Finance and Economics Discussion Series

Federal Reserve Board, Washington, D.C.
ISSN 1936-2854 (Print)
ISSN 2767-3898 (Online)

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2024-095

Please cite this paper as:
Christiano, Lawrence J., Martin Eichenbaum, and Benjamin K. Johannsen (2024). "Measured Inflation and the New-Keynesian Model," Finance and Economics Discussion Series 2024-095. Washington: Board of Governors of the Federal Reserve System, https://doi.org/10.17016/FEDS.2024.095.

Measured Inflation and the New-Keynesian Model*

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November 2024†

Abstract

Researchers typically compare inflation in the new Keynesian (NK) model to published inflation measures constructed from indices like the CPI. Inflation in the standard NK model without price indexation is bounded above. The model analogue of fixed-weight inflation measures, like the CPI, is not. When inflation is in the range of values observed after 2021, there is a substantial difference between model-based and fixed-weight measures of inflation. This finding poses a challenge to using linear approximations to the NK model in environments with moderately high inflation and implies that analysts should construct data-consistent model analogues when assessing the NK model.

*The views expressed here are those of the authors and not the Board of Governors of the Federal Reserve System, the FOMC, or anyone else associated with the Federal Reserve System.

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1 Introduction

The standard new-Keynesian (NK) model incorporates nominal rigidities via Calvo-style frictions according to which firms update prices with some exogenous probability. The Calvo (1983) model implies that inflation calculated using the model's consumption good price index (which is an ideal price index) is bounded (see Andreasen and Kronborg (2022)). The bound depends on two key model parameters: average markups and the probability of updating prices. The first parameter characterizes the constant-elasticity-of-substitution (CES) aggregator in households' utility functions. The second parameter characterizes the fraction of monopolist consumption good producers that cannot adjust their price in any given period.

We show that to a first-order approximation, the price of the consumption good in the Calvo (1983) model is the same as a fixed-weight price index with equal weights on all prices. In a first-order approximation, there is no upper bound on the level of inflation in the ideal price index. As pointed out by Andreasen and Kronborg (2022), the bound from the model's nonlinear equations implies that a first-order approximation must break down for high inflation levels. To examine how high inflation has to be before a first-order approximation breaks down, we use recent U.S. Consumer Price Index (CPI) data to show that the nonlinear dynamics of a fixed-weight index can differ meaningfully from the percent change in the price of the consumption good in the model. Our results offer an analyst one way to ascertain when the first-order approximation is no longer reasonably accurate. Our results also imply analysts using nonlinear models should construct data-consistent model analogues to assess the NK model when inflation is even moderately high.

Our results are distinct from a separate criticism of the Calvo model, which is that the price adjustment frequency changes at high inflation levels. It is an open question how high that level must be before the model breaks down. Our analysis indicates that, depending on parameter values, the Calvo model may not be able to generate that level of inflation in the ideal price index.

Our results are related to Kocherlakota (2024), who analyzes short-run differences between Laspeyres and ideal price indices in the standard NK model. By short-run, Kocherlakota (2024) means that people's expectations about future inflation and real marginal costs are held fixed at steady state values. In contrast, our analysis does not depend on production technology, government policy, or even rational expectations. Our results are also related to the trend inflation literature (see, for example, Ascari (2004)). The bound that we study differs from and can be lower than the bound on steady-state inflation analyzed in that literature, though the bound we study also applies to steady-state inflation.

The remainder of this paper is organized as follows. Section 2 presents the standard NK model. Section 3 presents the inflation bounds and calculates them for parameters used in the NK literature. Section 4 relates our analysis to the steady-state inflation bound of Ascari (2004). Section 5 discusses the properties of a fixed-weight price index and compares it to the model's ideal price index. We compare the behavior of these price indices since 2021 for parameters used in the NK literature. Finally, section 6 contains concluding remarks.

2 The standard NK model

In this section we lay out the standard NK model (see Clarida et al. (1999)). A representative household maximizes expected utility

$$E_t \sum_{j=0}^{\infty} \beta^j u\left(\left(\int_0^1 (C_{i,t+j})^{\frac{\varepsilon-1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}}, N_{t+j}\right), \; 0<\beta<1,\; 1<\varepsilon \tag{1}$$

where $C_{i,t}$ denotes the i^{th} consumption good and N_t denotes employment. The household's time t budget constraint is

$$\int_0^1 P_{i,t}C_{i,t}di + B_t \leq R_{t-1}B_{t-1} + W_tN_t + T_t. \tag{2}$$

Here, $P_{i,t}$ denotes the price of $C_{i,t}$, B_t denotes holdings of a one-period risk-free nominal bond with gross nominal return, R_t. Also, W_t denotes the wage rate, and T_t denotes lump-sum profits.

The household purchases $C_{i,t}$ directly from a monopolist producer of that good. A necessary condition for household optimization is

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} C_t, \tag{3}$$

where

$$P_t \equiv \left(\int_0^1 P_{i,t}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}, \; C_t \equiv \left(\int_0^1 (C_{i,t+j})^{\frac{\varepsilon}{\varepsilon-1}}\, di\right)^{\frac{\varepsilon-1}{\varepsilon}}. \tag{4}$$

Here, P_t denotes the ideal price index and C_t denotes the utility-relevant consumption bundle.

Monopolists produce differentiated consumption goods. The i^{th} monopolist sets its price, $P_{i,t}$, subject to (3), and the Calvo friction. In particular, θ randomly selected monopolists cannot change their price and set $P_{i,t} = P_{i,t-1}$ while the other monopolists can change their price. We denote the reset price by $\tilde{P}_t$.[1]

Our analysis does not require that we specify a functional form of u, the technology for producing $C_{i,t}$, the nature of government monetary policy, the monopolists' maximization problem, or even how E_t in (1) is formed. We do require that u satisfy the usual properties that it is increasing and concave in its first argument, decreasing and concave in the second, and continuously differentiable. Other than that, we only use the definition of the ideal index, P_t, the Calvo friction, the assumption that all monopolists that update their price choose the same reset price, and the fact that households maximize (1).

3 An upper bound on inflation in the ideal price index

Let $\pi_t = P_t/P_{t-1}$ denote inflation in the ideal price index. In this section, we derive an upper bound on π_t, as in Andreasen and Kronborg (2022). It follows from the definition of P_t and the Calvo friction that

$$P_t^{1-\varepsilon} = (1-\theta)\,\tilde{P}_t^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}. \tag{5}$$

Because $\varepsilon > 1$, P_t and π_t are increasing in the reset price, $\tilde{P}_t$. Consider the limit of P_t as $\tilde{P}_t \to \infty$. Because $\varepsilon > 1$ it follows that $(1-\theta)\tilde{P}_t^{1-\varepsilon} \to 0$, so $P_t^{1-\varepsilon} \to \theta P_{t-1}^{1-\varepsilon}$. After rearranging, we conclude that $\pi_t \to \theta^{\frac{1}{1-\varepsilon}}$ from below as $\tilde{P}_t \to \infty$, so that

$$\pi_t < \left(\frac{1}{\theta}\right)^{\frac{1}{\varepsilon-1}}, \tag{6}$$

for all t.

As pointed out by Andreasen and Kronborg (2022), the right-hand side of the inequality in (6) represents an upper bound on the level of period inflation in any equilibrium of the model. The upper bound arises because $\tilde{P}_t \to \infty$ means that the reset price is very high, and households shift demand away from those products toward lower-priced goods. To see this, note that the ideal

[1] We maintain the standard assumption that all monopolists that update their price choose the same reset price.

price index can be expressed as a weighted average over the individual consumption good prices. Combining equations (3) and (4), we obtain

$$P_t = \int_0^1 \frac{C_{i,t}}{C_t} P_{i,t} di. \tag{7}$$

The previous equation implies that the weight on $P_{i,t}$ in the ideal price index is the ratio of $C_{i,t}$ to C_t. That ratio is a decreasing function of $P_{i,t}$.

The upper bound in (6) depends on the parameters θ and ε. Different authors assume different values for these parameters. So, we calculate the upper bound for π_t in the standard NK model for various values of θ and ε, see Figure 1. The horizontal axis of each panel corresponds to the steady state markup implied by ε (which is given by $\varepsilon/(\varepsilon - 1) - 1$) expressed in percent. The vertical axis corresponds to the upper bound on π_t, expressed in annualized percent.

From Figure 1 we see that the bound on π_t is increasing in the size of the steady-state markup (lower ε). High values of the steady state markup mean that goods are less substitutable for each other. As $\tilde{P}_t$ rises, households are less willing to substitute toward lower-priced goods and π_t is higher. Not surprisingly, the same figure shows that the bound on π_t is increasing in the flexibility of prices (lower θ).

4 Relationship to bounds on steady-state inflation

King and Wolman (1996), Ascari (2004), and Ascari and Sbordone (2014) argue that the standard NK model, in which firms have rational expectations, has an upper bound on steady-state price inflation. Ascari and Sbordone (2014) show that for the firm problem to be well defined in steady state, it must be that $\beta\theta\pi^{\varepsilon} < 1$ and $\beta\theta\pi^{\varepsilon-1} < 1$.[2] These conditions implies that

$$\pi < \min\left\{ \left(\frac{1}{\beta\theta}\right)^{\frac{1}{\varepsilon}}, \left(\frac{1}{\beta\theta}\right)^{\frac{1}{\varepsilon-1}} \right\}. \tag{8}$$

Depending on parameter values, the bound on π in (8) may be higher or lower than the bound on

[2] King and Wolman (1996) argue that if the bound on trend inflation is not satisfied then the probability of not adjusting prices is high enough that firms would rather not operate than risk having their price be low for a very long time. Our analysis, (6), does not depend on optimal pricing by firms.

π_t given in (6). Because (6) must hold, the actual bound on π is given by

$$\pi < \min \left\{ \left(\frac{1}{\beta\theta} \right)^{\frac{1}{\varepsilon}}, \left(\frac{1}{\theta} \right)^{\frac{1}{\varepsilon - 1}} \right\}. \tag{9}$$

In the data, the level and volatility of inflation are often positively correlated (see Ball et al. (1988)). So, even if the average level of inflation satisfies (6), there may be long periods in which measured inflation exceeds the bound.

5 Measured inflation

Most researchers using NK models compare the percent change in the ideal price index to measured inflation. However, measured price indices are not perfect analogs to the ideal price index in the model. Indeed, over short horizons, measured price indices–like the CPI–are well approximated by a fixed-weight index. In this section, we analyze a fixed-weight price index in the standard NK model using equal weights on the intermediate goods to construct the price index. Our assumption of equal weights corresponds to using the time-series average of weights placed on the intermediate goods in the ideal price index. We use data on core CPI inflation as the data analogue for this fixed-weight index and show that it can differ meaningfully from inflation in the ideal price index when core CPI inflation is moderately high.[3]

We define the equal-weight price index, Z_t, by

$$Z_t = \int_0^1 P_{i,t} di = (1 - \theta) \tilde{P}_t + \theta Z_{t-1}. \tag{10}$$

In contrast to P_t, $Z_t \to \infty$ as $\tilde{P}_t \to \infty$ because the weights on high prices do not decline as consumers substitute toward lower-priced goods. Let z_t denote the ratio of the fixed-weight price index and the idealized price index (Z_t/P_t). In addition, let ζ_t denote inflation in the fixed-weight price index

[3]By contrast, Andreasen and Kronborg (2022) use core CPI inflation as the data analogue of the NK model's ideal price index.

(Z_t/Z_{t-1}). Then

$$z_t = (1-\theta)\,\tilde{p}_t + \theta\frac{z_{t-1}}{\pi_t}, \tag{11}$$

$$\zeta_t = (1-\theta)\,\tilde{p}_t\frac{\pi_t}{z_{t-1}} + \theta, \tag{12}$$

where $\tilde{p}_t = \tilde{P}_t/P_t$. Equations (11) and (12) imply that there is no upper bound on ζ_t.[4]

It is straightforward to show that in a first-order approximation to the model with zero steady state inflation, $\hat{z}_t = 0$ and $\hat{\zeta}_t = \hat{\pi}_t$.[5] However, in the nonlinear version of the model or in linearized versions of the model with positive steady-state inflation there can be substantial differences between ζ_t and π_t.

We take core CPI inflation as the data counterpart to the fixed-weight index discussed above (Z_t).[6] We use the nonlinear equations (5), (11), and (12) to construct a time series for π_t.[7] The black, solid line in Figure 2 corresponds to the core CPI inflation rate for 2021:Q1 through 2024:Q3. Figure 2 also displays the inflation rate in the ideal price index for different values of θ and ε. The figure shows that the series for π_t differs more from the CPI inflation rate when the bound on π_t is lower. Recall that the bound on π_t increases in the markup and decreases in θ. Moreover, the difference over the past few years can be substantial for values of θ and ε used in the literature.

6 Conclusion

We argue that, especially over short horizons, inflation in a fixed-weight price index is a closer model analogue to measured inflation than inflation in the NK model's ideal price index. We show that during a period of moderately high inflation, inflation in model-based fixed-weight prices indices may differ markedly from the model-based measure of inflation (using the ideal price index)

[4] Though, there is a lower bound equal to θ.

[5] Here, $\hat{x}_t$ denotes the log deviation of x_t from its steady-state value. To understand this result, note that $\hat{\tilde{p}}_t = \frac{\theta}{1-\theta}\hat{\pi}_t$. Then equation (11) implies $\hat{z}_t = 0$ (so long as $\hat{z}_0 = 0$) and equation (12) implies that $\hat{\zeta}_t = \hat{\pi}_t$. For there to be a well-defined steady state, it must be that the bound on π is satisfied so that $\tilde{p}$ is well defined. When $\pi > 1$, it is straightforward to show that $z > 1$ and $\zeta = \pi$. Intuitively, because Z_t is larger than P_t in such a steady state, the inflation rate in both indices is the same for the steady state real reset price, $\tilde{p}$, even though the weights on the reset price are different in the two indices.

[6] U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items Less Food and Energy in U.S. City Average [CPILFESL], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CPILFESL, November 7, 2024.

[7] We set $z_t = 1$ in 2020:Q4. Our results are little changed if we set z_t in 2020:Q4 equal to its steady state value under the assumption of 2 percent steady-state inflation.

typically used to empirically assess NK models. Our analysis indicates that researchers who use the standard NK model when inflation is moderately high (as it was in the post-Covid period) should use model-based measures of inflation that are consistent with the way inflation is measured in the data.

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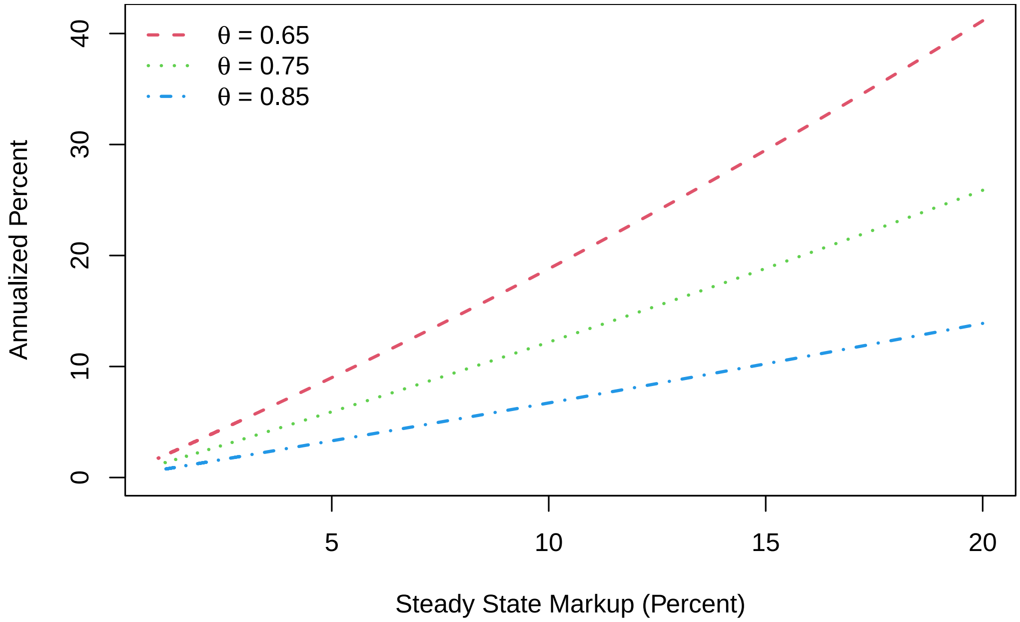
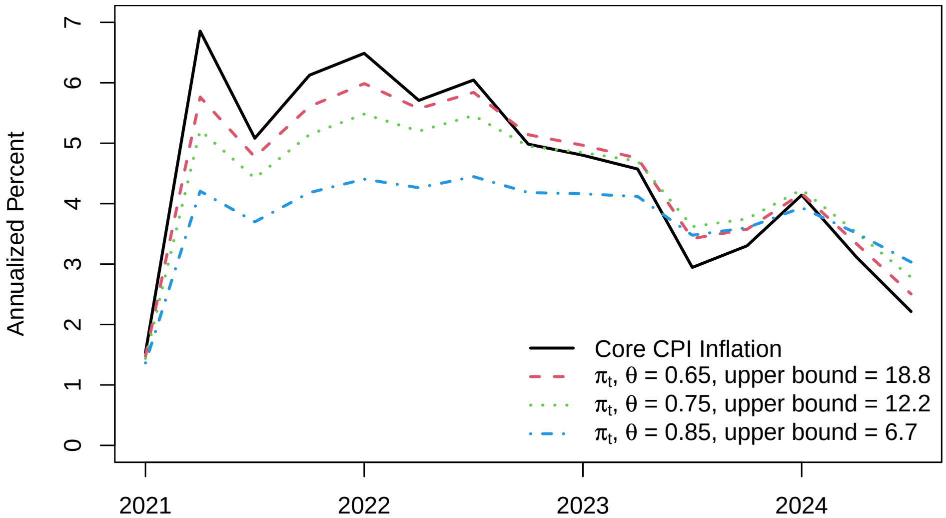
Figure 1: The maximum inflation rate is increasing in the steady state markup and decreasing in θ



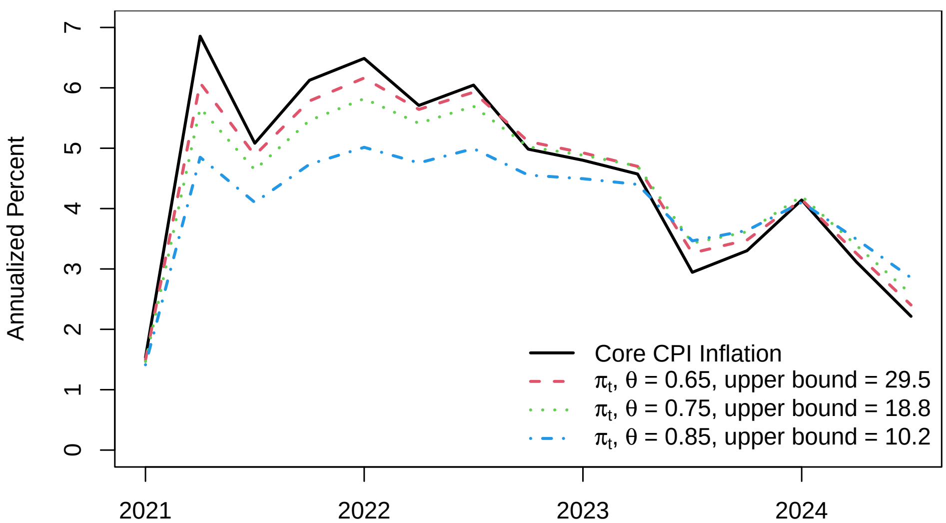
Source: Authors' calculations.

Figure 2: Measured inflation can be meaningfully different from π_t

(a) 10 percent steady state markup



(b) 15 percent steady state markup



Source: U.S. Bureau of Labor Statistics and Authors' calculations.