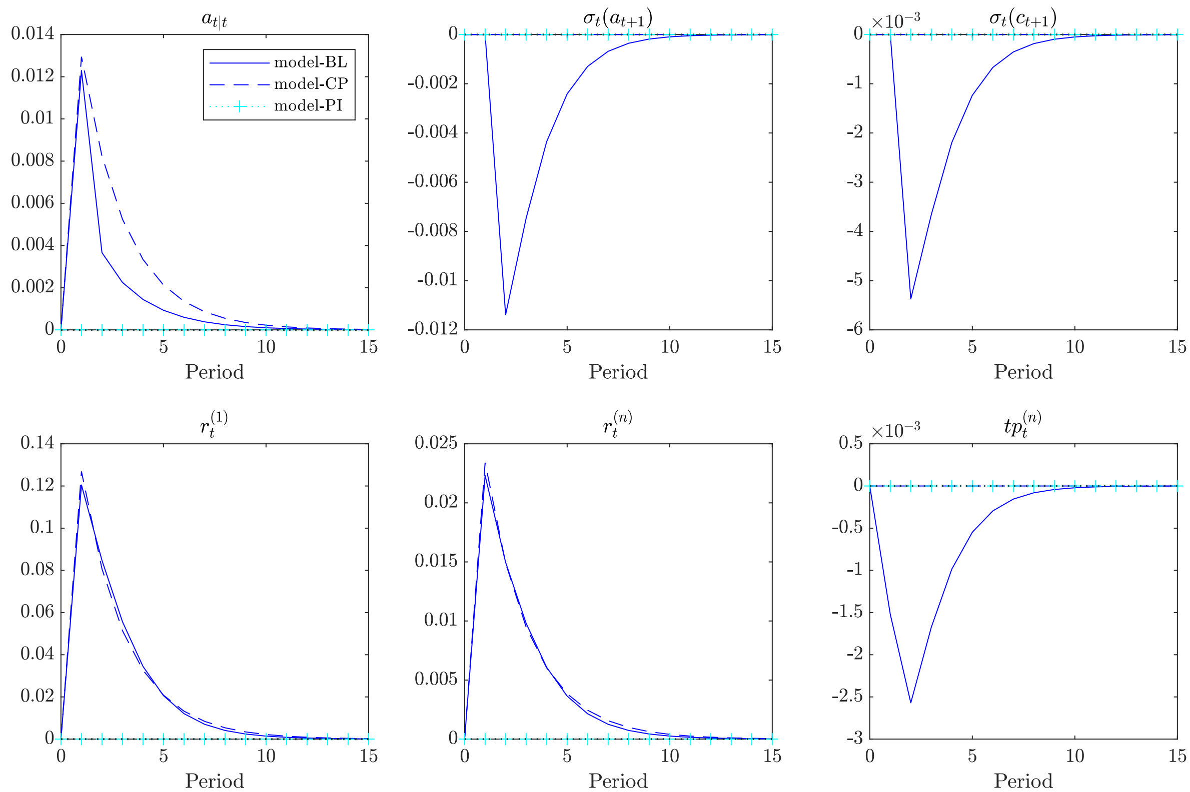

Figure 4: **Impulse Responses to $\varepsilon_{s,t}$**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).The yield maturity is $n = 5$.

Although the responses of $a_{t|t}$ and c_t (z_t) differ significantly from the responses to $\varepsilon_{a,t}$, the increase in h_t nevertheless lowers conditional uncertainty of $a_{t|t}$ (top middle panel) as the flow of information increases, which, in turn lowers the uncertainty about c_t, as in the case of a positive $\varepsilon_{a,t}$ (top right panel). Since the initial shock size is the same for both cases, the impulse responses for the conditional volatilities are in fact, equivalent.

Yields rise in response to $\varepsilon_{s,t}$ since the shock has no effect on current consumption, but yet increases beliefs about future consumption under imperfect information (model-BL and model-CP in Figure 4, bottom left and middle panels). However, consumption beliefs are still stationary, and the countercyclical uncertainty generated from $\varepsilon_{s,t}$ then leads to a countercyclical term premium for model-BL (bottom right). In contrast, both model-PI and CP cannot generate variation in term premia, similar to the responses to $\varepsilon_{a,t}$.

3.2.3 An Analytical Characterization

The model is stylized enough to make the key mechanism of countercyclical term premia transparent. Furthermore, the 2-period term premium has a simple analytical expression that confirms the intuition explained in the previous sections. The conditional log-normality of consumption implies that the term premium of a 2-period real bond is:

$$\begin{aligned} tp_t^{(2)} \equiv r_t^{(2)} - r_t^{(2)\mathbb{Q}} &\propto \mathbb{C}ov_t(m_{t+1}, r_{t+1}^{(1)}) \\ &= (1-\rho_a)\sigma_{a,t}^2 + \sigma_z^2. \end{aligned} \tag{9}$$

Note for simplicity, I assume $\chi_c = \theta_c = 1$. Then, $m_{t+1} = -\Delta c_{t+1}$ is the (real) stochastic discount factor (See Appendix D for the derivation).[15] $\sigma_{a,t}^2$ is the uncertainty about a_t given by (5). Since $\rho_a \in (0,1)$, (9) shows that the 2-period term premium is always positive. Intuitively, due to the (trend) stationarity of a_t, the 1-period bond provides a better hedge against economic fluctuations than the 2-period bond.

Recall from (5), $\frac{\partial \sigma_{a,t}^2}{\partial h_{t-1}} < 0$. This inequality and (9) further imply $\frac{\partial tp_t^{(2)}}{\partial h_{t-1}} < 0$. Therefore, the term premium is decreasing in h, and hence in both ε_a and ε_s, i.e., the term premium is countercyclical. The impulse response analysis above suggests that this intuition extends beyond two periods. Note that a_t is observable under perfect information, in which case:

$$tp_t^{(2)} \propto (1-\rho_a)\sigma_a^2 + \sigma_z^2. \tag{10}$$

Hence, there is no variation in risk quantity and the term premium is constant. By contrast, imperfect information generates endogenous time-varying term premium from homoskedastic shocks. Also, compared to the term premium with imperfect information, the term premium is smaller with perfect information, since $\sigma_{a,t}^2 > \sigma_a^2$ from (5). In this sense, imperfect information helps to increase the level of the term premium, in addition to its variability.

4 DSGE Term Structure Model with Imperfect Information

The analysis of the simple model shows that *both* the persistent productivity shock $\varepsilon_{a,t}$ and the noise shock $\varepsilon_{s,t}$ generate a decrease in the conditional volatility of productivity z_t, which leads to a drop in, or the countercyclicality of, the term premium. Importantly, this is the case though only $\varepsilon_{a,t}$ impacts z_t, and each shock leads to different belief dynamics.

[15] If the term premium was defined as the 1-period expected excess return of the 2-period bond (with a Jensen's correction), the final expression holds exactly.

However, the model remains fairly illustrative, and without incorporating inflation, it is difficult to interpret the shocks as "demand" or "supply" shocks. Hence, I next build a DSGE term structure model with imperfect information embedding the core of the simple model. This allows me to further assess the quantitative relevance of the featured mechanism of countercyclical term premia.

4.1 Model

The model has a mostly standard New-Keynesian core with nominal price rigidities and a monetary policy rule. The key departure is the inclusion of imperfect information. While features such as Epstein-Zin preferences with habit formation are added to improve the model's quantitative performance, I deliberately keep the rest of the model relatively simple and close to canonical models such as Rudebusch and Swanson (2012), so that the impact of imperfect information remains transparent. In this section, I elaborate mostly on the nonstandard features of the model, and relegate a complete characterization to Appendix E.

4.1.1 Households

The representative household has Epstein-Zin preferences (Epstein and Zin (1989)). Its value function V_t takes the following recursive form:

$$V_t = U_t(C_t, N_t) - \beta \left\{ \mathbb{E}_t \left[(-V_{t+1})^{1-\tilde{\gamma}} \right] \right\}^{\frac{1}{1-\tilde{\gamma}}}, \tag{11}$$

where the minus signs in (11) account for the fact that period utility $U_t \leq 0$ in my calibration. U_t takes a standard form with external habits: $U_t(C_t, N_t) \equiv \frac{\left(C_t - \chi_h \tilde{C}_{t-1}\right)^{1-\chi_c}}{1-\chi_c} - G_t^{1-\chi_c} \frac{N_t^{1+\chi_n}}{1+\chi_n}$, where $\chi_c, \chi_n > 0$. G_t is a deterministic trend in total factor productivity (TFP). C_t is the household's aggregate consumption of final goods, and $\tilde{C}_{t-1}$ is consumption in the previous period taken as given by the household. N_t is the labor supply in a frictionless labor market. The case $\tilde{\gamma} = 0$ corresponds to expected utility, with values of $\tilde{\gamma} < 0$ corresponding to risk aversion greater than expected utility (Swanson (2018)). The household faces complete financial markets, and maximizes (11) subject to a standard budget constraint. Thus, assets are priced by the unique pricing kernel M_{t+1}, consistent with (11).

4.1.2 Firms and Imperfect Information

There are a continuum of monopolistically competitive intermediate goods producers (firms) $i \in [0, 1]$ which faces nominal rigidities á la Calvo (1983). If a firm i cannot optimize its price $P_{i,t}$, it indexes the price to a weighted average of previous-period aggregate inflation $\Pi_{t-1} \equiv$

$\frac{P_{t-1}}{P_{t-2}}$ and steady-state inflation $\bar{\Pi}$. Each firm is also subject to demand: $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} Y_t$ and a production function: $Y_{i,t} = K_t^{1-\alpha}(G_t Z_t N_{i,t})^{\alpha}$, where Y_t is aggregate output taken as given by i, and $K_t = \bar{K} G_t$ is capital that grows deterministically with G_t.

TFP consists of two observable components G_t and Z_t. G_t grows at a deterministic rate of $\zeta \equiv \frac{G_t}{G_{t-1}}$. Z_t is a stationary component. Similar to the simple term structure model, $z_t \equiv \ln Z_t$ is composed of a "persistent" component a_t and a "transitory" component e_t:

$$z_t = a_t + e_t. \tag{12}$$

a_t and e_t are *unobservable* and follow independent AR(1) processes:

$$a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t} \tag{13}$$

$$e_t = \rho_e e_{t-1} + \sigma_e \varepsilon_{e,t}, \tag{14}$$

where $\varepsilon_{a,t}$ and $\varepsilon_{e,t}$ are (unobservable) i.i.d standard normal shocks, and $1 > \rho_a > \rho_e \geq 0$.[16] Note the state space system in the simple model was a special case where $\rho_e = 0$.

As in the simple model, firms also observe a continuum of noisy signals $s_{j,t}$ about a_t, where $j \in [0, J_t]$. By assuming $s_{j,t}$ are common knowledge across firms, the individual signals can be aggregated to a noisy public signal s_t. I further assume the mass of signals J_t is increasing in the log "output gap" from the previous period: $\tilde{y}_{t-1} \equiv \hat{y}_{t-1} - \bar{y}$, where $\hat{y}_{t-1} \equiv \ln \hat{Y}_t \equiv \ln \frac{Y_t}{G_t}$ and $\bar{y} \equiv \ln \bar{Y}$ is its steady state. Thus, $J_t = \phi(\tilde{y}_{t-1})$ with $\phi' > 0$, and s_t is given by:

$$s_t = a_t + \frac{\sigma_s}{\sqrt{\phi(\tilde{y}_{t-1})}} \varepsilon_{s,t}, \tag{15}$$

where $\varepsilon_{s,t}$ is i.i.d standard normal and unobservable to the firms (the noise shock). Following the simple model, I specify $\phi(\cdot)$ to be exponential, and hence $\ln J_t$ is linear in $\tilde{y}_{t-1}$, i.e., $\ln J_t = \xi \tilde{y}_{t-1}$. $\xi > 0$ is now the parameter that controls the amount of productivity signals generated by the output gap. Note J_t is increasing with respect to an endogenous variable (output gap) in contrast to the simple model, in which J_t was largely exogenous. That said, J_t is taken as given by each firm, and there is no active learning. The setup abstracts from heterogeneous expectations and higher-order beliefs, making it highly tractable.[17]

[16] While the assumption that a_t is stationary is standard in the macro literature, it is also important in generating positive real term premia (e.g., Rudebusch and Swanson (2012)). Nevertheless, a_t is estimated to be a near random walk, consistent with empirical evidence. Note that although I assume TFP does not have a stochastic trend, it still allows for a deterministic trend.

[17] In reality, agents can utilize various signals about productivity, including endogenous variables such as inflation. Incorporating these variables explicitly into the belief formation is out of the scope of this paper, but part of these effects may be captured through the parameters ξ and σ_s estimated from the data, as the

Firms form (common) beliefs about a_t and e_t by learning from observations of z_t and s_t via a Kalman filter. In other words, the beliefs are updated through:

$$\boldsymbol{x}_{t|t} \equiv \mathbb{E}_t[\boldsymbol{x}_t] = \boldsymbol{\rho}\boldsymbol{x}_{t-1|t-1} + \boldsymbol{K}_{t-1}(\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}), \tag{16}$$

where $\boldsymbol{x}_t = [\hat{a}_t, e_t]'$, $\boldsymbol{s}_t = [\hat{z}_t, \hat{s}_t]'$, $\boldsymbol{\rho} = [\rho_a, 0; 0, \rho_e]$ ("hat"s indicate "demeaned"). $\boldsymbol{K}_t$ is the Kalman gain matrix. Conditional expectations $\mathbb{E}[\cdot|\mathcal{I}_t]$ are defined over the information set $\mathcal{I}_t$ common across households and firms, which precludes $\{a_{t-\tau}, e_{t-\tau}, \varepsilon_{s,t-\tau}\}_{\tau\geq 0}$.

4.1.3 Monetary Policy/Aggregation/Term Structure of Interest Rates

The central bank sets the nominal 1-period interest rate, $R_t^{(1)}$, following a standard Taylor rule: $R_t^{(1)} = \left(R_{t-1}^{(1)}\right)^{\rho_r}\left(\bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\pi}\left[\frac{\hat{Y}_t}{\bar{Y}}\right]^{\phi_y}\right)^{1-\rho_r}$. The market clearing condition for final goods is: $Y_t = C_t + (\zeta + \delta - 1)\bar{K}G_t$, where δ is the capital depreciation rate.

The price of a n-period zero-coupon nominal bond that pays one dollar at maturity $P_t^{(n)}$ is derived recursively via the (nominal) pricing kernel M_{t+1}: $P_t^{(n)} = \mathbb{E}_t[M_{t+1}P_{t+1}^{(n-1)}]$, with $P_t^{(0)} = 1$. The yield to maturity then follows: $r_t^{(n)} = -\frac{1}{n}\ln P_t^{(n)}$ (note $R_t^{(1)} = \exp(r_t^{(1)})$). As described in Section 3.1, the n-period term premium $tp_t^{(n)}$ is computed from equation (8). Real yields and term premia are derived analogously using the real pricing kernel $M_{t+1}\Pi_{t+1}$.

4.1.4 Parameters

I determine the model parameters through a combination of calibration and estimation to fit key moments of macroeconomic variables and the term structure of interest rates in the U.S., over a sample period from the beginning of 1990 to the end of 2008. The end point is a conservative choice to avoid complications due to the ELB.[18] The chosen parameter values are summarized in Table 2.

signal structure (15) does not take a stand on the exact source of the signal.

[18]The data is mostly standard, and described in Appendix A. For an analysis on how the ELB affects yields and term premia using a DSGE term structure model, see for example, Nakata and Tanaka (2016).

Table 2: **Parameter Values for the DSGE Term Structure Model**

Parameter	Description	Value
Household		
$\tilde{\beta}$	Time discount rate	0.990
χ_c	1/EIS	5
χ_h	External habit	0.2
χ_n	1/Frisch elasticity	3
RRA	Risk aversion	55
Firm		
θ	Demand elasticity	6
φ	1 - price adjust. freq.	0.8
ι_p	Indexation weight	0.5
α	Labor share in prod.	0.67
δ	Capital depreciation rate	0.02
$\tilde{\xi}$†	Signal prod.	29.2

Parameter	Description	Value
Monetary Policy		
ϕ_π	Inflation gap coeff.	2.5
ϕ_y	Output gap coeff.	0.05
ρ_r	Interest-rate smoothing coeff.	0.5
$\bar{\Pi}$	Steady state inflation	1.009
Exogenous Processes†		
ρ_a	AR(1) of persistent TFP	0.992
ρ_e	AR(1) of transitory TFP	0.779
σ_a	Std of persistent TFP	0.010
σ_e	Std of transitory TFP	0.013
σ_s	Std of noisy signal	0.029

Notes: Parameters are calibrated except for those with a dagger (†), which are estimated by GMM.

χ_c, the inverse of the elasticity of intertemporal substitution (EIS, without habits), is set to 5. Accounting for habits, the EIS is 0.16.[19] Habit persistence (χ_h) is set to 0.2. These values are within the range found in previous macro studies.[20] χ_n is set such that the Frisch elasticity of labor supply is 1/3, in line with estimates from micro studies. The risk aversion parameter ($\tilde{\gamma}$) is set such that it implies a relative risk aversion of 55, based on the measure by Swanson (2018) that accounts for the household's ability to hedge risk by adjusting its labor supply.[21] This value is largely standard in the macro-finance literature, especially for production-based models designed to match yields and term premia. As discussed in Swanson (2019), the parameter can be interpreted as a stand-in for mechanisms that the model abstracts from, which increase the price or quantity of risk (e.g., ambiguity aversion, timing attitude or heterogeneous agents).[22] The trend growth in TFP is set to 0.9 percent

[19] Log-linearizing the Euler equation implies an elasticity of $(1-\chi_h\zeta^{-1})/\chi_c$.

[20] Many macro studies find EIS to be less than 1. For ranges of χ_h, see, e.g., Del Negro et al. (2015).

[21] Swanson's risk aversion measure for recursive utility with external habit can be computed as:

$$RRA = \frac{\chi_c}{1-\chi_h\zeta^{-1}}\frac{1}{1+\frac{\chi_c \bar{W}\bar{N}}{\chi_n(1-\chi_h\zeta^{-1})\bar{C}}} + \frac{\tilde{\gamma}(1-\chi_c)}{1-\chi_h\zeta^{-1}}\frac{1}{1+\frac{(\chi_c-1)\bar{W}\bar{N}}{(1+\chi_n)(1-\chi_h\zeta^{-1})\bar{C}}}.$$

$\bar{X}$ is the steady state of normalized X_t, i.e., X_t/G_t. Intuitively, the first and third terms on the right hand side constitute the traditional measure of relative risk aversion abstracting from the flexible labor margin, while the second and fourth terms scale down that measure when labor supply is determined endogenously.

[22] In fact, a wide range of values are used in the literature. For example, Rudebusch and Swanson (2012)

per year, consistent with Fernald (2014). The time discount rate $\tilde{\beta}$ is set to 0.99, implying an average 1-quarter real interest rate of 2.0 percent.

The parameters for the firms' problem are standard; the elasticity of substitution among intermediate goods (θ) is set to 6, and the probability with which a firm cannot readjust its price each period (φ) is set to 0.8. The firm uses price indexation which places equal weight (ι_p) on previous-period inflation and steady-state inflation. The labor-share parameter (α) is set to 0.67, and the capital depreciation rate (δ) to 0.02. The choices of the steady state capital stock ($\bar{K}$) and persistent component of technology ($\bar{a}$) mostly determine the capital-output ratio of 2.6, similar to Rudebusch and Swanson (2012).

For the monetary policy rule parameters, I set the coefficient on inflation (ϕ_π), the output gap (ϕ_y), and policy inertia (ρ_r) to be 2.5, 0.05, and 0.5, respectively. These values are largely in line with the literature.[23] Steady state inflation ($\bar{\Pi}$) is set to 1.009 to match an average annual inflation of 2.0 percent, close to the average core PCE inflation over the sample.

The parameters that characterize the aggregate signal, ξ and σ_s, as well as those for the unobserved productivity processes $\{\rho_a, \rho_e, \sigma_a, \sigma_e\}$ do not have obvious empirical counterparts, and are hard to calibrate from past studies. Thus, I estimate these parameters with the Generalized Method of Moments.[24]

As discussed above, the key implication of imperfect information is endogenous time-varying uncertainty about beliefs. Hence, to further assess the plausibility of the estimates, I compare the model-implied moments with measures of conditional uncertainty about GDP growth from the SPF, similar to Fajgelbaum et al. (2017). Specifically, I compute the standard deviation of the average forecast distribution of current-year GDP growth in the SPF as of the start of each fourth quarter, and check whether the model counterpart is in line with the mean and volatility of this standard deviation (results are in Section 4.2).[25]

While models with information frictions can face computational challenges and be hard to solve without linearization, my particular specification remains relatively tractable. I solve the model using a third-order perturbation method to account for time-variation in

uses 110, but values close to 200 can also be found (Bretscher et al. (2020)).

[23]See e.g., Del Negro et al. (2015). ϕ_π is somewhat larger to account for the low volatility of inflation relative to consumption over the sample.

[24]I target the unconditional volatility and first-order autocorrelation of consumption growth, inflation, and labor hours; the unconditional mean, volatility, and first-order autocorrelation of the 1-quarter and 5-to-10-year forward nominal rates, and the 5-to-10-year term premium, with a weighting matrix assigning equal weights to the distance between each model-generated (analytical) moment and its data counterpart.

In the table, I report $\tilde{\xi} \equiv \xi/4/(1+\sigma_{a,t}^{-}{}^2/\sigma_s^2)$ instead of ξ as a more intuitive measure that indicates the percentage increase in signal precision to an annualized percentage increase in the output gap.

[25]See Figure 2 for a visual of the series. I consider this standard deviation as a proxy for conditional volatility of year-over-year GDP growth 2-quarters ahead in the model, taking into account the uncertainty from future revisions of GDP data.

volatility and term premia. The state-space system is pruned, and the moments are computed analytically based on the method of Andreasen et al. (2018).

4.2 Moments

Table 3 summarizes the quantitative performance of the model by comparing model-implied moments with those of the data. The first column reports the moments from the data, and the second column reports the moments from the baseline model with imperfect information (model-BL). For reference, the last column reports the moments from the model with perfect information (model-PI), which shuts down the noise shock ($\sigma_s = 0$) while keeping the rest of the parameters unchanged from model-BL.

The standard deviations of macro variables (consumption growth, inflation, labor hours, and real wages) from model-BL are broadly in line with the data. The model also captures the negative correlation between consumption growth and inflation ($\rho[\Delta c, \pi]$). Model-PI generates a somewhat reduced standard deviation of consumption growth and labor hours. This is intuitive since imperfect information adds uncertainty to the economy by introducing a shock to the signal (though inflation volatility rises a bit). The correlation between consumption growth and inflation is less negative for the model-PI, but this is not necessarily obvious, and I discuss the mechanism in Appendix G.[26]

The table also reports the mean ($\mathbb{E}[\sigma[\Delta c]]$) and standard deviation ($\sigma[\sigma[\Delta c]]$) of conditional volatility of consumption (= GDP) growth 1-quarter ahead. The means of conditional volatility from both models are in line with the surveys. In terms of the standard deviation of conditional volatility, both models fall short of fitting the surveys. However, in terms of model-BL, I interpret this result favorably as the model fitting the large variation in yields and term premia without creating excess time-variation in consumption uncertainty from imperfect information. That said, model-BL generates a significantly larger variation than what is implied by model-PI. This ability to generate time-varying uncertainty in macro variables translates to better performance in fitting interest rates, as I explain below.

[26] I report the correlation of year-on-year changes as it is better linked with longer-term interest rates than the 1-quarter changes.

Table 3: **Selected Moments**

	Data	Model-BL (Imperfect Info.)	Model-PI (Perfect Info.)
Macro Variables			
$\sigma[\Delta c]$	2.07	1.86	1.67
$\sigma[\pi]$	0.97	1.07	1.23
$\sigma[n]$	3.71	4.35	3.98
$\sigma[w]$	2.52	3.51	3.42
$\rho_{-1}[\Delta c]$	0.23	0.22	0.22
$\rho_{-1}[\pi]$	0.79	0.91	0.91
$\rho_{-1}[n]$	0.98	0.96	0.95
$\rho_{-1}[w]$	0.90	0.80	0.74
$\rho[\Delta c, \pi]$	-0.24	-0.16	-0.12
$\mathbb{E}[\sigma[\Delta c]]$	0.66	0.65	0.63
$\sigma[\sigma[\Delta c]]$	0.18	0.07	0.01
Yields			
$\mathbb{E}[r^{(1)}]$	4.00	4.09	4.54
$\mathbb{E}[r^{(20\to 40)}]$	6.44	6.07	6.04
$\mathbb{E}[tp^{(20\to 40)}]$	2.03	2.07	1.60
$\sigma[r^{(1)}]$	1.82	2.12	2.53
$\sigma[r^{(20\to 40)}]$	1.29	1.20	1.00
$\sigma[tp^{(20\to 40)}]$	0.81	0.66	0.05
$\rho_{-1}[r^{(1)}]$	0.95	0.95	0.96
$\rho_{-1}[r^{(20\to 40)}]$	0.94	0.99	0.99
$\rho_{-1}[tp^{(20\to 40)}]$	0.90	0.99	0.99

Notes: Data is quarterly and in annualized percent. The sample period is from 1990.Q1 to 2008.Q4. $\mathbb{E}[\cdot]$, $\sigma[\cdot]$, and $\rho_{-1}[\cdot]$ indicate the unconditional mean, standard deviation, and 1-quarter autocorrelation, respectively.

Model-BL fits the nominal term structure moments well. As a proxy for long-term yields, I use the 5-to-10-year forward rate for ease of comparison with the empirical analysis in Section 2. The model broadly matches the average level of the yield curve data, as can be seen from the fit to the average 1-quarter rate ($\mathbb{E}[r^{(1)}]$) and the 5-to-10-year forward rate ($\mathbb{E}[r^{(20\to 40)}]$). The model also fits the standard deviations of the 1-quarter rate ($\sigma[r^{(1)}]$) and the 5-to-10-year forward rate ($\sigma[r^{(20\to 40)}]$) reasonably well. Conversely, the average slope in model-PI is notably smaller, and the yield volatility curve is much more downward sloping than what the data and model-BL suggest.

It is worth emphasizing the model's fit to empirical estimates of the nominal term premium.[27] In particular, model-BL can generate a sizable average term premium for the 5-to-10-year rate ($\mathbb{E}[tp^{(20\to 40)}]$). The term premium is positive because, in the model, (1) average real term premium is positive due to the (trend) stationarity of consumption, and (2) average inflation risk premium is positive due to the negative correlation between (longer-run) consumption growth and inflation. Additionally, while the term premium is not as volatile as what an average of the empirical estimates suggests, it still explains about 80 percent of the standard deviation of its empirical counterpart ($\sigma[tp^{(20\to 40)}]$). This is in clear contrast to model-PI, in which both the average and especially the volatility of term premia are significantly smaller. Importantly, the increase in term premium volatility by incorporating imperfect information is not simply a reflection of higher volatility in the macro variables. For example, the ratio of term premium volatility to consumption growth (inflation) volatility is 36 (61) percent in model-BL, but only 3 (4) percent for model-PI.

While I focused on the nominal term structure thus far, the model also has implications for the real term structure. In particular, model-BL can generate a real yield curve that has a sizable upward slope on average, with plausible variation in longer-term real yields. Further details of the real term structure moments can be found in Appendix F.

4.3 Impulse Responses

To understand the role of imperfect information further, I next turn to impulse responses, which can isolate the contribution of each shock to the dynamics of macro variables and interest rates. The format of Figures 5 and 6 closely follows the analysis for the simple term structure model, but here I plot the (nonlinear) generalized impulse responses of Koop et al. (1996). The dark blue lines show responses of model-BL. For reference, I also plot responses of model-PI (light blue lines), and responses of a model with imperfect information and constant precision ("model-CP", dashed dark blue lines) in which $\xi = 0$.

4.3.1 Impulse Responses to a Persistent Productivity shock

The first two rows of Figure 5 show responses of the macro variables to an (unobserved) positive $\varepsilon_{a,t}$ shock. Consumption (top left) rises as inflation drops (top middle), allowing $\varepsilon_{a,t}$ to be interpreted as a "supply" shock for all models. Consumption shows an empirically plausible hump-shaped response, which is partly due to habit formation (common across all models), but more importantly for model-BL and CP, generated from imperfect information which leads to a gradual learning of a_t. Moreover, consumption in model-BL shows a

[27] The moments of the 5-to-10-year term premium listed in the data column are the averages over the three term premium estimates analyzed in Section 2.

somewhat faster increase compared to model-CP, due to the mechanism similar to that in the simple model. Indeed, model-BL shows a decrease in the uncertainty of beliefs about a_t (top right) as well as z_t (2nd-row right)—responses that cannot be observed in model-CP and PI. In terms of matching consumption and the term structure data jointly, the endogenous hump-shaped consumption response due to learning is an appealing feature. While the trend stationarity of consumption ensures the average (real) term premium to be positive, the gradual increase in consumption after the response leads to a positive autocorrelation of consumption growth in the near-term, consistent with the data.

Compared with model-PI, inflation in model-BL decreases more, reflecting the slower pickup in consumption. The decrease in inflation is most pronounced for model-CP, since households learn about a_t the slowest, and thus demand is held back the most. Labor hours (2nd-row left) decrease upon the shock for all specifications, which is consistent with some leading (perfect information) DSGE models such as Smets and Wouters (2003).[28] Real wages (2nd-row middle) generally rise as productivity increases, but the rise is slower for model-BL and CP, in which it takes time to learn that the increase in productivity will persist.

The bottom two rows show the responses of interest rates and term premia. The 1-quarter nominal rate (policy rate) declines in model-BL (3rd-row left); a standard response to a positive supply shock as the central bank accommodates deflationary pressure. The degree of accommodation is most pronounced in model-CP, followed by model-BL, reflecting the size of the decline in inflation. The relatively large drop in the policy rate compared to longer-term yields leads to an increase in the nominal yield spread in model-BL and CP (3rd-row middle). These yield spread responses are consistent with similar models with perfect information. The 1-quarter real rate (bottom left) and the real yield spread (bottom middle) show a qualitatively similar pattern as their nominal counterparts.

[28] Labor hours can be made to respond positively by using a utility function proposed by Greenwood et al. (1988). The impulse responses for the other variables are qualitatively similar with this modification.

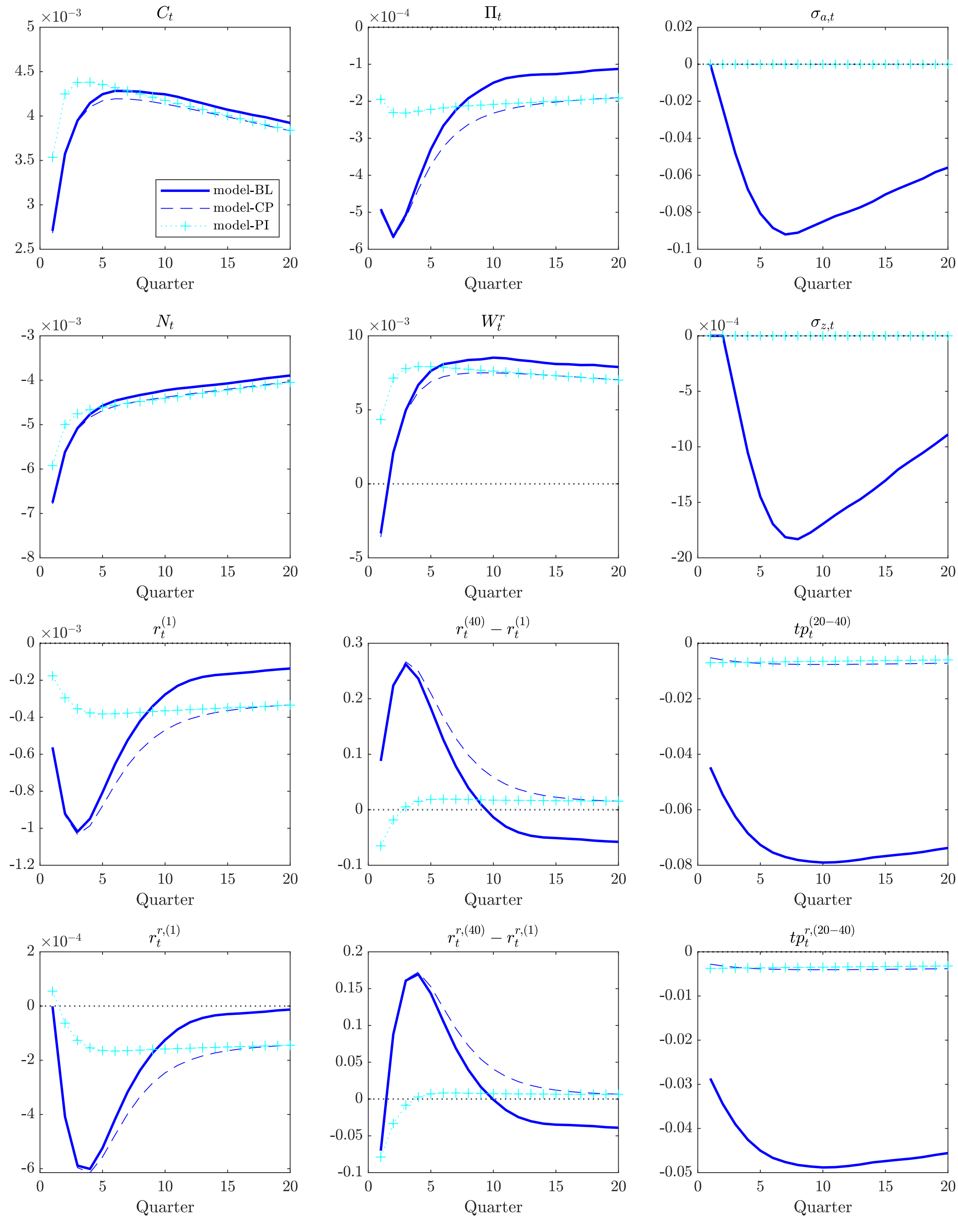

Figure 5: **Impulse Responses to $\varepsilon_{a,t}$**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).

The key feature of model-BL is the term premium responses, as both nominal (3rd-row right) and real (bottom right) term premia fall in response to $\varepsilon_{a,t}$. The countercyclicality of real term premia follows from countercyclical uncertainty about a_t, as in the simple model. Moreover, the nominal term premium shows similar countercyclicality, but with a larger magnitude. This is because the inflation risk premium is also countercyclical in the model, as I discuss further in Section 4.4. Conversely, since the mechanism for countercyclicality is absent in model-PI and CP, term premia vary much less in both models. Interestingly, the smaller term premium response in model-CP shows that time-varying signal precision is crucial in generating variation in term premia, and not imperfect information *per se*.

4.3.2 Impulse Responses to a Noise Shock

I next analyze the impulse responses to an (unobserved) positive $\varepsilon_{s,t}$ shock. The responses are summarized in Figure 6. Since this shock plays no role in model-PI, the relevant comparison with model-BL will only be model-CP.

A positive $\varepsilon_{s,t}$ shock increases consumption (top left) and inflation (top middle) in both models because under imperfect information, a positive $\varepsilon_{s,t}$ shock makes the consumer believe her present value of wealth has increased due to a persistent increase in TFP, boosting consumption. However, since TFP did not actually increase, supply cannot increase in tandem, creating upward pressure on inflation. The initial impact of a one standard deviation $\varepsilon_{s,t}$ shock is about 40 percent of the impact of a $\varepsilon_{a,t}$ shock of the same magnitude, and the impact on inflation is about 80 percent of a $\varepsilon_{a,t}$ shock in model-BL. The positive correlation between consumption and inflation suggests that the shock can be clearly interpreted as a "demand" shock, which confirms the results of related studies.[29]

Labor hours and real wages (2nd-row left and middle) rise following the shock. Similar to when a positive $\varepsilon_{a,t}$ shock hits, the increase in output lowers the uncertainty of beliefs about a_t (top right) and about z_t for model-BL (2nd-row right). Again, such a countercyclical response of volatility is absent in the other models.

The bottom two rows show the responses of interest rates and term premia. Both the policy rate (3rd-row left) and to a lesser extent, the real policy rate (bottom left) increase; a standard (monetary policy) response to a positive demand shock. The relatively large increase in the policy rate compared to longer-term yields narrows the nominal yield spread in both models (3rd-row middle). As the decrease in the yield spread is followed by a decrease in consumption and inflation, such a response is largely consistent with empirical evidence of the slope of the yield curve being a leading indicator of the business cycle.

[29] See, for example, Lorenzoni (2009) and Blanchard et al. (2013). Note these models fall into the class of model-CP, as they do not exhibit time-varying signal precision.

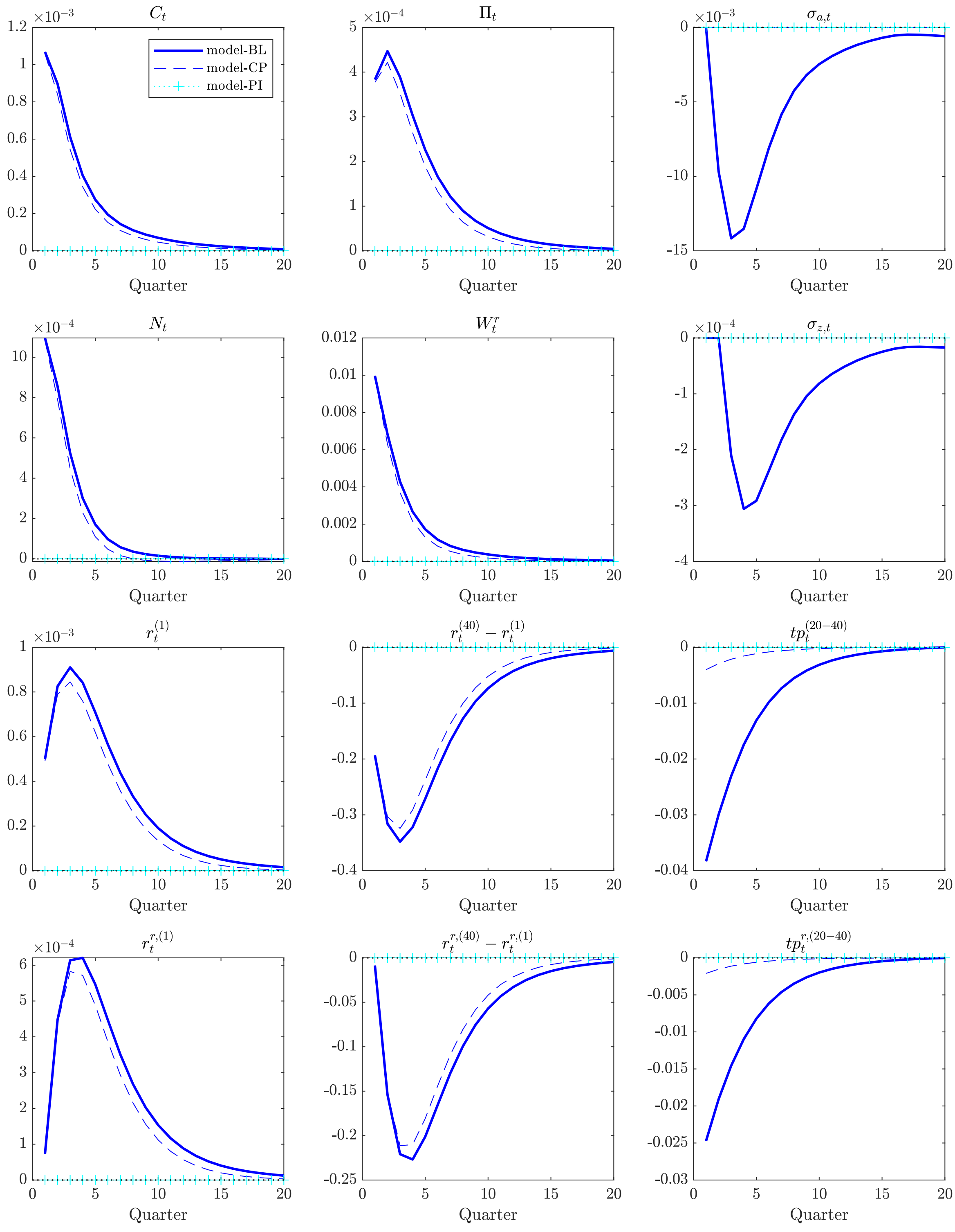

Figure 6: **Impulse Responses to $\varepsilon_{s,t}$**

Notes: All impulse responses are to a $+1\sigma$ shock. Dark blue lines indicate impulse responses of the baseline model with imperfect information (model-BL). Dashed dark blue lines indicate impulse responses of the model with imperfect information with constant precision (model-CP). Light blue lines indicate impulse responses of the model with perfect information (model-PI).

Similar to the case of a $\varepsilon_{a,t}$ shock, the countercyclical response of volatility to a $\varepsilon_{s,t}$ shock depresses both the nominal and real term premia in model-BL (3rd-row and bottom right). The key difference is that the drop in term premia is now associated with a demand shock that has distinct effects on the macroeconomy compared to a supply shock. While term premia in model-CP also show a countercyclical decline, the magnitude is significantly smaller.

In sum, my model with imperfect information offers an intuitive mechanism which significantly amplifies term premium variation due to a standard supply shock.[30] In addition, the model justifies a demand shock—the noise shock about productivity—as being an important driver of term premia. The model does not require an independent shock to the volatility of TFP, providing a deeper microfoundation to setups with exogenous stochastic volatility.

4.4 Further Discussion on the Effect of Imperfect Information

I further discuss how imperfect information affects the model dynamics, in particular, the variation of term premia, by isolating the roles of the two key parameters characterizing the information friction; the noise shock volatility (σ_s) and the signal production rate (ξ).

The top left panel of Figure 7 shows how the signal affects the model-implied volatility of the 5-to-10-year nominal term premium ($\sigma[tp^{(20\to 40)}]$) with respect to σ_s, and for different values of ξ. σ_s and ξ affect the term premium volatility in distinct ways; the volatility has a hump-shape with respect to σ_s, which becomes more pronounced as ξ increases. Importantly, a sufficiently large ξ is key in generating time variation in the term premium.

The top right panel further decomposes this effect into the effect on the real term premium volatility ($\sigma[tp^{r,(20\to 40)}]$, solid lines) and the inflation risk premium volatility ($\sigma[irp^{(20\to 40)}]$, dashed lines). Both volatilities show comparative statics similar to those of the nominal term premium, but the impact of both σ_s and ξ on the volatilities are outsized for the real term premium compared to the inflation risk premium.

The bottom left panel shows how σ_s and ξ affect the model-implied slope coefficient of a regression of the 5-to-10-year nominal term premium on year-over-year consumption growth ("beta"). In line with my analysis so far, beta becomes more negative, i.e., the term premium becomes more countercyclical, as ξ increases. Interestingly, the effects of σ_s and ξ are nonlinear, and beta is most negative for intermediate parameter values. While ξ generates countercyclicality of both real term premia and inflation risk premia, it impacts the real beta more strongly, similar to its impact on term premium volatility (bottom right).

[30] Thereby complementing models that generate countercyclical term premia from supply shocks through other mechanisms, e.g., Rudebusch and Swanson (2012), Andreasen et al. (2018), and Swanson (2019).

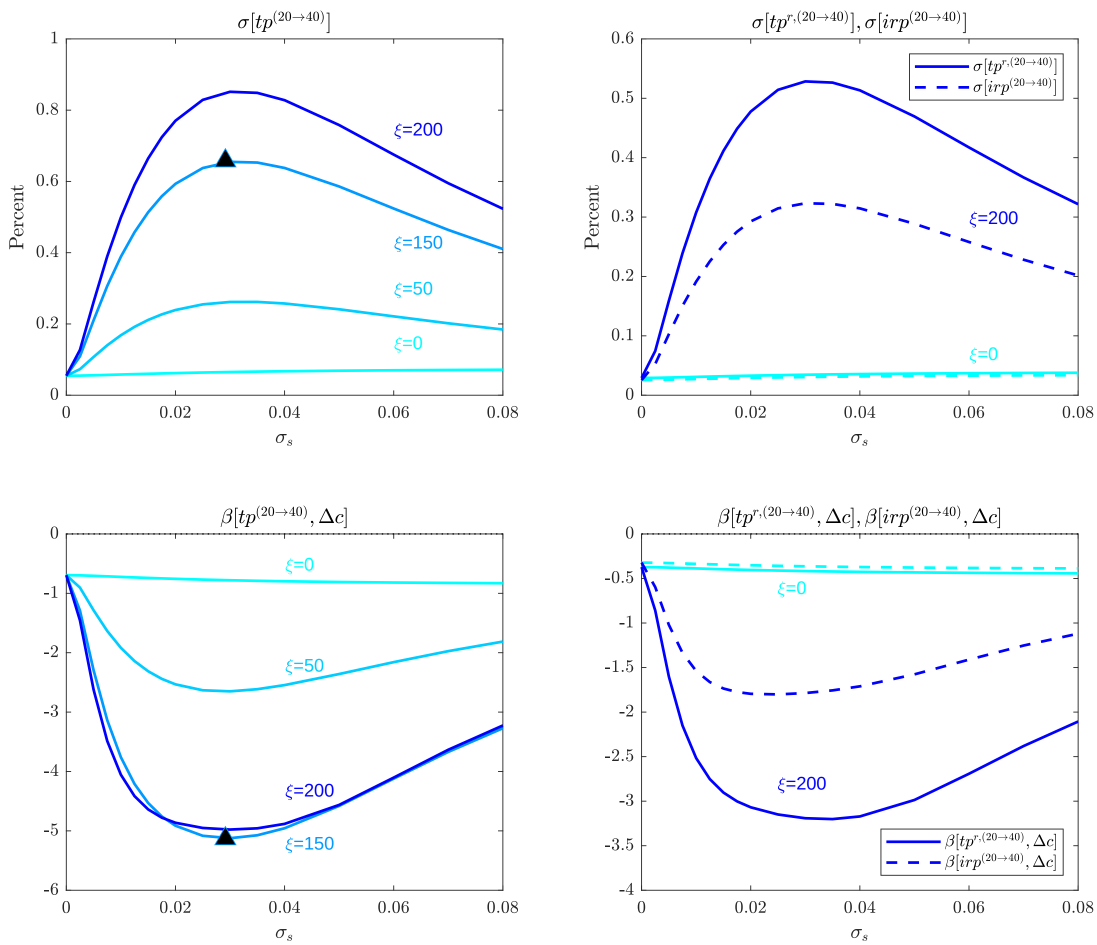

Figure 7: **Effect of the Noisy Signal on Term Premia**

Notes: Each line corresponds to the moments generated from the indicated value of ξ. The black triangle indicates the moment from the baseline model.

The intuition for the effects of σ_s and ξ on the real term premium can be understood clearly by revisiting the 2-period real term premium expression (9) in Section 3.2.3. From equations (9) and (5), the volatility of the real term premium is monotonically increasing in σ_s when $\xi = 0$, but is hump-shaped when $\xi > 0$. When $\xi > 0$, the term ϕ/σ_s^2 in (5) can become an important source of term premium variation, and ξ increases its variability, all else fixed. However, the effect of a time-varying ϕ disappears either as $\sigma_s \to 0$ or as $\sigma_s \to \infty$.[31]

The effects of σ_s and ξ on the inflation risk premium turns out to be qualitatively similar, and the mechanism is analogous to the effects on the real term premium. To guide intuition,

[31] This is because $\phi/\sigma_s^2 \to \infty$ as $\sigma_s \to 0$, and hence $\sigma_{a,t}^2 \to \sigma_a^2$ (constant), while $\phi/\sigma_s^2 \to 0$ as $\sigma_s \to \infty$, and hence $\sigma_{a,t}^2 \to \bar{\sigma_a}^2$ (another constant), where $\bar{\sigma_a}$ solves $\bar{\sigma_a}^2 = \rho_a^2 \left(\sigma_z^{-2} + \bar{\sigma_a}^{-2}\right)^{-1} + \sigma_a^2$.

consider a simple extension of the 2-period term premium analysis, where I add inflation π_t specified as $\pi_t = -\theta_\pi z_t$, where I assume $\theta_\pi > 0$ is an exogenous parameter. Then, the 2-period inflation risk premium ($irp_t^{(2)}$) is:

$$irp_t^{(2)} \propto \mathbb{C}ov_t(m_{t+1}, \pi_{t+2}) = \theta_\pi \rho_a \sigma_{a,t}^2. \tag{17}$$

In other words, the inflation risk premium is an increasing function of $\sigma_{a,t}$ just like the real term premium. Hence, the effects of σ_s and ξ work through $\sigma_{a,t}$ analogously.[32]

Imperfect information offers a channel that can increase the average and the volatility of nominal term premia by largely impacting real term premia. This emphasis on real term premia is a notable departure from the literature that stresses positive inflation risk premia as the primary factor behind positive nominal term premia. The result can be seen as lending theoretical support to studies such as Duffee (2018), which argues that a small portion of the variation in yield news can be explained by the variation in expected inflation news.

In Appendix G, I use the comparative statics to further discuss the increase in the correlation of consumption growth and inflation observed over the last few decades and its relation to term premia through the lens of my model.

5 Conclusion

In this paper, I studied the dynamics of default-free bond yields and term premia using a novel equilibrium term structure model which combined a New-Keynesian core with imperfect information about the persistence of shocks to productivity. I showed that imperfect information could justify a noise shock, which, incorporated in a DSGE term structure model with standard productivity shocks, generated term premia that are on average higher, with sizable countercyclical variation that arose endogenously. I argued that this feature helped reconcile the empirical evidence that term premia had been on average positive and countercyclical, with numerous studies pointing to demand shocks as being an important driver of business cycles over the last few decades. While the focus on a specific form of information frictions proved to be tractable and effective in understanding some important features of the yield curve, other, perhaps more elaborate variants could explain more aspects of agents' beliefs. I leave such an investigation for future research.

[32] θ_π is affected by σ_s and ξ in the DSGE model, and in particular, θ_π can decrease as consumption and inflation becomes more positively correlated under imperfect information. However, the simple example is useful in clarifying the channel through the effect on $\sigma_{a,t}$.

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Appendix

A Data

For the BR estimate of the term premium, I use an estimate of the trend real interest rate that is somewhat different from BR, constructed from the estimates of Holston et al. (2017) and Del Negro et al. (2017) (both the VAR-based and DSGE-based estimates), which are all publicly available. The estimate is a bit smoother than BR. Otherwise, the model is identical to their "observed shifting endpoint" model. I find that the correlation between the term premia from BR and my estimates over the sample period of BR (1971.Q4 to 2018.Q1) is nearly perfect, with a coefficient of over 0.98. The ACM and KW estimates are available on the websites of the Federal Reserve Bank of New York and the Board of Governors of the Federal Reserve System, respectively. The 1-year excess holding return of the 10-year (nominal) Treasury bond is computing using the zero-coupon yields from Gürkaynak et al. (2007) ($xhr_{t+4}^{(40)} \equiv -9r_{t+4}^{(36)} + 10r_t^{(40)} - r_t^{(4)}$). The FB estimate is the expected excess holding return ($\mathbb{E}_t[xhr_{t+4}^{(40)}]$) computed from the Fama and Bliss (1987) regression: $xhr_{t+4}^{(40)} = \beta_0 + \beta_1(r_t^{(36\to40)} - r_t^{(4)}) + u_{t+4}$, where $r_t^{(36\to40)}$ is the 9-to-10-year nominal Treasury forward rate, and $r_t^{(4)}$ is the 1-year nominal Treasury yield.

The macroeconomic data source used for the regression analysis in Section 2 is as follows. Nonfarm payroll, industrial production, capacity utilization, and real GDP data are taken from the FRED database. The GDP gap is the CBO measure of the output gap, from the Haver Analytics database, and the unemployment gap is the Civilian Unemployment Rate: 16 yr + (seasonally adjusted) minus the CBO measure of the natural rate of unemployment, also from Haver. I remove a linear trend from capacity utilization, estimated from monthly observations from January 1990 to December 2019. The VIX series is taken from the FRED database. The conditional volatility of TFP growth is computed by applying a GARCH(1,1) model similar to Bloom et al. (2018) to the unadjusted TFP growth data by Fernald (2014). The JLN macroeconomic uncertainty index is available on Sydney Ludvigson's website. I use the 3-month-ahead index for my calculations.

The patent data used in Figure 2 is compiled by Marco et al. (2015), and available on the website of the U.S. Patent and Trademark Office. I sum the monthly total application series for each quarter, and compute the annual growth rate for each quarter. I do not show growth rates that include data for 1995.Q2 and Q3, as well as 2013.Q1 and Q2 since there were large swings in applications due to regulatory changes, as described in Marco et al. (2015).

Additional data is used for calibrating the DSGE term structure model. For the short-term nominal interest rate, I use the 3-month T-bill rate from the Federal Reserve Board's H.15 statistical release. For nominal yields of 5-, and 10-year maturities, I use the zero-coupon yields from Gürkaynak et al. (2007).

For real rates, I rely on multiple sources to construct reference rates that cover 1990.Q1 to 2008.Q4. In particular, for the 5-to-10-year real forward rate, TIPS yields are not available from 1990.Q1 to 1998.Q4, so I use real yield estimates from D'Amico et al. (2018) and Chernov and Mueller (2012). From 1999.Q1 to 2008.Q4, the TIPS yield data by Gürkaynak et al. (2010) is available. However, it is well known that TIPS yields, especially around the inception of the TIPS market and during the financial crisis, likely contained a significant liquidity premium. Since the DSGE model-implied real yields do not contain any liquidity premia, the appropriate reference is yields that remove such premia. Thus, I consider the following two series: (1) from 1990.Q1 to 1998.Q4, estimated real yields from the model of D'Amico et al. (2018), and from 1999.Q1 to 2008.Q4, zero-coupon yields interpolated from TIPS data by Gürkaynak et al. (2010) subtracting the D'Amico et al. (2018) liquidity premium estimates, and (2) from 1990.Q1 to 1998.Q4, estimated real yields from the model of Chernov and Mueller (2012), and from 1999.Q1 to 2008.Q4, the same series as (1).[33] I compute the mean, standard deviation, and the 1-quarter autocorrelation of these two series, and average the respective moments (across the two series), which I take as the mean, standard deviation, and autocorrelation of the 5-to-10-year real forward rate. The data for the 1-quarter real rate also requires estimation, and for this series I use the estimates of Chernov and Mueller (2012) from 1990.Q1 to 1997.Q4, and the estimates of Aruoba (2020) from 1998.Q1 to 2008.Q4.

For consumption data, I compute per capita consumption from personal consumption expenditures (nondurables + services, seasonally adjusted). I use the quarterly change in the core CPI as a measure of inflation. Labor hours are calculated as the ratio of hours of all persons (seasonally adjusted) in the nonfarm business sector to the sum of the civilian labor force (16 years+) and the population not in the labor force (all seasonally adjusted). I use real compensation per hour of the nonfarm business sector (seasonally adjusted) as a measure of real wages, and detrend it using the method by Hamilton (2018). All of these measures are taken from Haver.

I use the Survey of Professional Forecasters (SPF), published by the Federal Reserve Bank of Philadelphia, to construct a measure of conditional uncertainty about GDP growth. In each quarterly survey, the SPF includes average forecast distributions of year-over-year GDP growth for the current year (and the next). Assuming the probability assigned to each bin represents the probability of the mid-point of that bin, I compute the standard deviation

[33] The real yield estimates from Chernov and Mueller (2012) are only available up to 2002.Q4.

of the average forecast distribution. The nature of the data is somewhat disconnected before and after 1992.Q1; the forecasts are based on real GNP before 1992.Q1 and on real GDP since then. The bins also vary across the two periods. I construct a time series using only surveys in the fourth quarter so that the forecast horizon is effectively constant at about a quarter.

B Additional Empirical Results

B.1 Empirical Literature on the Countercyclicality of Term Premia

Cochrane and Piazzesi (2005), Ludvigson and Ng (2009), and Piazzesi and Swanson (2008) are prominent examples establishing the countercyclicality of term premia using data before the financial crisis. Piazzesi and Swanson (2008) focus on excess returns of fed funds futures rates up to 6-months maturity. Further evidence of countercyclical term premia including the post-financial crisis observations is provided by Adrian et al. (2013) (although they do not provide a regression analysis), Wright (2011), Bauer et al. (2014), and Bauer and Rudebusch (2020), among others. Econometric analysis that include observations beyond 2009 appears relatively limited. Exceptions include Gargano et al. (2019), who show the countercyclicality of expected excess bond returns, and Bekaert et al. (2021), who find term premia implied from the Blue Chip surveys show countercyclicality with respect to a recession dummy. Bianchi et al. (2021) adapt machine learning techniques to forecast excess bond returns, and show that their measures of expected excess returns are countercyclical, and comove positively with measures of macroeconomic uncertainty. My analysis complements these studies, and find results that are broadly consistent with them, but uses different measures of term premia and business cycle indicators. In addition, while the aforementioned studies cover a longer sample period that includes the 1970s, my analysis focuses on a later period, which is arguably more relevant for understanding term structure dynamics in an environment where demand shocks are likely to be dominant.

B.2 Regression on Macroeconomic Indicators

In Table B.1, I show the results of the baseline regression on macroeconomic indicators discussed in the main text, using the sample period from 2000.Q1 to 2019.Q4. As described in the main text, I run a series of univariate regressions where each term premium series and the expected excess return are regressed separately on to one of the independent variables with a constant. The table reports the slope coefficient associated with the macroeconomic indicator for each individual regression.

Table B.1: **Regression of Nominal Term Premia on Macroeconomic Indicators (2000.Q1 - 2019.Q4)**

	ACM	BR	KW	FB
ΔNFP	−51.23***	−30.75***	−23.47***	−77.02***
	(10.34)	(6.57)	(5.04)	(18.82)
ΔIP	−8.49**	−3.71	−4.82***	−13.29**
	(3.64)	(2.89)	(1.83)	(6.26)
ΔGDP	−35.01***	−15.80***	−12.67***	−47.67***
	(7.23)	(4.77)	(4.41)	(15.44)
GDP gap	−28.75***	−11.23	−7.83	−91.23***
	(9.34)	(9.19)	(7.13)	(10.56)
UE gap (negative of)	−23.91**	−9.89	−4.15	−83.07***
	(11.00)	(11.26)	(8.83)	(12.76)
CU	−23.50***	−11.68***	−9.25***	−45.40***
	(3.74)	(3.05)	(2.39)	(8.98)

Notes: The sample period is 2000.Q1 through 2019.Q4 with quarterly frequency. Each value represents the slope coefficient from a univariate regression of a measure of term premium on to a macroeconomic indicator plus a constant. The ACM, BR, and KW measures are detrended using the method of Hamilton (2018). The FB measure is the expected excess return based on Fama and Bliss (1987). The first three regressors are year-on-year changes. The regression coefficients are in basis point units per one percentage point change in the regressor. ***, **, and * indicate 1%, 5%, and 10% significance based on Newey-West standard errors with 6 lags (in brackets), respectively.

Coefficients are negative regardless of the specification, implying countercyclicality of term premia. Many of the coefficients are statistically significant, and significance is particularly strong for the ACM and the expected excess return measures. Overall, the evidence of countercyclicality is about equally strong since the beginning of 2000 compared to the baseline regression results which used the sample period starting from 1990.[34]

In Table B.2, I show the results of the baseline regression on macroeconomic indicators, when the term premium measures (excluding the expected excess return) are not detrended. As described in the main text, I run a series of univariate regressions where each term premium series and the expected excess return are regressed separately on to one of the independent variables with a constant. The table reports the slope coefficient associated

[34]Splitting the sample periods further generally results in reduced statistical significance due to the smaller sample size, but evidence of countercyclicality can still be observed for the subsamples from 1990.Q1 to 1999.Q4, from 2000.Q1 to 2008.Q4, and from 2009.Q1 to 2019.Q4, respectively.

with the macroeconomic indicator for each individual regression. Note the results for the expected excess return is identical to what is reported in the main text, since it is not detrended in the first place.

Table B.2: **Regression of Nominal Term Premia on Macroeconomic Indicators (Without Detrending)**

	ACM	BR	KW	FB
ΔNFP	−36.29***	−18.37**	−12.82	−77.18***
	(9.83)	(8.32)	(9.76)	(13.33)
ΔIP	−2.96	1.13	1.59	−17.27***
	(3.75)	(3.35)	(3.12)	(4.67)
ΔGDP	−16.64**	−2.60	0.10	−45.54***
	(7.78)	(7.49)	(8.76)	(12.40)
GDP gap	−37.22***	−16.38*	−10.96	−89.14***
	(9.44)	(8.55)	(9.60)	(7.32)
UE gap (negative of)	−35.51***	−14.52	−4.59	−93.94***
	(11.48)	(10.23)	(10.26)	(12.11)
CU	−22.84***	−11.64***	−8.57*	−48.08***
	(5.42)	(4.27)	(4.77)	(7.34)

Notes: The sample period is 1990.Q1 through 2019.Q4 with quarterly frequency. Each value represents the slope coefficient from a univariate regression of a measure of term premium on to a macroeconomic indicator plus a constant. The first three regressors are year-on-year changes. The regression coefficients are in basis point units per one percentage point change in the regressor. ***, **, and * indicate 1%, 5%, and 10% significance based on Newey-West standard errors with 6 lags (in brackets), respectively.

Coefficients are negative regardless of the specification, except for a few cases in which the coefficients are fairly small and statistically insignificant, implying countercyclicality of term premia. Many of the coefficients are statistically significant, and significance is particularly strong for the ACM and the expected excess return measures, similar to when term premia are detrended. While the preferred specification adopted in the main text uses the detrended series, as they allow for a sharper focus on the business cycle dynamics of term premia, overall the results appear robust to whether or not we factor in the trend.

As an additional robustness check, I rerun the baseline analysis using the year-on-year differences of the three term premium measures and the expected excess return instead of their detrended counterparts, as an alternative way to deal with the downward trend in term premia observed over the sample period. I also take year-on-year differences of the gap measures and capacity utilization. Table B.3 summarizes the results. Similar to the baseline results, the coefficients are negative regardless of the specification and largely statistically significant.

Table B.3: **Regression of Nominal Term Premia on Macroeconomic Indicators (Difference Specification)**

	ΔACM	ΔBR	ΔKW	ΔFB
ΔNFP	−24.46***	−12.67***	−11.61***	−60.30***
	(6.37)	(4.51)	(3.25)	(17.69)
ΔIP	−7.94***	−3.69*	−4.28**	−20.04***
	(2.47)	(1.94)	(1.74)	(6.77)
ΔGDP	−18.63***	−5.21	−6.95*	−53.60***
	(6.27)	(5.52)	(4.00)	(15.38)
ΔGDP gap	−0.26***	−0.09	−0.10**	−0.73***
	(0.07)	(0.06)	(0.04)	(0.18)
ΔUE gap (negative of)	−0.50***	−0.30***	−0.24***	−1.17***
	(0.15)	(0.10)	(0.07)	(0.37)
ΔCU	−0.12***	−0.06**	−0.06***	−0.31***
	(0.04)	(0.03)	(0.02)	(0.10)

Notes: The sample period is 1990.Q1 through 2019.Q4 with quarterly frequency. Each value represents the slope coefficient from a univariate regression of a measure of term premium on to a macroeconomic indicator plus a constant. All variables are year-on-year changes. The regression coefficients are in basis point units per one percentage point change in the regressor. ***, **, and * indicate 1%, 5%, and 10% significance based on Newey-West standard errors with 6 lags (in brackets), respectively.

B.3 Regression on Uncertainty Indicators

In terms of the baseline regressions of term premia on measures of uncertainty, I use the first three of the four measures plotted in Figure 2 (left panel) of the main text: (1) the VIX, (2) the conditional volatility of TFP growth from a GARCH(1,1) model, and (3) the 1-quarter-ahead macroeconomic uncertainty index by Jurado et al. (2015) (JLN). I do not use the measure based on the SPF since it is constructed only at an annual frequency. The measure will nevertheless be useful to calibrate the term structure model in Section 4. While the conditional volatility of TFP growth is theoretically closest to the source of time-varying uncertainty in the term structure model I develop, the connection between the theoretical model and the other two measures is somewhat looser. Nevertheless, the VIX and JLN represent a fairly popular measure of financial and macroeconomic measure of uncertainty, respectively. Thus I take the 1-year moving average of these two variables and include them in the set of independent variables.

In Table B.4, I show the results of the baseline regression on uncertainty indicators discussed in the main text, using the sample period from 2000.Q1 to 2019.Q4. As described in the main text, I run a series of univariate regressions where each term premium series and the expected excess return are regressed separately on to one of the independent variables with a constant. The table reports the slope coefficient associated with the uncertainty indicator for each individual regression.

Table B.4: **Regression of Nominal Term Premia on Uncertainty Indicators (2000.Q1 - 2019.Q4)**

	ACM	BR	KW	FB
VIX	12.51***	6.40***	4.77***	19.62***
	(1.52)	(1.44)	(1.21)	(4.36)
TFP Vol	1.25***	0.62*	0.44*	3.12***
	(0.33)	(0.35)	(0.27)	(0.64)
JLN (Macro)	6.49***	3.99***	3.01***	5.97*
	(1.22)	(1.23)	(0.99)	(3.37)

Notes: The sample period is 2000.Q1 through 2019.Q4 with quarterly frequency. Each value represents the slope coefficient from a univariate regression of a measure of term premium on to an uncertainty indicator plus a constant. The ACM, BR, and KW measures are detrended using the method of Hamilton (2018). The FB measure is the expected excess return based on Fama and Bliss (1987). The regression coefficients are in basis point units per one percentage point change in the regressor. ***, **, and * indicate 1%, 5%, and 10% significance based on Newey-West standard errors with 6 lags (in brackets), respectively.

Coefficients are positive and statistically significant regardless of the specification, implying that a rise in uncertainty leads to an increase in term premia, as well as an increase in expected excess returns, consistent with the baseline results. Overall, the positive relation between term premia and uncertainty appears to remain equally strong since the beginning of 2000.

In Table B.5, I show the results of the baseline regression on uncertainty indicators discussed in the main text, when the term premium measures (excluding the expected excess return) are not detrended. As described in the main text, I run a series of univariate regressions where each term premium series and the expected excess return are regressed separately on to one of the independent variables with a constant. The table reports the slope coefficient associated with the uncertainty indicator for each individual regression. Note the results for the expected excess return is identical to what is reported in the main text, since it is not detrended in the first place.

Table B.5: **Regression of Nominal Term Premia on Uncertainty Indicators (Without Detrending)**

	ACM	BR	KW	FB
VIX	7.37**	4.06*	2.35	12.58**
	(3.15)	(2.33)	(2.43)	(5.64)
TFP Vol	1.65***	0.95***	0.77**	2.99***
	(0.37)	(0.31)	(0.39)	(0.51)
JLN (Macro)	2.90	1.03	−0.08	6.79**
	(1.80)	(1.40)	(1.48)	(3.24)

Notes: The sample period is 1990.Q1 through 2019.Q4 with quarterly frequency. Each value represents the slope coefficient from a univariate regression of a measure of term premium on to an uncertainty indicator plus a constant. The regression coefficients are in basis point units per one percentage point change in the regressor. ***, **, and * indicate 1%, 5%, and 10% significance based on Newey-West standard errors with 6 lags (in brackets), respectively.

Coefficients are positive regardless of the specification, except for one case in which the coefficient is fairly small and statistically insignificant, implying that a rise in uncertainty generally leads to an increase in term premia, as well as an increase in expected excess returns, consistent with the baseline results. Many of the coefficients are statistically significant.

As an additional robustness check, I rerun the baseline analysis using the year-on-year differences of the three term premium measures and the expected excess bond return instead of their detrended counterparts, as an alternative way to deal with the downward trend in term premia observed over the sample period. I also take year-on-year differences of the uncertainty measures. Table B.6 summarizes the results. I find that the coefficients are mostly positive, but the evidence is weaker compared to the baseline results. Nevertheless, several specifications remain significant. In addition, the uncertainty measure based on the conditional volatility of TFP—which, in theory, has the strongest connection to the term structure models I develop—appears to have a statistically significant correlation with various term premium measures.

Table B.6: **Regression of Nominal Term Premia on Uncertainty Indicators (Difference Specification)**

	ΔACM	ΔBR	ΔKW	ΔFB
ΔVIX	2.94	−0.47	0.23	8.23*
	(2.04)	(1.67)	(1.13)	(4.39)
ΔTFP Vol	0.72**	0.32	0.35*	1.95***
	(0.31)	(0.31)	(0.19)	(0.53)
ΔJLN (Macro)	0.96	−0.14	0.56	2.98
	(1.04)	(0.87)	(0.64)	(2.65)

Notes: The sample period is 1990.Q1 through 2019.Q4 with quarterly frequency. Each value represents the slope coefficient from a univariate regression of a measure of term premium on to an uncertainty indicator plus a constant. All variables are year-on-year changes. The regression coefficients are in basis point units per one percentage point change in the regressor. ***, **, and * indicate 1%, 5%, and 10% significance based on Newey-West standard errors with 6 lags (in brackets), respectively.

Lastly, I briefly consider the relationship between term premia and "disagreement"—a concept that is occasionally referred to as a proxy for uncertainty. Disagreement is commonly defined as the cross-sectional standard deviation of point forecasts (see, for example Clements et al. (2023)), and can be measured through survey forecasts. I take the disagreement measures of 1-year ahead real GDP growth and CPI inflation from the SPF, which are defined as the difference between the 75th and 25th percentile of the individual point forecasts of the respective variables. I run the baseline regressions replacing the uncertainty measures with the disagreement measures, and report the results in Table B.7.

Table B.7: **Regression of Nominal Term Premia on Disagreement Indicators**

	ACM	BR	KW	FB
ΔGDP	1.91***	1.33***	0.78***	2.00*
	(0.47)	(0.36)	(0.27)	(1.20)
CPI	2.20***	1.35***	0.71**	3.17**
	(0.66)	(0.45)	(0.35)	(1.33)

Notes: The sample period is 1990.Q1 through 2019.Q4 with quarterly frequency. Each value represents the slope coefficient from a univariate regression of a measure of term premium on to a disagreement measure plus a constant. The ACM, BR, and KW measures are detrended using the method of Hamilton (2018). The FB measure is the expected excess return based on Fama and Bliss (1987). The regression coefficients are in basis point units per one percentage point change in the regressor. ***, **, and * indicate 1%, 5%, and 10% significance based on Newey-West standard errors with 6 lags (in brackets), respectively.

Similar to the regressions using the uncertainty measures, the positive relation between disagreement measures and term premia appears statistically significant. These results are in line with Wright (2011) and Buraschi and Whelan (2022), who conduct a similar analysis but using somewhat different measures. However, whether disagreement can be a good proxy for uncertainty remains heavily contested in the literature (see, for example, the recent studies of Clements et al. (2023) and Zohar (2024)). Since my term structure model does not formalize an explicit role for disagreement, I treat the above regression results as only suggestive.

C The Simple Term Structure Model as an RBC Model

In this section, I show that the simple term structure model in Section 3.1 can be derived from a stylized real business cycle model without capital.

The representative household maximizes lifetime expected utility:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\chi_c}}{1-\chi_c}-\frac{N_t^{1+\chi_n}}{1+\chi_n}\right)\right], \tag{C.1}$$

subject to its budget constraint:

$$C_t+\mathbb{E}_t\left[M_{t+1}\mathcal{W}_{t+1}\right]\leq W_tN_t+\mathcal{W}_t. \tag{C.2}$$

C_t is consumption, N_t is labor, and W_t is (real) wages. Assuming complete financial markets, $\mathcal{W}_{t+1}$ is the payoff from the household's wealth portfolio of state contingent claims chosen by the end of period t. These claims are priced by the unique (real) stochastic discount factor $M_{t+1}\equiv\beta(C_{t+1}/C_t)^{-\chi_c}$ implied by the household's optimizing behavior. Assets are in zero net supply.

The perfectly competitive firm with a production function $Y_t=Z_tN_t$ maximizes its profits $Y_t-W_tN_t$ each period. Z_t is an exogenous productivity process. Market clearing imposes $C_t=Y_t$.

The model is simple enough to solve analytically. The equilibrium condition from the labor market implies:

$$C_t^{\chi_c}N_t^{\chi_n}=(Z_tN_t)^{\chi_c}N_t^{\chi_n}=Z_t. \tag{C.3}$$

Solving for N_t:

$$n_t=\frac{1-\chi_c}{\chi_c+\chi_n}z_t, \tag{C.4}$$

where small-case variables correspond to their log counterparts e.g., $n_t\equiv\ln(N_t)$. The decision rules for consumption immediately follows from (C.4) and the production function:

$$c_t = \frac{1+\chi_n}{\chi_c+\chi_n} z_t, \tag{C.5}$$

which corresponds to the consumption rule (6) in Section 3.1 with $\theta_c = \frac{1+\chi_n}{\chi_c+\chi_n}$.

Since this solution holds for any arbitrary exogenous process z_t, it must also hold for the state space system characterized by equation (1) up to the process for h_t in Section 3.1. Lastly, the stochastic discount factor of the household implies that default-free bonds are priced according to the Euler equation (7) where $\bar{r} = -\ln\beta$.

D Derivation of the 2-period Term Premium

This section shows the derivation of the 2-period term premium with imperfect information (equation (9) in Section 3.2.3).

$$\begin{aligned} tp_t^{(2)} \equiv r_t^{(2)} - r_t^{(2)\mathbb{Q}} &= -\frac{1}{2}\mathbb{C}ov_t(m_{t+1}, \mathcal{R}_{t+1}^{(2)}) \\ &= -\frac{1}{2}\mathbb{C}ov_t(-\chi_c\Delta c_{t+1}, \mathbb{E}_{t+1}[-\chi_c\Delta c_{t+2}] + \frac{1}{2}\mathbb{V}ar_{t+1}[\chi_c\Delta c_{t+2}]) \\ &= -\frac{1}{2}\chi_c^2\mathbb{C}ov_t(\Delta c_{t+1}, \mathbb{E}_{t+1}[\Delta c_{t+2}]) \\ &= -\frac{1}{2}\chi_c^2\theta_c^2\mathbb{C}ov_t(\Delta z_{t+1}, \mathbb{E}_{t+1}[\Delta z_{t+2}]) \\ &\propto -\mathbb{C}ov_t(\Delta z_{t+1}, \mathbb{E}_{t+1}[\Delta z_{t+2}]), \end{aligned} \tag{D.1}$$

where $\mathcal{R}_{t+1}^{(2)} = p_{t+1}^{(1)} - p_t^{(2)}$ is the (log) return from holding a 2-period (real) bond for 1-period. The second equality follows from $m_{t+1} \equiv -\bar{r} - \chi_c\Delta c_{t+1}$, and $p_t^{(1)} = -r_t^{(1)} = \mathbb{E}_t[m_{t+1}] + \frac{1}{2}\mathbb{V}ar_t[m_{t+1}]$. The third equality uses the fact that $\mathbb{V}ar_{t+1}[\chi_c\Delta c_{t+2}]$ is measurable at time t. The fourth equality follows from $c_t = \theta_c z_t$.

z_t follows:

$$z_t = \rho_a a_{t-1|t-1} + (\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}) \tag{D.2}$$

$$a_{t|t} = \rho_a a_{t-1|t-1} + \boldsymbol{K}_{t-1}(\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}), \tag{D.3}$$

where $\boldsymbol{K}_t$ is the Kalman gain matrix, and $\boldsymbol{s}_t \equiv [z_t, s_t]'$.

Generalizing Lemma 2 in Blanchard et al. (2013) to the case of time-varying coefficients, (D.3) and (D.2) are observationally equivalent to the system:

$$z_t = \rho_a \tilde{a}_{t-1} + \boldsymbol{\Sigma}_{s,t-1}\tilde{\varepsilon}_t \tag{D.4}$$

$$\tilde{a}_t = \rho_a \tilde{a}_{t-1} + \boldsymbol{K}_{t-1} \boldsymbol{\Sigma}_{s,t-1} \tilde{\varepsilon}_t, \tag{D.5}$$

where $\tilde{a}_t$ and $\tilde{\varepsilon}_t$ are observable, $\tilde{\varepsilon}_t$ are mutually independent, i.i.d. standard normal shocks, and $\boldsymbol{\Sigma}_{s,t-1}\boldsymbol{\Sigma}'_{s,t-1} = \mathbb{V}ar_{t-1}[\boldsymbol{s}_t]$. Substituting (D.5) and (D.4) into (D.1), and after some algebra, I obtain (9) in the main text.

E Details on the DSGE Term Structure Model

In this section, I fully describe the DSGE term structure model with imperfect information. The content partly overlaps with the description in the main text, but I keep it such that the section is self-contained.

E.1 Households

The representative household has Epstein-Zin (EZ) preferences (Epstein and Zin (1989)). Its value function V_t takes the following recursive form:

$$V_t = \begin{cases} U_t(C_t, N_t) + \beta \left\{ \mathbb{E}_t \left[V_{t+1}^{1-\tilde{\gamma}} \right] \right\}^{\frac{1}{1-\tilde{\gamma}}} & \text{for } U_t \geq 0 \\ U_t(C_t, N_t) - \beta \left\{ \mathbb{E}_t \left[(-V_{t+1})^{1-\tilde{\gamma}} \right] \right\}^{\frac{1}{1-\tilde{\gamma}}} & \text{for } U_t \leq 0. \end{cases} \tag{E.1}$$

β is the time discount rate. The period utility function $U_t(C_t, N_t)$ takes a standard form with external habits and separable labor disutility:

$$U_t(C_t, N_t) \equiv \frac{\left(C_t - \chi_h \tilde{C}_{t-1} \right)^{1-\chi_c}}{1-\chi_c} - G_t^{1-\chi_c} \frac{N_t^{1+\chi_n}}{1+\chi_n}, \tag{E.2}$$

where $\chi_c > 0$ captures the attitude towards intertemporal substitution of consumption (net of habits), and $\chi_n > 0$ is the inverse Frisch elasticity. G_t is a deterministic trend in total factor productivity (TFP) to be described later in the section. The scaling of labor disutility by G_t ensures the existence of a balanced growth path in equilibrium.

C_t is the household's aggregate consumption of final goods based on a CES aggregator of intermediate goods $C_t \equiv \left(\int_0^1 C_{i,t}^{1-\frac{1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the elasticity of demand for the intermediate goods. $\tilde{C}_{t-1}$ is aggregate consumption in the previous period which is taken as given by the household (external habits). $N_t = \int_0^1 N_{i,t} di$ denotes the household's total supply of labor, which is the integral of labor $N_{i,t}$ supplied to each intermediate good producer $i \in [0,1]$ in a perfectly competitive labor market. The household takes nominal wage W_t as given. $\tilde{\gamma}$ parameterizes the household's risk aversion. $\tilde{\gamma} = 0$ corresponds to the special case of expected utility. Note that a larger $\tilde{\gamma}$ implies higher risk aversion when $U_t \geq 0$ and lower risk aversion when $U_t \leq 0$. The calibrated model satisfies $U_t \leq 0$, so values of $\tilde{\gamma} < 0$ correspond to risk aversion greater than expected utility (Swanson (2018)).

The household maximizes (E.1) by choosing state contingent paths for C_t, N_t, and asset holdings subject to its initial wealth and the following sequence of flow budget constraints:

$$P_t C_t + \mathbb{E}_t \left[M_{t+1} \mathcal{W}_{t+1} \right] \leq W_t N_t + \mathcal{W}_t + D_t, \tag{E.3}$$

where the aggregate price level of the consumption basket $P_t \equiv \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ is implied by the household's cost minimization problem (or equivalently, the optimization of a perfectly competitive representative final good producer combining intermediate goods). Assuming complete financial markets, $\mathcal{W}_{t+1}$ is the payoff from the household's wealth portfolio of state contingent claims chosen by the end of period t. These claims are priced by the unique nominal pricing kernel M_{t+1} implied by the household's problem (for $U_t \leq 0$):

$$M_{t+1} = \beta \left(\frac{U_{C,t+1}}{U_{C,t}} \right) \left[\frac{-V_{t+1}}{\left[\mathbb{E}_t \left[(-V_{t+1})^{1-\tilde{\gamma}} \right] \right]^{\frac{1}{1-\tilde{\gamma}}}} \right]^{-\tilde{\gamma}} \frac{1}{\Pi_{t+1}}, \tag{E.4}$$

where $U_{C,t} = \left(C_t - \chi_h \tilde{C}_{t-1} \right)^{-\chi_c}$ and $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the (gross) aggregate inflation rate. The term with squared brackets is the additional term that appears by assuming EZ preferences, implying that the household is sensitive to the distribution of future consumption (and labor supply) in addition to current consumption growth. $D_t = \int_0^1 D_{i,t} di$ is aggregated firms' dividends rebated back to the household.

E.2 Firms

There are a continuum of intermediate goods producers (firms) indexed by $i \in [0, 1]$ who are monopolistically competitive, and maximize their equity value. Each firm faces nominal rigidities in the form proposed by Calvo (1983) where a firm can reoptimize the price of its good $P_{i,t}$ with only a fixed probability $1 - \varphi$ in each period. Firm i's equity value $V_{i,t}^f$ is then:

$$V_{i,t}^f = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \varphi^\tau M_{t+\tau} \left\{ P_{i,t} \left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} \right) Y_{i,t+\tau} - W_{t+\tau} N_{i,t+\tau} \right\} \right]. \tag{E.5}$$

When a firm cannot optimize its price, it indexes the price to a weighted average of inflation in the previous period and steady state inflation $\bar{\Pi}$. Each firm i is also subject to the demand

and production functions for its own good $Y_{i,t}$:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} Y_t \tag{E.6}$$

$$Y_{i,t} = K_t^{1-\alpha}(G_t Z_t N_{i,t})^{\alpha}, \tag{E.7}$$

where Y_t is aggregate output taken as given by each firm, and $K_t = \bar{K}G_t$ is the level of capital that grows deterministically with G_t (see next paragraph).

Productivity with imperfect information TFP consists of two observable components G_t and Z_t. G_t grows at a deterministic rate of $\zeta \equiv \frac{G_{t+1}}{G_t}$. Z_t is a stationary component. Similar to the simple term structure model, $z_t \equiv \ln Z_t$ is composed of a "persistent" component a_t and a "transitory" component e_t:

$$z_t = a_t + e_t. \tag{E.8}$$

a_t and e_t are *unobservable*, and follow independent AR(1) processes:

$$a_t = (1-\rho_a)\bar{a} + \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t} \tag{E.9}$$

$$e_t = \rho_e e_{t-1} + \sigma_e \varepsilon_{e,t}, \tag{E.10}$$

where $\varepsilon_{a,t}$ and $\varepsilon_{e,t}$ are (unobservable) i.i.d standard normal shocks, and $1 > \rho_a > \rho_e \geq 0$. Note the state space system in the simple model was a special case where $\rho_e = 0$.

As in the simple model, firms also observe a continuum of noisy signals $s_{j,t}$ about a_t, where $j \in [0, J_t]$. By assuming $s_{j,t}$ are common knowledge across firms, the individual signals can be aggregated to a noisy public signal s_t. I further assume the mass of signals J_t is increasing in the log "output gap" from the previous period: $\tilde{y}_{t-1} \equiv \hat{y}_{t-1} - \bar{y}$, where $\hat{y}_{t-1} \equiv \ln \hat{Y}_t \equiv \ln \frac{Y_t}{G_t}$, and $\bar{y} \equiv \ln \bar{Y}$ is its steady state. Thus, $J_t = \phi(\tilde{y}_{t-1})$ with $\phi' > 0$, and s_t is given by:

$$s_t = a_t + \frac{\sigma_s}{\sqrt{\phi(\tilde{y}_{t-1})}}\varepsilon_{s,t}, \tag{E.11}$$

where $\varepsilon_{s,t}$ is i.i.d standard normal and unobservable to the firms (the noise shock). Following the simple model, I specify $\phi(\cdot)$ to be exponential, and hence $\ln J_t$ is linear in $\tilde{y}_{t-1}$, i.e., $\ln J_t = \xi \tilde{y}_{t-1}$. $\xi > 0$ is now the parameter that controls the amount of productivity signals generated by the output gap. Note J_t is increasing with respect to an endogenous variable (output gap) in contrast to the simple model, in which J_t was largely exogenous. That said, J_t is taken as given by each firm, and there is no active learning. The setup abstracts from heterogeneous expectations and higher-order beliefs, making it highly tractable.

Firms form beliefs about the unobservable components a_t and e_t by learning from observations on z_t and s_t via a Kalman filter. In other words, the beliefs are updated through the following system of equations (E.12) through (E.15):

$$\boldsymbol{s}_t = \boldsymbol{\Psi}\boldsymbol{\rho}\boldsymbol{x}_{t-1|t-1} + (\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}) \tag{E.12}$$

$$\boldsymbol{x}_{t|t} \equiv \mathbb{E}_t[\boldsymbol{x}_t] = \boldsymbol{\rho}\boldsymbol{x}_{t-1|t-1} + \boldsymbol{K}_{t-1}(\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}) \tag{E.13}$$

$$\boldsymbol{K}_{t-1} = \boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}'(\boldsymbol{\Psi}\boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}' + \boldsymbol{\Sigma}_{s,t-1}\boldsymbol{\Sigma}'_{s,t-1})^{-1} \tag{E.14}$$

$$\boldsymbol{V}_{t+1|t} = \boldsymbol{\rho}(\boldsymbol{V}_{t|t-1} - \boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}'(\boldsymbol{\Psi}\boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}' + \boldsymbol{\Sigma}_{s,t-1}\boldsymbol{\Sigma}'_{s,t-1})^{-1}\boldsymbol{\Psi}\boldsymbol{V}'_{t|t-1})\boldsymbol{\rho}' + \boldsymbol{\Sigma}_x\boldsymbol{\Sigma}'_x, \tag{E.15}$$

where $\boldsymbol{x}_t = [\hat{a}_t, e_t]'$, $\boldsymbol{s}_t = [\hat{z}_t, \hat{s}_t]'$, $\boldsymbol{\rho} = [\rho_a, 0; 0, \rho_e]$, $\boldsymbol{\Psi} = [1, 1; 1, 0]$, $\boldsymbol{\Sigma}_x = [\sigma_a, 0; 0, \sigma_e]$, $\boldsymbol{\Sigma}_{s,t} = [0, 0; 0, \sigma_s/\sqrt{\phi(\tilde{y}_t)}]$. The "hat" variables indicate the demeaned versions. $\boldsymbol{K}_t$ is the Kalman gain matrix, and $\boldsymbol{V}_{t|t-1}$ is the forecast variance matrix of $\boldsymbol{x}_t$ (not to be confused with the value function V_t).

The timeline of events within a period for each firm is summarized in Figure E.1. After the unobserved shocks $\varepsilon_{a,t}$, $\varepsilon_{e,t}$, and $\varepsilon_{s,t}$ are realized, firms update their beliefs about productivity based on signals $\boldsymbol{s}_t$, the precision of which is affected by the output gap in the previous period $\tilde{y}_{t-1}$. Then goods are produced based on the beliefs and dividends are paid out to the household.

Figure E.1: **Timeline of Events for the Firm**

E.3 Monetary Policy

The central bank sets the (gross) nominal 1-period interest rate, $R_t^{(1)}$, following a standard Taylor rule:

$$R_t^{(1)} = \left(R_{t-1}^{(1)}\right)^{\rho_r} \left(\bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\pi}\left[\frac{\hat{Y}_t}{\bar{Y}}\right]^{\phi_y}\right)^{1-\rho_r}, \tag{E.16}$$

where $\bar{R}$ and $\bar{Y}$ denote the steady state of $R_t^{(1)}$ and normalized output $\hat{Y}_t \equiv \frac{Y_t}{G_t}$, respectively. I abstract from monetary policy shocks for simplicity.

E.4 Market Clearing

In equilibrium, the goods market, labor market, and asset market must clear at all dates and states. The clearing condition for final goods is:

$$Y_t = C_t + (\zeta + \delta - 1)\bar{K}G_t. \tag{E.17}$$

Aggregating the supply of intermediate goods by integrating each producer's supply leads to:

$$Y_t = \frac{1}{\Delta_t} K_t^{1-\alpha}(G_t Z_t N_t)^\alpha, \tag{E.18}$$

where Δ is a measure of cross-sectional price dispersion, which follows:

$$\Delta_t^{\frac{1}{\alpha}} = (1-\varphi)\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta}{\alpha}} + \varphi\left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}}\right)^{\frac{\theta}{\alpha}} \Delta_{t-1}^{\frac{1}{\alpha}} \tag{E.19}$$

where P_t^* is the price set by the optimizing firm. For the asset market, I make a standard assumption that state contingent claims are in zero net supply.

E.5 Term Structure of Interest Rates

Given the equilibrium under complete markets, the price of a $n-$period zero-coupon nominal bond that pays one dollar at maturity $P_t^{(n)}$ can be derived recursively using the nominal stochastic discount factor (E.4):

$$P_t^{(n)} = \mathbb{E}_t[M_{t+1}P_{t+1}^{(n-1)}], \tag{E.20}$$

where $P_t^{(0)} = 1$ for $\forall t$. The continuously compounded yield to maturity of this bond follows directly from its price: $r_t^{(n)} = -\frac{1}{n}\ln P_t^{(n)}$. Note $r_t^{(1)}$ is the 1-period nominal risk-free rate, and $R_t^{(1)} = \exp(r_t^{(1)})$ in the monetary policy rule.

As described in Section 3.1, the $n-$period nominal term premium $tp_t^{(n)}$ is computed from equation (8) with the risk neutral yield $r_t^{\mathbb{Q}(n)}$ computed by recursively discounting cashflows using the nominal risk-free rate (instead of the real risk-free rate). The yield to maturity and the term premium of a $n-$period zero-coupon real bond can be derived analogously, by simply replacing the nominal stochastic discount factor with the real stochastic discount factor $M_{t+1}\Pi_{t+1}$, and replacing the nominal 1-period interest rate used for discounting the risk-neutral prices with their real counterparts.

E.6 Equilibrium Characterization

Given the initial condition $\{R_{-1}^{(1)}, C_{-1}, \Pi_{-1}, \Delta_{-1}, \boldsymbol{x}_{-1|-1}, \boldsymbol{V}_{0|-1}\}$ and the exogenous processes $\{G_t, a_t, e_t, \varepsilon_{s,t}\}_{t\geq 0}$, a monopolistically competitive rational expectations equilibrium is defined in a standard way as a set of stochastic processes for quantities and prices such that (1) households maximize utility, (2) firms set prices and maximize profits, (3) the central bank conducts monetary policy according to the interest rate rule, and (4) goods, labor, and asset markets clear. Conditional expectations $\mathbb{E}[\cdot|\mathcal{I}_t]$ are defined over the information set $\mathcal{I}_t$ common across households, firms, and the central bank, which precludes $\{a_{t-\tau}, e_{t-\tau}, \varepsilon_{s,t-\tau}\}_{\tau\geq 0}$.

To obtain a stationary equilibrium I follow the standard procedure of normalizing all relevant variables by the deterministic trend G_t.

E.7 Summary of the Equilibrium Conditions

In this section, I list the equilibrium conditions of the DSGE term structure model in full (excluding the equations for the term structure to save space). Defining the normalized variables using hats (e.g., $\hat{C}_t \equiv \frac{C_t}{G_t}$), the normalized equilibrium conditions are (for $U_t \leq 0$):

$$\hat{V}_t = \frac{\hat{\mathbb{C}}_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} - \tilde{\beta}\zeta \left\{\mathbb{E}_t\left[(-\hat{V}_{t+1})^{1-\tilde{\gamma}}\right]\right\}^{\frac{1}{1-\tilde{\gamma}}} \tag{E.21}$$

$$M_{t+1} = \tilde{\beta}\left(\frac{\hat{\mathbb{C}}_{t+1}}{\hat{\mathbb{C}}_t}\right)^{-\chi_c}\left[\frac{-\hat{V}_{t+1}}{\left[\mathbb{E}_t\left[(-\hat{V}_{t+1})^{1-\tilde{\gamma}}\right]\right]^{\frac{1}{1-\tilde{\gamma}}}}\right]^{-\tilde{\gamma}}\frac{1}{\Pi_{t+1}} \tag{E.22}$$

$$\hat{W}_t^r = N_t^{\chi_n}\hat{\mathbb{C}}_t^{\chi_c} \tag{E.23}$$

$$\mathbb{E}_t\left[M_{t+1}R_t^{(1)}\right] = 1 \tag{E.24}$$

$$\hat{F}_t = \frac{\theta}{\theta-1}\hat{\lambda}_t^r \hat{Y}_t + \mathbb{E}_t\left[\varphi M_{t+1}\Pi_{t+1}\left(\frac{\Pi_{t+1}}{\Pi_t^{\iota_p}\bar{\Pi}^{1-\iota_p}}\right)^{\frac{\theta}{\alpha}}\hat{F}_{t+1}\zeta\right] \tag{E.25}$$

$$\hat{H}_t = \hat{Y}_t + \mathbb{E}_t\left[\varphi M_{t+1}\Pi_{t+1}\left(\frac{\Pi_{t+1}}{\Pi_t^{\iota_p}\bar{\Pi}^{1-\iota_p}}\right)^{\theta-1}\hat{H}_{t+1}\zeta\right] \tag{E.26}$$

$$\left(\frac{P_t^*}{P_t}\right)^{1+\frac{\theta(1-\alpha)}{\alpha}} = \frac{\hat{F}_t}{\hat{H}_t} \tag{E.27}$$

$$\hat{\lambda}_t^r = \frac{\hat{W}_t^r}{\alpha Z_t \bar{K}^{\frac{1-\alpha}{\alpha}}\hat{Y}_t^{\frac{\alpha-1}{\alpha}}} \tag{E.28}$$

$$\frac{P_t^*}{P_t} = \left(\frac{1-\varphi\left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}}\right)^{\theta-1}}{1-\varphi}\right)^{\frac{1}{1-\theta}} \tag{E.29}$$

$$\Delta_t^{\frac{1}{\alpha}} = (1-\varphi)\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta}{\alpha}} + \varphi\left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}}\right)^{\frac{\theta}{\alpha}}\Delta_{t-1}^{\frac{1}{\alpha}} \tag{E.30}$$

$$R_t^{(1)} = \left(R_{t-1}^{(1)}\right)^{\rho_r}\left(\bar{R}\left[\frac{\Pi_t}{\bar{\Pi}}\right]^{\phi_\pi}\left[\frac{\hat{Y}_t}{\bar{Y}}\right]^{\phi_y}\right)^{1-\rho_r} \tag{E.31}$$

$$\hat{Y}_t = \frac{1}{\Delta_t}\bar{K}^{1-\alpha}(Z_t N_t)^{\alpha} \tag{E.32}$$

$$\hat{Y}_t = \hat{C}_t + (\zeta+\delta-1)\bar{K} \tag{E.33}$$

$$\boldsymbol{s}_t = \boldsymbol{\Psi}\boldsymbol{\rho}\boldsymbol{x}_{t-1|t-1} + (\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}) \tag{E.34}$$

$$\boldsymbol{x}_{t|t} \equiv \mathbb{E}_t[\boldsymbol{x}_t] = \boldsymbol{\rho}\boldsymbol{x}_{t-1|t-1} + \boldsymbol{K}_{t-1}(\boldsymbol{s}_t - \boldsymbol{s}_{t|t-1}) \tag{E.35}$$

$$\boldsymbol{K}_{t-1} = \boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}'(\boldsymbol{\Psi}\boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}' + \boldsymbol{\Sigma}_{s,t-1}\boldsymbol{\Sigma}_{s,t-1}')^{-1} \tag{E.36}$$

$$\boldsymbol{V}_{t+1|t} = \boldsymbol{\rho}(\boldsymbol{V}_{t|t-1} - \boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}'(\boldsymbol{\Psi}\boldsymbol{V}_{t|t-1}\boldsymbol{\Psi}' + \boldsymbol{\Sigma}_{s,t-1}\boldsymbol{\Sigma}_{s,t-1}')^{-1}\boldsymbol{\Psi}\boldsymbol{V}_{t|t-1}')\boldsymbol{\rho}' + \boldsymbol{\Sigma}_x\boldsymbol{\Sigma}_x', \tag{E.37}$$

where $\hat{\mathbb{C}}_t \equiv \hat{C}_t - \frac{\chi_h}{\zeta}\hat{C}_{t-1}$, $\hat{V}_t \equiv \frac{V_t}{G_t^{1-\chi_c}}$, $\tilde{\beta} \equiv \beta\zeta^{-\chi_c}$, and all other variables are defined above.

F Moments of the Real Term Structure

In this section, I discuss how the model-implied moments with respect to the real term structure of interest rates fare against their data counterparts. The results are summarized in Table F.1. The construction of the data-implied moments is discussed in A. I do not report data counterparts for the real term premium since it is difficult to find consensus estimates over the sample period.

The baseline model with imperfect information (model-BL) fits the real term structure reasonably well, given that the moments are not targeted in the estimation. In addition, the moments of model-BL improve notably in some aspects compared to the moments of the model with perfect information (model-PL). For example, the average yield slope is steeper and closer to the data in model-BL, and yield volatility declines more gradually in maturity for model-BL, consistent with the data. These results are partly due to the model-implied real term premia—the average and volatility of the real term premium ($\mathbb{E}[tp^{r,(20\to40)}]$ and $\sigma[tp^{(20\to40)}]$) are larger in model-BL, compared to those in model-PL. In particular, the volatility of the real term premium is negligible for model-PI, similar to the nominal term premium.

Table F.1: **Moments on the Real Term Structure**

	Data	Model-BL (Imperfect Info.)	Model-PI (Perfect Info.)
Yields			
$\mathbb{E}[r^{r,(1)}]$	1.52	1.95	2.25
$\mathbb{E}[r^{r,(20\to40)}]$	3.01	2.98	2.97
$\mathbb{E}[tp^{r,(20\to40)}]$	—	1.06	0.76
$\sigma[r^{r,(1)}]$	1.54	1.28	1.55
$\sigma[r^{r,(20\to40)}]$	0.73	0.52	0.44
$\sigma[tp^{(20\to40)}]$	—	0.41	0.03
$\rho_{-1}[r^{r,(1)}]$	0.92	0.94	0.94
$\rho_{-1}[r^{r,(20\to40)}]$	0.88	0.99	0.99
$\rho_{-1}[tp^{(r,20\to40)}]$	—	0.99	0.99

Notes: The real yield data is adjusted for liquidity premia in TIPS yields; see Appendix A for details. The data is in quarterly frequency and in annualized percent. The sample period is from 1990.Q1 to 2008.Q4. $\mathbb{E}[\cdot]$, $\sigma[\cdot]$, and $\rho_{-1}[\cdot]$ indicate the unconditional mean, standard deviation, and 1-quarter autocorrelation, respectively.

Of note, Duffee (2018) proposes an alternative way to assess a macro-finance term structure model's fit to the data, by looking at the ratio of the variance of news about expected inflation to the variance of yield shocks. Specifically, the inflation variance ratio for yield maturity m is computed as $\frac{\mathbb{V}\text{ar}[\eta^{(m)}_{\pi,t}]}{\mathbb{V}\text{ar}[\tilde{r}^{(m)}_t]}$, where $\eta^{(m)}_{\pi,t} \equiv \mathbb{E}_t\left[\frac{1}{m}\sum_{i=1}^{m}\pi_{t+i}\right] - \mathbb{E}_{t-1}\left[\frac{1}{m}\sum_{i=1}^{m}\pi_{t+i}\right]$ and $\tilde{r}^{(m)}_t \equiv r^{(m)}_t - \mathbb{E}_{t-1}[r^{(m)}_t]$. Duffee argues that at a quarterly frequency, this ratio is between 10 to 20 percent in the data. It turns out that the model with imperfect information implies a variance ratio of 12 and 13 percent at the 5- and 10-year maturity, respectively, which is consistent with Duffee's observation and my finding that the model reasonably fits other moments of the data. The model with perfect information under my specification implies a higher variance ratio, but the increase turns out to be modest.

G Term Premia and the Changing Correlation of Consumption Growth and Inflation

While a detailed study is out of the scope of this paper, the comparative statics can offer some insight into the determinants of a longer-term relationship between term premia and the increasing correlation of consumption growth and inflation (or similarly, the correlation of output gap and inflation) that has been documented in the literature (e.g., Song (2017), Campbell et al. (2020)). Figure G.1 shows the model-implied correlation of consumption growth and inflation with respect to σ_s and ξ. Recall a noise shock can be interpreted as a demand shock, in the sense that it moves consumption and inflation in the same direction, as shown by the impulse responses. Hence, it may be natural to expect an *increase* in the correlation of consumption and inflation by adding a noise shock. However, the figure shows that this is not necessarily the case. In particular, when the signal precision is constant ($\xi = 0$), the correlation becomes more *negative* as σ_s increases from the perfect information case when there are only TFP shocks ($\sigma_s = 0$). Correlation can increase only when the signal precision is sufficiently time-varying, i.e., ξ is large, but even in this case, the correlation is not monotonically increasing with respect to σ_s.

The reason why a larger volatility of the noise shock can lead to a (further) negative correlation between consumption and inflation is that a noise shock changes how a TFP shock affects consumption and inflation by influencing agents' beliefs about productivity. For instance, under imperfect information, an increase in TFP due to an increase in the persistent component of TFP is perceived to be partly driven by an increase in the transitory component. Hence, compared with the case under perfect information, the increase in consumption is dampened, exacerbating the deflationary pressure of the TFP shock and resulting in a stronger negative correlation.[35]

[35] By assuming no habit formation, no monetary policy reaction to the output gap, $\rho_a = 1$ and $\rho_e = 0$ for the TFP process, the linearized solution of the DSGE model admits an analytical expression where inflation is orthogonal to ε_a under perfect information, but negatively correlated with ε_a under imperfect information.

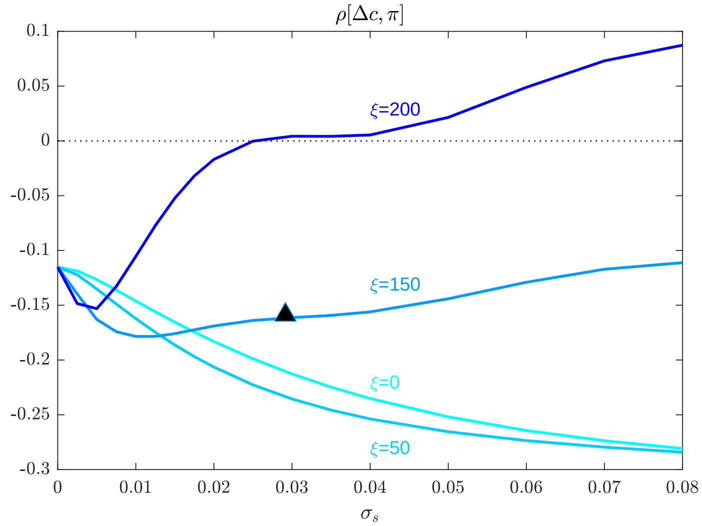

Figure G.1: **Effect of the Noisy Signal on the Correlation of Δc and π**

Notes: Each line corresponds to the correlations generated from the indicated value of ξ. The black triangle indicates the statistic from the baseline model.

The time-variation in signal precision is critical in reversing this mechanism. If signal precision is procyclical, this dampening effect is offset as the increase in consumption leads to an increased flow of information.[36] This result shows that the increase in the correlation may not be simply due to an increase in the volatility of a demand shock, but rather, the origin may be traced to a combination of moves in σ_s and ξ. This point can actually be made without considering the term structure of interest rates, but it is an interesting byproduct of introducing time-varying signal precision to improve the DSGE model's fit to the term structure, and a point that appears overlooked in the literature.

As discussed in Section 2 and Appendix B, the countercyclicality of term premia did not necessarily weaken after 2000, while there was an increase in the correlation of consumption (growth) and inflation. Through the lens of my model, this phenomenon would be qualitatively consistent with an increase in σ_s or ξ, especially when ξ is sufficiently large. Alternatively, and perhaps more realistically, the mechanism through which imperfect information affects the term premium can be combined with other possible explanations in the literature. For instance, the model of Nakata and Tanaka (2016) suggests that an increase in the volatility of a demand shock can depress term premia while increasing the correlation of consumption and inflation. In this case, however, term premia end up being procyclical.

[36]As $\sigma_s \to \infty$, the correlation asymptotes to a unique level that is more negative than the case with perfect information, regardless of ξ. This is because, as s_t becomes completely uninformative, the only effective signal is productivity itself, the precision of which does not vary over time.

Imperfect information could reverse this procyclicality, potentially offering an explanation that brings the model more in line with the data.

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