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Inflation Expectations with Finite Horizon Planning*

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Abstract

Under finite horizon planning, households and firms evaluate a full set of state-contingent paths along which the economy might evolve out to a finite horizon but have limited ability to process events beyond that horizon. We show–analytically and empirically–that such a model accounts for an initial underreaction and subsequent overreaction of inflation forecasts. A planning horizon of four quarters can account for the evidence on the predictability of inflation forecast errors and macroeconomic data. Our identification and estimation strategies combine full-information methods based on aggregate data with regression-based estimates that directly use inflation expectations data.

JEL Classification: C11, E52, E70
Keywords: Finite horizon planning, inflation expectations, bayesian estimation.

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1 Introduction

Motivated by limitations in the cognitive ability of people to understand and process information, macroeconomists have increasingly incorporated behavioral elements into their models as an alternative to rational expectations. A novel approach in this regard is the finite horizon planning (FHP) framework developed in Woodford (2018), in which agents are boundedly rational, as their ability to evaluate the full set of state-contingent paths along which the economy might evolve is limited to a finite horizon. To highlight the appeal of this approach, Woodford (2018) embeds FHP into a New Keynesian (NK) model and shows that monetary policy does not suffer from a "forward guidance" puzzle in which a credible promise to keep the policy rate unchanged in the distant future produces counterfactually large effects on current inflation and output.[1]

Our previous research, Gust et al. (2022), provides new evidence that a NK model with FHP offers a compelling framework for understanding aggregate output, inflation, and interest-rate dynamics. Our estimates show that the model is able to generate substantial inflation persistence and realistic costs to an anticipated disinflation announced by a central bank. In addition, the model fits the macroeconomic time series substantially better than other behavioral models and mirrors the "hybrid" NK model featuring rational expectations, habit persistence in consumption, and exogenous price indexation.[2]

While the FHP model has demonstrated success in explaining macroeconomic time series, its empirical properties regarding expectations formation are almost fully unexplored. In particular, it remains an open question how well it accounts for some key stylized facts revealed by empirical studies using surveys of inflation expectations. This literature finds that inflation forecast errors are systematically predictable in a way that is difficult to rationalize with macroeconomic models that feature full information and rational expectations (FIRE). This research emphasizes that survey data on expectations help to discriminate across alternative models of expectation formation. Several influential papers point to a set of stylized facts that may be difficult to reconcile with some simple versions of the behavioral models including those emphasizing diagnostic expectations or cognitive discounting.[3] Important contributions to this literature include Coibion and Gorodnichenko (2015) (hereafter CG (2015)), Angeletos et al. (2020) (hereafter AHS (2020)), and Kohlhas and Walther (2021) hereafter KW (2021).[4] CG (2015) study the correlation between consensus forecast errors and forecast revisions of inflation and find evidence consistent with an *underreaction* of forecasts to revisions. The evidence in KW (2021) also supports the finding of

[1] For other applications of the finite horizon planning framework to policy analysis, see, for example, Woodford and Xie (2022) and Dupraz et al. (2023).

[2] The specific behavioral models that we compare to the FHP model are the models of Angeletos and Lian (2018) and Gabaix (2020). Herbst (2023) provides evidence on the out-of-sample forecasting performance of the FHP model.

[3] See, for example, Kohlhas and Walther (2021), who point to some evidence that they suggest is challenging to explain with simple formulations of diagnostic expectations. Similarly, Angeletos et al. (2020) present evidence that seems at odd with simple formulations of cognitive discounting.

[4] We focus on the evidence from these papers because they are directly relevant to the macroeconomic models that we investigate. Another important branch of this literature, including Bordalo et al. (2018), Fuhrer (2018), and Broer and Kohlhas (2018), examines the predictability of forecasts errors of individual forecasters instead of average or consensus forecasts.

an underreaction of forecasts to revisions, but KW (2021) also provide evidence of an *overreaction* of average forecasts to recent data. AHS (2020) study the impulse responses of inflation forecasts from the survey of professional forecasters (SPF) and find that the average forecast underreacts to shocks initially *but overreacts later on*.

To understand the model's implications for inflation expectations and forecast predictability, we begin by examining the price-setting behavior of FHP firms in a partial equilibrium setting, which allows us to analytically characterize the dynamics of aggregate inflation and firms' inflation forecasts. We provide formal conditions under which inflation forecasts under FHP are consistent with findings of AHS (2020), CG (2015), and KW (2021). In particular, we show that this evidence from the predictability literature requires two key features. First, firms' forecasts are based on a finite planning horizon. Second, FHP firms update their beliefs about events outside their planning horizons based on past data. We show that these two features imply that inflation expectations respond sluggishly at first but later on overreacts to shocks, consistent with the evidence in CG (2015), AHS (2020), and KW (2021).[5]

We formalize these results in three propositions and show that there is a wide range of parameter values for which inflation expectations under FHP are qualitatively consistent with the aforementioned patterns of inflation forecast errors. These propositions also highlight an attractive property of the FHP model — its parsimony. The model adds only two new parameters to an otherwise standard NK model of firm price setting. These two parameters are easily understandable. The first parameter is the length of agents' planning horizon or how far into the future they evaluate the full set of state-contingent paths along which the economy might evolve. The second parameter governs how responsive firms' longer-run beliefs (i.e., those outside their planning horizons) are to past observations of inflation. These two parameters, and the persistence of aggregate shocks, are enough to formalize the predictions of the model for generating: (i) positive serial correlation of forecast errors, (ii) the sluggishness in expectations following an aggregate shock with forecasts underreacting in the short run but overreacting later on.

In the second part of the paper, we build on this analysis and study FHP in the context of dynamic, general equilibrium model developed by Woodford (2018) and estimated by Gust et al. (2022). We show that the modelling of expectations formation using FHP is a fruitful and empirically-relevant approach, as the model is capable of generating inflation forecast errors in line with the patterns observed in the empirical literature while remaining parsimonious and easy to solve and use for macroeconomic analysis. Moreover, the general equilibrium version of the FHP model introduces richer dynamics into inflation and inflation expectations. For instance, inflation and inflation expectations are influenced by an array of aggregate shocks as well as by other aspects of the economy such as the central bank's reaction function. These features, as well as the degree of price stickiness and the intertemporal elements of aggregate demand, influence the model's

[5]While Woodford (2018) incorporates learning about events outside agents planning so that the FHP model has desirable theoretical properties regarding the response of output and inflation to long-lasting economic shocks, our results suggest that this learning is also important from an empirical perspective in order to account for the predictability observed in the data regarding inflation forecast errors.

implications for the predictability regressions of CG (2015) and KW (2021) and predictability impulse responses of AHS (2020), which, in turn, highlights the importance of examining this evidence using a dynamic, general equilibrium model. Thus, an important lesson drawn from the general equilibrium analysis is that other structural parameters, auxiliary to expectations formation, influence the predictability of inflation forecast errors. For this reason, it is difficult to take a limited information approach that uses *only* the empirical estimates from the predictability regressions and impulse responses of inflation forecast errors to shocks to identify the two key parameters governing expectations formation in the FHP model.

Instead, we leverage the approach in our previous research, Gust et al. (2022), and estimate the model with a Bayesian, full-information likelihood-based approach using U.S. data on output growth, inflation, and nominal interest rates from 1966:Q1 through 2007:Q4, a time period for which there were notable changes in trends in both inflation and output growth.[6] Armed with these estimates, we investigate the FHP model's implications for the behavior of inflation expectations. Specifically, we formalize the statistics of CG (2015), KW (2021), and AHS (2020) as posterior predictive checks of our estimated DSGE model. These predictive checks are a coherent way to assess the model's performance along these dimensions using parameterizations that can also rationalize the behavior of key macroeconomic aggregates. We use these predictive checks in two ways. First, the predictive checks help refine the estimate of a key parameter, the length of the planning horizon. While macroeconomic data points strongly towards short planning horizons, it is less helpful at distinguishing between planning horizons of one quarter or one year. Second, we use the suite of predictive checks to evaluate the FHP model more generally. Because the information coming from these empirical moments is not directly used to estimate the model, these predictive checks can be viewed as external model validation.

With regard to estimating the length of agents' planning horizons, a key insight of our analysis is that combining information on inflation forecast predictability at two different forecast horizons is useful in identifying the length of a firm's planning horizon. In particular, we use the predictability regression of CG (2015)—which emphasizes the relationship between inflation forecast errors and forecast revisions— at forecast horizons of one quarter and one year to help identify agents' planning horizons. We show that a planning horizon on the order of one year fits this evidence and the macroeconomic data quite well. In contrast, a short planning horizon on the order of a quarter, while yielding a slightly better fit of the macroeconomic data, implies near-term forecast errors that are overly predictable. This result reflects that, as the planning horizon increases in the FHP model, forecasts become increasingly rational and thus forecasts errors become increasingly unpredictable. Accordingly, too short of a planning horizon can lead to overly predictable forecast errors, while a more moderate planning horizon yields a degree of predictability in line with the evidence.

[6] A perceptive reader will notice that our estimation period excludes the last fifteen years of data. Including these years will require a treatment of the zero lower bound as well as the volatility in aggregate data induced by the pandemic. While an interesting and important extension, we concentrate on the shorter sample period to reduce the complexity that the treatment of these years would introduce in the model and in the adjustment of agents' expectations.

The FHP model with a one-year planning horizon also performs effectively with regard to other nontargeted moments in our predictive checks: one year ahead predictability of inflation forecast errors documented in KW (2021) and the impulse responses of inflation and expected inflation presented in AHS (2020). Notably, the FHP model with a planning horizon of a year generates the reversal of the sign of inflation forecast errors following an aggregate shock—that is, inflation forecasts underreact initially and then overreact later on.

As a point of contrast to the FHP model, we also investigate a canonical NK model featuring sticky information (SI), another form of imperfect expectations popular for characterizing the behavior of aggregate expectations. Like the FHP model, the SI model can generate predictable forecast errors in line with the evidence in CG (2015). However, the SI model cannot achieve the sign reversal highlighted in the impulse responses of AHS (2020). Overall, our results support the view that modelling expectations formation using finite horizon planning merits further attention as part of an agenda incorporating behavioral features into macroeconomic models.

Our paper is related to earlier work, including Del Negro and Eusepi (2011), that uses survey data on inflation expectations in the context of the estimation of DSGE models. While we also estimate a DSGE model, our emphasis is on evaluating the model's ability to account for the predictability regressions of CG (2015) and KW (2021) and the predictability impulse responses of AHS (2020). We pointedly exclude inflation expectations data from our model estimation, ensuring that the parameterizations used in our exercise are generated solely to fit non-expectational data, though are results are materially unchanged when we include expectations data. Our paper is also related to papers such as Milani (2007), Slobodyan and Wouters (2012), and Ormeno and Molnar (2015) that estimate behavioral models with learning as well as Eusepi and Preston (2018), and Carvalho et al. (2023), which emphasize learning about long-run trends. A key difference between all of these papers and the finite-horizon approach used here is that expectation formation in these papers is backward looking while expectation formation under FHP has both a backward-looking and forward-looking component. We find that both components are important for explaining the predictability results of CG (2015), KW (2021), and AHS (2020).

The rest of the paper proceeds as follows. The next section presents the properties of aggregate inflation when firms' set prices with finite horizon plans and the analytical results regarding the predictability of inflation forecasts under FHP. Section 3 describes the general equilibrium version of the model that we estimate. Section 4 discusses the estimation results of that model, including its fit of the predictability regressions of CG (2015) and KW (2021) and the predictability impulse responses of AHS (2020). It also compares the FHP's empirical performance with alternative models including a NK model emphasizing sticky information. Section 5 concludes.

2 Finite Horizon Planning and Inflation Forecast Predictability

A key finding in the empirical literature using survey data is that inflation forecast errors are systematically predictable. CG (2015) emphasize this predictability by running regressions and showing that average inflation forecast errors in the SPF are correlated with forecast revisions. AHS (2020) show that the impulse response of the average inflation forecast in the SPF underreacts to aggregate shocks in the short run before overreacting later on. Building on CG (2015), KW (2021) find evidence of both an underreaction of the average forecast to new information but a simultaneous overreaction to recent data. In this section, we follow Woodford (2018) and assume firms, setting prices according to Calvo (1983) contracts, have finite planning horizons. We study the inflation forecasts of these firms and derive conditions under which they are consistent with the empirical results of CG (2015), AHS (2020), and KW (2021).

Finite horizon planning. Before discussing the economy's price-setting firms, we first define the expectations operator of an agent who is a finite-horizon planner. As discussed in Woodford (2018), such an agent making decisions at date t can only look forward and formulate plans that take into account the model's relationships and all possible realizations of shocks occurring between periods t and $t+k$, where k denotes the length of an agent's planning horizon. Let $\mathbb{E}_t^k$ denote the subjective expectations of a finite-horizon planner. Then, for any endogenous variable in periods $t+k-j$, Z_{t+k-j}, with $j = 0, 1, 2, ..., k$ (i.e., j indexes the number of periods remaining within the planning horizon), the following relationship holds:

$$\mathbb{E}_t^k Z_{t+k-j} = E_t Z_{t+k-j}^j, \tag{1}$$

where E_t denotes the model-consistent expectations operator conditional on time t information and the variable Z_{t+k-j}^j reflects the subjective expectations of a finite-horizon planner. Because an agent has a limited understanding of events outside of its planning horizons, its expectations are not fully model consistent. However, within its planning horizon a firm makes fully state-contigent plans using all of the model's relatonships and thus expression (1) provides a mapping between an agent's subjective expectations operator and the model-consistent expectations operator.

Price setting. The price-setting firms are monopolistic competitors whose prices are staggered á la Calvo (1983). When resetting their price, each firm is assumed to have a finite planning period of length k. As shown in Woodford (2018), under these assumptions, firms' price-setting behavior implies a log-linearized relationship for inflation given by:

$$\pi_\tau^j = \beta E_\tau \pi_{\tau+1}^{j-1} + \kappa y_\tau, \tag{2}$$

where $\tau = t+k-j$ denotes the planning period and $1 < j \leq k$. The variable π_τ^j denotes the (log-linearized) inflation rate implied by firms' plans in period τ. The parameter β is the discount rate, and the parameter κ is a function of the Calvo price-setting parameter, θ_p, and parameters that affect the link between firms' real marginal costs and aggregate output. The variable y_τ represents the (log-linearized) output gap, which is assumed to follow an AR(1) process:

$$y_t = \rho y_{t-1} + e_t \tag{3}$$

with $\rho \geq 0$. For now, we assume that the output gap evolves exogenously. This simplification allows us to derive analytical results, providing a better understanding of the implications that FHP has for the predictability impulse responses of AHS (2020) and the predictability regressions of CG (2015) and KW (2021). Later, we modify this assumption, and allow the output gap to be endogenously determined in the context of the general equilibrium model that we estimate.

Equation (2) reflects the behavior of firms who have the opportunity to change their prices at date t and it holds in each period of those firms' planning horizons except the last period. Iterating forward on the expressions implied by equation (2) results in an expression that determines aggregate inflation:

$$\pi_t^k = \kappa E_t \sum_{i=0}^{k-1} \beta^i y_{t+i} + \beta^k E_t \pi_{t+k}^0 \tag{4}$$

where $\pi_t^k = \pi_t$ denotes aggregate inflation (in log deviation from steady state). According to equation (4), aggregate inflation depends on the expected path of the output gap and on the expected inflation rate at the end of firms' planning horizons ($E_t\pi_{t+k}^0$). The NK Phillips curve under RE arises as a special case: as $k \to \infty$, the planning horizon extends over a firm's infinite lifetime and inflation depends on the entire future path of the output gap.

Firms with the opportunity to reset their price at date t make a fully state-contingent plan through $t+k$. They use their knowledge of the model's structural equations to do so. However, these firms use continuation value functions to assign value to events outside of their planning horizons (i.e., the longer-run from their viewpoint). These value functions affect the expected inflation rate at the end of firms' planning horizons (π_{t+k}^0). Formally, the (log-linearized) equilibrium condition associated with firms' pricing plans at the end of their horizon is:

$$\pi_{t+k}^0 = \kappa y_{t+k} + \beta(1-\theta_p)v_{pt}, \tag{5}$$

where v_{pt} is the (log-linearized) *continuation value* to the plans of firms with the opportunity to reset their prices at date t and $1-\theta_p$ is the fraction of firms that have the opportunity to re-optimize their price at date t.

Learning. While firms are sophisticated in thinking about events within their planning horizon, they are less so when thinking about events further in the future. In this regard, we consider two different situations. In the no learning case, firms' beliefs about longer-run events (i.e., outside their planning horizon) are fixed at their steady state values so that $v_{pt} = 0 \ \forall t$. Alternatively, we allow firms to learn and update their beliefs based on *past experience*. In this case, the value function v_{pt} evolves according to:

$$v_{pt+1} = (1-\gamma_p)v_{pt} + \gamma_p v_{pt}^e, \tag{6}$$

where v_{pt}^e is a firm's *new* estimate of its value function. The parameter γ_p determines how much weight they place on that new estimate and satisfies $0 < \gamma_p < 1$. The new estimate of the value function is determined by firms who can re-optimize their prices at time t, as v_{pt}^e is chosen optimally taking its current value function as given. Optimization by a price-setting firm, as shown in Woodford (2018) implies that in equilibrium v^e_{pt} satisfies:

$$v^e_{pt} = (1-\theta_p)^{-1}\pi^k_t. \tag{7}$$

According to equation (7), v^e_{pt} depends on aggregate inflation scaled by the fraction of firms who can re-optimize their prices at date t.[7] Combining equations (6) and (7), it follows that firms' beliefs about events outside their planning horizon (i.e., v_{pt}) depend on past realizations of inflation.

With v^e_{pt} chosen optimally by firms, the adaptive learning scheme introduces only one extra parameter, γ_p, relative to the version of the model with no learning. As discussed in Woodford (2018), the model with learning is particularly advantageous relative to the no-learning model in response to long-lasting changes in policy or other fundamental economic changes. Without learning, for instance, a firm will continue to use an outdated value function if, for example, there is a permanent change in a central bank's inflation target. In contrast, with learning, a firm's value function will change and eventually fully reflect the change in a central bank's inflation target. While the model with learning is more appealing from a theoretical perspective in response to long-lasting economic changes, in this paper we emphasize how firms' learning about their value functions affects the dynamics of inflation and inflation expectations in the model. In particular, firms' learning based on past experience introduces stickiness in inflation and inflation expectations.

Inflation Dynamics. The equilibrium dynamics of inflation can be characterized analytically. Equations (4), (5), and (6) imply that aggregate inflation is the sum of a component that reflects a firm's future beliefs about the output gap over its finite horizon and a component that reflects firm's beliefs about longer-run events outside of its planning horizon:

$$\pi^k_t = A(k)\kappa y_t + \beta^{k+1}(1-\theta_p)v_{pt}, \tag{8}$$

where the parameter $A(k) = \frac{1-(\beta\rho)^{k+1}}{1-\beta\rho}$.[8] Both the parameters affecting the response of inflation to changes in the output gap and changes in their longer-run beliefs depend on k, the length of a firm's planning horizon.

The dynamics of inflation under RE correspond to the case in which $k \to \infty$. In that case, a firm's longer-run beliefs, v_{pt}, become irrelevant and $A(\infty) = \frac{1}{1-\beta\rho}$ so that inflation evolves according to

$$\pi^{RE}_t = \frac{\kappa}{1-\beta\rho}y_t.$$

Inflation dynamics under FHP expectations involves two deviations from the dynamics of inflation under RE. First, since $0 < 1-(\beta\rho)^{k+1} \leq 1$, inflation in the FHP model is less responsive to fluctuations in the output gap. This muted responsiveness of inflation is a function of ρ and the

[7]More specifically, to a first order approximation, Woodford (2018) shows $v^e_{pt} = p^{*k}_t$, where p^{*k}_t denotes the optimal contract price chosen by firms with an opportunity to reset their price at date t. Equation (7) then reflects the equilibrium relationship between aggregate inflation and the contract price: $\pi^k_t = (1-\theta_p)p^{*k}_t$.

[8]The term on the output gap reflects that firms know the process for y_t so that $\mathbb{E}^k_t y_{t+i} = E_t y_{t+i} = \rho^i y_t$.

length of the planning horizon. A shorter planning horizon or a more persistent shock imply a more muted response of inflation to movements in the output gap relative to the RE solution. The second deviation from inflation dynamics under RE is that a firm's longer-run beliefs about inflation, as discussed above, depend on past inflation and thus inflation under FHP expectations displays an excess sensitivity to past inflation.

Forecasting. To understand the implications of FHP expectations for forecast predictability, we characterize a firm's one-step ahead forecast for inflation and the associated forecast error. A firm with a planning horizon of length $k > 0$ has a one-step ahead forecast given by:

$$\mathbb{E}_t^k \pi_{t+1} = \rho A(k-1)\kappa y_t + \beta^k (1-\theta_p) v_{pt} = [1-(\beta\rho)^k] \frac{\kappa\rho}{1-\beta\rho} y_t + \beta^k (1-\theta_p) v_{pt}. \tag{9}$$

Like inflation, a firm's one-step ahead forecast is sticky, since a firm's longer-run beliefs about inflation affect $\mathbb{E}_t^k \pi_{t+1}$ and these beliefs depend on lagged inflation.[9] This stickiness diminishes as $k \to \infty$. In that case, expression (9) converges to the forecast under rational expectations: $\mathbb{E}_t^\infty = E_t \pi_{t+1} = \frac{\kappa\rho}{1-\beta\rho} y_t$.

Under FHP expectations, a firm will make systematic forecast errors. To see this, define the one-step ahead forecast error under FHP as $\mathbb{F}_{t+1}^k \equiv \pi_{t+1}^k - \mathbb{E}_t^k \pi_{t+1}$. Using expressions (8) and (9), the one-step ahead forecast error evolves according to:

$$\mathbb{F}_{t+1}^k = \left[\beta^{k+1}\gamma_p A(k) + \rho(\beta\rho)^k\right] \kappa y_t - \beta^k \left[1-\beta(1-\widetilde{\gamma}_p)\right](1-\theta_p) v_{pt} + O_{t+1}. \tag{10}$$

where $\widetilde{\gamma}_p = \gamma_p(1-\beta^{k+1})$ and O_{t+1} represents omitted terms that depend on the innovation in the output gap at date $t+1$, e_{t+1}.[10]

Equation (10) is a key equation for determining the forecasting properties of inflation in the FHP model. Firms' forecast errors for inflation are the sum of an unpredictable component (O_{t+1}) and two predictable components. One of those predictable components relates to errors associated with firms underpredicting the responsiveness of inflation to movements in the output gap: In response to an increase in the output gap, the inflation forecast error rises because realized inflation responds more than expected inflation. Accordingly, firms' forecasts underreact to changes in the output gap. The other predictable component relates to changes in the value function governing firms' longer-run beliefs. Because $0 < \beta < 1$ and $0 < \widetilde{\gamma}_p < 1$, the forecast error falls in response to an increase in the value function, indicating that firms' forecasts overreact to changes in their longer-run beliefs. This overreaction reflects that a firm is closer to the end of its planning horizon when forming expectations of future inflation, making the sensitivity of the one-step ahead forecast to a firm's (continuation) value function greater than that of realized inflation.

[9] As equation (9) highlights, the learning framework in Woodford (2018) uses the "anticipated utility" approach of Kreps (1998) and a firm's forecast of future inflation ignores the fact that v_{pt} will change over time.

[10] To obtain expression (10), note that $A(k) = A(k-1) + (\beta\rho)^k$.

2.1 Impulse Response Predictability

Equations (9) and (10) can be used to characterize the impulse responses of inflation forecasts and forecast errors to an innovation in e_t, allowing us to relate the model's implications to the empirical work of AHS (2020). Using data from the SPF, AHS (2020) compute the impulse response of the median respondent's inflation forecast and forecast error from the shock that maximizes the business cycle variation in inflation. Their results are striking, as they show there is a sign switch in the impulse response of the inflation forecast error: it underreacts before overreacting later on. We show that the FHP model is capable of generating this sign switch, and the following proposition established conditions under which it does so.

Proposition 1. *(IRFs of Inflation Forecasts and Forecast Errors). Let* $\frac{\partial \mathbb{E}^k_{t+i}\pi_{t+1+i}}{\partial e_t}$ *and* $\frac{\partial \mathbb{F}^k_{t+1+i}}{\partial e_t}$ *for* $i \geq 0$ *be the impulse response functions for a firm's one-step ahead inflation forecast and forecast error, respectively.*

1. *Without learning:* $\frac{\partial \mathbb{E}^k_{t+i}\pi_{t+1+i}}{\partial e_t} \geq 0$ *and* $\frac{\partial \mathbb{F}_{t+1+i}}{\partial e_t} \geq 0$, $\forall i \geq 0$ *and* $k > 0$.
2. *With learning: If* $\gamma_p \leq \frac{1-\rho}{1-\beta^{k+1}}$, *there is a threshold forecast horizon,* $i^\star$, *such that:*
   (a) $\frac{\partial \mathbb{E}^k_{t+i}\pi_{t+1+i}}{\partial e_t} \geq 0$ *for* $i \geq 0$,
   (b) $\frac{\partial \mathbb{F}_{t+1}}{\partial e_t} > 0$ *and* $\frac{\partial \mathbb{F}_{t+1+i}}{\partial e_t} < 0$ *for* $i \geq i^\star$,

Proof: See the appendix.

Proposition 1 indicates that, in the model in which firms do not update their longer-run beliefs about inflation (i.e., no learning), both the inflation forecast and inflation forecast error respond positively to an innovation to the output gap. Accordingly, the impulse responses are characterized by a systematic underreaction — there is no flip in the sign of the impulse response function at any horizon. This underreaction reflects that without learning $v_{pt} = 0$ $\forall t$. As a result, only changes in the output gap affect a firm's inflation forecast. Moreover, a firm with finite-planning horizon neglects changes in the output gap that occur outside its planning horizon, implying that its forecast underreacts to such changes. Thus, the FHP model without learning, similar to other behavioral models emphasizing cognitive discounting, can not account for the empirical evidence in AHS (2020).

While incorporating learning into the model may be a necessary condition to account for the evidence in AHS (2020), it is not a sufficient condition. For the forecast error to change signs from an underreaction to an overreaction, Proposition 1 also provides a sufficient condition of an upper bound on γ_p, the speed at which firms update their longer-run beliefs using past data. Focusing on $\beta \approx 1$, this condition only imposes a restriction on γ_p for shocks that are highly persistent (i.e., values of ρ close to 1.) For more moderate values of ρ and relatively short planning horizons, this condition is easily satisfied so that all values of γ_p between zero and one are consistent with the result in AHS (2020). Intuitively, if the output-gap shock is not very persistent, the impulse responses of the forecast error in later periods will be mostly determined by changes in a firm's longer-run inflation beliefs. With the effect of the value function growing more important for the impulse responses in later periods, a firm's forecast will eventually display an overreaction, since firms' forecasts are excessively sensitive to changes in v_{pt}.

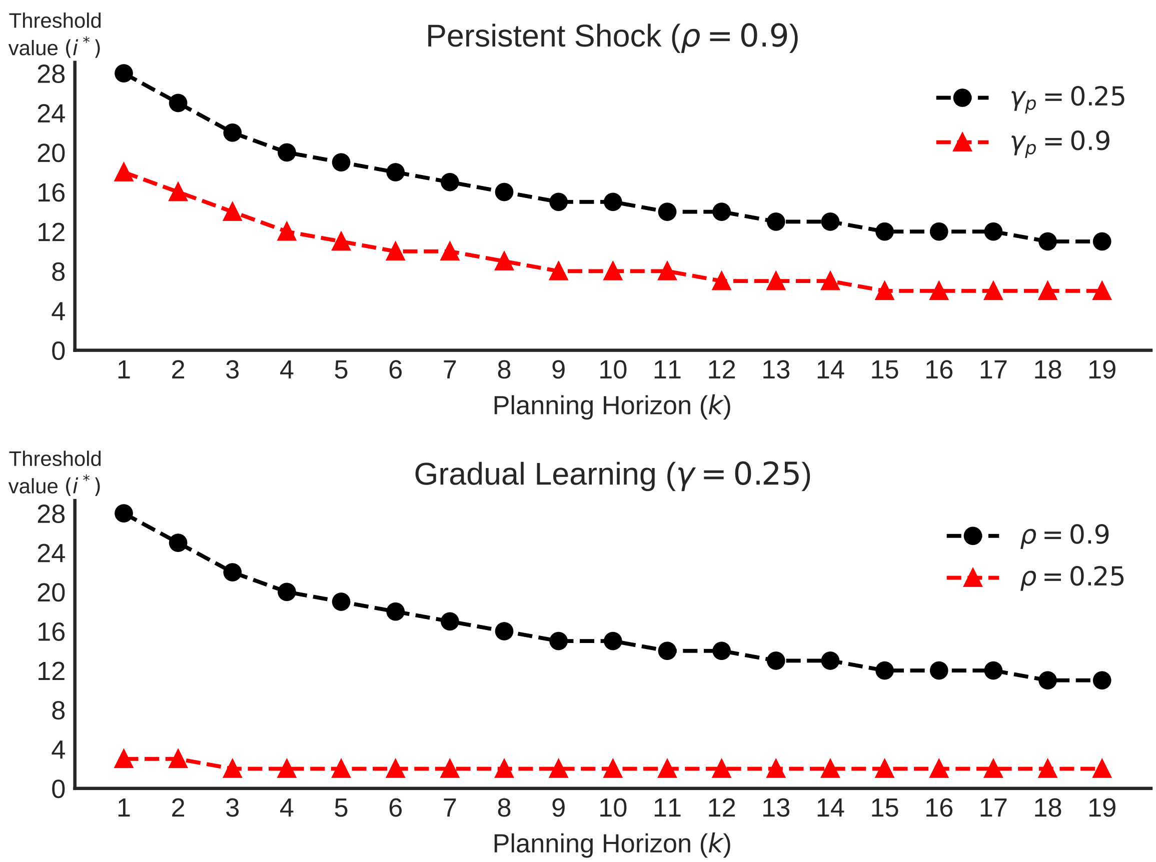
Proposition 1 establishes the existence of a threshold horizon at which the impulse response of the inflation forecast error in the FHP model switches signs. Figure 1 illustrates how this threshold horizon depends on the model's structural parameters. The upper panel plots the threshold horizon ($i^\star$) as a function of a firm's planning horizon (k) for two different values of γ_p. The threshold horizon is higher for shorter planning horizons. For instance, when ($\gamma_p = 0.25, \rho = 0.9$) and $k = 12$, the impulse response switches from an underreaction to an overreaction after 14 quarters, while when $k = 1$, it takes 28 quarters. A shorter planning horizon has this effect because, all else equal, it makes the magnitude of the underreaction of the forecast to the change in the output gap larger, delaying the eventual overshoot. Figure 1 also highlights that a higher value of γ_p or lower value of ρ results in an earlier overreaction of a firm's inflation forecast. These results reflect that a higher value of γ_p speeds up the learning process so that the model's eventual overreaction occurs earlier. A lower value of ρ has a similar effect, since it implies a smaller and less persistent underreaction of a firm's inflation forecast. Overall, proposition 1 indicates that the sign switch in the impulse response function of the forecast error is a robust, qualitative feature of the FHP model with learning. Later, we conduct a more rigorous empirical evaluation of the FHP model, pinning its parameters down using macroeconomic time series while using the impulse response of AHS (2020) and the predictability regressions of CG (2015) and KW (2021) as additional tests regarding the nature of expectation formation embedded in finite horizon planning.

2.2 Inflation Predictability Regressions

To discriminate across alternative models of expectation formation, CG (2015) and KW (2021) emphasize the predictability of forecast errors from regressions using survey data on expectations. CG (2015) regress the median forecast error of inflation on the median forecast revision and show that there is a positive correlation between the forecast error and forecast revision, implying an underreaction of forecasts to new information. Although, KW (2021) emphasize the underreaction of forecasts to new information, they also provide additional evidence that forecasts in survey data also involve an overreaction to recent data. In particular, they regress average forecast errors from survey data on the forecasted variable and show a negative correlation between the forecast error and the forecasted variable, implying an overreaction to recent data. They argue that a wide class of models of expectation formation are unable to account for this simultaneous underreaction to new information implied by the CG (2015) regression and overreaction to recent data implied by their regression. In this section we investigate the implications of FHP expectations for the predictability regressions of CG (2015) and KW (2021).

To examine the implication of the FHP model for the predictability result of CG (2015), a firm's

Figure 1: Delayed Overreaction of Inflation Forecasts in the FHP Model



Note: The figure shows the threshold date at which the impulse response of the inflation forecast in the FHP model switches from an underreaction to an overreaction.

inflation forecast revision is defined as $\mathbb{R}_t^k = \left[\mathbb{E}_t^k - \mathbb{E}_{t-1}^k\right]\pi_{t+1}$. A firm's forecast at $t-1$ satisfies:

$$\mathbb{E}_{t-1}^k \pi_{t+1} = \rho^2 A(k-2)\kappa y_{t-1} + \beta^{k-1}(1-\theta_p)v_{pt-1} \tag{11}$$

for $k > 1$. At time $t-1$ a firm's expectation for π_{t+1} differs from its expectations at time t because it has less information than at time t. In addition, a firm is looking an extra period ahead and is closer to the end of its planning horizon. Because, it is close to the end of its planning horizon, its forecast of π_{t+1} at time $t-1$ puts more weight on a firm's value function and less weight on the output gap than a firm's one-step ahead forecast. Proposition 2 characterizes the relationship between inflation forecast errors and revisions for a firm with FHP expectations.

Proposition 2. *(Forecast Error and Revision Correlation). Let* $\beta_{CG} = \frac{cov(\mathbb{R}_t^k,\ \mathbb{F}_{t+1}^k)}{var(\mathbb{R}_t^k)}$ *denote the univariate regression coefficient from regressing the one-step ahead forecast error on the forecast revision in the FHP model.*

1. *Without learning: If $\rho > 0$, then $\beta_{CG} > 0$, for any finite planning horizon $k > 0$.*

2. *With learning: If $\rho = 0$ and $\gamma_p < \frac{1-\beta}{1-\beta^{k+1}}$, then $\beta_{CG} > 0$.*

Proof: See the appendix.

Proposition 2 shows that for positively correlated shocks, the FHP model without learning always results in a positive value of β_{CG}, in line with the empirical estimates of CG. Without learning, a persistent increase in the output gap ($\rho > 0$) leads firms with FHP expectations to revise up their forecasts of inflation. From equation (10), it follows that their inflation forecast errors rise persistently, and there is a persistent underreaction of the inflation forecast to new information.

With learning, the dynamics of inflation are richer (and more complex) and the correlation between the forecast error and forecast revision can be either positive or negative depending on the length of a firm's planning horizon (k), how quickly firms update their longer-run beliefs (γ_p), and the persistence of the shock (ρ). When the shocks are uncorrelated and if firms do not update their longer-run beliefs too quickly, Proposition 2 indicates that the FHP model implies $\beta_{CG} > 0$. Specifically, there is an upper bound on γ_p that grows increasingly tight as the length of the planning horizon increases. For instance, with short-horizon planning ($k = 1$) and with learning occurring at relatively sluggish rate, (i.e., $\gamma_p < 0.5 < \frac{1}{1+\beta}$), the model generates $\beta_{CG} > 0$.

Table 1: β_{CG} in the FHP Model

	$h=1$		$h=4$	
	$\rho = 0.25$	$\rho = 0.90$	$\rho = 0.25$	$\rho = 0.90$
$k = 1$				
$\gamma_p = 0.25$	0.74	1.99	0.32	3.90
$\gamma_p = 0.9$	0.44	1.38	0.28	3.42
$k = 4$				
$\gamma_p = 0.25$	0.60	0.71	0.28	4.91
$\gamma_p = 0.9$	0.45	0.91	0.18	3.24

Note: The entries report population coefficients from a regression of one-step ($h = 1$, left columns) and four-step ($h = 4$, right columns) ahead inflation forecast errors on forecast revisions. The values of β and κ were set to 0.99 and 0.05, respectively.

Table 1 provides some numerical results for β_{CG}, varying the persistence of the shock, the length of firms' planning horizons, the learning speed, as well as the forecast horizon (h). The table highlights that $\beta_{CG} > 0$ for firms' planning horizons of 1 to 4 quarters and that β_{CG} is positive for a wide range of values of ρ and γ_p. In particular, when $\rho = 0.9$, β_{CG} is positive even for high values of γ_p. The table also highlights that for persistent shocks ($\rho = 0.9$) the coefficient β_{CG} is larger for longer forecast horizons. This result reflects that a high value of ρ strengthens the underreaction of a firm's forecast, and this underreaction for persistent shocks can grow as h increases to approach and even exceed a firm's planning horizon.[11] Overall, we conclude that the

[11] In making their decisions, firms do not need to forecast inflation outside of their planning horizons. Accordingly,

FHP model can be qualitatively consistent with the empirical evidence of CG (2015). Below we investigate this question further in the context of an estimated, general equilibrium model.

Our final proposition considers the regression statistic of KW (2021). While KW (2021) mainly focus on forecasts of output growth, they show that several survey measures of inflation expectations, including average forecasts of consumer price inflation from the SPF, display a negative correlation between the forecasted variable and survey respondents' forecast errors. Propositon 3 characterizes the relationship between inflation and forecast errors under FHP expectations.

Proposition 3. *(Forecast Error and Inflation Correlation). Let* $\beta_{KW} = \frac{cov(\pi_t^k,\, \mathbb{F}_{t+h}^k)}{var(\pi_t^k)}$ *denote the univariate regression coefficient from regressing the h-step ahead forecast error on inflation in the FHP model.*

1. *Without learning: If* $\rho > 0$*, then* $\beta_{KW} > 0$*, for any finite planning horizon* $k \geq h \geq 1$*.*

2. *With learning: If* $\rho = 0$ *and* $k \geq h > 1$*, then* $\beta_{KW} < 0$ *if and only if:*

$$\frac{\beta^{k+1}(1-\widetilde{\gamma}_p)}{(1-\beta^{k+1})(2-\widetilde{\gamma}_p)}\left[\beta^{-h}(1-\widetilde{\gamma}_p)^{-h}-1\right] > 1.$$

Proof: See the appendix.

Proposition 3 indicates that the correlation between inflation and the forecast error is positive in the FHP model without learning. However, when the FHP model includes learning, the correlation can be negative for forecasts beyond a quarter (i.e., $h > 1$). For uncorrelated shocks, the condition in proposition 3 indicates that for a fixed value of k, a longer forecast horizon or a larger value of γ are more likely to imply that $\beta_{KW} < 0$. This negative correlation is possible because firms learn and update their longer-run beliefs about events outside their planning horizons by extrapolating from past data on inflation. Such behavior implies that a firm's forecasts can overreact to movements in inflation, and faster learning in which firms' beliefs depend more on recent inflation data exacerbates this overreaction.

Table 2 shows some numerical results for β_{KW} varying the persistence of the shock, the length of firms' planning horizons, the learning speed, as well as the forecast horizon. For forecasts of horizon $h = 1$, the table highlights that $\beta_{KW} > 0$ for firms' planning horizons of 1 to 4 quarters and that β_{KW} is positive for a wide range of values of ρ and γ_p. In particular, when $\rho = 0.9$, β_{KW} is positive even for high values of γ_p. For a forecast horizon of four quarters, the table highlights that β_{KW} can be positive or negative depending on how firms update their beliefs about their value functions. When $\gamma_p = 0.9$, the sign of β_{KW} is negative, as firm's beliefs about their value functions responds relatively quickly to past changes in inflation. However, when $\gamma_p = 0.25$, firms' beliefs depend relatively more on inflation in the distant past, and the sign of β_{KW} is positive or near zero in those cases.

we need to make an additional assumption when a firm's forecast horizon exceeds its planning horizon, which is the case since we are forecasting inflation four quarters ahead with $k = 1$. In that case, we assume a firm uses its beliefs at the end of its planning horizon to make its forecast, taking into account its knowledge regarding the persistence of shocks. The appendix provides more details regarding this assumption.

Table 2: β_{KW} in the FHP Model

	$h = 1$		$h = 4$	
	$\rho = 0.25$	$\rho = 0.90$	$\rho = 0.25$	$\rho = 0.90$
$k = 1$				
$\gamma_p = 0.25$	0.01	0.0	0.00	0.00
$\gamma_p = 0.9$	0.00	0.0	-0.04	-0.01
$k = 4$				
$\gamma_p = 0.25$	0.03	0.0	-0.00	0.00
$\gamma_p = 0.9$	0.02	0.0	-0.10	-0.03

Note: The entries report population coefficients from a regression of one-quarter ($h = 1$, left columns) and four-quarter ($h = 4$, right columns) ahead inflation forecast errors on inflation. The values of β and κ were set to 0.99 and 0.05, respectively.

3 Dynamic General Equilibrium Model

The previous section establishes that inflation expectations implied by the FHP model are broadly consistent with key stylized facts that have emerged from the empirical literature using survey data on expectations. With this result established, we next turn to investigating whether the FHP model in Woodford (2018) is jointly consistent with the survey data on inflation expectations as well as the fluctuations in output, inflation, and interest rates in U.S. data. To address this question, this section extends the analysis in the previous section to include households with finite planning horizons and monetary policy that is specified to follow an interest rate rule.

As in the previous section, it is assumed that all agents have the same planning horizon of length k. The model's inflation dynamics are determined from similar expressions to those shown in equations (4) and (5) except that now the output gap no longer follows an exogenous process but is endogenously determined. With an endogenously determined output gap, the NK Phillips curve becomes:

$$\pi_t^k = \kappa E_t \sum_{i=0}^{k} \beta^i (y_{t+i}^{k-i} - y_{t+i}^{\star}) + \beta^{k+1}(1-\theta_p)v_{pt} \tag{12}$$

where $y_{t+i}^{\star}$ is an exogenous shock to aggregate supply and y_{t+i}^{k-i} is a firm's beliefs about the output gap in period $t+i$ which as discussed below is determined by the level of household expenditures.

Households. There is a large number of identical, infinitely-lived households. Each household makes a consumption/savings decision but like the economy's firms only has the ability to plan k periods ahead. Households also supply their labor services to firms in a perfectly competitive labor market. As shown in the appendix, optimization by households gives rise to a (log-linearized) relationship that relates household expenditures at time t to future interest rates that occur over their planning horizon:

$$y_t^k = -\sigma E_t \sum_{i=0}^{k-1} \left(i_{t+i}^{k-i} - \pi_{t+i+1}^{k-i-1} - r_{t+i}^{\star} \right) + E_t y_{t+k}^0 \tag{13}$$

where y_t^k are a household's demand for expenditures at time t and i_{t+i}^j denotes a household's beliefs about the setting of the policy rate in period $t+i$. The parameter σ is the inverse of a household's relative risk aversion, and the variable $r_t^\star$ is an exogenous shock to preferences.[12] This shock as well as the supply shock, $y_t^\star$, are assumed to follow AR(1) processes with persistence parameters, ρ_y, for the supply shock, and ρ_r for the demand shock. A household's expenditures at time t, y_t^k, also depend on its plans for expenditures at the end of their planning horizion, y_{t+k}^0, which are given by:

$$y_{t+k}^0 = -\sigma\left(i_{t+k}^0 - r_{t+k}^\star\right) + v_{ht} \tag{14}$$

where the variable v_{ht} is the value that household assigns to events that occur outside of their planning horizons and reflects that households, like firms, have a limited ability to understand and evaluate situations that occur in the distant future.

Similar to firms, households update v_{ht} based on past events and do so in a way that is consistent with their optimal finite-horizon plan. In particular, when they decide on their expenditures, y_{t+k}^k, they form a new estimate of their value function, v_{ht}^e, and use it to update their beliefs according to:

$$v_{ht+1} = (1-\gamma)v_{ht} + \gamma v_{ht}^e, \tag{15}$$

where $0 < \gamma < 1$ determines how much weight they place on their new estimate. This new estimate is chosen optimally by households and as shown in the appendix reflects outcomes for both expenditures and inflation:

$$v_{ht}^e = y_t^k + \sigma\pi_t^k \tag{16}$$

Substituting equation (16) into equation (15), it follows that v_{ht} depends on past realizations of household expenditures and inflation. Accordingly, a household's longer-run beliefs are determined in a backward-looking manner and because they depend in particular on lagged expenditures, these longer-run beliefs can give rise to persistence in household expenditures. Relative to the model without learning (i.e., $v_{ht} = 0\ \forall t$), household learning about their value functions introduces one extra parameter, γ, and allows households' value functions to reflect the effect of long-lasting economic changes. As shown in Woodford (2018), this property ensures that the Fisher equation is satisfied in the long run despite the finite planning horizon that households have.

Trend-Cycle Decomposition. As discussed in Woodford (2018), an interesting feature of the FHP model is that its variables can be decomposed output into a "cyclical" component—reflecting the effect of the model's shocks—and a "trend" component—reflecting changes in household and firm beliefs' about their longer-run continuation values. Specifically, the "trend" components (denoted by $\bar{\pi}_t^j$, $\bar{y}_t^j$, and $\bar{i}_t^j$, respectively, for $j = 0, 1, ..., k$) reflect the learning that households and firms do about their value functions and can be defined by abstracting from the effect of shocks in equations (12) and (13). Accordingly, the evolution of these trends can be written as functions of

[12] As shown in the appendix, this shock affects a household's discount factor and differs from the preference shock used in Woodford (2018).

the continuation values of households and firms decisions, v_{ht} and v_{pt}:

$$\bar{\pi}_t^k = \kappa \sum_{i=0}^{k} \beta^i \bar{y}_t^{k-i} + \beta^{k+1}(1-\theta_p)v_{pt}$$
$$\bar{y}_t^k = -\sigma \left[\sum_{i=0}^{k} \bar{i}_t^{k-i} - \sum_{i=0}^{k-1} \bar{\pi}_t^{k-i-1} \right] + v_{ht}$$

where $\left\{\bar{\pi}_t^k, \bar{y}_t^k\right\}_{j=0}^k$ denote the effect of the continuation value functions on the plans of households and firms. (For these variables, we denote the effects of the v_{ht} and v_{pt} on household and firm plans with only a t subscript since v_{ht} and v_{pt} are fixed at time t.) We use the trend variables to help characterize the model's dynamics and understand the role of household and firms' longer-run beliefs in generating endogenous persistence and influencing inflation expectations.

Monetary Policy. Monetary policy at each date t is specified as an interest-rate rule of the form:

$$i_t^k = \bar{i}_t^k + \phi_\pi(\pi_t^k - \bar{\pi}_t^k) + \phi_y(y_t^k - \bar{y}_t^k) + i_t^\star \tag{17}$$

where $i_t^\star$ is an exogenous shock to the rule assumed to follow a first-order autoregressive process with persistence parameter, ρ_i. We assume that the intercept of the policy rule depends on the evolution of the model's trends. In particular, $\bar{i}_t^k$ is given by:

$$\bar{i}_t^k = \bar{\phi}_\pi \bar{\pi}_t^k + \bar{\phi}_y \bar{y}_t^k \tag{18}$$

The time-varying intercept in the interest rate rule is intended to capture two aspects of monetary policy. First, it acknowledges that policymakers do not necessarily view the "equilibrium" or longer-run real interest rate as a constant.[13] Second, it also allows for the possibility that policymakers may respond more aggressively to persistent deviations of inflation from their inflation target, as captured by $\bar{\pi}_t^k$, than they do to temporary deviations. In that case, $\bar{\phi}_\pi > \bar{\phi}$, and as shown in our empirical analysis in Gust et al. (2022), we find that such a monetary policy response fits the data substantially better than a rule in which monetary policy responds equi-proportionately to cyclical and trend inflation.

4 Empirical Analysis

In this section, we assess the dynamic, general equilibrium model's ability to match the impulse response predictability results of AHS (2020) and predictability regressions of CG (2015) and KW (2021). The model and methodology closely follow Gust et al. (2022), where we estimated the FHP model employing a Bayesian, full-information likelihood-based approach using U.S. data on

[13]This formulation is consistent with policymakers' efforts to inform their decisions distinguishing trend factors—such as demographic or productivity changes—from cyclical variations in output and inflation. It implicitly assumes that policymakers are no better at separating trend from cycle as the private sector.

output growth, inflation, and interest rates.[14] In addition, we use the predictability regression of CG (2015) to inform estimate of a key model parameter, the length of the planning horizon k. Finally, as detailed below, we implement predictive checks to examine the model's performance on the predictability statistics discussed in AHS (2020), CG (2015), and KW (2021).

For this assessment, we use plausible parameter configurations from the posterior distribution of the FHP model. The use of a full-information estimation as the basis for assessing these moments is a bit of a departure from the previous literature, which typically has relied on regression analysis or partially identified VAR models. The full information strategy employed here offers several advantages. First, as the propositions in Section 2 indicate, the consistency of the FHP model with particular stylized facts of inflation expectations depends not only on parameters directly governing finite horizon planning, but additional structural parameters auxiliary to the planning horizon. The presence of these auxiliary parameters can substantially complicate the evaluation of these moments. By using the full posterior distribution from the model, we incorporate likely values of these auxiliary parameters into our evaluation. The methodology here also ensures that the assessment of particular moments related to inflation and inflation expectations is done conditional on parameterizations that also can rationalize the realized time series of output growth, inflation, and interest rates. Finally, our use of a fully specified model allows us to examine the behavior of these moments conditional on specific structural shocks. This is in important because—as Section 2 highlights—the persistence of the shock is often critical for determining the values of β^h_{CG} and β_{KW} and the impulse responses of inflation forecasts to shocks.

The FHP model described in Section 3 is estimated using data on output growth, GDP deflator inflation, and the federal funds rate in the United States from 1967-2007 for different planning horizon lengths, k. The priors and computational strategy for eliciting the posterior distribution of the parameters, $p(\theta|Y)$, closely resemble Gust et al. (2022). Thus, we relegate the estimation details to the appendix.

4.1 Predictive checks for assessing inflation expectation predictability

While the models have been estimated to jointly account for fluctuations in output growth, inflation, and interest rates, the estimation strategy of Gust et al. (2022) does not use information on inflation expectations or the predictability statistics emphasized by AHS (2020), CG (2015), and KW (2021). To incorporate evidence on inflation predictability to inform the model's parameter estimates, we use the framework of *predictive checks*. These checks involves comparing a given statistic or moment from the data to the predictive distribution of that statistic under a given model. Here we use the posterior distribution of the estimated FHP model. Let Y^{DSGE} denote the set of observables used to estimate the DSGE model—output growth, inflation, and interest rates—and let Y be an expanded set of observables which includes inflation expectations data. Formally, let $\mathcal{S}(Y)$ be some statistic of this data, where $\mathcal{S}(Y)$ can be a scalar—like the regression coefficient as in CG

[14] A minor difference in the model described in Section 3 from the one in Gust et al. (2022) is that the demand shock is specified slightly differently. This change affects the estimation results minimally.

(2015)—or a vector—like the impulse responses of AHS (2020). For a specific model, $\mathcal{M}$, we can draw from the posterior distribution of parameter estimates to a obtain a simulated counterpart to Y—called $\widetilde{Y}$—and compute the predictive distribution for $\widetilde{Y}$ as

$$p(\widetilde{Y}|\mathcal{M}) = \int p(\widetilde{Y}|\theta, \mathcal{M})p(\theta|Y^{DSGE}, \mathcal{M})d\theta. \tag{19}$$

Using (19), one can compare where the observed statistic $\mathcal{S}(Y)$ lies in the predictive distribution for $S(\widetilde{Y}|\mathcal{M})$. If $\mathcal{S}(Y)$ falls in the tail of a particular model's predictive distribution, it suggests a deficiency of the model in representing that aspect of the data. To compute $\mathcal{S}(Y)$ for the predictability IRF of AHS (2020), we estimate a VAR(4) on the observed data and construct impulse responses identifying a shock, as they do, that maximizes the forecast error variance of inflation over the medium term. We follow the same approach to compute the model analogues of these impulse responses from simulated data. For the predictability regression of CG (2015), we run regressions of inflation forecast errors on forecast revisions using survey data and model simulated data. For the predictability regression of KW (2021), we run regressions of inflation forecast errors on lagged inflation. Algorithm 1 provides more details for these posterior predictive checks. Following CG (2015) and others, we use the mean forecast for four-quarter-ahead (GDP deflator) inflation expectations ("Expected Inflation") from the Survey of Professional Forecasters (SPF) as the actual inflation expectations data.

Algorithm 1 Predictive Checks

For $i = 1, \ldots, N$:

1. *Construct* $\widetilde{Y}$. Draw $\theta^i \sim p(\theta|Y^{DSGE})$, and simulate a single trajectory of $\tilde{Y} = \{\Delta y_t, \pi_t, i_t, \mathbb{E}_t^k[\pi_{t+4}^A], \mathbb{E}_{t-1}^k[\pi_{t+4}^A]\}_{t=1}^T$, where $T = 168$, length of the actual observables The variable $\pi_t^A = \pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}$ is the four-quarter inflation rate.

2. *Compute* $\mathcal{S}(Y)$. Compute the statistic of interest, $\mathcal{S}(Y)$, from the actual data, Y, and from the simulated data, $\widetilde{Y}$.

Notes: We use $N = 200$ draws from the posterior.

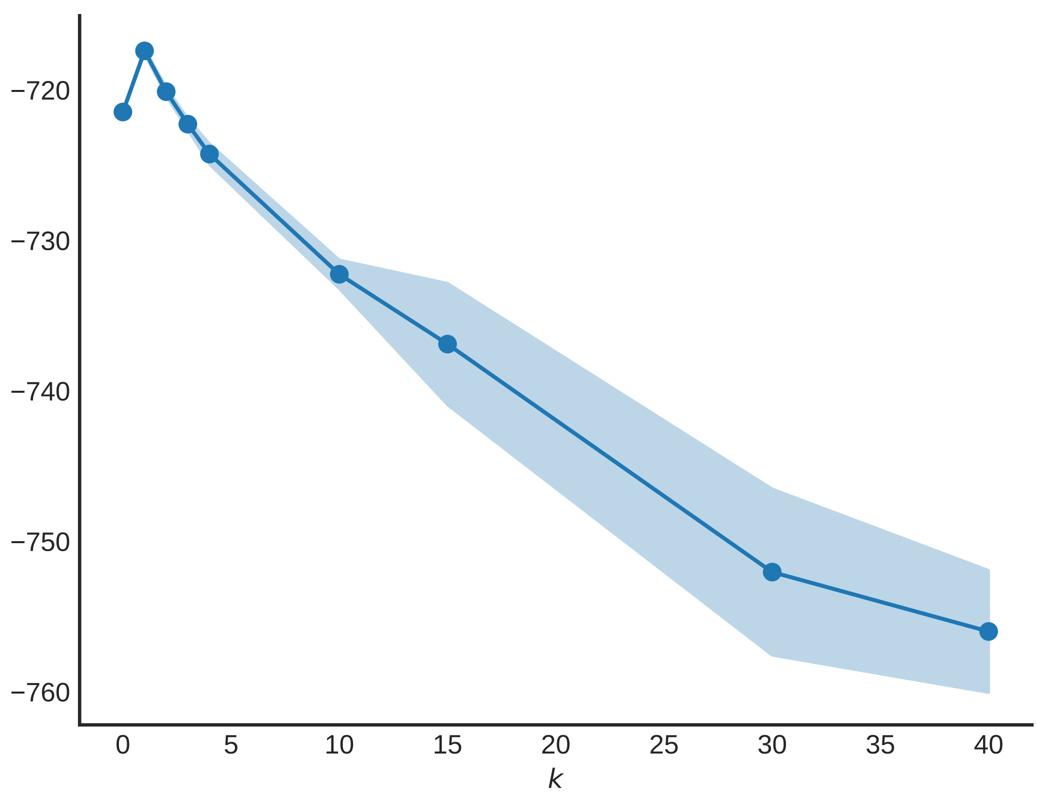
4.2 Identifying the Planning Horizon Using Evidence on Inflation Predictability

The planning horizon k is a key parameter for determining the behavior of expectations in the FHP model. Gust et al. (2022) argue that a smaller value of k is necessary to fit the time series behavior of output growth, inflation, and interest rates. They find that $k = 1$ maximizes the overall fit of these macroeconomic time series, measured using log marginal data densities (MDDs).[15] Figure 2 displays both the point estimates and 95 percent uncertainty bands for the log of the estimated

[15]The MDD of a Bayesian model with data Y and parameters θ is given by $p(Y) = \int p(Y|\theta)p(\theta)d\theta$, i.e., the integral of the likelihood over the prior distribution. This quantity, central to Bayesian model comparison, is a summary measure of model fit.

MDD (MDDs) for the model in Section 3 for different values of k, with larger numbers indicating a better fitting model. Consistent with Gust et al. (2022), the model's fit is maximized at $k = 1$. The log MDD is substantially lower for large values of k relative to $k = 1$, but the difference in the log MDDs for $0 \leq k \leq 4$ is only a few log points. While a strict application of the Bayesian calculus would favor $k = 1$ to other small k models, changes to, say, the prior distribution or the inclusion an additional reference model could affect these odds. Therefore, we do not interpret them literally. Instead, we conclude that the FHP models with low values of k do not differ dramatically in terms of their fit of macro data and we use the predictive check framework to help guide our choice of k.

Figure 2: Log MDD Across k



Notes: Figure displays point estimates (blue line) of the log MDD for the FHP for different values of k with the shaded blue region giving approximate 95 percent uncertainty bands. Appendix D gives computational details.

To do so, we use the predictability regression of CG (2015) which emphasizes the relationship between inflation forecast errors and forecast revisions. We focus on two variants of the FHP-NK model with planning horizons $k = 1$ and $k = 4$. For each model variant, run the predictability regression of CG (2015) at two different forecast horizons, h:

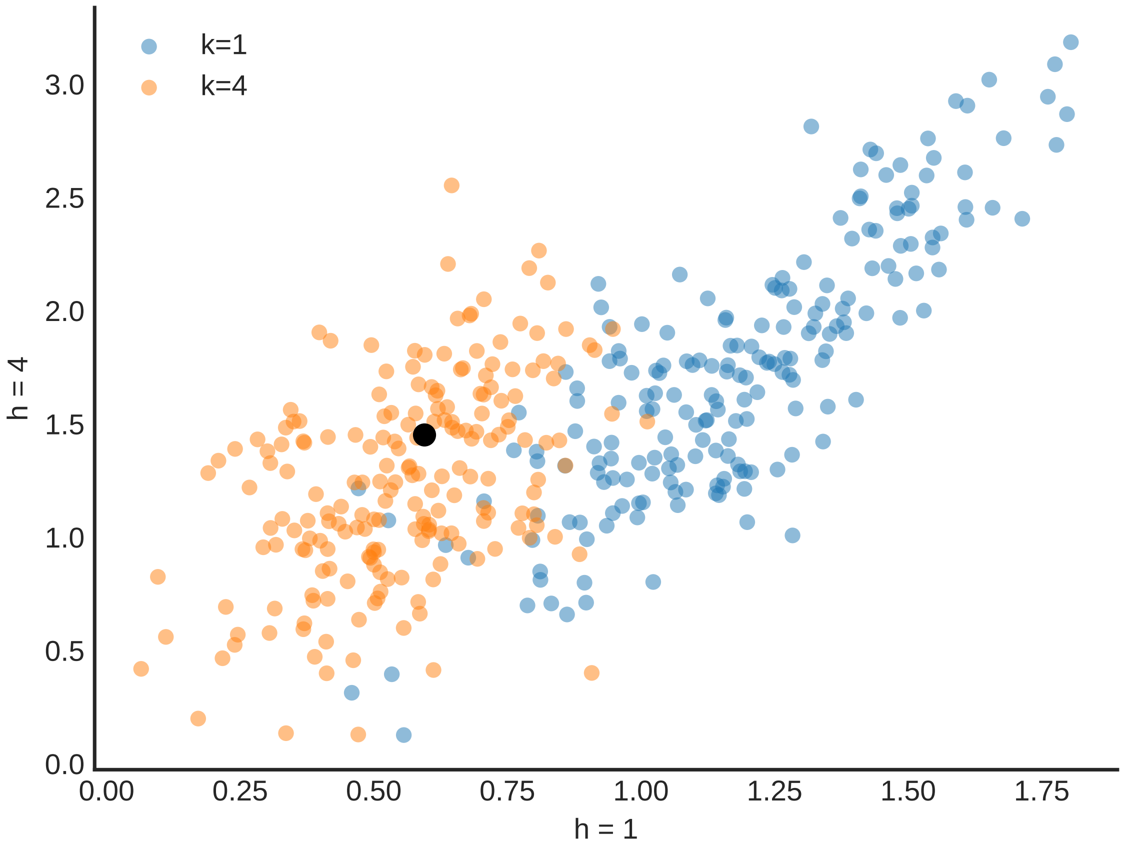
$$\pi_t^A - \mathbb{E}_{t-h}^k[\pi_t^A] = \alpha + \beta_{CG}^h \left(\mathbb{E}_{t-h}^k[\pi_t^A] - \mathbb{E}_{t-h-1}^k[\pi_t^A] \right) + u_{t,h}, \tag{20}$$

where the statistic of interest is the OLS point estimate of β^h_{CG}, $\mathcal{S}(\tilde{Y}) = \widehat{\beta}^h_{CG}$. for $h = 1$ and $h = 4$. In equation (20), π^A_t corresponds to four-quarter average inflation when $h = 4$, and it corresponds to quarterly inflation, π_t, when $h = 1$. Running these two regressions, we obtain a sample of regression coefficients, $\{(\widehat{\beta}^1_{CG}, \widehat{\beta}^4_{CG})\}$.

Figure 3 displays scatter plots of these two statistics implied by the FHP model with $k = 1$ (blue dots) and $k = 4$ (orange dots). The values obtained by running these regression using the SPF data correspond to the large black circle in the figure. The point estimate for β^4_{CG} using the SPF survey data is close to 1.5, while the point estimate for β^1_{CG} is only about one-half. The data sits squarely inside the orange cloud, indicating that the FHP model with a planning horizon of $k = 4$ fits the observed forecast error predictability for $h = 1$ and $h = 4$ well. In contrast, the FHP model with $k = 1$ badly overestimates the predictability for one-quarter ahead forecast errors (the x-axis). While the FHP model with $k = 4$ results in a slight deterioration in fit of the macro time series relative to the model with $k = 1$, Figure 3 highlights its superior performance in accounting for the evidence regarding the predictability of inflation forecasts.[16] Taking into account this evidence, we judge that the FHP model with $k = 4$ yields the best overall fit of the macroeconomic time series and the predictability regression of CG (2015). Thus, for the remainder of our analysis, we focus on the FHP model with a planning horizon of $k = 4$.

[16]Figure 3 focuses on distinguishing between planning horizons of one and four quarters. We could, in principle, consider larger values for k but are hesitant to do so for two reasons. First, these predictability regressions use data on (at most) one-year ahead inflation expectations and, as k becomes sufficiently large, these regression statistics would tend toward zero as the FHP model approaches the rational expectations benchmark. Thus, larger values of k would imply a deterioration in fit relative to what we show in the figure. Second, the evidence from the log marginal data densities shown in Figure 2 indicates that the macroeconomic fit deteriorates substantially for $k > 4$.

Figure 3: Joint Distribution of Predictability Regression Coefficients



Note: The figure shows scatter plots of $\{(\widehat{\beta}^1_{CG}, \widehat{\beta}^4_{CG})\}$ for the FHP $k = 1$ (blue dots) and $k = 4$ (orange dots) models. The x-axis ($h = 1$) shows the values of $\widehat{\beta}^1_{CG}$, the regression coefficient from regressing one-step ahead inflation forecast errors on forecast revisions. The The y-axis ($h = 4$) shows the values of $\widehat{\beta}^4_{CG}$, the regression coefficient from regressing four-quarter ahead inflation forecasts on forecast revisions. The values observed in the data are given by a large black circle.

Table 3 describes key features of the posterior distribution for the FHP($k = 4$) model. The posterior mean estimates of the learning rates for the households and firms are 0.54 and 0.21, respectively, indicating that firms weight recent data less in forming their beliefs about events outside their planning horizons than households. This relatively slower speed of belief updating by firms tends to lengthen out the period over which inflation forecasts underreact to shocks, as discussed in Section 2. The monetary policy rule has the same features as in Gust et al. (2022): it displays a strong response to trend inflation and cyclical output but essentially no response to trend output. Both the demand shock and monetary policy shocks are estimated to exhibit a high level of persistence, while the posterior mean for the autoregressive coefficient for the supply shock is only 0.36. As discussed further below, the estimated persistence of these shocks has important implications for the model's ability to account for the predictability of inflation forecasts.

Table 3: FHP($k = 4$) Model: Selected Posterior Statistics

	Description	Mean	[0, 95]
γ	Household learning rate	0.54	[0.33, 0.78]
γ_f	Firm learning rate	0.21	[0.13, 0.30]
κ	Slope of the Phillips curve	0.01	[0.01, 0.01]
σ	Coef. of relative risk aversion	2.27	[1.52, 3.15]
ϕ_π	Int. rule response to $\tilde{\pi}_t$	1.08	[0.80, 1.39]
ϕ_y	Int. rule response to $\tilde{y}_t$	1.29	[0.91, 1.77]
$\bar{\phi}_\pi$	Int. rule response to $\overline{\pi}_t$	1.57	[1.30, 1.89]
$\bar{\phi}_y$	Int. rule response to $\overline{y}_t$	0.07	[0.03, 0.14]
ρ_ξ	AR coeff. for demand shock	0.90	[0.84, 0.96]
ρ_i	AR coeff. for monetary policy shock	0.95	[0.90, 0.99]
ρ_y	AR coeff. for supply shock	0.36	[0.23, 0.50]

Note: The table shows estimates of the posterior means, 5th, and 95th percentiles of the model parameters computed from output of the SMC sampler. See appendix for details.

4.3 Impulse Response Predictability

With the choice of $k = 4$ in hand, we now turn to providing external model validation using the the methodology of AHS (2020) as a predictive check. To apply their methodology, we construct $\mathcal{S}(\tilde{Y})$ as the point estimates of the impulse response coefficents of π_t^A, defined as four-quarter average inflation, and $\mathbb{E}_{t-4}^k[\pi_t^A]$ to an AHS "inflation shock" in a VAR(4) model for $\left[\Delta y_t, \pi_t, i_t, \mathbb{E}_t^k[\pi_{t+4}^A]\right]$. The VAR's inflation shock is identified as the shock that maximizes the variance in inflation over frequencies associated with periods of length 6 to 32 quarters.[17] Figure 4 displays the impulse responses of inflation, the inflation forecast, and the inflation forecast error from the AHS-style VAR in the FHP model. The solid black lines in Figure 4 display the impulse responses of inflation, the inflation forecast, and the inflation forecast error for the inflation shock using the identification scheme of AHS on the actual data[18]. The blue lines show the (pointwise) mean estimates using simulated data from the FHP model to compute these impulse responses, and the shaded blue regions correspond to the 90 percent pointwise credible ranges. The mean impulse responses from the FHP model for inflation and the forecast of inflation track those in the data, especially in the first five years. In both the model and the data, the response of the inflation forecast is more muted than actual inflation so that the forecast error rises on impact. The mean date at which the inflation forecast error flips from an underreaction to an overreaction, i^*, is nearly 8 quarters in the model and at 11 quarters is a little longer in the data. The overreaction that follows this underreaction in the data is somewhat larger in the data than implied by the model though in both cases the overreaction is persistent. Overall, the FHP model is successful in capturing the predictability of the impulse responses of inflation forecast errors described in AHS (2020) and in particular their

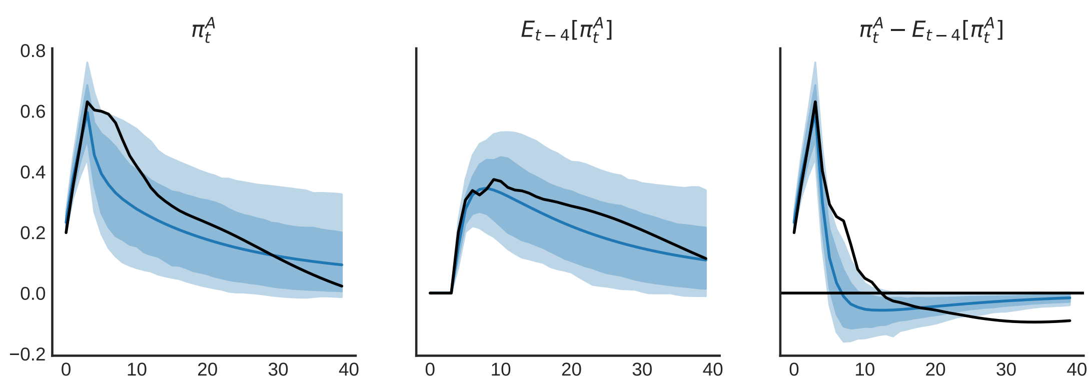
[17] Appendix Section H describes this procedure in detail.

[18] For the years before 1974, there are missing observations in the inflation expectations series constructed using the SPF data. To avoid this, we start the sample in 1974Q4 for the estimation of the VAR.

empirical finding that the inflation forecast underreacts initially and then overreacts later on.

To understand this result, it is important to realize that the AHS inflation shock is an amalgamation of the model's three structural shocks. However, the AHS shock is heavily influenced by the supply shock with the monetary policy shock playing a secondary role. This reflects that these two shocks explain most of the variation in inflation at business cycle frequencies (i.e., those with periods between 6 and 32 quarters), and those frequencies are used to identify the AHS inflation shock. Figure 5 shows the fraction of variance explained by each shock over the frequency domain, where the frequency has been mapped into quarters. For shorter frequencies, the fluctuations in inflation are almost exclusively explained by supply shocks, while the importance of the monetary policy shock rises so that it accounts for about a quarter of the variability in inflation at a horizon of 32 quarters. With the impulse responses of the AHS inflation shock driven by these two shocks, Table 4 displays the mean date at which the impulse responses for each of the model's shocks imply that an initial underreaction of the inflation forecast flips to an overreaction, i^*. Consistent with the discussion in Section 2, the period of underreaction of inflation forecasts in response to a supply shock is relatively short, because the estimated persistence of supply shocks is fairly low, as shown in Table 3. In particular, the mean date for i^* conditional on a supply shock is about 6 quarters, while the mean dates for i^* conditional on a monetary policy shock and an aggregate demand are 23 and 30 quarters, respectively. Accordingly, the mean date for i^* conditional on either monetary policy or aggregate demand shocks would be considerably larger than the 11 quarters observed in the data, but with the supply shocks playing a large role in accounting for the AHS inflation shock the model accounts for the date of the flip from an inflation forecast underreaction to overreaction reasonably well.

Figure 4: Impulse Response to an AHS Inflation Shock



Notes: The figure shows the impulse response of inflation, the inflation forecast, and the inflation forecast error from the AHS-style VAR in the FHP model. The solid blue line denotes the (pointwise) mean across the predictive checks, while the shaded regions denote the ninety (light blue) and sixty-eight (dark blue) percent bands (across the means of the predictive checks). The black lines correspond to the impulse responses constructed using the actual data.

Figure 5: Frequency-based Variance Decomposition for Inflation

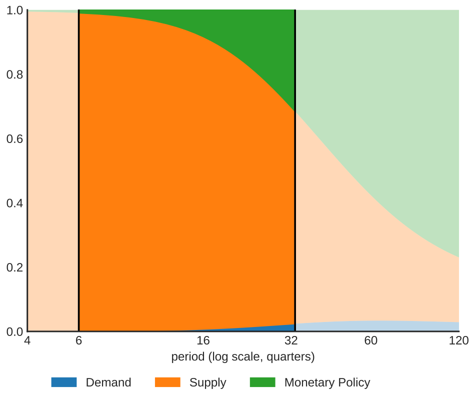


Table 4: Average Threshold Date for Impulse Response Sign Switch in the FHP Model

Shock	$E[i^*\|i^* < 40]$
AHS	7.50
Supply	6.02
Monetary Policy	23.61
Demand	15.83

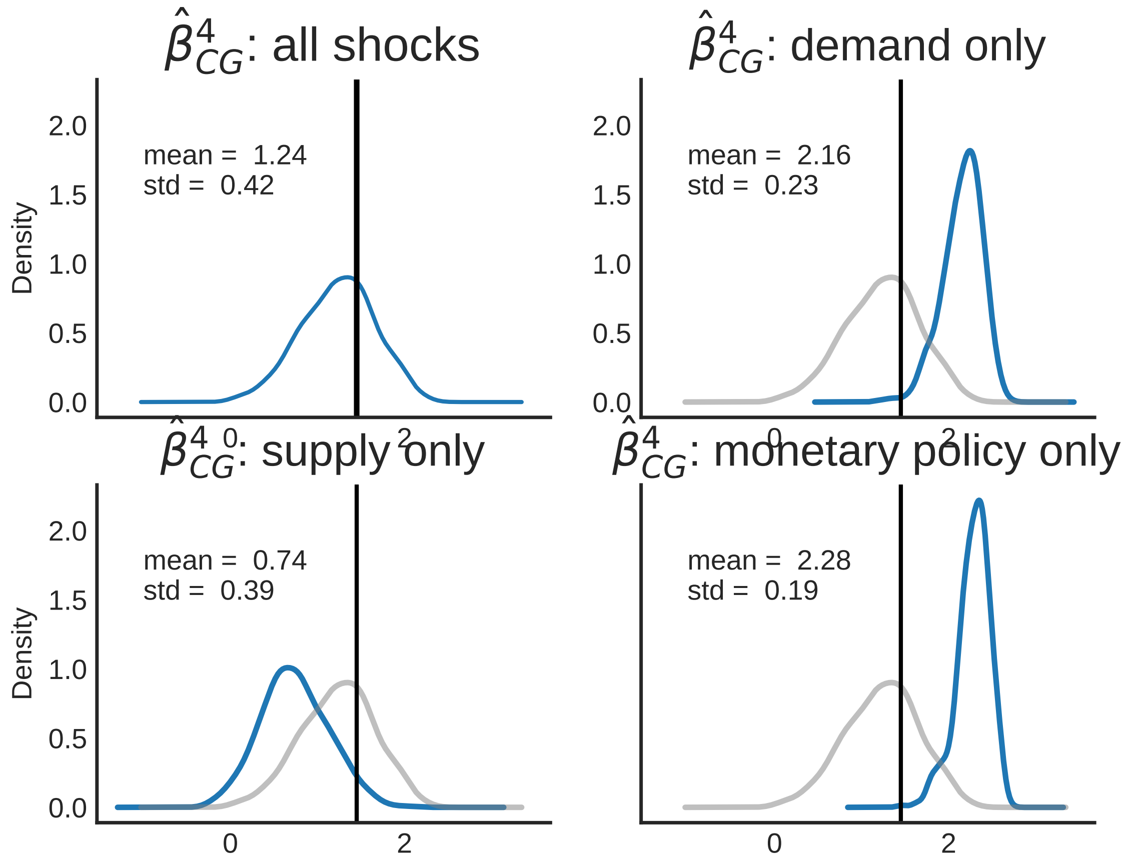
Notes: The figure displays the fraction of variance of inflation attributable to demand (blue), supply (orange), and monetary policy (green) for different frequencies, using the posterior mean parameter estimates. The table displays the average threshold date in which a sign switch occurs in the impulse response of the inflation forecast error for the model's implied AHS inflation shock and its three structural shocks.

4.4 Inflation Predictability Regressions Revisited

Figure 3 established that the FHP model with a planning horizon of four quarters is capable of matching the predictability regression of CG (2015) reasonably well. In this section, we decompose the model's predictive distribution of $\widehat{\beta}^4_{CG}$ into its contributions from each of the model's shocks and perform an additional predictive check using the KW (2021) regression statistic.

The upper left panel of Figure 6 shows the predictive distribution of $\widehat{\beta}^4_{CG}$ implied by the FHP($k = 4$) model. This distribution reflects the effects of all three of the model's shocks. To help decompose the effects of each shock on $\widehat{\beta}^4_{CG}$, the remaining panels show the predictive distributions of $\widehat{\beta}^4_{CG}$ conditional on an individual shock. Conditional on only demand and monetary policy shocks, the distribution of model estimates lies above the point estimate in the data. As shown in Table 3, these two shocks are highly persistent so that the initial underreaction that occurs in response to these two shocks lasts a long time, generating a stronger positive relationship between FHP agents' forecast errors and forecast revisions. As noted earlier, the supply shock is considerably less persistent than these two shocks so that the initial underreaction is less persistent and the overreaction that occurs later on in response to supply shocks is relatively more important for this shock. Accordingly, the distribution conditional on this shock lies to the left of the model's other two shocks and helps the model generate a distribution for $\widehat{\beta}^4_{CG}$ conditional on all of the model's shocks in line with the data.

Figure 6: Distribution of CG (2015) coefficients



Note: The figure shows the predictive densities for $\hat{\beta}_{CG}$ simulated using all the shocks (upper left panel and grey lines in the other panels), and conditional only on demand (blue line in upper right panel), supply (blue line in bottom left panel), and monetary policy (blue line in bottom right panel) shocks. The black vertical line indicates the point estimate using the SPF data.

Finally, we provide additional external validation using the KW (2021) regression. In this case, the statistic of interest is the point estimate of β_{KW}, the coefficient on lagged inflation in the regression of inflation forecast errors on lagged inflation:

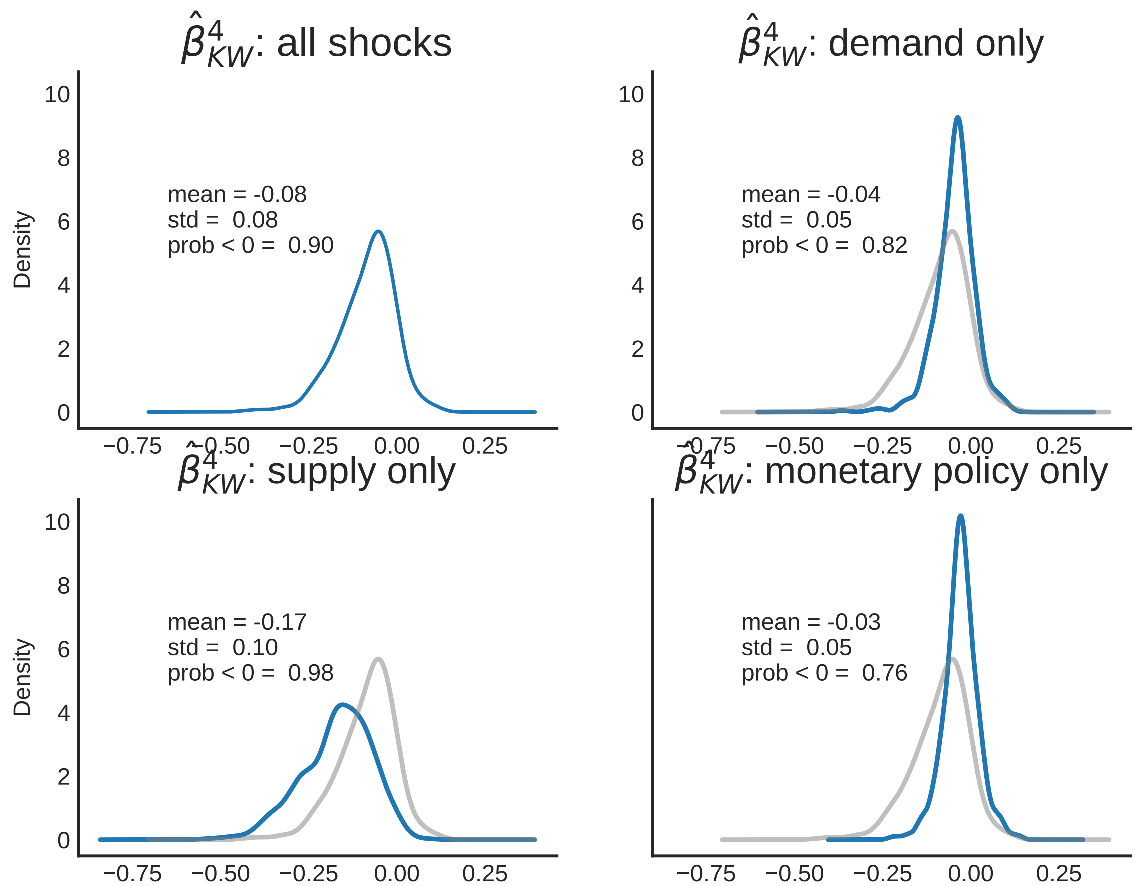
$$\pi_t^A - \mathbb{E}_{t-4}^k[\pi_t^A] = \alpha + \beta_{KW}\pi_{t-4}^A + u_t, \tag{21}$$

with $\mathcal{S}(\tilde{Y}) = \widehat{\beta}_{KW}$. Figure 7 shows the predictive distributions of $\widehat{\beta}_{KW}$. As shown in the upper left panel, using all of the shocks, the mean of $\widehat{\beta}_{KW}$ is about -0.06, with about 80 percent of the simulations featuring a $\widehat{\beta}_{KW} < 0$.[19] The posterior mean conditional on supply shocks is about

[19] While KW (2021) present convincing evidence on extrapolation present in inflation forecasts (leading to $\beta_{KW} < 0$), this finding is not present in the average GDP deflator-based inflation forecast in the SPF—see Table C.7 of the Online Appendix of KW (2021). Therefore, we omit the estimate of $\widehat{\beta}_{KW}$ from the plots. We note that though the FHP model generally predicts a negative KW coefficient, there is meaningful mass on $\beta_{KW} > 0$, which we view as consistent with the mixed evidence in the SPF data.

−0.10, while it is closer to zero for the demand and monetary policy shocks. The mean conditional on supply shock is lower than for the other two shocks, because this lower persistence of this shock implies that the overreaction of inflation forecasts occurs earlier and relatively sooner than for the other shocks, as highlighted in Table 4.

Figure 7: Distribution of KW coefficients



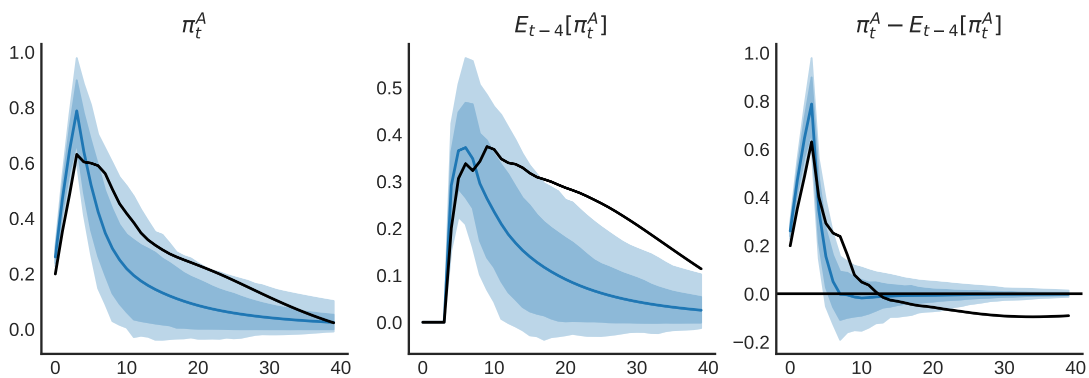
Note: The figure shows the posterior predictive densities for $\hat{\beta}_{KW}$ simulated using all the shocks (upper left panel and grey lines in the other panels), and conditional only on demand (blue line in upper right panel), supply (blue line in bottom left panel), and monetary policy (blue line in bottom right panel) shocks.

Overall, the FHP($k = 4$) model is consistent with the predictability statistics of CG(2015), AHS (2020) and KW (2021). Section I in the Appendix re-estimates the FHP model incorporating SPF inflation expectations data as an additional observable along with the data on output growth, inflation, and interest rates. The parameter estimates are very similar to those shown in Table 3, and so there are little implications for the predictability statistics. Using inflation expectations as an observable also leads to some additional time series evidence in favor of planning horizons of $k = 4$.

4.5 Comparison with a Sticky Information Model

In this section, we examine the implications of an estimated sticky information (SI) model for the predictability statistics of CG (2015), AHS (2020), and KW (2021). In the SI model, the formation of inflation expectations is imperfect due to the presence of sticky information, as firms' pricing decisions are not always based on current information. SI models are an attractive point of comparison, as a number of researchers have found these models to fit macro time series at least as well as models emphasizing sticky prices.[20] Moreover, as emphasized in CG (2015), SI models can successfully account for the correlation between consensus forecast errors and revisions observed in the SPF. The SI model that we use is described in detail in the appendix and to estimate it we follow the same strategy as we used for the FHP model in which we use data on output growth, inflation, and interest rates as observables.

Figure 8: Impulse Response to an AHS Inflation Shock



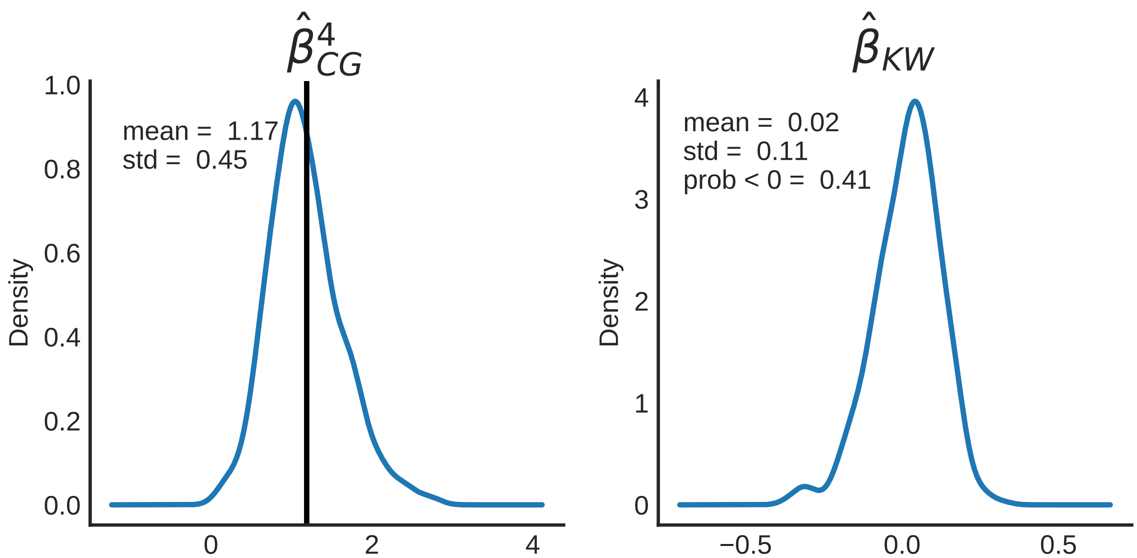
Notes: The figure shows the impulse response of inflation, the inflation forecast, and the inflation forecast error from the AHS-style VAR in the SI model. The solid blue line denotes the (pointwise) mean across the predictive checks, while the shaded regions denote the ninety (light blue) and sixty-eight (dark blue) percent bands (across the means of the predictive checks). The black lines correspond to the impulse responses constructed using the actual data.

With estimates of the SI model's parameters in hand, we perform predictive checks of the model using the predictability of impulse responses of AHS (2020) and the predictability regressions of CG (2015) and KW (2021). Figure 8 displays the impulse response of inflation, inflation expectations, and the inflation forecast error in response to an AHS "inflation" shock from a VAR constructed as in Section 4.3. While the inflation forecast error initially rises in response in response to the shock–that is, we see an initial underreaction of inflation expectations–the mean impulse response of the forecast error under the SI model does not subsequently show an overreaction. Instead, the mean inflation forecast error monotonically converges back to zero. It is true that the some of trajectories exhibit overreaction–the shaded region in the forecast error panel contains negative values, but this owes only to finite sample considerations. If the length of the simulated trajectories used in the predictive checks was increased substantially more than the sample length of $T = 168$,

[20]For estimated models with sticky information, see Andrés et al. (2005) and Chung et al. (2014).

the probability of overreaction reflected in the impulse responses would approach zero. In fact, in the appendix, we study a partial equilibrium version of the sticky information model. In the partial equilibrium model, it is possible to show that there is always an underreaction–never an overreaction–of the impulse response of the one-step ahead forecast in the SI model. Overall, the SI model, unlike the FHP model has difficulty generating the sign flip in the impulse response of the inflation forecast error that AHS (2020) document.

Figure 9: Distribution of CG and KW coefficients



Notes: The figure shows the posterior predictive densities for $\hat{\beta}^4_{CG}$ and $\hat{\beta}^4_{KW}$ simulated for the SI model. On the left panel, the black vertical line indicates the point estimate using the SPF data.

The densities for the coefficients of the predictability regressions of CG (2015) and KW (2021) are shown in Figure 9. Consistent with the initial underreaction of expectations present in Figure 8, the posterior predictive distribution for $\widehat{\beta}^4_{CG}$ is positive, with a mean of about 1.2, close to the point estimate in the actual data. Thus, as in CG (2015), the SI model is consistent with the evidence on the relationship between forecast errors and forecast revisions.[21] The right panel of Figure 9 shows the distribution of the $\widehat{\beta}_{KW}$. For the KW regression, the sticky information model produces both positive and negative estimates with a mean estimate near zero, implying that, on average, there is little correlation between inflation and its forecast error across the model's parameter estimates.

[21]The SI parameter, λ, in the SI model has a posterior mean of about 0.6, implying a coefficient of $\beta^4_{CG} = \lambda/(1-\lambda) \approx 1.3$, in line with the central tendency of the posterior predictive checks.

5 Conclusion

In this paper, we used survey data on inflation expectations as well as aggregate data on output, inflation, and short-term interest rates to estimate and evaluate a NK model featuring FHP. We found that the FHP model can account for the predictability of inflation forecast errors as well as the empirical evidence that the average inflation forecast in the SPF typically underreacts relative to realized inflation but overreacts later on. We also showed that the FHP model can account for survey measures of inflation expectations while also providing a reasonable fit of output, inflation, and interest-rate dynamics over the business cycle. In doing so, we found that planning horizons of about a year fit the survey data on inflation expectations best. In addition, we found that the learning that households and firms do to form beliefs about events outside of their planning horizons was crucial to the successful performance of the FHP model in terms of its ability to account for aggregate time series as well as the evidence on forecast error predictability. Overall, our results support the view that modelling expectations formation using finite horizon planning merits further attention as part of an agenda incorporating behavioral features into macroeconomic models.

The short-planning horizons and the inertia in private sector beliefs about events outside of their planning horizons have important implications for monetary policy. Notably, forward guidance policies are much less effective than when households and firms have lengthy planning horizons and disinflations are much more costly than in the canonical NK model in which households and firms have full information, rational expectations. Given these notable differences, an important avenue of future research is studying optimal monetary policy when households and firms have finite planning horizons to investigate how optimal policy depends on agents' planning horizons as well as the evolution of their beliefs regarding events outside of their planning horizons.

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Appendix for
Inflation Expectations with Finite Horizon Planning

Christopher Gust, Edward Herbst, and David Lopez-Salido

A Finite-Horizon Household Planning

In this section, we describe the optimal finite-horizon plan set by households and derive equations (13) and (16). We focus on the optimal plan chosen by a finite-planning houesehold and abstract from the static labor supply decision that a household makes. A household chooses a state-contingent plan for consumption and bond holdings, $\{C_\tau, B_{\tau+1}\}_{\tau=t}^{t+K}$ to maximize:

$$\mathbf{E}_t^K \left\{ \sum_{\tau=t}^{t+K} \beta^{\tau-t} Q_\tau U(C_\tau) + \beta^{K+1} Q_{t+K+1} V_t(B_{t+K+1}) \right\} \tag{A-1}$$

where $0 < \beta < 1$ and $V_t(B_{t+K+1})$ is the value function a household uses to assign continuation values to its plans over the remainder of its infinite lifetime. As discussed further below, this value function varies over time but is fixed at time t when a household chooses its finite-horizon plans. The value function depends on a household's financial position at the end of its planning period, B_{t+K+1}. However, households are assumed to have limited ability to understand events that occur in the distant future and thus, the value function is not the model consistent one that reflects all possible contingencies that a household may face in the future. The variable Q_τ reflects that the discount factor is stochastic. For $t \leq \tau \leq t+K+1$, it evolves according to:

$$Q_\tau = \prod_{i=0}^{\tau-t-1} \xi_{t+i}, \tag{A-2}$$

where the variable ξ_t is an exogenous shock that affects a household's rate of time preference between periods t and $t+1$. According to equation (A-2), $Q_t = 1$, $Q_{t+1} = \xi_t$, and Q_{t+K+1} reflects that a household contemplates all possible contingencies of the shocks, $\xi_t, \xi_{t+1}, ...\xi_{t+K}$, that take place over its finite-planning horizon.

A household takes its initial bond holdings, B_t, as given and faces a per-period budget constraint given by:

$$B_{\tau+1} = (1+i_\tau)\left[\frac{B_\tau}{\Pi_\tau} + Y_\tau - C_\tau\right], \tag{A-3}$$

where i_τ is the policy rate, Π_τ is the (gross) inflation rate, and Y_τ denotes a household's disposable income which includes labor income as well as the profits a household receives from ownership of the economy's firms.

The first order conditions from a household's finite horizon plan are given by:

$$\frac{U_{C\tau}}{1+i_\tau} = \beta \xi_\tau \mathbf{E}_\tau^K \frac{U_{C\tau+1}}{\Pi_{\tau+1}} \qquad \text{for } t \leq \tau \leq t+K-1, \tag{A-4}$$

$$\frac{U_{Ct+K}}{1+i_{t+K}} = \beta \xi_{t+K} V_{Bt}(B_{t+K+1}) \tag{A-5}$$

where $V_{Bt}(B_{t+K+1})$ denotes a household's marginal value function with respect to its financial position at the end of its planning horizon (i.e., $V_{Bt}(B_{t+K+1}) = \frac{\partial V_t(B_{t+K+1})}{\partial B_{t+K+1}}$). A household's marginal utility of consumption, $U_{c\tau}$, satisfies:

$$U_{C\tau} = C_\tau^{\frac{-1}{\sigma}}.$$

We log-linearize a household's first order condtions around a non-stochastic steady state in which aggregate output and consumption satisfy $Y = C = 1$. Also, $\xi = 1$ and the nominal interest rate in steady state satisfies $1 + i = \frac{\Pi}{\beta}$ where Π denotes the (gross) inflation rate. Log-linearizing equation (A-4) implies:

$$\mathbb{E}_{\tau}^{k} \left\{ c_{\tau} - c_{\tau+1} + \sigma \left[i_{\tau} - \pi_{\tau+1} - r_{\tau}^{\star} \right] \right\} = 0 \tag{A-6}$$

for $t \leq \tau \leq t + K - 1$. We use lower case variables to denote the log-linearized variables so that $i_{\tau} = \log(1 + i_{\tau}) - \log(1 + i)$ and $\pi_{\tau} = \log(\Pi_{\tau}) - \log(\Pi)$. Also, $r_{\tau}^{\star}$ is defined as $= r_{\tau}^{\star} = -\log(\xi_{\tau})$. Using (1), the subjective expectations operator in this expression can be replaced by the rational expectations operator with redefined variables that reflect an agent's subjective expectations:

$$c_{t+K-j}^{j} = E_t c_{t+K-j+1}^{j-1} - \sigma \left[i_{t+K-j}^{j} - E_t \pi_{t+K-j+1}^{j-1} - r_{t+K-j}^{\star} \right] \tag{A-7}$$

for $0 \leq j \leq K$. We also log-linearize a household's terminal condition. This condition requires that we approximate $V_{Bt}(B_t)$. To do so, we log-linearize it around its non-stochastic steady state value of $\frac{1}{\Pi}$ and parameterize it as a linear function whose slope and intercept coefficients can potentially change over time as household's learn and update their longer-run beliefs based on their past experience:

$$V_{Bt}(B_t) \approx -\sigma^{-1} \left[v_{ht} + \chi_t b_t \right] \tag{A-8}$$

When making their optimal plan at time t, a household treats v_t and χ_t as fixed and hence their optimal consumption and savings decisions at time t depends on these parameters as well as their initial net asset position, B_t. In expression (A-8), $b_t = \frac{B_t}{\Pi_t}$ and we linearize around b_t, since in steady state $B = 0$. Using equation (A-8), the log-linearized version of equation (A-5) is:

$$c_{t+K}^{0} = -\sigma \left[i_{t+K}^{0} - r_{t+K}^{\star} \right] + \left[v_t + \chi_t b_{t+K+1}^{0} \right] \tag{A-9}$$

We also linearize a household's budget constraint:

$$b_{t+K-j+1}^{j} = \beta^{-1} \left[b_{t+K-j}^{j+1} + y_{t+K-j}^{j} - c_{t+K-j}^{j} \right] \tag{A-10}$$

where with this notation b_t^{K+1} denotes a household's initial net asset position and $b_{t+K-j+1}^{j}$ for $0 \leq j \leq K$ denotes a household's plans for the evolution of its net assets.

Households do not know the model-consistent value functions, but they learn adaptively and update their value functions based on observed data. Specifically, a household computes a new estimate of their value function at the same time as choosing its optimal state-contingent plan. This new estimate is consistent with their optimal plan, as it satisfies the envelope condition associated with maximizing equation (A-1):

$$V_{Bt}^{E}(B_t) = E_t^{K} \frac{U_{ct}\left(C_t(B_t) \right)}{\Pi_t}, \tag{A-11}$$

where $V_{Bt}^{E}(B_t)$ denotes a household's new estimate of its value function. In expression (A-11), $C_t(B_t)$ is a household's optimal consumption decision taking v_t, χ_t, and B_t as given. A household uses $V_{Bt}^{E}(B_t)$ to form their continuation value function at date $t + 1$ by combining it with their current continuation value function according to:

$$V_{Bt+1}(B_t) = (1 - \gamma) V_{Bt}(B_t) + \gamma V_{Bt}^{E}(B_t), \tag{A-12}$$

We linearize the functions in expressions (A-11) and (A-12). The latter linearization implies:

$$v_{ht+1} = (1 - \gamma) v_{ht} + \gamma v_{ht}^{e} \tag{A-13}$$

$$\chi_{t+1} = (1 - \gamma) \chi_t + \gamma \chi_t^{e} \tag{A-14}$$

where v_{ht}^e and χ_t^e are the intercept and slope coefficients to the linear approximation to $V_{Bt}^E(B_t)$:

$$V_{Bt}^E(B_t) \approx -\frac{1}{\sigma}\left(v_{ht}^E + \chi_t^E b_t\right) = -\frac{1}{\sigma}\left(c_t^K(b_t; v_{ht}, \chi_t) + \sigma\pi_t^K\right) \tag{A-15}$$

The linearized consumption function, $c_t^K(b_t; v_{ht}, \chi_t)$, depends on the parameters of the value function as well as their initial net asset position. The linearized solution to the optimal consumption function can be determined through recursive substitution using equations (A-7), (A-10), and (A-9). Since it is a linear function, it is convenient to write it in terms of an intercept term and a slope term:

$$c_t^K(b_t; v_{ht}, \chi_t) = c_t^K(0; v_{ht}) + g^K(\chi_t)b_t \tag{A-16}$$

where $c_t^K(0; v_{ht})$ is the intercept term associated with setting $b_t = 0$ and $g^K(\chi_t)$ is given by:

$$g^K(\chi_t) = \frac{\chi_t\beta^{-(K+1)}}{1 + \chi_t\sum_{i=1}^{K+1}\beta^{-i}} = \frac{\chi_t}{\beta^{K+1} + \chi_t\frac{1-\beta^{K+1}}{1-\beta}} \tag{A-17}$$

With optimal consumption defined in this way, equation (A-15) implies that $v_t^e = c_t^K(0; v_{ht}) + \sigma\pi_t^K$ and $\chi_t^e = g^K(\chi_t)$.

In equilibrium, $b_t = 0$, and $y_t^K = c_t^K$ so we can simplify v_{ht}^e further:

$$v_{ht}^e = y_t^K + \sigma\pi_t^K \tag{A-18}$$

which is equation (16) in the main text. In addition, setting $b_t = 0$ in equation (A-9) allows us to determine $y_t^K = c_t^K = c_t^K(0; v_{ht})$ through recursive substitution using equation (A-7):

$$y_t^K = -\sigma E_t\sum_{i=0}^{K}\left(i_{t+i}^{K-i} - r_{t+i}^\star\right) + \sigma E_t\sum_{i=0}^{K-1}\pi_{t+i+1}^{K-i} + v_{ht} \tag{A-19}$$

which is equation (13) in the main text. With a representative household and bonds in fixed supply, the evolution of χ_t is irrelevant for the economy's aggregate dynamics. In particular, only v_{ht} affects aggregate expenditures and thus it is sufficient to use only equations (A-13) and (A-18) to characterize the evolution of longer-run beliefs of households.

B Analytical Results for the FHP Model

In this section, we provide proofs of the paper's three propositions. Proposition 1 characterizes the impulse response functions of inflation forecasts and forecast errors in the FHP model, and the proposition 2 characterizes the FHP model's properties for the predictability regression of CG (2015). Proposition 3 characterizes the impulse response of the average inflation forecast in the SI model.

Proof of Proposition 1. Consider the first part of Proposition 1 that characterizes the impulse response functions of the the model with no learning. In that case, $v_{pt} = 0$ and we need only focus on the effect of changes in the output gap on the impulse responses. The impulse response of the output gap at date $t + i$ is given by:

$$\frac{\partial y_{t+i}}{\partial e_t} = \rho^i \tag{A-20}$$

which reflects that firms know the process for y_t. Using this expression in equation (9) and differentiating it at $t + i$ with respect to e_t implies that the impulse respose function of agents' one:

$$\frac{\partial \mathbb{E}_{t+i}^k \pi_{t+1+i}}{\partial e_t} = \rho^{i+1}A(k-1)\kappa \tag{A-21}$$

Because $\rho \geq 0$, $0 < \beta < 1$, and $\kappa > 0$, this expression is non-negative for any finite value of $k > 0$. We can also use expression (10) to determine the impulse response of the one-step ahead forecast error. Differentiating this expression at $t + i$ with respect to e_t and using (A-20) yields:

$$\frac{\partial \mathbb{F}_{t+1}}{\partial e_t} = \rho^{i+1}(\beta\rho)^k \kappa \geq 0 \tag{A-22}$$

which completes the proof for the FHP model without learning.

The second part of the proposition characterizes the impulse response function of the model with learning. To characterize the impulse responses of expected inflation and the inflation forecast error, it is convenient to first characterize the impulse response of the value function. The impulse response of the value function at $t + i$ is:

$$\frac{\partial v_{pt+i}}{\partial e_t} = \frac{\gamma_p}{1-\theta} \sum_{j=0}^{i-1} (1-\gamma)^{i-1-j} \frac{\partial \pi^k_{t+j}}{\partial e_t} \tag{A-23}$$

for $i > 0$. We also know that $\frac{\partial v_{pt}}{\partial e_t} = 0$ since the value function is predetermined at time t. We can use the impulse response of inflation to rewrite the impulse response of the value function in terms of the model's parameters. The impulse response of inflation is given by:

$$\frac{\partial \pi^k_{t+i}}{\partial e_t} = A(k)\kappa\rho^i + \beta^{k+1}(1-\theta)\frac{\partial v_{pt+i}}{\partial e_t} \tag{A-24}$$

Using equation (A-24), expression (A-23) can be rewritten as:

$$\frac{\partial v_{pt+i}}{\partial e_t} = (1-\widetilde{\gamma}_p)\frac{\partial v_{pt+i-1}}{\partial e_t} + \frac{\gamma_p}{1-\theta_p} A(k)\kappa\rho^{i-1} \tag{A-25}$$

where $\widetilde{\gamma}_p = \gamma_p(1-\beta^{k+1})$. This expression can be rewritten as:

$$\frac{\partial v_{pt+i}}{\partial e_t} = \frac{\gamma_p}{1-\theta_p} A(k)\kappa \sum_{j=0}^{i-1} (1-\widetilde{\gamma}_p)^{i-1-j} \rho^j \tag{A-26}$$

which holds for $i > 0$. From this expression, we can see that $\frac{\partial v_{pt+i}}{\partial e_t} \geq 0 \; \forall i$, which implies that the impulse response of expected inflation is always non-negative:

$$\frac{\partial \mathbb{E}^k_{t+i}\pi_{t+1+i}}{\partial e_t} = \rho^{i+1} A(k-1)\kappa + \beta^k(1-\theta)\frac{\partial v_{pt+i}}{\partial e_t} \geq 0. \tag{A-27}$$

For the change in the sign of the impulse response of the one-step ahead inflation forecast error, note that at $i = 0$, $\frac{\partial v_{pt}}{\partial e_t} = 0$ and

$$\frac{\partial \mathbb{F}_{t+1}}{\partial e_t} = \left[\rho(\beta\rho)^k + \beta^{k+1}\gamma_p A(k)\right]\kappa > 0 \tag{A-28}$$

Accordingly, on impact the forecast error rises. For $i > 0$, the impulse response of the forecast error is given by:

$$\frac{\partial \mathbb{F}_{t+1+i}}{\partial e_t} = \left[\rho(\beta\rho)^k + \beta^{k+1}\gamma_p A(k)\right]\kappa\rho^i - \beta^k\left[1-\beta(1-\widetilde{\gamma}_p)\right](1-\theta)\frac{\partial v_{pt+i}}{\partial e_t} \tag{A-29}$$

For the impulse response of the forecast error to be negative at $i > 0$ and $\rho > 0$ requires that:

$$[1-\beta(1-\widetilde{\gamma}_p)]\sum_{j=0}^{i-1}\left(\frac{1-\widetilde{\gamma}_p}{\rho}\right)^{i-1-j} > \left[\beta+\frac{\rho^{k+1}}{\gamma_p A(k)}\right]\rho \tag{A-30}$$

If expression (A-30) holds at response $i^\star$, then it will also hold at $i > i^\star$ given that the sum on left hand side grows over time. If $1-\widetilde{\gamma}_p > \rho$, then the forecast error is unbounded as $i \to \infty$ and there must exist an $i^\star$ for which expression (A-30) is satisfied. Substituting the expression for $\widetilde{\gamma}_p$ into the condtion, $1-\widetilde{\gamma}_p > \rho$, yields the expression used in the proposition. Note that if $\rho = 0$, this condition does not apply and expression (A-29) implies $i^\star = 1$.

Proof of Proposition 2. To show that $\beta_{CG} > 0$, it is sufficient to show that the covariance between the inflation forecast error and forecast revision is positive. To show this in the no learning case, note that in the case of no learning equation (10) can be simplified to:

$$\mathbb{F}^k_{t+1} = \rho(\beta\rho)^k\kappa(\rho y_{t-1}+e_t) \tag{A-31}$$

Also, without learning, the forecast revision at date t is given by:

$$\mathbb{R}^k_t = \rho^2\kappa(\beta\rho)^{k-1}y_{t-1}+\rho A(k-1)\kappa e_t \tag{A-32}$$

Using these two expressions, the covariance between forecast errors and revisions is:

$$\text{cov}(\mathbb{R}^k_t,\mathbb{F}^k_{t+1}) = \rho^4(\beta\rho)^{2k-1}\kappa^2\text{var}(y_t)+\rho^2(\beta\rho)^k A(k-1)\kappa^2\text{var}(e_t) \tag{A-33}$$

Expression (A-33) implies that if $\rho > 0$, then $\text{cov}(\mathbb{R}^k_t,\mathbb{F}^k_{t+1}) > 0$, which completes the proof for the no learning case.

With learning, we focus on the case in which $\rho = 0$. In that case, equation (10) can be written as:

$$\mathbb{F}^k_{t+1} = \beta^{k+1}\gamma_p A(k)\kappa e_t - \beta^k\left(1-\beta(1-\widetilde{\gamma}_p)\right)(1-\theta)v_{pt} \tag{A-34}$$

With $\rho = 0$, the forecast revision at time t is given by:

$$\mathbb{R}^k_t = (1-\theta)\beta^{k-1}(\beta v_{pt}-v_{pt-1}) \tag{A-35}$$

Both v_{pt} and v_{pt-1} are uncorrelated with e_t since they are determined before e_t is realized. This implies that:

$$\text{cov}(\mathbb{R}^k_t,\mathbb{F}^k_{t+1}) = -\beta^{2k-1}\left(1-\beta(1-\widetilde{\gamma}_p)\right)(1-\theta)^2\left[\beta\text{var}(v_{pt})-\text{cov}(v_{pt},v_{pt-1})\right] \tag{A-36}$$

With $\rho = 0$, we can write the covariance between the value function at time t and $t-1$ as:

$$\text{cov}(v_{pt},v_{pt-1}) = (1-\widetilde{\gamma}_p)\text{var}(y_t)$$

Substituting this expression into equation (A-36), the covariance between forecast error and revision is:

$$\text{cov}(\mathbb{R}^k_t,\mathbb{F}^k_{t+1}) = -\beta^{2k-1}\left(1-\beta(1-\widetilde{\gamma}_p)\right)(1-\theta)^2\left[\gamma_p(1-\beta^{k+1})-(1-\beta)\right]\text{var}(v_{pt}) \tag{A-37}$$

This expression implies that if $1-\beta > \gamma_p(1-\beta^{k+1})$, then $\text{cov}(\mathbb{R}^k_t,\mathbb{F}^k_{t+1}) > 0$, which is the condition given in Proposition 2.

Proof of Proposition 3. We need to consider a firm's forecast h quarters ahead along with the associated forecast error. For $k \geq h$, a firm's forecast is given by:

$$\mathbb{E}_t^k \pi_{t+h} = \rho^h A(k-h)\kappa y_t + \beta^{k-h+1}(1-\theta_p)v_{pt}. \tag{A-38}$$

A firm's forecast error is given by:

$$\mathbb{F}_{t+h}^k = \left\{ \left[\rho^h + \beta^{k+1}\gamma_p B_h(k) \right] A(k) + \rho^h A(k-h) \right\} \kappa y_t + \left[\beta^{k+1}(1-\widetilde{\gamma}_p) - \beta^{k-h+1} \right] (1-\theta_p)v_{pt} + O_{t+h}. \tag{A-39}$$

where O_{t+h} is an unpredictable component consisting of innovations in the shock from periods $t+1$ to $t+h$. The term $B_h(k)$ satisfies

$$B_h(k) = \sum_{i=0}^{h-1} \rho^{h-1-i}(1-\widetilde{\gamma}_p)^i$$

The first part of the proposition considers the case of no learning. In that case, a firm's forecast error satisfies:

$$\mathbb{F}_{t+h}^k = \frac{(\beta\rho)^{k-h+1}\left[1-(\beta\rho)^h\right]}{1-\beta\rho}\kappa y_t \tag{A-40}$$

Under no learning, equation (8) implies that inflation evolves according to:

$$\pi_t^k = A(k)\kappa y_t \tag{A-41}$$

where $A(k)\kappa > 0$. With $\rho > 0$, the forecast error's coefficient on the output gap is positive for any $h > 1$. Because the coefficient on the output gap for inflation is also positive, the covariance between the forecast error at horizon $h > 1$ and inflation will be positive when $\rho > 0$. Accordingly, under no learning, $\beta_{KW} > 0$ with $\rho > 0$.

The second part of the proposition considers the case of learning when the output-gap shock is *iid*. With $\rho = 0$, the h-step ahead forecast error with learning simplifies to:

$$\mathbb{F}_{t+h}^k = \left[\beta^{k+1}\gamma_p(1-\widetilde{\gamma}_p)^{h-1} \right] A(k)\kappa e_t + \left[\beta^{k+1}(1-\widetilde{\gamma}_p)^h - \beta^{k-h+1} \right] (1-\theta_p)v_{pt} + O_{t+h}. \tag{A-42}$$

Using equation (8), the covariance between the forecast error and inflation is given by:

$$\text{cov}(\pi_t^k,\ \mathbb{F}_{t+h}^k) = \left[\beta^{k+1}\gamma_p(1-\widetilde{\gamma}_p)^{h-1} \right] (A(k)\kappa)^2 \text{var}(e_t) + +\beta^{k+1} \left[\beta^{k+1}(1-\widetilde{\gamma}_p)^h - \beta^{k-h+1} \right] \text{var}(\widetilde{v}_{pt}) \tag{A-43}$$

where $\widetilde{v}_{pt} = (1-\theta_p)v_{pt}$. Using equation (6), the variance of the value function satisfies:

$$\text{var}(\widetilde{v}_{pt}) = \frac{\gamma_p \kappa^2 \text{var}(e_t)}{(1-\beta^{k+1})(2-\widetilde{\gamma}_p)} \tag{A-44}$$

Using this expression in equation (A-43), the covariance between the forecast error and inflation can be rewritten as:

$$\text{cov}(\pi_t^k,\ \mathbb{F}_{t+h}^k) = \beta^{k+1}\gamma_p(1-\widetilde{\gamma}_p)^{h-1}\kappa^2 \left\{ 1 - \frac{\beta^{k+1}(1-\widetilde{\gamma}_p)}{(1-\beta^{k+1})(2-\widetilde{\gamma}_p)} \left[\beta^{-h}(1-\widetilde{\gamma}_p)^{-h} - 1 \right] \right\} \text{var}(e_t)$$

This expression implies that the covariance will be negative if and only if:

$$\frac{\beta^{k+1}(1-\widetilde{\gamma}_p)}{(1-\beta^{k+1})(2-\widetilde{\gamma}_p)} \left[\beta^{-h}(1-\widetilde{\gamma}_p)^{-h} - 1 \right] > 1.$$

which is the expression shown in proposition 3.

C Computing Inflation Expectations for $h > k$

We need to make an additional assumption about agents' beliefs to compute their forecasts of inflation for forecast horizons h that exceed agents' k-period ahead planning horizons. The assumption that we make in this case is that agents use their beliefs at the end of their planning horizons and do so taking into account their knowledge of the persistence of the shocks.

To understand this assumption, we first consider its implications in the partial equilibrium model. An agent's expectations for inflation at the end of its planning horizon are:

$$E_t^k \pi_{t+k} = \kappa E_t^k y_{t+k} + \beta(1-\theta_p)v_{pt}. \tag{A-45}$$

In the partial equilibrium model, we can use equation (1) and the fact that the output gap follows an exogenous, AR(1) process to write:

$$E_t^k \pi_{t+k} = \kappa E_t y_{t+k} + \beta(1-\theta_p)v_{pt} = \kappa\rho^k y_t + \beta(1-\theta_p)v_{pt} \tag{A-46}$$

Unlike $E_t^k \pi_{t+k}$, an agent's expectations for $h > k$ do not affect their decisions and thus are not needed to solve the FHP model. However, in our empirical exercise to compare the model's implications to the empirical moments in the data, we need to compute $E_t^k \pi_{t+h}$ for $h > k$. We do so assuming agents forecast applying equation (A-45) to periods beyond their planning horizon:

$$E_t^k \pi_{t+h} = \kappa E_t^k y_{t+h} + \beta(1-\theta_p)v_{pt} \tag{A-47}$$

where $h > k$. In this expression, we still need to compute $E_t^k y_{t+h}$ and do so applying equation (1) and the fact that in partial equilibrium the output gap follows an AR(1) process. This implies

$$E_t^k \pi_{t+h} = \kappa E_t y_{t+h} + \beta(1-\theta_p)v_{pt} = \kappa\rho^h y_t + \beta(1-\theta_p)v_{pt} \tag{A-48}$$

for $h > k$. Accordingly, when $h > k$, agents use their beliefs about events outside of their planning horizons to compute $E_t^k \pi_{t+h}$ as well as their knowledge about the persistence of the output gap.

We use the same approach in general equilibrium and compute $E_t^k \pi_{t+h}$ in an analagous manner. The difference, however, is that $E_t^k \pi_{t+h}$ and $E_t^k y_{t+h}$ are simultaneously determined, respond to more shocks, and depend on v_{ht} as well as v_{pt}. Using equation (1), an agent's expectations k-periods ahead in this case are given by:

$$E_t X_{t+k}^0 = A_0^{-1} B_0 P^k S_t + A_0^{-1} B_v V_t \tag{A-49}$$

where $X_{t+k} = (y_t, \pi_t)'$ and $S_t = (r_t^\star, y_t^\star, i_t^\star)'$. The vector $V_t = (v_{ht}, v_{pt})'$ and the matrices A_0, B_0, B_v are functions of the model's parameters. The matrix P is a diagonal matrix whose elements along the diagonal consist of the AR(1) coefficients of the three shocks. Equation (A-49) can be used to determine $E_t^k \pi_{t+k}$ and is the analagous expression to equation A-46. We assume that agents apply the same knowledge in making forecasts in which $h > k$ and assume that:

$$E_t^k X_{t+h} = A_0^{-1} B_0 P^h S_t + A_0^{-1} B_v V_t \tag{A-50}$$

for $h > k$, which is the analagous expression to equation (A-48).

D Estimation of the FHP Model

The solution to the system of equations describing the equilibrium jointly with the observations equations define the measurement and state transition equations of a linear Gaussian state-space system. The state-space representation of a DSGE model yields a likelihood function, $p(Y|\theta)$, where Y is the observed data and θ is a vector comprised of the model's structural parameters. We estimate θ using a Bayesian approach in which the object of interest is the posterior distribution of the parameters θ. The posterior distribution is calculated by combining the likelihood and prior distribution, $p(\theta)$, using Bayes theorem:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}.$$

Because we can only characterize the solution to our model numerically, following Herbst and Schorfheide (2014), we use sequential Monte Carlo (SMC) techniques to generate draws from the posterior distribution. In addition to draws from the posterior $\{\theta^i\}_{i=1}^{N_{part}}$, the SMC algorithm computes as a by-product an estimate of the marginal data density (MDD), $\widehat{p(Y)}$. The log of this estimate is used in Figure 2. To approximate each posterior, we run the SMC algorithm 10 times. Each SMC sampler uses the following settings: $N_{part} = 16000$, $N_\phi = 500$, $N_{blocks} = 3$, $\lambda = 2.1$. Herbst and Schorfheide (2015) provide further details on these hyperparameters and Bayesian estimation of DSGE models more generally.

We estimate the FHP model as well as several alternative DSGE models using U.S. data on output growth, inflation, and nominal interest rates from 1966:Q1 through 2007:Q4, a time period for which there were notable changes in trends in inflation and output. The observation equations for the other variables are:[22]

$$\text{Output Growth}_t = \mu^Q + y_t - y_{t-1} \quad \text{(A-51)}$$

$$\text{Inflation}_t = \pi^A + 4 \cdot \pi_t \quad \text{(A-52)}$$

$$\text{Interest Rate}_t = \pi^A + r^A + 4 \cdot i_t, \quad \text{(A-53)}$$

where π^A and r^A are parameters governing a model's steady state inflation rate and real rate, respectively. Also, μ^Q is the growth rate of output, as we view the DSGE models as having been detrended from an economy growing at a constant rate, μ^Q. Thus, we are using the DSGE models to explain low frequency trends in the data but not the average growth rate or inflation rate which are exogenous.

E Sticky Information Model

In this section of the appendix, we describe the sticky information model that we estimate and compare to the FHP model. Under sticky information, price-setting firms do not face costs to adjusting their prices but instead firms infrequently update the information set upon which their price decisions are based. In particular, following Mankiw and Reis (2002), we assume that price-setters update their information sets in a staggered fashion in which there is a constant probability, $1 - \lambda$, that a firm setting a new price will revise its information set. Accordingly, a fraction, λ, of firms adjust their prices on the basis of previous information.[23] This setup gives rise to a log-linearized Phillips curve of the form:

$$\pi_t = (1 - \lambda)\lambda^{-1} mc_t + \mathbb{E}_{t-1}^{\lambda} \left[\pi_t + \Delta mc_t\right], \quad \text{(A-54)}$$

[22] We reparameterize β to be written in terms in the of the annualized steady-state real interest rate: $\beta = 1/(1 + r^A/400)$.

[23] Reis (2009) shows how this time-dependent updating of information can arise when firms face a fixed cost to updating their information.

where mc_t denotes a firm's real marginal cost and $\mathbb{E}_{t-1}^{\lambda}$ representing the average time $t-1$ forecast across agents. This forecast is a weighted average of past RE forecasts (E_{t-j-1}):

$$\mathbb{E}_{t-1}^{\lambda} = (1-\lambda)\sum_{j=0}^{\infty} \lambda^j E_{t-j-1}. \tag{A-55}$$

Because the average inflation forecast depends on past expectations of inflation, sticky information induces inertia in inflation with the degree of inertia depending on the information rigidity parameter, λ. Higher values of λ correspond to firms updating their information sets more slowly, which reduces the responsiveness of inflation to marginal cost and increases the importance of past expectations of inflation.

Given the focus of our paper on inflation, we only model price-setting firms as having sticky information. Households are assumed to use current information in their consumption-savings decisions though we allow for habit persistence in consumption. Accordingly, the (log-linearized) aggregate demand relationship in the model is:

$$[1+\zeta]\, y_t = \zeta y_{t-1} + E_t y_{t+1} - \sigma(1-\zeta)\left[i_t - E_t \pi_{t+1} - r_t^{\star}\right]. \tag{A-56}$$

The presence of habits formation in consumption ($\zeta > 0$) affects the determination of real marginal cost, which satisfies:

$$mc_t = \frac{1}{1-\zeta}\left[y_t - \zeta y_{t-1} - y_t^{\star}\right].$$

As in the NK model with FHP, $r_t^{\star}$ and $y_t^{\star}$ are AR(1) shocks to the equilibrium real rate and aggregate supply, respectively. Finally, in the SI model, monetary policy is specified to follow a Taylor rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + i_t^{\star}. \tag{A-57}$$

where, as in the NK model with FHP, $i_t^{\star}$ is an AR(1) shock to the monetary policy rule.

F Results for the Sticky Information Model

This section section contains analytical and empirical results for the SI model.

Analytical Results. To understand the implications of SI for the predictability IRFs and predictability regressions, we consider a partial equilibrium version of the model in which a firm's marginal cost is exogenous and governed by an AR(1) process:

$$mc_t = \rho_m mc_{t-1} + e_{mt}$$

In that case, we can show analytically that the IRF of the average forecast error across agents to such a shock underreacts relative to realized inflation at each point of the IRF. Moreover, as shown in CG (2015), under SI, there is a positive relationship between the average forecast error and forecast revision:

$$\mathbb{F}_{t+1}^{\lambda} \equiv \pi_{t+1} - E_t^{\lambda}\pi_{t+1} = \frac{\lambda}{1-\lambda}\left(E_t^{\lambda}\pi_{t+1} - E_{t-1}^{\lambda}\pi_{t+1}\right) + \epsilon_{t+1} \tag{A-58}$$

where ϵ_{t+1} is a function of the white noise process, e_{mt+1}. Because e_{mt+1} is unforecastable at date t, βCG, the univariate regression of the SI forecast error on revision satisfies $\beta_{CG} = \frac{\lambda}{1-\lambda}$. Accordingly, in the SI model, this regression coefficient is positive and depends only on the information rigidity parameter, λ. The sticky information model implies a positive relationship between forecast errors and revisions, because only a fraction $1 - \lambda$ update their information set to a shock at date t. Accordingly, for a shock that increases marginal cost at date t, the average forecast is not revised up that much, inducing positive co-movement between the average forecast revision and forecast error. The extent of this underreaction of the forecast to the shock depends entirely on the information rigidity parameter, λ, with larger values of λ implying a more sizeable underreaction of the forecast.

While CG (2015) prove this result for the SI model for the predictability regressions that they run, they do not study the implications of sticky information for the preditability impulse responses of AHS (2020). Proposition 4 establishes that the impulse response of the SI inflation forecast to changes in marginal cost underreacts relative to realized inflation at each date of the response. Accordingly, there is no eventual overreaction, as documented by AHS (2020).

Proposition 4. *(Underreaction of IRFs of SI Inflation Forecasts). Let* $\frac{\partial \mathbb{E}_{t+i}\pi_{t+1+i}}{\partial e_{mt}}$ *and* $\frac{\partial \mathbb{E}^{\lambda}_{t+i}\pi_{t+1+i}}{\partial e_{mt}}$ *for* $i \geq 0$ *be the impulse response to an innovation in marginal cost at date* t *for realized inflation and the average inflation forecast across agents in the sticky information model, respectively. Then,* $\frac{\partial \mathbb{E}^{\lambda}_{t+i}\pi_{t+1+i}}{\partial e_{mt}} = (1 - \lambda^{i+1})\frac{\partial \mathbb{E}_{t+i}\pi_{t+1+i}}{\partial e_{mt}}$, $\forall i \geq 0$.

Proposition 4 establishes that the impulse response of the average forecast across firms is proportional to the response of realized inflation at each date. Moreover, the response of the average forecast is proportionately smaller than the response of realized inflation at date $t + i$ by a factor, $0 \leq 1 - \lambda^{i+1} < 1$ so that there is never an overreaction of the average forecast. The extent of the underreaction depends on λ with higher values implying a slower updating of firms' information sets and a greater underreaction of the response of the average inflation forecast.

Proof of Proposition 4. To prove proposition 4, note that with exogenous marginal cost, the solution to the SI model can be determined analytically. In particular, inflation evolves according to:

$$\pi_t = \sum_{j=0}^{\infty} b_j mc_{t-j} \tag{A-59}$$

With $mc_t = \rho_m mc_{t-1} + e_{mt}$, these coefficients satisfy:

$$b_0 = \frac{1-\lambda}{\lambda} \tag{A-60}$$

and for $j > 0$:

$$b_j = \frac{1-\lambda}{\lambda}\left[\sum_{i=0}^{j-1} \rho_m^{j-i} b_i + \rho_m^{j-1}(\rho_m - 1)\right] \tag{A-61}$$

Using equation (A-60) in equation (A-61) for $j = 1$ and repeating this substitution pattern, we can show that for $j > 0$:

$$b_j = \frac{1-\lambda}{\lambda}\left(\frac{\rho_m}{\lambda}\right)^{j-1}\left(\frac{\rho_m}{\lambda} - 1\right) \tag{A-62}$$

Note that for a non-explosive solution to exist, the persistence of the marginal cost shock can not be too large. In particular, the persistence of the shock is bounded by the parameter λ so that $\rho_m < \lambda$.

With this solution in hand, the impulse response of realized inflation one-period ahead as well as the average forecast across firms can also be characterized analytically. The impulse response of realized inflation next period is given by:

$$\frac{\partial E_{t+i}\pi_{t+i+1}}{\partial e_{mt}} = A_{i+1} \tag{A-63}$$

where $A_{i+1} = A_i\rho_m + b_{i+1}$ and $A_1 = (b_0\rho_m + b_1)$. The impulse response of average inflation is given by:

$$\frac{\partial E^{\lambda}_{t+i}\pi_{t+i+1}}{\partial e_{mt}} = (1-\lambda)\sum_{j=0}^{i}\lambda^j \frac{\partial E_{t+i-j}\pi_{t+i+1}}{\partial e_{mt}} \tag{A-64}$$

Note that because we are taking the impulse response at date t with respect to e_{mt}, it is true that:

$$\frac{\partial E_{t+i}\pi_{t+i+1}}{\partial e_{mt}} = \frac{\partial E_t\pi_{t+i+1}}{\partial e_{mt}} \tag{A-65}$$

We can rewrite this expression in expression (A-65) and rewrite the response of the average forecast as:

$$\frac{\partial E^{\lambda}_{t+i}\pi_{t+i+1}}{\partial e_{mt}} = (1-\lambda)\frac{\partial E_t\pi_{t+i+1}}{\partial e_{mt}}\sum_{j=0}^{i}\lambda^j = (1-\lambda^{i+1})A_{i+1} \tag{A-66}$$

Accordingly, the response of the average inflation forecast at each date is proportional to the response of realized inflation, as described in Proposition 4.

Empirical Results. Like the FHP model, the SI model is estimated using a Bayesian approach with the same set of observables. The model is solved using the synthetic method described in Meyer-Gohde (2010) and the likelihood function is constructed using the covariance matrix implied by the MA solution. An approximate sample from the posterior is obtained using the SMC sampler. For parameters common to both the SI and FHP models, the priors are identical. The prior for the sticky information parameter λ is a Beta distribution with mean 0.5 and standard deviation 0.1. The prior for the habit parameter ζ is a Uniform distribution on $[0, 1]$. Table A-1 displays select posterior statistics for SI model. The posterior mean for λ is about one half, indicating about 50 percent of firms update their information set each quarter. The posterior distribution for the habits parameter, ζ, indicates a minimal role for habits. Finally, the supply shock is estimated to be extremely persistent, in contrast with its estimated behavior in the FHP model. The estimate of the log MDD, at around -753, indicates that the SI model fits the macroeconomic data substantially worse than FHP models with low k.

Table A-1: SI Model: Selected Posterior Statistics

	Description	Mean	[0, 95]
λ	Sticky Information Parameter	0.56	[0.48, 0.64]
σ	Coef. of relative risk aversion	0.37	[0.28, 0.49]
ζ	Habit Formation	0.27	[0.14, 0.42]
ϕ_π	Int. rule response to $\tilde{\pi}_t$	2.26	[1.85, 2.69]
ϕ_y	Int. rule response to $\tilde{y}_t$	0.08	[0.01, 0.18]
ρ_ξ	AR coeff. for demand shock	0.85	[0.81, 0.91]
ρ_i	AR coeff. for monetary policy shock	0.80	[0.71, 0.90]
ρ_y	AR coeff. for supply shock	1.00	[0.99, 1.00]
Log MDD		-753.73	

Note: The table shows estimates of the posterior means, 5th, and 95th percentiles of the model parameters as well the estimate of the log MDD as computed from output of the SMC sampler.

G CG Regressions

In this section, we reproduce the main regression from Coibion and Gorodnichenko (2015). We use data from the Survey of Professional Forecasters (SPF). The SPF is a quarterly panel collecting various economic forecasts from professional forecasters. Our focus will be on forecasts for four-quarter GDP deflator inflation. Specifically, we report OLS estimates of the coefficient β in the regression:

$$\pi_{t+h} - E_t\pi^A_{t+h} = c^h_{CG} + \beta^h_{CG}(E_t\pi^A_{t+h} - E_{t-1}\pi^A_{t+h}) + error_{t+h}. \tag{A-67}$$

Where $E_t\pi^A_{t+h}$ is the time t consensus (mean) forecast of annual inflation at time $t+h$. The actual inflation π_{t+h} is constructed using the vintage available one year after $t+h$ from the Philadelphia Fed's realtime data set. Table A-2 shows the regression results using data from 1969Q4-2007Q4. (Results are similar for other sample periods and design choices.)

Table A-2: CG Regression Results

$\widehat{c}_{CG}$	0.056
	(0.148)
$\widehat{\beta}_{CG}$	1.30
	(0.50)
n	148
R-squared	0.21

Note: The table shows point estimates and HAC standard errors (parentheses) from the OLS regression of A-67 along with the sample size and adjusted R-squared. The HAC standard errors are Newey-West standard errors with a Bartlett kernel with trunction equal to 4.

H AHS VAR

This section describes the algorithm for computing the "shock" as in AHS. Our starting point is the p-lag vector autoregression for the n dimensional vector y_t:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + u_t, \quad u_t \overset{iid}{\sim} N(0, \Sigma).$$

Write the VAR in companion form:

$$\xi_t = F_0 + F_1 \xi_{t-1} + \nu_t,$$

with

$$\xi_t = [y_t', \ldots, y_{t-p-1}']', \quad \nu_t = [u_t', 0, \ldots, 0]', E[\nu_t \nu_t'] = \Omega, \quad \text{and } F_1 = \begin{bmatrix} \Phi_1 & \Phi_2 & \ldots & \Phi_p \\ I & 0 & \ldots & 0 \\ 0 & I & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 \end{bmatrix}.$$

Define the $n \times np$ selection matrix M such that $y_t = M\xi_t$. Consider the variance of y_t over the frequency range $[\omega_0, \omega_1]$:

$$V(\omega_0, \omega_1) = \int_{w_0}^{w_1} M \left(I - F_1 e^{-i\omega}\right)^{-1} \Omega \left(I - F_1 e^{-i\omega}\right)^{-1'} M' d\omega.$$

Consider now identifing a single structural shock ϵ_{1t}, given (estimates for) $\Phi_0, \ldots, \Phi_p$ and Σ. Decompose the covariance matrix Σ as

$$\Sigma = \Sigma_{tr} \begin{bmatrix} \alpha_1 & \cdots & \alpha_n \end{bmatrix},$$

where Σ_{tr} is the lower cholesky factorization of Σ and $\{\alpha_1, \ldots, \alpha_n\}$ is a collection of $n \times 1$ orthonormal vectors (i.e., $\alpha_i \alpha_j' = 1$ if $i = j$ and 0 otherwise.) Identifying the structural shock ϵ_{1t} is equivalent to finding α_1. The variance of innovations attributable to the first structural shock is

$$\Sigma_1(\alpha_1) = \Sigma_{tr} \alpha_1 \alpha_1' \Sigma_{tr}'.$$

Following AHS, we identify α_1 by maximizing contribution of the shock ϵ_{1t} over a particular frequency band. variance of y_t attributable to the first structural shock is given by:

$$S(\omega_0, \omega_1, \alpha_1) = \int_{w_0}^{w_1} M \left(I - F_1 e^{-i\omega}\right)^{-1} \Omega_1(\alpha_1) \left(I - F_1 e^{i\omega}\right)^{-1'} M' d\omega.$$

where $\Omega_1(\alpha_1)$ is defined analogously to $\Sigma_1(\alpha_1)$. Let i be the index which corresponds to the inflation observable. Then α_1 is such that

$$\alpha_1^* = \text{argmax}_{|\alpha_1|=1} [S(\omega_0, \omega_1, \alpha_1)_{ii} / V(\omega_0, \omega_1)_{ii}].$$

Following AHS, we set the frequencies ω_0 and ω_1 to corresponds to periods of length 32 and 6, respectively. In our computations, the integrals are replaced by sum over 100 grid points.

I Fitting Observed Inflation Expectations

The model featured in the paper did not use inflation expectations as an observable. In some sense, this makes the fact that the model can match the inflation expectation predictability statistics more impressive, as these moments are not implicity contained in the likelihood function. That said, it is not obvious that the model can track the time series of inflation expectations and continue to match these moments (and continue to fit output growth, inflation, and interest rates well.) In this subsection, we evaluate the FHP model's ability to fit an additional observable, inflation expectations. We use, as in the previous subsection, the SPF to construct our inflation expectations series.

Recall that $\mathbb{E}_t^k[\pi_{t+h}]$ denotes the expectations of a h-period ahead inflation for an agent with a k-period planning horizon. We link this model variable to observed inflation expectations series using the following measurement equation:

$$\text{Expected Inflation}_t = \pi^A + \mathbb{E}_t^k[\pi_{t+1} + \pi_{t+2} + \pi_{t+3} + \pi_{t+4}] + \eta_t \qquad \text{(A-68)}$$

The parameter π^A is the steady state annual inflation rate and $\mathbb{E}_t^k$ denotes the forecast of economic agents with planning horizon of length k. We follow Del Negro and Eusepi (2011) and allow for measurement error, η_t, when including inflation expectations as an observable. The measurement error follows an AR(1) process:

$$\eta_t = \rho_\eta \eta_{t-1} + \epsilon_{\eta,t}, \text{ with } \epsilon_{\eta,t} \overset{iid}{\sim} N\left(0, \sigma_\eta^2\right).$$

As discussed in Del Negro and Eusepi (2011) there are number of reasons why it may be important to include measurement error when adding the SPF measure of inflation expectations to our estimation. One reason is that the information sets of the SPF forecasters and those of the economic agents in the model may not correspond exactly. Indeed, the SPF is produced in the middle of the quarter, while the model-based forecasts are made at the start of every quarter as leading to an information mismatch.[24]

To evaluate how the inflation data affects the FHP model estimates, Table A-3 shows the log marginal data density (MDD), a summary measure of fit, for different values of agents' planning horizon, k. The first column shows the MDD using only the macroeconomic data and thus in logs is defined as:

$$\log p(Y) = \log\left(\int p(Y|\theta)p(\theta)d\theta\right).$$

where Y is the observed data consisting of the "standard" macroeconomic observables of output growth, inflation, and interest rates. As shown in this column, a planning horizon in which agents only make state-contingent plans one-quarter ahead ($k = 1$) fits the macroeconomic data better than the models with longer planning horizons.

The second and third columns of the table emphasize the role of the inflation forecast data, which we denote as $E\pi$, as an observable. In particular, these columns show the log predictive data density as well as the MDD inclusive of the inflation forecast data. The log predictive data density provides a measure of the model's fit on the inflation forecast data and is defined as conditional on the standard macreconomic data:

$$\log p(E\pi|Y) = \log\left(\int p(E\pi|Y,\theta)p(\theta|Y)d\theta\right).$$

[24] We follow Del Negro and Eusepi (2011) as well as others by using the current vintage of data for the other observables while the series on SPF inflation forecasts is a real-time measure. This difference leads to an information mismatch between the econometrician and forecaster and hence is another reason for the inclusion of measurement error.

Table A-3: Log MDD Estimates

Model	$\log p(Y)$	$\log p(E\pi\|Y)$	$\log p(Y, E\pi)$
$k = 0$	-720.42	-56.02	-776.44
$k = 1$	-716.54	-49.37	-765.91
$k = 2$	-718.91	-49.53	-768.44
$k = 3$	-721.46	-48.95	-770.41
$k = 4$	-723.52	-48.46	-771.99
$k = 5$	-724.11	-51.47	-775.57

Note: The table shows point estimates of the log MDDs and the log predictive data density computed using the output of the SMC samplers. See appendix for details.

The log predictive data density allows us to uncouple the model's fit of the survey data of inflation expectations from its fit of the standard macroeconomic data.[25] As shown in the second column, the estimate of the planning horizon changes significantly if we focus on fitting only the survey data on inflation expectations. In that case, the estimate of the planning horizon would include the next four quarters as well as the current quarter ($k = 4$). Such a change has important implications for monetary policy, as it would considerably strengthen the economic effects of forward guidance that policymakers gave about the policy rate.

While focusing exclusively on fitting inflation forecast data implies that the estimates of agents' planning horizons are considerably longer, the third column shows the MDD using both sets of observables, which satisfies:

$$\log p(Y, E\pi) = \log p(Y) + \log p(E\pi|Y).$$

This relationship is highlighted in Table A-3, as the values in the third column of the table are the sum of the first two columns. Table A-3 indicates that the improved fit of the inflation forecast from the FHP model with $k = 4$ is more than offset by the deterioration in fit, shown in column 1, of the standard macroeconomic data. Because of this deterioration in fit in the standard macroeconomic time series, the estimated planning horizon using this data jointly with the survey data on inflation expectations involves planning only one quarter ahead ($k = 1$).

The inflation expectations data when used jointly as an observable with the other macroeconomic data does not change the estimated planning horizon in the FHP model and we find similar results for the model's other parameters. Table A-4 compares the posterior estimates of other structural parameters of the FHP model with $k = 4$ using both sets of observables to their estimates when the observables do not include the inflation expectations data. The posterior distribution of the firm learning rate, γ_p, shifts slightly, with the posterior mean decreasing to 0.16 from 0.20. The estimates of the parameters, κ and σ, which determine the sensitivity of inflation to the output gap and the sensitivity of aggregate demand to changes in the policy rate, respectively, are little changed by the inclusion of the survey data on inflation expectations. The same is true for the persistence of the exogenous shocks.

[25] In addition, this object is less sensitive to a researcher's prior distribution, as $p(\theta)$ is replaced by $p(\theta|Y)$.

Table A-4: FHP($k = 4$) Model: Comparison of Selected Posterior Statistics

	With Expectations Data		Without Expectations Data	
r^A	1.93	[0.88, 3.08]	2.00	[0.93, 3.15]
π^A	3.62	[2.38, 4.87]	3.67	[2.41, 4.97]
γ^Q	0.44	[0.42, 0.46]	0.44	[0.41, 0.46]
κ	0.01	[0.01, 0.01]	0.01	[0.01, 0.01]
σ	2.39	[1.63, 3.30]	2.28	[1.52, 3.15]
ϕ_π	1.08	[0.80, 1.39]	1.08	[0.80, 1.40]
ϕ_y	1.31	[0.94, 1.78]	1.29	[0.91, 1.78]
σ_ξ	0.30	[0.27, 0.33]	0.30	[0.27, 0.33]
σ_y	16.56	[12.29, 22.36]	18.44	[11.09, 29.01]
σ_i	0.97	[0.70, 1.33]	0.96	[0.68, 1.33]
ρ_ξ	0.91	[0.85, 0.96]	0.90	[0.84, 0.96]
ρ_i	0.95	[0.91, 0.99]	0.95	[0.90, 0.98]
ρ_y	0.35	[0.29, 0.41]	0.36	[0.23, 0.50]
γ	0.59	[0.37, 0.84]	0.54	[0.33, 0.78]
γ_f	0.16	[0.13, 0.20]	0.21	[0.13, 0.30]
$\bar{\phi}_\pi$	1.67	[1.40, 1.97]	1.57	[1.30, 1.90]
$\bar{\phi}_y$	0.07	[0.03, 0.13]	0.07	[0.02, 0.14]
σ_η	0.07	[0.07, 0.08]		
ρ_η	0.92	[0.87, 0.97]		

Note: The table shows estimates of the posterior means, 5th, and 95th percentiles of the model parameters computed from output of the SMC sampler. See appendix for details.