Finance and Economics Discussion Series

Federal Reserve Board, Washington, D.C.
ISSN 1936-2854 (Print)
ISSN 2767-3898 (Online)

Optimal Design of Contingent Capital

Lionel Melin, Ahyan Panjwani

2024-051

Please cite this paper as:
Melin, Lionel, and Ahyan Panjwani (2024). "Optimal Design of Contingent Capital," Finance and Economics Discussion Series 2024-051. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/FEDS.2024.051.

NOTE: Staff working papers in the Finance and Economics Discussion Series (FEDS) are preliminary materials circulated to stimulate discussion and critical comment. The analysis and conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors. References in publications to the Finance and Economics Discussion Series (other than acknowledgement) should be cleared with the author(s) to protect the tentative character of these papers.

Optimal Design of Contingent Capital*

Lionel Melin† Ahyan Panjwani‡

May 7, 2024

Abstract

This paper proposes a parsimonious framework for designing contingent capital contracts (CoCos). CoCos designed this way (i) are either optimal or incentive compatible for equity holders, (ii) implement a unique equilibrium, and (iii) result in an optimal capital structure for the firm. We consider CoCos with equity conversion and write-down modalities. Equity conversion CoCos are optimal; write-down CoCos are incentive-compatible. Both types of CoCos can be implemented by exogenously specifying a capital ratio rule that triggers conversion and, hence, qualify as additional tier 1 (AT1) capital. A policymaker can use a normative criterion, e.g., capital ratio after conversion, to determine the desired capital ratio rule *ex-ante*. Given the policymaker's choice of the capital ratio rule, our model pins down the CoCo that respects (i), (ii), and (iii). We show that including such a CoCo in the firm's capital structure increases its optimal levered value while making it more resilient to bankruptcy. Lastly, CoCos in this framework are time-consistent. This characteristic alleviates the risk of renegotiation by stakeholders and removes the uncertainty of a discretionary trigger: precisely what spooked markets during the run on Credit Suisse in March 2023.

Keywords: Contingent convertible debt, Bail-in debt, Capital structure, Capital requirements, Bank regulation, Bank capital

JEL classification: G12, G13, G23, G32

*We thank Anil Kashyap, Charles Calomiris, Skander Van den Heuvel, David Arseneau, Matt Darst, Francesca Carapella, David Rappoport, Antonis Kotidis, Chase Ross, and Riyuchiro Izumi for comments and feedback. The analysis and conclusions set forth are those of the authors and do not indicate concurrency by members of the Board of Governors of the Federal Reserve System, or their staffs.

†MacroLucid Research. Email lionel.melin@macrolucid.com

‡Board of Governors of the Federal Reserve System. Email ahyan.panjwani@frb.gov (corresponding author)

1 Introduction

In March 2023 UBS acquired Credit Suisse for $3 billion. The deal, orchestrated by the Swiss Financial Market Supervisory Authority (FINMA), was aimed at stopping the loss of trust in and the run on Credit Suisse. As part of the deal, shareholders of Credit Suisse retained the $3 billion in equity value while FINMA wrote-down contingent convertible bonds (CoCos) with a face value of $17 billion to zero.[1] On the one hand, this arrangement appears to be in line with the language in the CoCos' prospectus.[2] On the other hand, the arrangement seems contrary to the traditional priority order that debt is senior to equity which is the basis of Chapter 11 corporate reorganizations and the European Bank Recovery and Resolution Directive (Paz Valbuena and Eidenmüller, 2023).[3] This *prima facie* contradiction within the arrangement led to significant stress in the CoCo market at the time, see Figure 1, and has since renewed questions on how to design CoCos such that they act as a line of defense, not add fuel to the existing fire during a crisis.

In this paper, we ask how to design CoCos that i) are either optimal or, at least, incentive compatible, ii) implement a unique equilibrium, and iii) result in an optimal capital structure. The first two of these design aspects merit a brief discussion. First, the CoCos we design are incentive-compatible in that equity holders are indifferent to the conversion at the trigger. Moreover, CoCo holders do not hasten a conversion as they benefit more if the firm recovers and they remain creditors. Such time-consistent contracts alleviate incentives for stakeholders to renegotiate their terms. This incentive compatibility also removes the need for ad hoc interventions from the regulator to decide when the CoCo should be converted. We consider CoCos with two different modalities; equity conversion and write-down.[4] For the equity conversion modality, our CoCo design is optimal: the contract globally maximizes the value of the equity holders' claim. Second, a common concern in the literature is that CoCos may give rise to multiple equilibria (Sundaresan and Wang, 2015; Pennacchi and Tchistyi, 2019b). Our CoCo design implements a unique equilibrium despite using a market-based trigger. These two design aspects together ensure there is no incentive for accounting, market manipulations, or erroneous conversion. In doing so, our paper connects the literature on CoCo design and the multiplicity of equilibria.

[1] List of Credit Suisse's CoCos written-down from FINMA.

[2] Credit Suisse's information memorandum for CoCos issued in 2022 states "If a Write-down Event occurs, [...] the full principal amount of each Note will automatically and permanently be written-down to zero."

[3] For a host of reasons including questions of efficacy and tax treatment, CoCos have been issued by international banks in general and Euro-area banks in particular while US banks have not issued such instruments. See Admati and Hellwig (2014); Wang (2023) for further discussion.

[4] In equity conversion, CoCo holders are issued equity when conversion is triggered. In write-down, the firm writes-down the CoCo holders by paying them a fraction of the debt's face value.

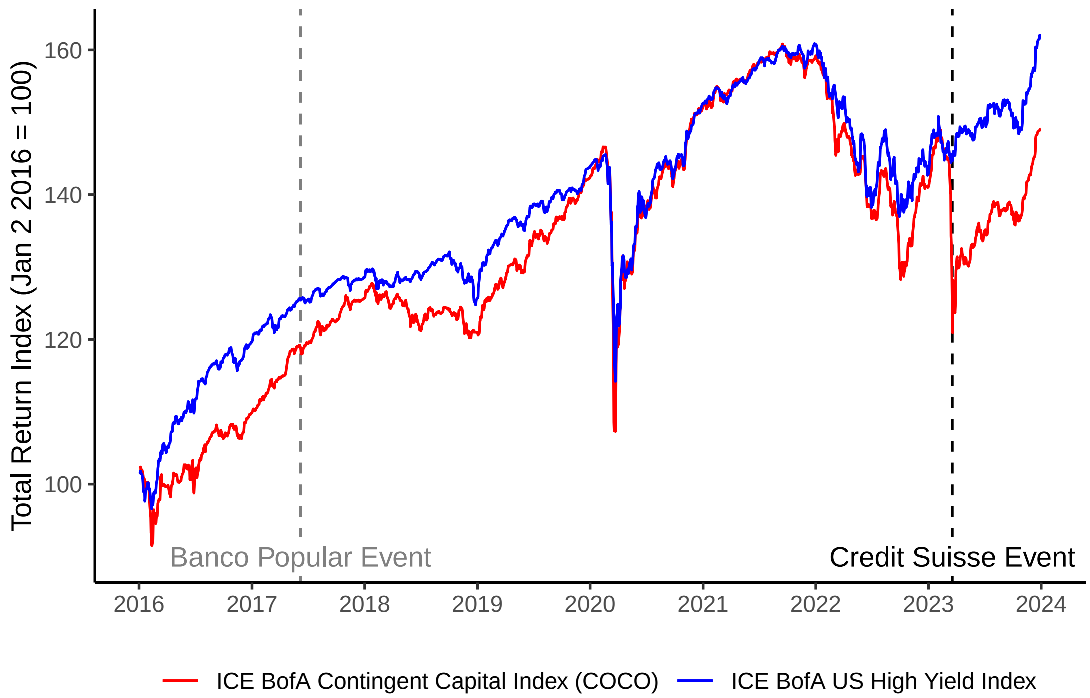

Figure 1: CoCo versus high yield indices This figure shows the ICE BofA Contingent Capital (CoCo) index (red) and the ICE BofA US High Yield Index (blue) between Jan 2016 and Jan 2024, with the indices re-based to 100 on Jan 2 2016. The two indices track each other closely until UBS's announcement to acquire Credit Suisse on March 19 2023 (black, dashed line). Since then, a yawning gap has emerged between the two indices. Such divergence between the two indices was not observed following Banco Santander's acquisition of Banco Popular in June 2017, despite a complete write-down of the latter's CoCos.
Source: ICE Data Indices, LLC, used with permission.

Our first main result shows that in the equity conversion modality there is an endogenously optimal conversion threshold for the firm's assets, A_c^*, that can be uniquely implemented. This conversion threshold is optimal since it globally maximizes the value of the equity holders' claim. For a CoCo with this asset level, A_c^*, at conversion, the equity holders are indifferent (by smooth pasting) to conversion versus the status quo despite being diluted as a result of the conversion. Moreover, CoCo holders prefer to wait and see if the firm's fortunes can improve (in the vicinity of the conversion threshold) as they benefit from remaining creditors to the firm. How to implement this endogenously optimal conversion threshold level of assets, A_c^*? We find a unique mapping from the capital ratio rule that triggers conversion, specified when the CoCo is issued, to a dilution factor such that if the conversion was triggered, the firm's asset level would be equal to the endogenously optimal conversion threshold, A_c^*.

This optimally designed CoCo would still qualify for AT1 rating. To qualify for the rating, CoCos need to specify a conversion trigger at issuance.[5] Our result shows that when such CoCos are issued, equity holders and CoCo holders agree on a capital ratio rule that would trigger conversion if the firm's capital ratio fell below the preset threshold in the capital ratio rule. If conversion is triggered then CoCo holders would receive equity in lieu of their bonds. The resulting dilution for existing equity holders is common knowledge ex-ante. And if and when the conversion does happen, it happens at an asset level that equity holders have no incentive to block while CoCo holders do not force an erroneous conversion in haste.

Our second main result shows that in the write-down modality, the contract can be designed to implement a unique and incentive-compatible equilibrium. In general, the write-down modality does not admit a well-defined optimal trigger, unlike the equity conversion modality. Intuitively, if the contract stipulates that the firm can write-down the CoCo holders for a fraction of what the latter is owed, equity holders will choose to do so right away. Conversely, CoCo holders would rather delay the conversion as much as possible to benefit from the coupon payment, knowing that they will receive a fixed amount upon conversion regardless of the value of the firm. We pin down the CoCo contract that i) is triggered by a preset threshold in a capital ratio rule so that the CoCo is AT1-eligible, ii) is incentive compatible for equity holders in that they are indifferent between the status quo and conversion (by smooth pasting), iii) ensures (weakly) positive equity at conversion, and iv) ensures a unique equilibrium by preserving a one-to-one mapping between asset and equity value so that the conversion is not triggered erroneously. Our approach for (iv) complements that of Pennacchi and Tchistyi (2019a) who also contend with potential multiplicity of equilibria.

Our result in the write-down modality also explains the market stress following the regulator's decision to write-down Credit Suisse's AT1-rated CoCos in March 2023. We find that, ex-ante, if a CoCo stipulates complete write-down then for almost all conversion thresholds equity holders would not be indifferent to conversion at the trigger (smooth pasting violated at the trigger threshold).[6] Consequently, when the pre-specified conversion threshold is reached, equity holders may prefer one outcome over the other which may be time inconsistent. Intuitively, if equity holders could write-down CoCo holders entirely while retaining positive value for themselves ex-post, they would of course take the deal. Which is precisely the violation of the customary resolution waterfall that occurred in the case of Credit Suisse and was bound to disrupt markets. And it did. Figure 1 shows that the ICE BofA Contingent Capital Index sharply diverged from the ICE BofA High Yield Index the day the Credit

[5] For a discussion of prerequisites for AT1 rating, see Avdjiev, Kartasheva, and Bogdanova (2013).

[6] At best, a sufficiently high asset level for the trigger threshold could ensure a unique equilibrium.

Suisse-UBS deal was announced whereas they previously tracked each other very closely; even when Banco Santander acquired Banco Popular for €1 in June 2017 and the latter's CoCos were completely written-down the CoCo market response was muted.[7,8] Against this backdrop of market stress, Bank of England and the European Central Bank, along with other EU regulators, had to reaffirm that AT1 instruments rank ahead of common equity as part of the resolution waterfall (Bolton, Jiang, and Kartasheva, 2023).

Our third main result completes the specification of the CoCo contract by pinning down the optimal coupon for the CoCo such that the resulting capital structure of the firm is optimal. This result is important for two reasons. First, as Pennacchi and Tchistyi (2019a) note, almost all CoCos (including Credit Suisse's) are perpetuities so their pricing hinges critically on the coupon in addition to the risk and modality of conversion. In fact, Bolton, Jiang, and Kartasheva (2023) discuss the possible role of higher coupon rates for Credit Suisse's CoCos in attracting investors in a low interest rate environment through the 2010s. Conversely, Glasserman and Nouri (2016) note that the market panicked around Deutsche Bank's CoCos in 2016 because market participants were concerned about the firm's ability to make the coupon payment, not because a conversion was likely.

A by-product of this result is that we also pin down the optimal coupon for consol bonds, which is directly proportional to the coupon in Leland (1994). This mapping from optimal coupon for consol bonds in our setting to that in Leland (1994) underscores how closely our model tracks the seminal model in the literature while adding CoCos with various modalities. This simple modeling approach à-la Leland allows us to find closed-form solutions that highlight the key mechanisms at play. Importantly, these optimal coupons maximize the firm's levered value when then identifies CoCos' contribution to improving broad economic social welfare by increasing value of the firm and alleviating collateral scarcity. Our approach can be extended to more advanced capital structure models which would allow for precise calibration to market data.[9]

We then simulate the optimal capital structure of the firm and compare outcomes across modalities. We find that firms substitute towards CoCos and away from consol bonds as doing so improves their resilience by pushing the bankruptcy threshold lower. Moreover, we show that write-down CoCos lead to earlier conversion and delays bankruptcy, creating a larger buffer than equity conversion CoCos. Of course, this increased resilience comes

[7]Source: ICE Data Indices, LLC, used with permission.

[8]For contemporaneous press coverage regarding the muted response, see Financial Times: Banco Popular Serves as a Harsh Lesson for CoCo Debt Holders.

[9]We present closed-form results with infinite maturity bonds and purely diffusive dividend processes. Although adding jump and rollover risk features would enrich the model and help calibrate the model to data, it would not affect the fundamental logic at play.

at a cost: optimal coupons are bigger for write-down CoCos than their equity conversion counterparts. Lastly, we show that write-down CoCos lead to higher recapitalization after conversion. The policymaker can use such a normative criterion—capital ratio after conversion—to back out the capital ratio rule they should require for AT1 eligibility. Once the policymaker chooses the capital ratio rule, our machinery determines the optimal capital structure of the firm and the conversion and bankruptcy thresholds.

Related Literature Our paper contributes to two strands of literature on CoCos: one on contract design, in particular the choice of conversion trigger, and one concerning the multiplicity of equilibria. The literature on CoCo design stems from the seminal paper of Flannery (2005) which proposed reverse convertible debentures that would convert from debt to equity at a given equity price.[10] Following the 2008 financial crisis, CoCos received significant attention from both the research and policy circles: e.g, Kashyap, Rajan, and Stein (2008); French et al. (2010); Hanson, Kashyap, and Stein (2011) and Pazarbasioglu et al. (2011); Avdjiev, Kartasheva, and Bogdanova (2013). In fact, the Basel III regulatory framework incentivizes CoCos by allowing them to be included in AT1 or AT2 capital depending on the modalities of the contract.

Since then the literature on CoCo design and the choice of trigger has blossomed. A trigger could be based on three different types of metrics. First, triggers based on accounting measures of capital adequacy, e.g, Madan and Schoutens (2011) and Glasserman and Nouri (2012). Second, triggers based on market measures like decline in stock prices, e.g., the dual-triggers in Pennacchi (2010); De Martino et al. (2010); Calomiris and Herring (2013) and McDonald (2013), or increases in CDS premiums, e.g., Hart and Zingales (2011).[11] Last, triggers based on regulatory discretion allow regulators to mandate the conversion of CoCos into equity during a banking crisis. Bolton and Samama (2012) develop a framework based on Coasean bargaining that strikes a balance between issuer's, investor's, and the regulator's objectives. See Oster (2020) for a detailed survey.

Our paper builds on this literature by pinning down an endogenously optimal market-based trigger such that the conversion is incentive compatible and not susceptible to market manipulation nor erroneously implemented; concerns commonly voiced about market-based triggers. To assuage these concerns about market-based triggers, Pennacchi, Vermaelen, and Wolff (2014) design a new type of contingent capital: one where new shares are issued upon conversion but existing equity holders have the right to buy them at the bond's par value. Himmelberg and Tsyplakov (2020) investigate other aspects of contingent capital in rather

[10]Relatedly, Flannery (2016) proposes contingent capital certificates.

[11]Relatedly, Avdjiev et al. (2020) show that CoCo issuance may reduce CDS spreads.

different models with a focus on the incentive effects of the conversion haircut level.

However, a related strand of literature has raised concerns regarding the multiplicity of equilibria arising due to market-based triggers, e.g., Sundaresan and Wang (2015) and Prescott (2012). Glasserman and Nouri (2016) show that while multiple equilibria arise in discrete time, there is a unique equilibrium in continuous time while Pennacchi and Tchistyi (2019a) and Pennacchi and Tchistyi (2019b) show that unique equilibria can exist under most likely circumstance, particularly when CoCos are perpetuities which is often the case. We build on this literature by showing that our endogenously optimal conversion threshold, which is a market-based trigger, implements a unique trigger in both the equity conversion and the write-down modalities.[12]

In terms of modeling, Chen et al. (2017) and Albul, Jaffee, and Tchistyi (2015) are closest to our paper. Both of these papers develop a structural model with endogenous bankruptcy beyond conversion.[13] The latter makes use of consol debt claims and assumes an exogenous conversion trigger and focuses on static replacement—of debt or equity by CoCos—scenario analysis. The former enriches their modeling with jumps and finite maturity debt claims and also assumes an exogenous conversion trigger. Our paper adds to this literature along several dimensions by keeping analytical tractability while modeling an incentive-compatible design of CoCo modalities (trigger rules and haircuts).

Outline Section 2 introduces the baseline model and derives the value of the firm's assets and liabilities (including CoCos) before and after conversion. Section 3 presents the endogenously optimal conversion threshold and the capital ratio rule that uniquely implements the threshold under the equity conversion modality. Section 4 shows that a unique conversion ratio can be implemented under the write-down modality despite conflicting incentives between equity- and CoCo-holders. Section 5 derives the optimal coupons, highlights the connection with Leland (1994), and—having completed the specification of CoCos—shows results from simulations. Section 6 concludes. All proofs are contained in the Appendix.

2 Model

The asset portfolio (A) of the representative firm produces dividends according to a geometric Brownian motion process

$$dA_t/A_t = \mu dt + \sigma dZ_t,$$

[12] In the write-down modality, we show that the equilibrium is unique under realistic assumptions.

[13] Following Leland (1994), Leland and Toft (1996), Goldstein, Ju, and Leland (2001), Chen and Kou (2009) and He and Xiong (2012)

with drift μ and volatility σ in an economy with risk free rate r. The firm's capital structure is composed of three instruments: consol bonds (B), CoCos (CC), and equity claims (E). On the asset side of the balance sheet, beyond the assets' intrinsic value (A), the firm derives benefits from the tax-deductibility of both types of debt claims which we refer to as a tax shield (TS), while facing bankruptcy costs (BC) in case of default.

Conversion of the CoCos occurs when asset value reaches the threshold A_c while bankruptcy occurs when asset value reaches the threshold A_b, hence defining the corresponding stopping time $\tau_i = \inf\{s : A_s < A_i\}, i \in \{b, c\}$. We assume that conversion is triggered before bankruptcy so that $A_c > A_b$ and $\tau_c < \tau_b$. Conversion takes place without possible write-back, i.e., once the A_c threshold has been crossed, CoCos are definitely and entirely traded for equity rights (or cash) according to the conversion rule.

We proceed by backward induction: first, section 2.1 discusses the after-conversion capital structure, and then section 2.2 details pre-conversion pricing, together with conversion modalities.

2.1 After conversion

Similar to that in Leland (1994). The firm's levered value (W) corresponds to the value of its assets and the tax shield provided by the debt contracts net of bankruptcy costs. The firm's balance sheet implies that its levered value combines the value of debt and equity. Hence,

$$W_t \equiv A_t + TS_t - BC_t = B_t + E_t, \ \forall t \in [\tau_c, \tau_b].$$

where A_t is the *unlevered* value of the firm's assets, TS_t is the tax shield benefit allowed at rate θ_b of coupon payments on the outstanding debt, and BC_t is the bankruptcy cost amounting to a fraction α of the value of the firm when bankruptcy is endogenously triggered by the equity holders. On the liability side, consol bondholders, B_t, receive the promised coupons C_b and the potential liquidation value of the firm at bankruptcy while equity shares E_t are valued as residual claims to the firm.[14]

Given the ownership structure, equity holders optimally default at the asset threshold

[14] As derived in appendix $\forall t \in [\tau_c, \tau_b]$

$$TS_t = \theta_b \frac{C_b}{r}\left[1 - \left(\frac{A_t}{A_b}\right)^{-\gamma}\right] \quad , \quad B_t = \frac{C_b}{r}\left[1 - \left(\frac{A_t}{A_b}\right)^{-\gamma}\right] + (1-\alpha)A_b\left(\frac{A_t}{A_b}\right)^{-\gamma},$$

$$BC_t = \alpha A_b \left(\frac{A_t}{A_b}\right)^{-\gamma} \quad \text{and} \quad E_t = A_t - (1-\theta_b)\frac{C_b}{r} + \left[(1-\theta_b)\frac{C_b}{r} - A_b\right]\left(\frac{A_t}{A_b}\right)^{-\gamma}$$

where $\gamma = \left[\mu - \frac{\sigma^2}{2} + \sqrt{\left[\mu - \frac{\sigma^2}{2}\right]^2 + 2r\sigma^2}\right] / \sigma^2$

A_b derived in the appendix and common in the literature:

$$A_b = \frac{\gamma}{1+\gamma}(1-\theta_b)\frac{C_b}{r}, \tag{1}$$

which satisfies the smooth pasting condition $\frac{\partial}{\partial A}E_t\big|_{A=A_b} = 0$. It is straightforward to check that the optimality condition for endogenous default $A_b < (1-\theta_b)\frac{C_b}{r}$ is verified.

2.2 Before conversion

Before conversion happens, the levered value of the firm corresponds to the asset value, net of bankruptcy costs, but is now augmented by the tax shield provided by both types of debt contracts.[15] The levered firm value equates on the liability side of the balance sheet to all three instruments: consol bonds (B_t), CoCos (CC_t), and equity (E_t)

$$W_t \equiv A_t + TS_t - BC_t = B_t + CC_t + E_t, \quad \forall t < \tau_c$$

where the CoCo are worth the entitled coupon payments until the conversion trigger is reached in which case its holders receive recovery R

$$CC_t = \mathbb{E}_t\left[\int_t^{\tau_c} e^{-r(s-t)}C_c ds\right] + \mathbb{E}_t\left[e^{-r(\tau_c-t)}R\right] = \frac{C_c}{r}\left[1-\left(\frac{A_t}{A_c}\right)^{-\gamma}\right] + R\left(\frac{A_t}{A_c}\right)^{-\gamma}$$

The recovery R at trigger depends on whether the CoCos entail (i) an equity conversion (EC), i.e., the exchange of the CoCos for newly issued shares hence diluting preexisting equity holders by a share δ, or (ii) a write-down (WD), i.e., the repayment of a fraction λ of the CoCos' face value $\frac{C_c}{r}$. Concretely,

$$\begin{aligned} CC_t &= \frac{C_c}{r}\left[1-\left(\frac{A_t}{A_c}\right)^{-\gamma}\right] + R^k\left(\frac{A_t}{A_c}\right)^{-\gamma}, k \in \{EC, WD\} \\ R^{EC} &= \delta E_{\tau_c^+} \text{ and } R^{WD} = \lambda\frac{C_c}{r}. \end{aligned}$$

where $E_{\tau_c^+}$ is the value of the equity instantaneously after conversion.

[15] Now $TS_t = \theta_b\frac{C_b}{r}\left[1-\left(\frac{A_t}{A_b}\right)^{-\gamma}\right] + \theta_c\frac{C_c}{r}\left[1-\left(\frac{A_t}{A_c}\right)^{-\gamma}\right]$, where θ_c denotes the tax rate allowed for deductibility of coupon payments on CoCos and is not necessarily equal to θ_b.

Equity valuation is deduced as the residual claim to the firm

$$E_t = A_t - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_t}{A_b}\right)^{-\gamma} - (1-\theta_c)\frac{C_c}{r} + \left[(1-\theta_c)\,\frac{C_c}{r} - R\right]\left(\frac{A_t}{A_c}\right)^{-\gamma}.$$

The last two terms of this expression correspond to the cost-benefit balance of CoCo claims for equity holders. In particular, equity holders derive value from the existence of the CoCos in the firm's capital structure only if

$$R \leq (1-\theta_c)\,\frac{C_c}{r}, \tag{2}$$

i.e., the conversion costs not be so large that they outweigh the tax advantage. Notably, this condition parallels the endogenous default condition (1) above.

Additionally, in the hypothetical case where the CoCos were simply consol bonds with coupon C_c without any conversion optionality, then the default threshold would be

$$A_{bc} \equiv \frac{\gamma}{\gamma+1}\left[(1-\theta_b)\,\frac{C_b}{r} + (1-\theta_c)\,\frac{C_c}{r}\right],$$

mirroring the endogenous default condition (1) above. This hypothetical threshold, A_{bc}, will serve as a useful benchmark: it is the lowest potential asset value for CoCo conversion since below this point a standard default would be triggered.

3 CoCos with Equity Conversion Modality

3.1 Arithmetic of CoCos' conversion rule

With equity conversion, new shares are issued to CoCo holders in lieu and place of their CoCos. CoCo holders receive a guaranteed share δ of the firm's equity ownership post-conversion at the expense of existing equity holders who get diluted by a factor of δ as a result. Several modalities for equity conversion are possible. In the case where CoCos are exchanged for a preset number of shares (independent of the CoCos' face value) the dilution factor, δ, is a pre-determined constant.

Alternatively, the most common modality for the conversion of CoCos into equity stipulate a given conversion equity share price, say p^c. Then, if N is the number of equity shares outstanding before conversion and C_c/r the face value of CoCos, the number of conversion shares issued is C_c/rp^c, and the fraction of the firm owned post-conversion by the original

CoCo investors

$$\delta = \frac{C_c/r}{p^c N + C_c/r}.$$

A related alternative is that conversion occurs at the market price of equity shares $p^m = E_{\tau_c^-}/N$ prevailing at the time of trigger, instead of being exogenously set. The resulting dilution factor is $\delta = \frac{C_c/r}{E_{\tau_c^-}+C_c/r}$. Combining the value conservation equation $E_{\tau_c^-} + \delta E_{\tau_c^+} = E_{\tau_c^+}$ and the expression for the dilution factor, δ, we derive that

$$E_{\tau_c^+} = C_c/r + E_{\tau_c^-}$$

i.e., in this case, CoCos are exchanged for a number of shares exactly worth in total the debt par value, hence a conversion with zero haircut. Hence, for contracts where the committed conversion price is defined as the maximum of the prevailing market price and a fixed conversion price, the market value of equity shares CoCo holders receive is at most the face value of their contracts.

The rest of the paper focuses on two equity conversion (henceforth 'EC') scenarios: (1) fixed number of shares, (2) fixed share price. Scenario (3) market share price, being somewhat degenerate, gets a side treatment.

3.2 Optimal conversion and smooth pasting conditions

In scenarios (1) and (2), at conversion, CoCo holders receive a number of shares worth a fraction $\delta \in [0, 1]$ of the equity value of the firm, independent of the trigger threshold A_c so that

$$R^{EC} = \delta E_{\tau_c^+} = \delta \left\{ A_c - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma} \right\}.$$

Theorem 1 *From the point of view of the firm's equity holders, the endogenously optimal conversion threshold is*

$$A_c^* = \frac{\gamma}{\gamma+1}\left[(1-\theta_b)\frac{C_b}{r} + \frac{1}{\delta}(1-\theta_c)\frac{C_c}{r}\right]. \tag{3}$$

This conversion trigger satisfies the smooth pasting condition on equity holdings.

Proof. See the Appendix. ■

This endogenously optimal trigger level is higher than the default thresholds $A_c^* \geq A_{ab} \geq A_b$. The larger the ownership transfer at conversion, the later conversion would occur.

Nevertheless, optimal conversion is guaranteed to trigger before a joint debt-liability-induced collapse could take place endogenously.

The smooth pasting condition for equity holders when taking into account dilution at conversion

$$\left.\frac{\partial E}{\partial A}\right|_{A\downarrow A_c} = (1-\delta)\left.\frac{\partial E}{\partial A}\right|_{A\uparrow A_c}$$

is verified for $A_c = A_c^*$. Meeting the smooth pasting condition for equity holders guarantees the incentive compatibility of the threshold: equity holders are indifferent between the status quo and conversion so do not have an incentive for accounting or market manipulations.

Lemma 2 *From the point of view of CoCo holders, a conversion trigger based on smooth pasting condition* $\left.\frac{\partial CC}{\partial A}\right|_{A\downarrow\tilde{A}_c} = \delta \left.\frac{\partial E}{\partial A}\right|_{A\uparrow\tilde{A}_c}$ *for their claims would command a conversion threshold*

$$\tilde{A}_c = \frac{\gamma}{\gamma+1}\left[(1-\theta_b)\frac{C_b}{r} + \frac{1}{\delta}\frac{C_c}{r}\right],$$

which is not impacted by the tax deductibility of θ_c *and hence higher than* A_c^*.

A conversion threshold defined as $\tilde{A}_c$ *would actually minimize the value of the CoCos at any point in time.*

Proof. See the Appendix. ■

Indeed, Coco contract holders do not benefit directly from the tax deductibility of the coupons they receive, and Coco claims admit a local minimum (in terms of A_c) at $\tilde{A}_c$.

Hence, with a trigger A_c^* the valuation profile of CoCos is kinked at that conversion point, see Figure 2. CoCo holders are better off after conversion since their post-conversion claims lose value at a slower pace afterward. As a result, CoCo holders do not have an incentive to try and block conversion but, on the contrary, support it, which is preferable from a market structure standpoint.

The determination and implementation of the optimal threshold just derived ensures that CoCo contractual terms are time-consistent, consequently alleviating incentives for stakeholders to renegotiate their terms.

3.3 How to implement the optimal conversion threshold?

To be eligible for AT1 rating, the CoCo conversion trigger i) must be set exogeneously at issuance of the contracts, and ii) guarantee a Capital Equity Tier 1 (CET1) ratio of at least 4.5%. We show that, given any capital ratio rule with a preset threshold κ that triggers conversion, a *unique* dilution factor δ^* can be chosen such that the asset value at the trigger is exactly equal to the endogenously optimal conversion threshold, A_c^*. In practice, the capital ratio rule states that conversion takes place when the ratio of equity value to risk weighted assets (RWA) reaches a preset threshold κ. For tractability, our model assumes a market-based trigger. But the two measures are not drastically divergent since assets are marked to market. Hence we replace the denominator of this ratio with the firm's assets, and the asset value triggering conversion A_c solves

$$(1-\delta)\left[A_c - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right] = \kappa A_c. \tag{4}$$

Theorem 3 *For any preset threshold in a capital ratio rule, $\kappa \in [0,1]$, there exists a unique dilution factor, $\delta^*(\kappa)$, such that a conversion is triggered when the firm's capital ratio reaches the preset threshold in the capital ratio rule, defined by (4). At this conversion trigger, the firm's asset value is equal to the endogenously optimal conversion threshold, A_c^*, given by (3).*

Proof. See the Appendix. ■

Two key takeaways from Theorem 3 warrant discussion. First, for any preset threshold in a capital ratio rule, κ, the theorem guarantees a dilution factor such that the firm's asset value is equal to A_c^*. By smooth pasting, equity holders are indifferent to a conversion at this asset value and do not have an incentive to oppose or manipulate the market. And although CoCo holders prefer the conversion at this point, Lemma 2 shows this conversion is not triggered they pushed for it and hastened the outcome. Second, the dilution factor resulting from (4) is unique, sidestepping concerns around multiple equilibria common in the literature.

Corollary 4 *The implicit function $\delta^*(\kappa)$ is strictly decreasing from $[0,1]$ onto itself.*

Proof. See the Appendix. ■

The proof for Theorem 3 also leads to Corollary 4, see Appendix B.2, which implies that a higher CET1 ratio comes at the expense of existing equity holders as it requires a higher dilution factor and, consequently, an earlier trigger. Importantly, however, Theorem 3 ensures incentive compatibility for all stakeholders at the conversion trigger regardless of the magnitude of the preset threshold in the capital ratio rule.

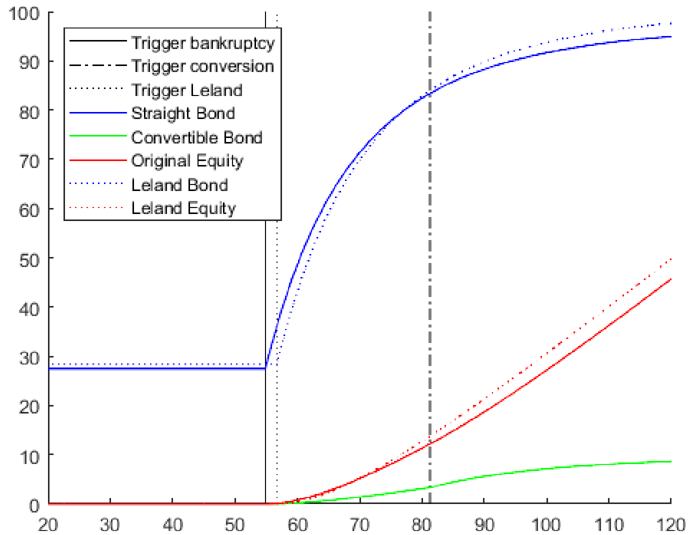

Figure 2: Equity conversion This figure shows equity (E_t), CoCo (CC_t), and consol bond (B_t) values (vertical axis) as a function of the firm's asset value (A_t, horizontal axis) with optimal trigger and coupons and CoCos with equity conversion modality where CoCo holders receive a fixed number of shares at conversion.

4 CoCos with Write-Down Modality

Instead of converting CoCo holders into equity holders, a firm could alternatively write-down the CoCo holders by paying them a fraction λ of their CoCos' face value.[16] We refer to λ as the conversion ratio. Contrary to the equity conversion modality, the write-down modality does not admit a well-defined, finite, optimal trigger threshold (for either equity or CoCo holders). If at any point in time, the coupon payment to CoCo holders (net of the tax shield) was larger than the partial lump-sum needed to write them down

$$\lambda < 1 - \theta_c, \tag{5}$$

then the equity holders would find it optimal that conversion was triggered at the maximal possible value, leading to immediate triggering.[17] On the other hand, CoCo holders would prefer to wait and see the conversion happen at the smallest possible asset value as they would receive a fixed coupon in the meantime and a fixed recovery amount when a conversion did eventually occur.

In this section, we first specify conversion ratios that ensure (weakly) positive equity for

[16] We only model permanent write-downs, leaving optional write-up clauses aside.

[17] $\max_{\{A_c\}} A_c^{\gamma}(1 - \theta_c - \lambda)\frac{C_c}{r}$ leads to optimal $A_c = +\infty$

equity holders at conversion and implement a unique (and endogenous) conversion threshold, A_c. We then refine the CoCo design by ensuring AT1-eligibility by creating a mapping between the preset threshold in the capital ratio rule, κ, to the conversion ratio, λ and incentive compatibility (by smooth pasting).

4.1 Conversion trigger and ratio

The write-down modality leads to the equity valuation formula

$$E_t = A_t - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_t}{A_b}\right)^{-\gamma} - (1-\theta_c)\frac{C_c}{r} + (1-\theta_c-\lambda)\frac{C_c}{r}\left(\frac{A_t}{A_c}\right)^{-\gamma} \tag{6}$$

which is convex in the underlying asset value.

First, equity value cannot fall into negative territory before conversion since equity being negative would trigger immediate liquidation of the firm. For such a scenario to be ruled out until conversion, it should be the case that $E_{\tau_c^-} \geq 0$. Rearranging this condition, and observing that CoCo holders have been written down by this point in (6), gives

$$\lambda^{PO}(A_c) \equiv \left[A_c - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right] / \left[\frac{C_c}{r}\right], \tag{7}$$

where λ^{PO} is the largest possible conversion ratio for which equity is positive.

Second, we model the asset-based trigger such that it can be implemented on a market value basis, similar to our approach in the equity conversion modality above. But equity valuation pre-conversion has a non-monotonic profile with respect to the underlying asset value, see Figure 3. Thus, if the preset threshold for asset value that would trigger a conversion was below the inflection point (dashed red line in Figure 3), then the corresponding equity value would be reached for another—higher—level of the underlying asset value (dashed blue line in Figure 3). This non-monotonicity, as a result, threatens a conversion earlier than warranted. To rule out such an outcome, we restrict attention to the part of the convex segment of the curve where the mapping between market and capital measures at conversion is one-to-one. This one-to-one mapping can be ensured if and only if $\frac{\partial E}{\partial A}\big|_{A\downarrow A_c} \geq 0$ which is represented by the thick blue segment of the curve in Figure 3. By (6), this condition translates into requiring λ to be greater than or equal to

$$\lambda^{UN}(A_c) \equiv \left[(1-\theta_c)\frac{C_c}{r} - \frac{A_c}{\gamma} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right] / \left[\frac{C_c}{r}\right]. \tag{8}$$

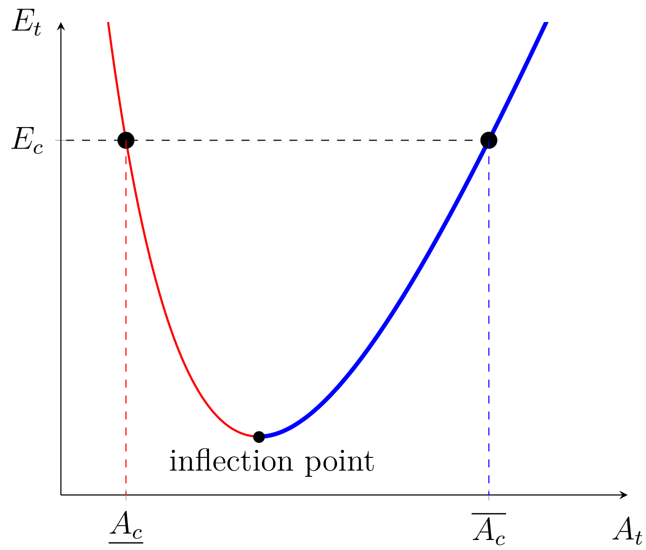

Figure 3: Non-monotonicity of equity in assets Without any restrictions on the domain, the same equity level can be reached for two distinct asset levels on either side of the inflection point. While the contract may have been designed for conversion to happen at the higher asset level, $\underline{A_c}$ (red), it will actually happen at a higher asset level, $\overline{A_c}$ (blue). Consequently, we restrict attention to the curve on the right side of the inflection point (in blue) where equity is increasing in assets.

where λ^{UN} is the lowest possible conversion ratio that guarantees a unique mapping between equity value and asset value.

By equating (7) and (8), we can pin down the lowest possible conversion threshold for asset value, A_c. The resulting conversion threshold is equal to A_{bc}: the asset level corresponding to the endogenous default threshold that would prevail if the convertibility option were to never be exercised, and CoCos behaved like plain debt contracts. The conversion ratio that also respects these constraints for a trigger asset level of A_{bc} is

$$\lambda_{bc} \equiv \frac{\gamma}{\gamma+1}(1-\theta_c) + \frac{1-\theta_b}{\gamma+1}\frac{C_b}{C_c}\left[\left(\frac{A_{bc}}{A_b}\right)^{-\gamma} - 1\right].$$

Figure 4 shows equilibrium outcomes in the (A_c, λ)-space with respect to the constraints (5), (7) and (8). CoCos in regions (1) and (2) see (theoretical) conversion occur for negative equity levels so that conversion would not have a chance to take place before bankruptcy. For equilibria in region (3), the conversion ratio is such that equity value is positive upon conversion but multiplicity arises. Region (3') is singled out as for these parameters equity value would have crossed into the negative region before reaching conversion which is incompatible with the model's assumption and lead to early bankruptcy similar to that in

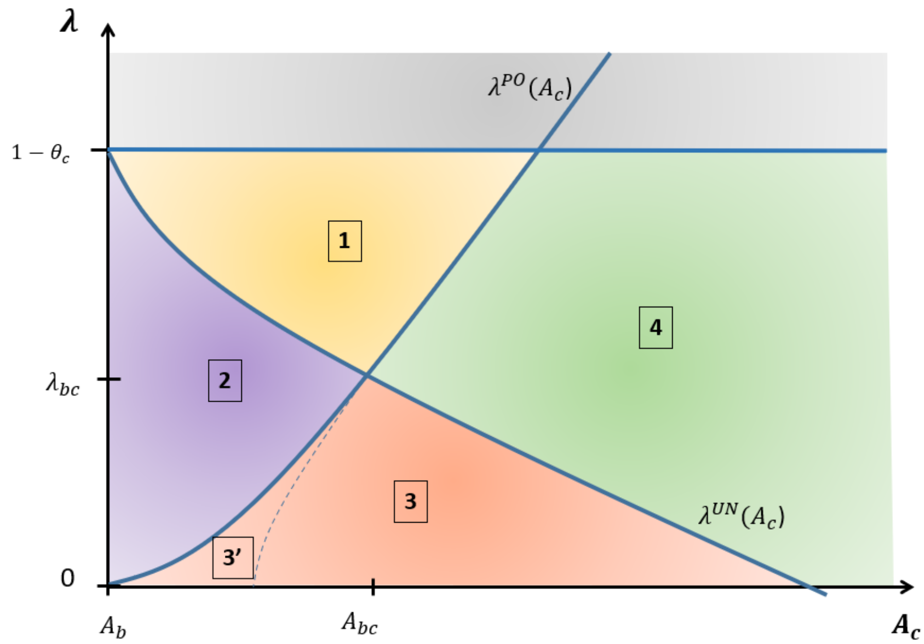

Figure 4: Write-down conversion space This figure shows equilibria in the (A_c, λ)-space. Region 1 shows the set of equilibria that are unique but equity at conversion is not positive hence bankruptcy would be triggered before conversion. Region 2 shows the set of equilibria that are neither unique nor is equity positive at conversion. Region 3 shows the set of equilibria where equity is positive at conversion but equilibria are not unique. Region 4 is the set of unique equilibria with positive equity at conversion.

regions (1) and (2). Region (4) encompasses the equilibria where conversion takes place at positive equity value, and equity-based triggering of conversion admits a one-to-one mapping to the endogenous optimal threshold asset level. The frontier between regions (1) and (4) corresponds to the bail-in scenario where at conversion, shareholders are wiped out.

Our strategy for determining a conversion ratio that implements a unique conversion threshold complements the approach in Pennacchi and Tchistyi (2019a) who also contend with the non-monotonicity of equity in assets. They show that non-monotonicity only arises when volatility is low or close to zero ($\sigma \approx 0$) and monotonicity can be insured under mild regularity conditions.[18] In our environment, as volatility decreases to zero ($\sigma \to 0$) γ becomes large and the linear A_c term dominates in (8).[19] As a result, the locus traced by $\lambda^{UN}(A_c)$ becomes a flatter and straighter downward sloping line. In turn, the region below the locus of $\lambda^{UN}(A_c)$—regions 2 and 3 in Figure 4—becomes bigger while the region above shrinks. Equivalently, the range of conversion ratios for which the conversion threshold is non-unique increases. However, as long as $\sigma > 0$ and $\lambda < 1 - \theta_c$, our environment admits CoCos that have a unique conversion threshold, A_c, i.e., regions 1 and 4 in Figure 4 are well-defined. Thus, a pair (A_c, λ) that satisfies (8) characterizes a CoCo that implements a unique conversion threshold.

The question then arises: how to design a CoCo that guarantees a unique conversion threshold, A_c? First, we create a mapping from the conversion ratio, λ, to a capital ratio rule with a preset threshold, κ, at which conversion is triggered and is specified when the CoCo is issued per AT1-eligibility requirements. At conversion, the equation $E_{\tau_c^-} = \kappa A_{\tau_c^-}$ defines the asset level that triggers conversion, A_c, for a corresponding conversion ratio, λ. Using (6), we derive the implicit formula

$$\lambda^{\kappa}\left(A_c\right) \equiv \left[A_c\left(1-\kappa\right) - \left(1-\theta_b\right)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right] / \left[\frac{C_c}{r}\right], \tag{9}$$

which can be simply linked to $\lambda^{PO}(A_c)$ as $\lambda^{\kappa}\left(A_c\right) = \lambda^{PO}(A_c) - \frac{r\kappa}{C_c}A_c$.

How to then fund the write-down? The firm has two options: either issue more equity or sell assets.

4.1.1 Equity issuance to fund the write-down

At conversion, the CoCo's tax shield benefit vanishes, and given the assets Brownian diffusion payoffs, the levered value of the firm crosses the threshold continuously. Consequently, the

[18]See Lemma 3 and Theorem 2 in Pennacchi and Tchistyi (2019a) for regularity conditions on parameters for the write-down modality.

[19]The power term, $A_c^{-\gamma}$, in (8) tends to zero as γ becomes large.

change in the valuation of equity across the conversion threshold must be exactly equal to the payout to contingent claim holders at conversion, translating as $E_{\tau_c^+} = E_{\tau_c^-} + CC_{\tau_c^-}$. Equity ownership is hence diluted by the (percentage) change in equity outstanding and the dilution factor is given by

$$\delta \equiv \frac{E_{\tau_c^+} - E_{\tau_c^-}}{E_{\tau_c^+}} = \frac{E_{\tau_c^+} - \left(E_{\tau_c^+} - \lambda C_c/r\right)}{E_{\tau_c^+}} = \frac{\lambda C_c/r}{E_{\tau_c^+}}.$$

Constructing a CoCo contract that entails smooth pasting of equity value at conversion would alleviate manipulation incentives since the equity holder would be indifferent between the status quo and conversion. Here again, time-consistency in the CoCo contract alleviates the risk of renegotiation by equity holders. For the existing equity holders, who get diluted at conversion, smooth pasting means $\frac{\partial E}{\partial A}\Big|_{A\downarrow A_c} = (1-\delta)\,\frac{\partial E}{\partial A}\Big|_{A\uparrow A_c}$. This condition is met when

$$\lambda^{SP}(A_c) \equiv (1-\theta_c)\left[A_c \frac{\gamma + (A_b/A_c)^{\gamma+1}}{\gamma+1} - A_b\right] / \left[A_c - A_b\right], \tag{10}$$

where λ^{SP} is the conversion ratio that ensures that smooth pasting is satisfied. This ratio is decreasing in A_b while increasing and concave in A_c. At the limit $\lim_{A_c \to +\infty} \lambda^{SP}(A_c) = \frac{\gamma}{\gamma+1}(1-\theta_c)$. The pair $\left(A_{bc}, \lambda^{bc}\right)$ verifies the equity smooth pasting conditions. Hence for all $A_c \geq A_{bc}$ a write-down factor that respects the smooth pasting condition necessarily belongs to $[\lambda^{bc}, \frac{\gamma}{\gamma+1}(1-\theta_c)]$.

Figure 5 shows the crossing between the conversion ratio that satisfies smooth pasting, λ^{SP}, and the conversion ratio mapped to the preset threshold, κ, in capital ratio rule, λ^{κ}. Unsurprisingly, at the crossing, the asset level that triggers conversion is exactly equal to $A_c^*(\kappa)$ identified in the equity conversion modality previously.[20] Simulations corresponding to the three scenarios highlighted in Figure 5 are presented in the appendix.

4.1.2 Asset sale to fund the write-down

Alternatively, the firm could fund the write-down by asset sales. The question that arises immediately is that of a potential fire sale which would impair asset valuation. Impairment may be suffered only at the time of conversion, or impact the firm's asset valuation going forward. The latter risk is already captured by the bankruptcy cost, while the former should be anticipated.

Indeed, at conversion, to face up to the $\lambda \frac{C_c}{r}$ liquidity need, the firm facing up to fire

[20] The conversion threshold, $A_c^*(\kappa)$, is the same across the equity conversion and write-down funded by equity sales modalities only if the coupons, C_c and C_b, across the two modalities are the same.

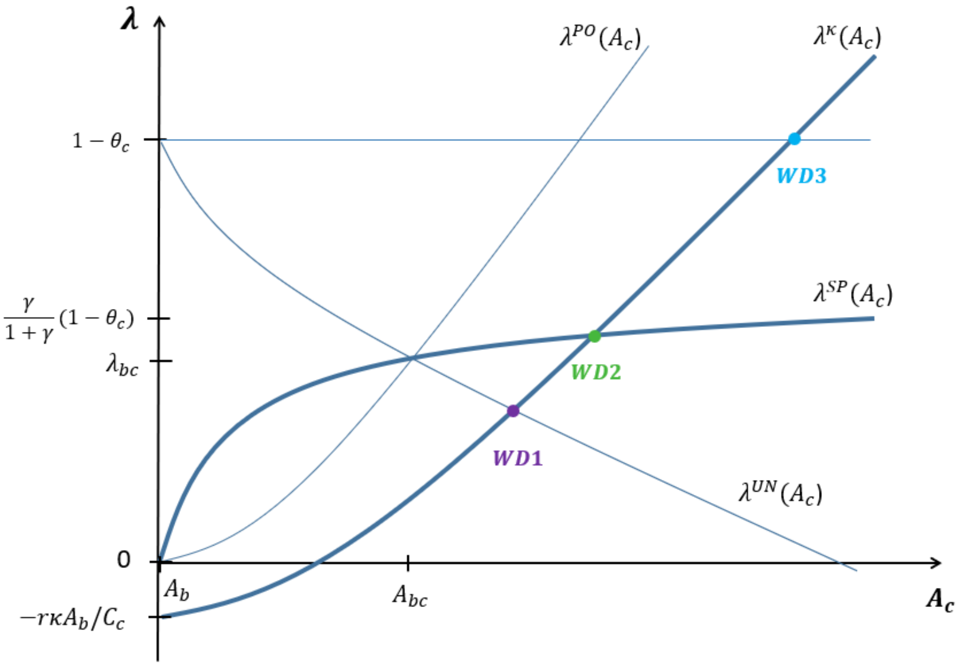

Figure 5: Write-Down conversion space with equity issuance This figure shows equilibria in the (A_c, λ)-space in the write-down modality with equity issuance. Equilibria to the right of $\lambda^{PO}(A_c)$ and $\lambda^{UN}(A_c)$ entail positive equity at conversion and uniqueness, respectively. Equilibria along the locus of points traced by $\lambda^{\kappa}(A_c)$ and $\lambda^{SP}(A_c)$ entail AT1-eligibility and smooth pasting, respectively.

sales would see its asset stock reduced by $\lambda(1+\eta)\frac{C_c}{r}$, where η is the impairment fraction. Capturing the expectation of this fire sale risk, we augment the levered firm value equation with the fire-sale (FS_t) cost anticipation term.

$$W_t \equiv A_t + TS_t - BC_t - FS_t = B_t + CC_t + E_t, \quad \forall t < \tau_c$$

where

$$FS_t = \mathbb{E}_t\left[e^{-r(\tau_c - t)}\eta\lambda\frac{C_c}{r}\right] = \eta\lambda\frac{C_c}{r}\left(\frac{A_t}{A_c}\right)^{-\gamma}.$$

This impairment risk we detail only here, yet it could also apply in the equity-issuance funding scenario as investors may require a rebate to fund a firm under regulatory scrutiny.

Then, denoting $\hat{\lambda} = \lambda(1+\eta)$ the impaired repayment fraction, as assets are sold, the value of the equity claim increases by $\frac{A_b^{\gamma+1}}{\gamma}[(A_c - \hat{\lambda}\frac{C_c}{r})^{-\gamma} - A_c^{-\gamma}]$. The jump in equity value stems from the very nature of asset sales and not due to the impairment of assets, $\eta > 0$.

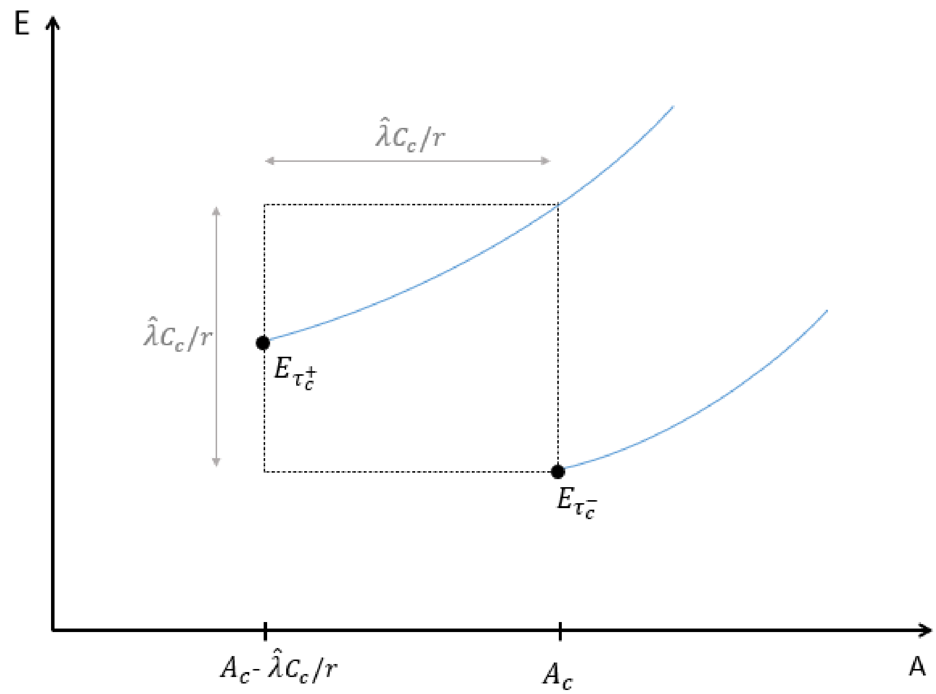

Figure 6: Write-down with asset sale equity impact This figure shows equity value as a function of asset value A_t in the write-down modality with asset sale. Across the conversion threshold (moving right to left along the horizontal axis), assets discretely jump down as some are sold to finance CoCo write-down. As a result, equity value also jumps discretely.

Figure 6 decomposes the three phenomena at play during the asset sale funding of conversion. First, the y-axis jump from lower blue to higher (blue) locus stems from the equity value anticipation of the sale. Second, the x-axis downward jump reflects the sale itself. Eventually, the convex nature of the equity claim valuation dictates the post-conversion equity valuation on the upper locus.

4.2 The Curious Case of Credit Suisse

We briefly characterize how our framework speaks to the complete write-down of Credit Suisse's CoCos by the regulator in March 2023. A complete write-down implies $\lambda = 0$ which in Figure 5 would be along the horizontal axis. For almost all values of the asset conversion threshold A_c—which is set ex-ante—a CoCo with conversion ratio $\lambda = 0$ would not satisfy the smooth pasting condition or, equivalently, it will not lie on the locus of points traced by $\lambda^{SP}(A_c)$, meaning equity holders will either strongly prefer conversion or the status quo when A_c is reached ex-post.[21] This preference when A_c is reached ex-post may not be consistent with the customary resolution waterfall which gives priority to debt over equity. In fact, the only scenario in which a complete write-down entails smooth pasting is when equity is valued at zero (the origin in Figure 5) which, in turn, would trigger bankruptcy and, hence, CoCos would effectively serve as pure debt.

Credit Suisse's CoCos stipulated a complete write-down. As discussed above, smooth pasting could prevail in this situation only if equity were valued at zero at conversion, otherwise equity holders may prefer one outcome over the other. However, as part of UBS's acquisition of Credit Suisse, equity holders retained positive value yet CoCos were written-down completely. While accepting this outcome was rational for equity holders ex-post, doing so was inconsistent with the customary waterfall of banking resolution, which inevitably led to market stress, see Figure 1. For comparison, Figure 1 shows a muted response from the CoCo market to the complete write-down of Banco Poular's CoCos when Banco Santander purchased the former for €1 in June 2017. In the model, a CoCo with complete write-down is incentive compatible (by smooth pasting) only when equity is valued at zero at conversion, i.e., the customary waterfall of banking resolution is preserved. Thus, a contributing factor towards the market stress may have been the difference in treatment of equity across the two events.[22]

[21] For a sufficiently large A_c the outcome will be unique.

[22] This insight from the model does not preclude the role Credit Suisse's G-SIB status and size relative to Banco Popular may have played, among other contributing factors, in the ensuing market stress.

5 Optimal Capital Structure and Simulations

Why is the equity conversion modality distinct from the write-down modality when equity sales are used to fund the write-down? If we limit attention to when the conversion is triggered then the two modalities might appear similar. However, these two outcomes represent very different opportunities for CoCo holders. In the equity conversion modality, they effectively receive an option on the firm which may increase in value later. Alternatively, in the write-down setup, they receive a fixed cash-out without any option value. This difference in payout must be reflected in the CoCo coupons set at the time of issuance. In this section, we pin down the optimal coupon for CoCos (and consol bonds) which captures the difference in payouts ex-ante.

We can determine optimal coupons for both types of debt that maximize the firm's levered value. Cocos hence contribute to improving the broad economic social welfare by increasing corporate value. Doing so completes our specification of the optimal capital structure which we then simulate and highlight key results.

5.1 Optimal Coupon

The time-0 levered firm value

$$W_0 = A_0 + \theta_b \frac{C_b}{r}\left[1 - \left(\frac{A_0}{A_b}\right)^{-\gamma}\right] + \theta_c \frac{C_c}{r}\left[1 - \left(\frac{A_0}{A_c}\right)^{-\gamma}\right] - \alpha A_b \left(\frac{A_0}{A_b}\right)^{-\gamma}$$

is strictly decreasing in A_c at any level of asset value, and is independent of the conversion ratios λ or δ. Hence from the point of view of the firm itself, the optimal conversion threshold (conditional on a given set of coupons) would be as low as possible. The firm levered value is maximized[23] for coupons (C_b^*, C_c^*) that solve the first order conditions $\frac{\partial W_0}{\partial C_b} = 0$ and $\frac{\partial W_0}{\partial C_c} = 0$.

Proposition 5 *The optimal CoCo coupon which maximizes the ex-ante firm value is determined by*

$$C_c^* = \frac{1}{\gamma}\frac{A_c}{\partial A_c / \partial C_c}\left[\left(\frac{A_0}{A_c}\right)^{\gamma} - 1\right].$$

While the optimal plain debt coupon payments with a capital structure with contingent claims stands at $C_b^* = \zeta C_b^{Leland}$, with $C_b^{Leland} = \left(1 + \gamma + \alpha\gamma\frac{1-\theta_b}{\theta_b}\right)^{-1/\gamma} \frac{1+\gamma}{\gamma}\frac{rA_0}{1-\theta_b}$ the optimal

[23] A_c being increasing and convex in C_c is a sufficient condition.

coupon payment for a capital structure without contingent convertible debt, and

$$\zeta = \left(1 - \frac{\theta_c}{\theta_b}\left[1 - \left(\frac{A_0}{A_c}\right)^{-\gamma}\right]\frac{\partial A_c/\partial C_b}{\partial A_c/\partial C_c}\right)^{1/\gamma}.$$

Proof. See the Appendix. ■

5.2 Optimal capital structure simulations

We simulate the optimal capital structure for a range of capital ratio rules up to 15%. We present simulation results for the three incentive-compatible cases: i) equity conversion with fixed number of shares (EC scenario 1), ii) equity conversion with fixed share price (EC scenario 2), and iii) write-down with equity sales to fund it (WD with equity smooth pasting). Table 1 summarizes the parameters used to calibrate the model.

Parameter		Value	Description
Asset features			
	A_0	100	Optimization level
	r	6%	Risk free rate
	μ	1%	Drift
	σ	10%	Volatility
Capital structure features			
	α	50%	Loss Given Default
	θ_b	30%	Plain debt Tax Shield
	θ_c	30%	Coco Tax Shield

Table 1: Baseline parameters This table summarizes the parameters used simulate the model. Importantly, these parameter values are similar to Leland (1994).

The right panels in Figure 7 show that compared to a hypothetical bank with no CoCos à la Leland (1994), a bank that issues CoCos issues less plain debt via consol bonds (small C_b) and is more resilient as the bankruptcy threshold, A_b, is lower regardless of the CoCo modality. Thus, the substitution towards CoCos has two effects: first, the direct change in the optimal capital structure (less consol bonds), second, a larger buffer before bankruptcy due to recapitalization in stressful times.

Comparing the two equity conversion scenarios, we find that CoCos are more attractive and plain debt leverage is lower in scenario 2 than scenario 1.[24] This difference in capi-

[24] In the Equity Conversion scenarios, the marginal impact of C_c on A_c differs depending on the conversion modality. In scenario (1), where CoCos are converted to a fixed number of shares, $\frac{\partial A_c}{\partial C_c} = \frac{\gamma}{\gamma+1}\frac{1-\theta_c}{r}\frac{1}{\delta}$, while

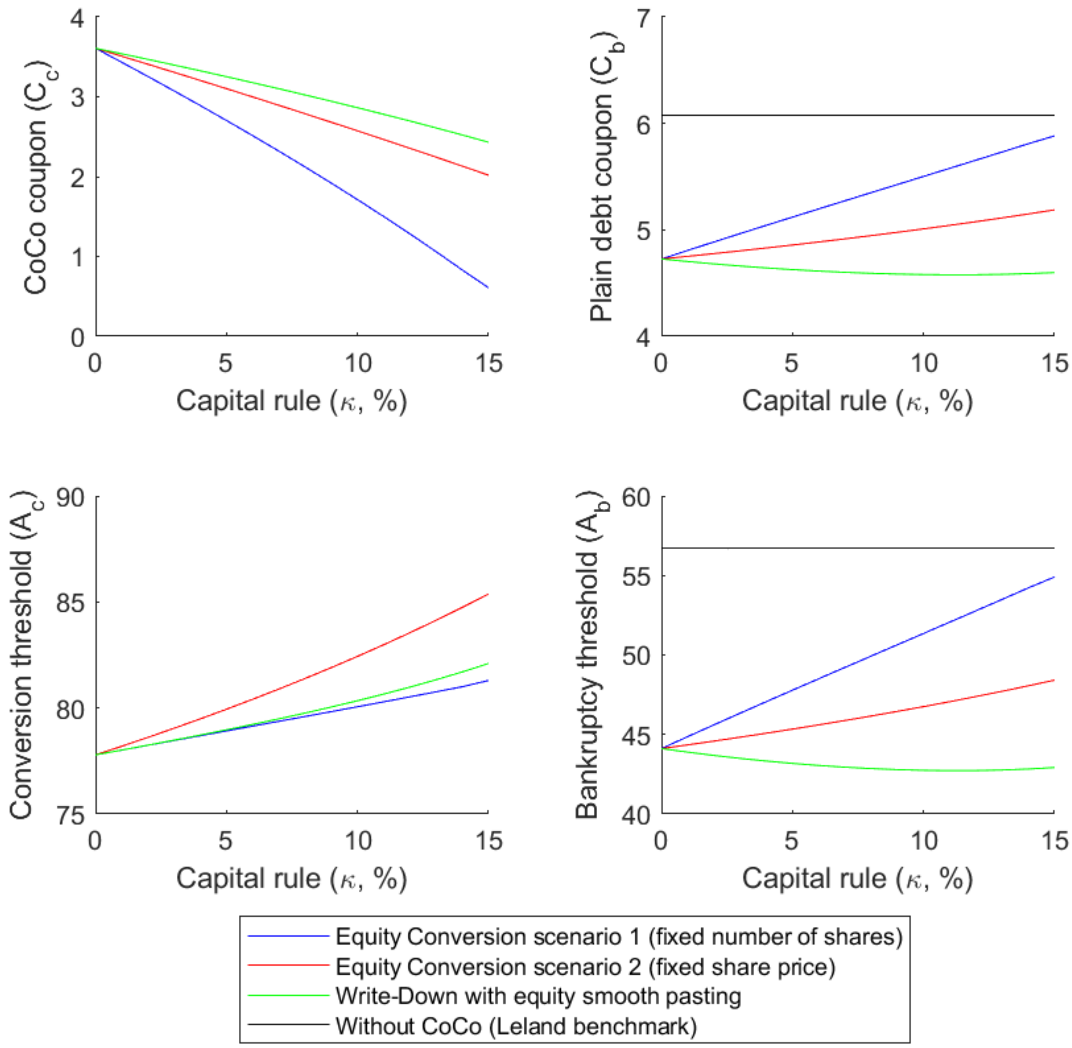

Figure 7: Optimal coupons and thresholds This figure shows the optimal coupons for CoCos and consol bonds (top panel) and thresholds for conversion and bankruptcy (bottom panel) as a function of the capital ratio rule specified for conversion ex-ante for the three incentive compatible scenarios. Results include consol bond coupons and bankruptcy threshold à la Leland (1994) as benchmarks for comparison.

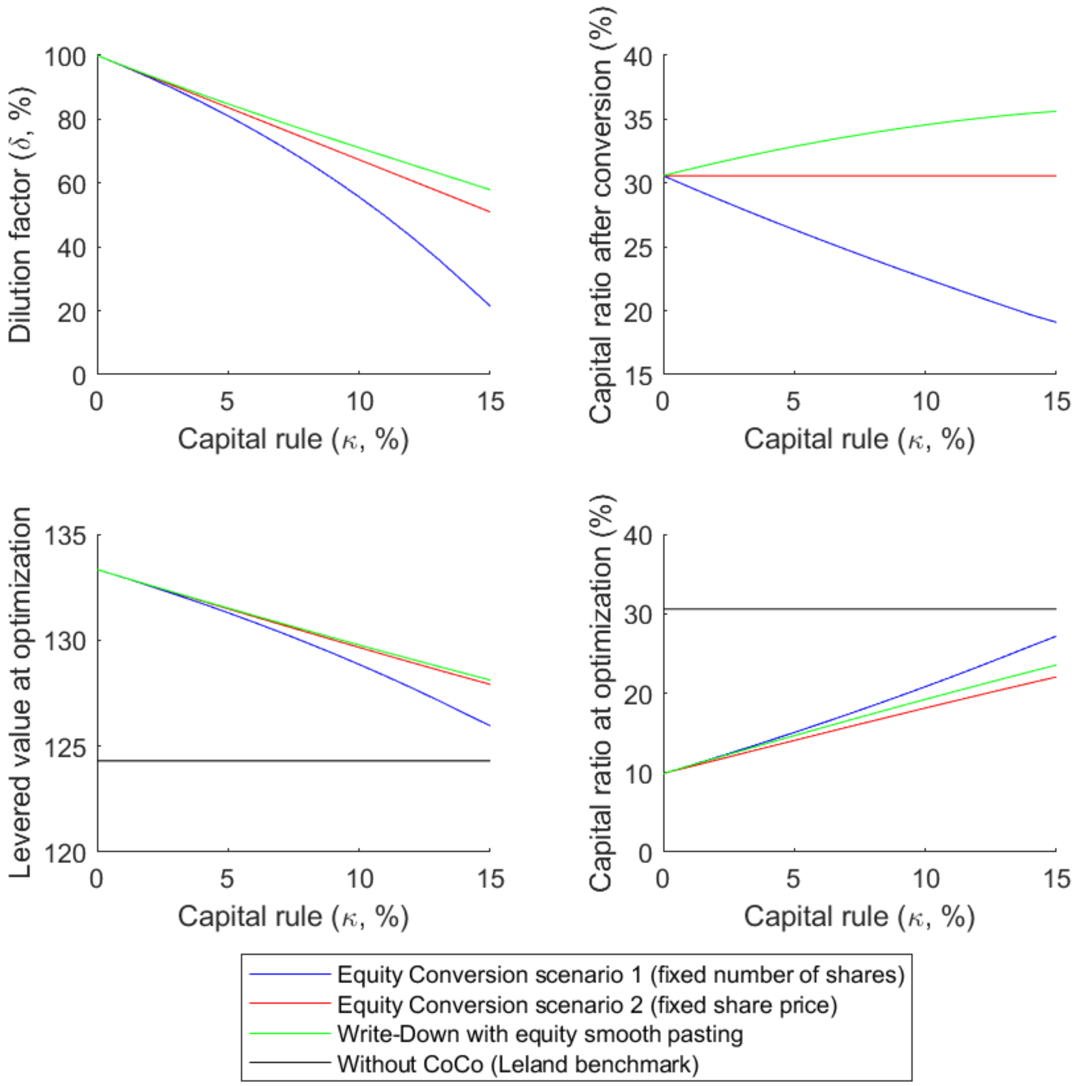

Figure 8: Optimal value, capital ratio, and conversion patterns This figure shows the dilution ratio and capital ratio after conversion (top panel) and the levered value of the firm and capital ratio at optimization (bottom panel) as a function of the capital ratio rule specified for conversion ex-ante for the three incentive compatible scenarios. Results include levered value and capital ratio at optimization for a firm without CoCos, à la Leland (1994), as benchmarks for comparison.

tal structure results in an earlier conversion threshold (higher A_c) and more resilience to bankruptcy (lower A_b) in scenario 2, see the lower panel in Figure 7. Moreover, in scenario 2 dilution is less acute than in scenario 1 and the capital ratio after conversion is fixed, see Figure 8.

Lastly, comparing the equity conversion and write-down modalities shows that the firm is more resilient if it issues write-down CoCos, see the bottom-right panel in Figure 7. The policymaker can normatively choose a capital ratio rule, κ, and ensure the bank is resilient to a satisfactory level. Given this choice of κ, (9) and (10) determine the conversion threshold A_c (by pinning down the conversion ratio, λ). For small values of κ the conversion threshold is the same between the write-down equity conversion scenario 1 modalities, but markedly lower than the conversion threshold for equity conversion scenario 2. Moreover, write-down leads to milder dilution of existing equity holders and a higher recapitalization, see Figure 8.

However, there is no free lunch: the optimal coupon for write-down CoCos is higher than the coupon for both equity conversion modalities, see the top-left panel in Figure 7. This larger coupon reflects two features of the write-down CoCo: first, its holders have to wait longer before the conversion threshold is reached, and second, they receive a fixed cash payout at conversion instead of equity which may increase in value if the firm's fortunes improve later. Moreover, there are two important caveats to the write-down results. First, structurally the conversion threshold for write-down CoCos is not optimal, only incentive compatible, for equity holders. Second, these simulations do not include the impact of impairment that may affect the firm's asset value during fire sales.

6 Conclusion

In this paper, we presented a tractable structural model for CoCo design à la Leland (1994). We analyzed two modalities: equity conversion and write-down. We showed that our proposed design is optimal for equity holders in the equity conversion case. In the write-down case, our proposed design implements a CoCo that is incentive-compatible for equity holders. In both settings, we rule out the multiplicity of equilibria and ensure that the resulting capital structure is optimal. Such CoCo contracts are time-consistent, hence alleviating the risk of renegotiation by the various stakeholders.

Crucially, we show that the above properties can be achieved such that the CoCo is AT1-eligible. In fact, for any recapitalization ratio chosen by the policymaker on normative grounds, there is a corresponding capital ratio rule that can be used to determine the optimal

in scenario (2), where CoCos' conversion hinges on a preset share price, $\frac{\partial A_c}{\partial C_c} = \frac{\gamma}{\gamma+1}\frac{1-\theta_c}{r}$.

or incentive-compatible CoCo contract. This way, our framework minimizes the uncertainty stemming from the role of discretion in determining if and when conversion ought to be triggered. This is precisely the kind of uncertainty that spooked markets when FINMA announced the write-down of all of Credit Suisse's CoCos in March 2023.

Eventually, we determine optimal coupon payouts to both types of debt holders that maximize firm-levered value. This result anchors CoCo as an asset class that can contribute to increasing economic and social welfare by improving corporate value.

Future research could consist of the empirical investigation of our design. Indeed, the suggested framework can easily be enriched to more advanced capital structure models which would allow for precise calibration to market data.

References

Admati, A., and M. Hellwig. 2014. *The bankers' new clothes: What's wrong with banking and what to do about it*. Princeton University Press.

Albul, B., D. M. Jaffee, and A. Tchistyi. 2015. Contingent Convertible Bonds and Capital Structure Decisions. *Working Paper* .

Avdjiev, S., B. Bogdanova, P. Bolton, W. Jiang, and A. Kartasheva. 2020. CoCo issuance and bank fragility. *Journal of Financial Economics* 138:593–613.

Avdjiev, S., A. Kartasheva, and B. Bogdanova. 2013. CoCos: a primer. *BIS Quarterly Review* .

Bolton, P., W. Jiang, and A. Kartasheva. 2023. The Credit Suisse CoCo wipeout: Facts, misperceptions, and lessons for financial regulation. *Journal of Applied Corporate Finance* 35:66–74.

Bolton, P., and F. Samama. 2012. Capital access bonds: contingent capital with an option to convert. *Economic Policy* 27:275–317.

Calomiris, C. W., and R. J. Herring. 2013. How to Design a Contingent Convertible Debt Requirement That Helps Solve Our Too-Big-to-Fail Problem. *Journal of Applied Corporate Finance* 25:39–62.

Chen, N., P. Glasserman, B. Nouri, and M. Pelger. 2017. Contingent Capital, Tail Risk, and Debt-Induced Collapse. *The Review of Financial Studies* 30:3921–69.

Chen, N., and S. G. Kou. 2009. Credit Spreads, Optimal Capital Structure, and Implied Volatility with Endogenous Default and Jump Risk. *Mathematical Finance* 19:343–78.

De Martino, G., M. Libertucci, M. Marangoni, and M. Quagliariello. 2010. Countercyclical Contingent Capital (CCC): Possible Use and Ideal Design. *Banca D'Italia Working Paper* .

Flannery, M. J. 2005. No Pain, No Gain? Effecting Market Discipline via "Reverse Convertible Debentures". *Working Paper* .

———. 2016. Stabilizing Large Financial Institutions with Contingent Capital Certificates. *The Quarterly Journal of Finance* 06:1650006–.

French, K., M. Baily, J. Campbell, J. Cochrane, D. Diamond, D. Duffie, A. Kashyap, F. Mishkin, R. Rajan, D. Scharfstein, R. Shiller, H. S. Shin, M. Slaughter, J. Stein, and R. Stulz. 2010. The Squam Lake Report: Fixing the Financial System. *Journal of Applied Corporate Finance* 22:8–21.

Glasserman, P., and B. Nouri. 2012. Contingent Capital with a Capital-Ratio Trigger. *Management Science* 58:1816–33.

———. 2016. Market-Triggered Changes in Capital Structure: Equilibrium Price Dynamics. *Econometrica* 84:2113–53.

Goldstein, R., N. Ju, and H. Leland. 2001. An EBIT-Based Model of Dynamic Capital Structure. *The Journal of Business* 74:483–512.

Hanson, S. G., A. K. Kashyap, and J. C. Stein. 2011. A Macroprudential Approach to Financial Regulation. *Journal of Economic Perspectives* 25:3–28.

Hart, O., and L. Zingales. 2011. A New Capital Regulation for Large Financial Institutions. *American Law and Economics Review* 13:453–90.

He, Z., and W. Xiong. 2012. Rollover Risk and Credit Risk. *The Journal of Finance* 67:391–430.

Himmelberg, C. P., and S. Tsyplakov. 2020. Optimal terms of contingent capital, incentive effects, and capital structure dynamics. *Journal of Corporate Finance* 64:101635–.

Kashyap, A., R. Rajan, and J. Stein. 2008. Rethinking capital regulation. *Proceedings - Economic Policy Symposium - Jackson Hole* 431–71. Institution: Federal Reserve Bank of Kansas City.

Leland, H. E. 1994. Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. *The Journal of Finance* 49:1213–52.

Leland, H. E., and K. B. Toft. 1996. Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads. *The Journal of Finance* 51:987–1019.

Madan, D. B., and W. Schoutens. 2011. Conic coconuts: the pricing of contingent capital notes using conic finance. *Mathematics and Financial Economics* 4:87–106.

McDonald, R. L. 2013. Contingent capital with a dual price trigger. *Journal of Financial Stability* 9:230–41.

Oster, P. 2020. Contingent Convertible bond literature review: making everything and nothing possible? *Journal of Banking Regulation* 21:343–81.

Paz Valbuena, J., and H. Eidenmüller. 2023. Bailout blues: the write-down of the at1 bonds in the credit suisse bailout. *European Corporate Governance Institute Working Paper* .

Pazarbasioglu, C., J. Zhou, V. L. Lesle, and M. Moore. 2011. Contingent capital: Economic rationale and design features. *Staff Discussion Notes* 2011:A001–.

Pennacchi, G. 2010. A Structural Model of Contingent Bank Capital. *Working Paper* Institution: Federal Reserve Bank of Cleveland.

Pennacchi, G., and A. Tchistyi. 2019a. Contingent Convertibles with Stock Price Triggers: The Case of Perpetuities. *The Review of Financial Studies* 32:2302–40.

———. 2019b. On Equilibrium When Contingent Capital Has a Market Trigger: A Correction to Sundaresan and Wang. *The Journal of Finance* 74:1559–76.

Pennacchi, G., T. Vermaelen, and C. C. P. Wolff. 2014. Contingent Capital: The Case of COERCs. *Journal of Financial and Quantitative Analysis* 49:541–74.

Prescott, E. S. 2012. Contingent capital: The trigger problem. *FRB Richmond Economic Quarterly* 98:33–50.

Sundaresan, S., and Z. Wang. 2015. On the Design of Contingent Capital with a Market Trigger. *The Journal of Finance* 70:881–920.

Wang, Z. 2023. Coco bonds: Are they debt or equity? do they help financial stability? — lessons from credit suisse nt1 bonds. *ECGI Blog* .

Appendix

A Detailed derivations

We remind two technical results in the Brownian diffusion setting of this paper. For (A_i, τ_i) a given couple of threshold and corresponding stopping time

$$\begin{aligned}
\mathbb{E}_t\left[e^{-r(\tau_i-t)}\right] &= \left(\frac{A_t}{A_i}\right)^{-\gamma}, \text{ and} \\
\mathbb{E}_t\left[\int_t^{\tau_i} e^{-r(s-t)}ds\right] &= \mathbb{E}_t\left[\frac{1-e^{-r(\tau_i-t)}}{r}\right] = \frac{1}{r}\left[1-\left(\frac{A_t}{A_i}\right)^{-\gamma}\right], \\
\text{with} \quad \gamma &= \left[\mu-\frac{\sigma^2}{2}+\sqrt{\left[\mu-\frac{\sigma^2}{2}\right]^2+2r\sigma^2}\right]/\sigma^2.
\end{aligned}$$

A.1 After conversion, ie for $t \in [\tau_b, \tau_c]$

After conversion the firm capital structure is exactly similar to that in Leland (1994). As such, the firm levered value corresponds to the value of its assets, net of bankruptcy costs, and added to the tax shield provided by the debt contracts. The firm balance sheet implies that its levered value combines the value of debt and equity

$$A_t + TS_t - BC_t = B_t + E_t,\ t \in [\tau_b, \tau_c].$$

On the asset side of the equation, the levered value of the firm is equal to the sum of the unlevered firm value -or assets A_t, added to the tax shield stemming from tax-deductibility of coupon payments (for a tax rate θ_b)

$$TS_t = \mathbb{E}_t\left[\int_t^{\tau_b} e^{-r(s-t)}\theta_c C_b ds\right] = \theta_b \frac{C_b}{r}\left[1-\left(\frac{A_t}{A_b}\right)^{-\gamma}\right]$$

and net of bankruptcy cost (with loss-given-default ratio of α)

$$BC_t = \mathbb{E}_t\left[e^{-r(\tau_b-t)}\alpha A_b\right] = \alpha A_b \left(\frac{A_t}{A_b}\right)^{-\gamma}.$$

On the liability side, bonds are priced as the discounted claims to coupon payments and recovery value of the bankrupt firm

$$B_t = \mathbb{E}_t\left[\int_t^{\tau_b} e^{-r(s-t)} C_b ds + e^{-r(\tau_b - t)}(1-\alpha)A_b\right] = \frac{C_b}{r}\left[1 - \left(\frac{A_t}{A_b}\right)^{-\gamma}\right] + (1-\alpha)A_b\left(\frac{A_t}{A_b}\right)^{-\gamma},$$

so that equities are valued as the residual claims[25]

$$E_t = A_t + TS_t - BC_t - B_t = A_t - (1-\theta_b)\frac{C_b}{r} + \left[(1-\theta_b)\frac{C_b}{r} - A_b\right]\left(\frac{A_t}{A_b}\right)^{-\gamma}.$$

Given the ownership structure, equity holders default at the level of asset A_b which satisfies the smooth pasting condition $\frac{\partial}{\partial A} E_t\big|_{A=A_b} = 0$, ie

$$A_b = \frac{\gamma}{1+\gamma}(1-\theta_b)\frac{C_b}{r}.$$

A.2 Before conversion, ie for $t < \tau_c$

Before conversion is triggered, the levered value of the firm still corresponds to the asset value, net of bankruptcy costs, but now augmented by the tax shield provided by both types of debt contracts. θ_c denotes the tax rate allowed for the deductibility of coupon payments on a CoCo contract, and is not necessarily equal to θ_b. The levered firm value corresponds now on the liability side of the balance sheet to all three instruments: straight debt, contingent convertible debt, and straight equity.

$$A_t + TS_t - BC_t = B_t + CC_t + E_t, \quad t < \tau_c$$

On the asset side, the unlevered firm value A_t, is diminished by the bankruptcy costs (which depends only on the bankruptcy threshold since $A_b < A_c$)

$$BC_t = \alpha A_b \left(\frac{A_t}{A_b}\right)^{-\gamma},$$

[25] Equivalently, with d_t the dividend process generated by the assets,

$$E_t = \mathbb{E}_t\left[\int_t^{\tau_b} e^{-r(s-t)}\left\{d_t - (1-\theta_b)c_b\right\} ds\right].$$

and augmented by the dual tax shield

$$TS_t = \theta_b \frac{C_b}{r} \left[1 - \left(\frac{A_t}{A_b} \right)^{-\gamma} \right] + \theta_c \frac{C_c}{r} \left[1 - \left(\frac{A_t}{A_c} \right)^{-\gamma} \right].$$

On the liability side, straight debt pricing is identical to that detailed in the previous section

$$B_t = \frac{C_b}{r} \left[1 - \left(\frac{A_t}{A_b} \right)^{-\gamma} \right] + (1-\alpha) A_b \left(\frac{A_t}{A_b} \right)^{-\gamma}.$$

CoCos on their end are worth the entitled coupon payments until the conversion trigger is reached. The recovery R at trigger consists of either (i) a conversion to equity (EC), ie. the exchange of CoCo contracts for newly issued shares, or (ii) a write-down (WD), ie. the repayment of a fraction λ of the CoCo's face value $\frac{C_c}{r}$. One can read that both scenarios lead to the creation of an additional equity segment at trigger: its ownership (and risk profile) is internalized in the price of the contingent contracts in case (i), but not in case (ii) where the CoCo holders are guaranteed a recovery compensation.

$$\begin{aligned} CC_t &= \mathbb{E}_t \left[\int_t^{\tau_c} e^{-r(s-t)} C_c ds \right] + \mathbb{E}_t \left[e^{-r(\tau_c - t)} R \right] \\ &= \frac{C_c}{r} \left[1 - \left(\frac{A_t}{A_c} \right)^{-\gamma} \right] + R \left(\frac{A_t}{A_c} \right)^{-\gamma} \\ R^{EC} &= \delta E_{\tau_c^+} \text{ and } R^{WD} = \lambda \frac{C_c}{r}. \end{aligned}$$

Equity is hence valued as a residual claim to the firm

$$\begin{aligned} E_t &= A_t - (1-\theta_c) \frac{C_c}{r} \left[1 - \left(\frac{A_t}{A_c} \right)^{-\gamma} \right] - R \left(\frac{A_t}{A_c} \right)^{-\gamma} - (1-\theta_b) \frac{C_b}{r} \left[1 - \left(\frac{A_t}{A_b} \right)^{-\gamma} \right] - A_b \left(\frac{A_t}{A_b} \right)^{-\gamma} \\ &= A_t - (1-\theta_b) \frac{C_b}{r} + \frac{A_b}{\gamma} \left(\frac{A_t}{A_b} \right)^{-\gamma} - (1-\theta_c) \frac{C_c}{r} + \left[(1-\theta_c) \frac{C_c}{r} - R \right] \left(\frac{A_t}{A_c} \right)^{-\gamma}. \end{aligned}$$

The last two terms of this expression correspond to the cost-benefit balance of CoCo claims for equity holders. In particular, shareholders derive value from the existence of CoCo in the firm's capital structure only if

$$R \leq (1-\theta_c) \frac{C_c}{r}.$$

B Equity-Conversion

B.1 Optimal endogenous conversion and smooth pasting conditions

At conversion, CoCo holders receive an amount of shares worth a fixed fraction $\delta \in [0,1]$ of the equity value of the firm, ie

$$R^{EC} = \delta E_{\tau_c^+} = \delta \left\{ A_c - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma} \right\}.$$

While equity before conversion is hence valued $\forall t < \tau$ as

$$E_t = A_t - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_t}{A_b}\right)^{-\gamma} - (1-\theta_c)\frac{C_c}{r} + \left[(1-\theta_c)\frac{C_c}{r} - \delta E_{t^+}\right]\left(\frac{A_t}{A_c}\right)^{-\gamma}.$$

Theorem 3 states that from the point of view of the firms' shareholders the endogenously optimal conversion threshold is

$$A_c^* = \frac{\gamma}{\gamma+1}\left[(1-\theta_b)\frac{C_b}{r} + \frac{1}{\delta}(1-\theta_c)\frac{C_c}{r}\right].$$

Smooth pasting conditions that take into account dilution at conversion lead to the same result.

At any point in time, equityholders would find it optimal that the conversion trigger A_c be set so as to maximize, for any given A_t, the value of their claim. The corresponding optimization program

$$\max_{\{A_c\}} A_c^{\gamma}\left[(1-\theta_c)\frac{C_c}{r} - \delta\left\{A_c - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right\}\right]$$

admits first-order conditions that determine the optimal threshold to be

$$A_c^* = \frac{\gamma}{\gamma+1}\left[(1-\theta_b)\frac{C_b}{r} + \frac{1}{\delta}(1-\theta_c)\frac{C_c}{r}\right].$$

This endogenously optimal trigger level is higher than the default threshold $A_b = \frac{\gamma}{\gamma+1}(1-\theta_b)\frac{C_b}{r}$ by $\frac{1}{\delta}\frac{\gamma}{\gamma+1}(1-\theta_c)\frac{C_c}{r}$ and higher than the threshold A_{bc} by $\frac{1-\delta}{\delta}\frac{\gamma}{\gamma+1}(1-\theta_c)\frac{C_c}{r}$. As reads naturally, the larger the ownership transfer at conversion, the later conversion would occur. Nevertheless, optimal conversion is guaranteed to trigger before a joint debt-liability-induced collapse could take place endogenously.

The smooth pasting condition for equity holders when taking into account dilution at conversion

$$\left.\frac{\partial E}{\partial A}\right|_{A\downarrow A_c} = (1-\delta)\left.\frac{\partial E}{\partial A}\right|_{A\uparrow A_c}$$

deliver the same threshold.

Proof. We verify here that the optimization does deliver a maximum in scenarios (1) and (2). The optimization program has

$$\begin{aligned} FOC &= \gamma A_c^{\gamma-1}\left[(1-\theta_c)\frac{C_c}{r} + \delta(1-\theta_b)\frac{C_b}{r} - \delta A_c\frac{1+\gamma}{\gamma}\right] \\ SOC &= \gamma(\gamma-1)A_c^{\gamma-2}\left[(1-\theta_c)\frac{C_c}{r} + \delta(1-\theta_b)\frac{C_b}{r} - \delta A_c\frac{1+\gamma}{\gamma}\right] - \delta A_c^{\gamma}(1+\gamma) \end{aligned}$$

where the second order condition is clearly negative $\forall A_c > A_c^*$.

In scenario (3) the maximization program simplifies to

$$\max_{\{A_c\}} (-1)\, A_c^{\gamma}\theta_c\frac{C_c}{r}$$

which finds its maximum at $A_c = 0$ ■

On the other hand, CoCo contract holders do not benefit directly from the tax deductibility of coupon payments, and smooth pasting conditions on their claims would demand earlier conversion.

Lemma 2 states that a conversion trigger based on the smooth-pasting condition for the CoCo contracts $\left.\frac{\partial CC}{\partial A}\right|_{A\downarrow A_c} = \delta\left.\frac{\partial E}{\partial A}\right|_{A\uparrow A_c}$ would command a conversion threshold

$$\widetilde{A_c} = \frac{\gamma}{\gamma+1}\left[(1-\theta_b)\frac{C_b}{r} + \frac{1}{\delta}\frac{C_c}{r}\right],$$

not impacted by the tax deductibility of θ_c and hence higher than A_c^*.

Hence, with a trigger A_c^* the valuation profile of CoCos is kinked at that conversion point.

B.2 Replication with capital ratio trigger

With a capital ratio κ, the trigger rule

$$(1-\delta)\left[A_c - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right] = \kappa A_c$$

determines the conversion threshold A_c (given δ and κ).

Corollary 4 states that for any capital ratio $\kappa \in [0,1]$ there exists a unique $\delta^*(\kappa)$ that will imply that a conversion taking place when the capital ratio rule (4) is met coincides with the endogenously optimal asset level A_c^* defined in (B.1).

Proof. Let's define the implicit function κ of δ

$$\begin{aligned} \kappa &= (1-\delta)\left[A_c - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right]/A_c \text{ with} \\ A_c &= \frac{\gamma}{\gamma+1}\left[(1-\theta_b)\frac{C_b}{r} + \frac{1}{\delta}(1-\theta_c)\frac{C_c}{r}\right] \end{aligned}$$

Immediately $\kappa \geq 1-\delta$.

Then we have for $\delta = 1$ that $\kappa = 0$, while for $\delta \to 0$ the optimal trigger A_c is infinitely large, and κ tends to 1.

We then show that $\kappa(\delta)$ is strictly decreasing. After computation

$$A_c^2 \frac{\partial \kappa}{\partial \delta} = (1-\delta)\frac{\partial A_c}{\partial \delta}\left\{(1-\theta_b)\frac{C_b}{r} - (1+\gamma)\frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma} - \frac{A_c}{(1-\delta)\frac{\partial A_c}{\partial \delta}} E_{\tau_c^+}\right\}$$

Together, the first two terms in the curved brackets are positive by definition of A_b. The third term is also positive thanks to $\frac{\partial A_c}{\partial \delta} < 0$. We conclude that the mapping between κ and δ is a bijection. ■

C Write-Down

The Write-Down (WD) specification leads to the equity valuation formula

$$E_t = A_t - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_t}{A_b}\right)^{-\gamma} - (1-\theta_c)\frac{C_c}{r} + (1-\theta_c-\lambda)\frac{C_c}{r}\left(\frac{A_t}{A_c}\right)^{-\gamma}.$$

At conversion, the CoCo's tax shield benefit vanishes, and given the assets Brownian diffusion payoffs, the levered value of the firm crossed the threshold continuously. Consequently, the change in the valuation of equity across the conversion threshold must be exactly equal to the payout to contingent claim holders at conversion, translating as

$$E_{\tau_c^+} = E_{\tau_c^-} + CC_{\tau_c^-}.$$

Equity ownership is hence diluted by

$$\delta \equiv 1-\frac{E_{\tau_c^-}}{E_{\tau_c^+}}=1-\frac{E_{\tau_c^+}-\lambda C_c/r}{E_{\tau_c^+}}=\frac{\lambda C_c/r}{E_{\tau_c^+}}.$$

C.1 Conversion trigger and ratio

First, equity value falling in negative territory before conversion would trigger immediate liquidation of the firm. For such a scenario to be ruled out until conversion, it should be the case that $E_{\tau_c^-} \geq 0$. Re-writing this condition in terms of conversion ratio, positivity is assured for all λ smaller or equal to

$$\lambda^{PO}\left(A_c\right) \equiv \left[A_c-\left(1-\theta_b\right)\frac{C_b}{r}+\frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right] / \left[\frac{C_c}{r}\right].$$

Second, we like to ensure in our modeling that the asset-based trigger can be implemented on a market-value basis. But) equity valuation pre-conversion has a convex profile with respect to the underlying asset value. If the trigger asset-defined-level is lower than the convexity inflation point of the equity function, the equity value at that point has been reached for another -higher- level of underlying asset value, threatening a conversion earlier than warranted. The one-to-one mapping between the market and capital measures at conversion is thus secured if and only if $\frac{\partial E}{\partial A}\big|_{A\downarrow A_c} \geq 0$. which we translate as requiring λ to be greater or equal to

$$\lambda^{UN}\left(A_c\right) \equiv \left[\left(1-\theta_c\right)\frac{C_c}{r}-\frac{A_c}{\gamma}+\frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right] / \left[\frac{C_c}{r}\right].$$

The lowest possible conversion threshold to possibly meet the previous two sets of conditions is A_{bc} This asset level corresponds exactly to the endogenous default threshold that would prevail if the convertibility option were to never be exercised, and CoCo behaved like plain debt contracts. The conversion ratio that also respects these constraints for a trigger asset level of A_{bc} is

$$\lambda_{bc} \equiv \frac{\gamma}{\gamma+1}\left(1-\theta_c\right)+\frac{1-\theta_b}{\gamma+1}\frac{C_b}{C_c}\left[\left(\frac{A_{bc}}{A_b}\right)^{-\gamma}-1\right].$$

C.2 Optimal trigger from shareholders perspective in cases of proportional write down

Contingent convertible contracts have been designed to address a demand for a bail-in solution, ie automatic recapitalization of a financial institution facing acute solvability issues. The newest regulatory reference points for assessing the proper capitalization of such institutions are the Basel III capital ratio requirements guidelines, which recommend maintaining a core tier I capital to risk-weighted assets (RWA) ratio of 4.5% and a total capital to RWA of 6%. Most of the CoCos issued so far all take as trigger rule the Basel ratio just mentioned, so that we do follow the same logic in our design and denote by κ the reference regulatory capital ratio. As such, to respect at conversion

$$\frac{E_{\tau_c^-}}{A_{\tau_c^-}} = \kappa,$$

CoCo modalities would verify that

$$\lambda^{\kappa}\left(A_c\right) = \left[A_c\left(1-\kappa\right) - \left(1-\theta_b\right)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma}\right] / \left[\frac{C_c}{r}\right].$$

C.2.1 Equity issuance to fund the write-down and smooth pasting

Constructing a CoCo contract that entails smooth pasting of equity value at conversion would alleviate manipulation incentives. For the existing equity holders, who get diluted at conversion, smooth pasting means

$$\left.\frac{\partial E}{\partial A}\right|_{A\downarrow A_c} = \left(1-\delta\right)\left.\frac{\partial E}{\partial A}\right|_{A\uparrow A_c}.$$

This condition is met when

$$\lambda^{SP}\left(A_c\right) \equiv \left(1-\theta_c\right)\left[A_c\frac{\gamma + \left(A_b/A_c\right)^{\gamma+1}}{\gamma+1} - A_b\right] / \left[A_c - A_b\right].$$

C.2.2 Asset sale to fund the write-down

Alternatively, the firm could fund the write-down by asset sales. The question that arises immediately is that of a potential fire sale which would impair asset valuation. Impairment may be suffered only at the time of conversion, or impact the firm's asset valuation going forward. The latter risk is captured by the bankruptcy cost.

At conversion, to face up to the $\lambda\frac{C_c}{r}$ liquidity need, the firm reduces asset stock by $\lambda(1+\eta)\frac{C_c}{r}$ where η is the impairment fraction. Capturing the expectation of this fire sale risk, the levered firm value equation is augmented with the fire-sale (FS_t) cost anticipation term.

$$W_t \equiv A_t + TS_t - BC_t - FS_t = B_t + CC_t + E_t, \quad \forall t < \tau_c$$

where

$$FS_t = \mathbb{E}_t\left[e^{-r(\tau_c - t)}\eta\lambda\frac{C_c}{r}\right] = \eta\lambda\frac{C_c}{r}\left(\frac{A_t}{A_c}\right)^{-\gamma}.$$

At conversion, denoting $\hat{\lambda} = \lambda(1+\eta)$, as assets are sold, equity value adjusts from

$$E_{\tau_c^-} = A_c - \hat{\lambda}\frac{C_c}{r} - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c}{A_b}\right)^{-\gamma} \quad \text{to}$$

$$E_{\tau_c^+} = A_c - \hat{\lambda}\frac{C_c}{r} - (1-\theta_b)\frac{C_b}{r} + \frac{A_b}{\gamma}\left(\frac{A_c - \hat{\lambda}\frac{C_c}{r}}{A_b}\right)^{-\gamma},$$

so that the value of the equity claim increases by $\frac{A_b^{\gamma+1}}{\gamma}[(A_c - \hat{\lambda}\frac{C_c}{r})^{-\gamma} - A_c^{-\gamma}]$, see Figure 6.

D Optimal coupons

Proposition 5 states that the optimal CoCo coupon which maximizes the ex-ante firm value is determined by

$$C_c^* = \frac{1}{\gamma}\frac{A_c}{\partial A_c/\partial C_c}\left[\left(\frac{A_0}{A_c}\right)^{\gamma} - 1\right].$$

While the optimal straight debt coupon payments with a capital structure with contingent claims stands at $C_b^* = \zeta C_b^{Leland}$, with $C_b^{Leland} = [1+\gamma+\alpha\gamma(1-\theta_b)/\theta_b]^{-1/\gamma}(1+\gamma)\,rA_0/[\gamma(1-\theta_b)]$ the optimal coupon payment for a capital structure without contingent convertible debt, and

$$\zeta = \left[1 - \frac{\theta_c}{\theta_b}\left[1 - \left(\frac{A_0}{A_c}\right)^{-\gamma}\right]\frac{\partial A_c/\partial C_b}{\partial A_c/\partial C_c}\right]^{1/\gamma}.$$

The time-0 levered firm value

$$W_0 = A_0 + \theta_b\frac{C_b}{r}\left[1 - \left(\frac{A_0}{A_b}\right)^{-\gamma}\right] + \theta_c\frac{C_c}{r}\left[1 - \left(\frac{A_0}{A_c}\right)^{-\gamma}\right] - \alpha A_b\left(\frac{A_0}{A_b}\right)^{-\gamma}$$

is strictly decreasing in A_c at any level of asset value, and is independent of the conversion ration λ. Hence from the point of view of the aggregate firm, the optimal conversion threshold (conditional on a given set of coupons) would be as low as possible. It reaches its maximum the coupon levels (C_b^*, C_c^*) that solve the first order conditions

$$\frac{\partial W_0}{\partial C_b} = 0 \text{ and } \frac{\partial W_0}{\partial C_c} = 0.$$

leading to

$$\begin{aligned} C_c^* &= \frac{1}{\gamma} \frac{A_c}{\partial A_c / \partial C_c} \left[\left(\frac{A_0}{A_c} \right)^\gamma - 1 \right] \\ C_b^* &= \zeta C_b^{Leland} \end{aligned}$$

where the baseline coupon $C_b^{Leland} = \left[1 + \gamma + \alpha\gamma \frac{1-\theta_b}{\theta_b}\right]^{-1/\gamma} \frac{1+\gamma}{\gamma} \frac{rA_0}{1-\theta_b}$ is scaled by

$$\zeta = \left[1 - \gamma C_c^* \frac{\theta_c}{\theta_b} \left(\frac{A_0}{A_c} \right)^{-\gamma} \frac{\partial A_c / \partial C_b}{A_c} \right]^{1/\gamma} = \left[1 - \frac{\theta_c}{\theta_b} \left[1 - \left(\frac{A_0}{A_c} \right)^{-\gamma} \right] \frac{\partial A_c / \partial C_b}{\partial A_c / \partial C_c} \right]^{1/\gamma}.$$

Proof. In the Equity Conversion case, we prove that, if A_c is increasing and weakly convex in A_c then (C_b^*, C_c^*) is optimal. Focusing on A_c and C_c, and denoting for simplification $W' = \frac{\partial W}{\partial C_c}$

$$\begin{aligned} W &\propto -C_c A_c^\gamma \\ W' &\propto -A_c^\gamma - \gamma C_c A_c' A_c^{\gamma-1} \\ W'' &\propto -2A_c' A_c - A_c'' A_c - (\gamma - 1) A_c' \end{aligned}$$

■

E Simulations of three Write-Down scenarios

For illustration purposes, we present simulation results for three Write-Down scenarios (depicted in Figure 5), all relying on equity issuance to fund the conversion, and for a range of capital ratio rule: i) at the uniqueness frontier, ii) with equity smooth pasting, and iii) at the CoCo-worthiness frontier for equity holders.

Scenario (iii) leads to limited reliance on CoCos in the optimal capital structure, early

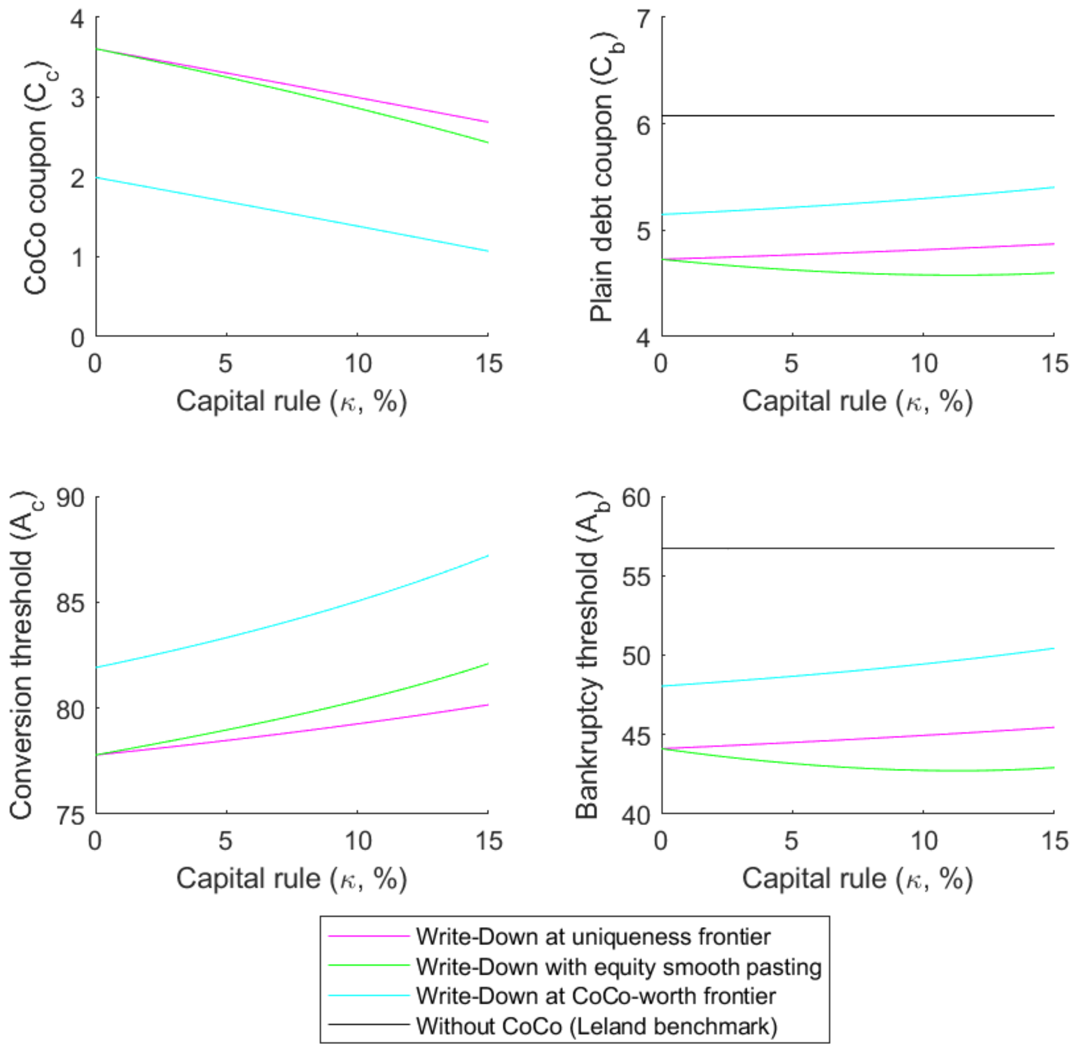

Figure 9: Optimal coupons and thresholds. Three Write-Down scenarios. Values as a function of the capital rule on the x-axis.

conversion and limited resilience improvement. Scenario (i) adds more CoCos and straight debt than scenario (ii), later conversion, but earlier default. Scenario (ii) leads to the best recapitalization at conversion and the latest bankruptcy.

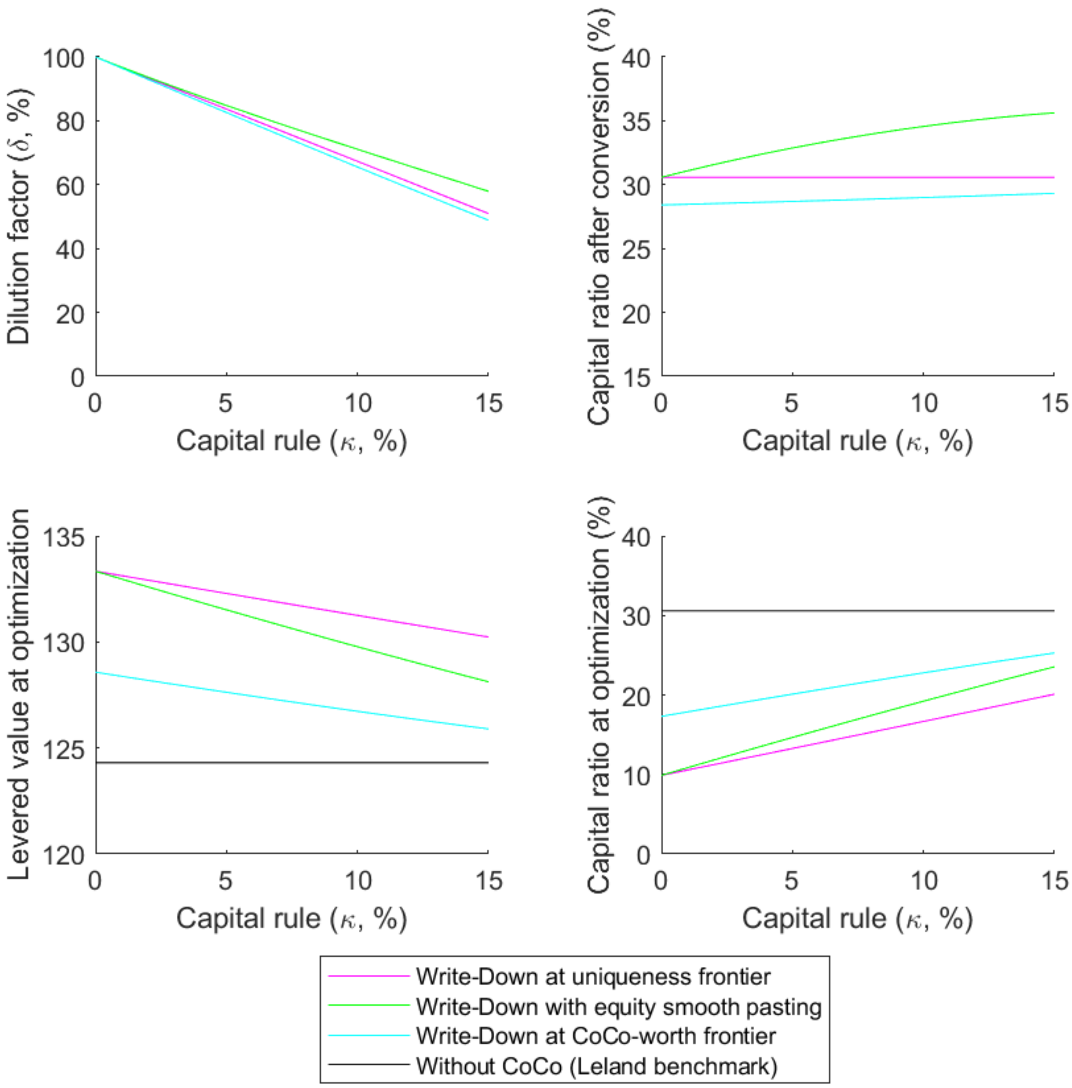

Figure 10: Optimal value, capital ratio and conversion patterns. Three Write-Down scenarios. Values as a function of the capital rule on the x-axis.