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Regulating Bank Portfolio Choice Under Asymmetric Information

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Abstract

Regulating bank risk-taking is challenging since banks know more than regulators about the risks of their portfolios and can make adjustments to game regulations. To address this problem, I build a tractable model that incorporates this information asymmetry. The model is flexible enough to encompass many regulatory tools, although I focus on taxes. These taxes could also be interpreted as reflecting the shadow costs of other regulations, such as capital requirements. I show that linear risk-sensitive taxes should *not* generally be set more conservatively to address asymmetric information. I further show the efficacy of three regulatory tools: (1) not disclosing taxes to banks until after portfolio selection, (2) nonlinear taxes that respond to information contained in banks' portfolio choice, and (3) taxes on banks' realized profits that incentivize banks to reduce risk.

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1 Introduction

Modern banking regulation generally aims to be risk sensitive. For example, under the current Basel III framework, the primary determinants of capital requirements are risk-based measures that aim to assign higher requirements to riskier portfolios. Risk weights are a major input into these measures, which reflect regulators' best guess of the risks of different exposures within banks' portfolios.

However, regulators typically know less about risk than banks, which can expose these frameworks to potential gaming. When regulators underestimate an asset's risk, banks can take advantage by overweighting it in their portfolios. Therefore, a common heuristic among regulators is to be conservative on average, under the theory that such conservatism will counterbalance this gaming.[1] However, it is unclear under which circumstances this heuristic should work or whether there are better ways of addressing banks' information advantage. Furthermore, there has been surprisingly little theoretical work on this topic, despite its practical importance. In this paper, I build a tractable model to address the question of how to regulate banks' portfolio choice in the presence of asymmetric information, which takes into account that banks will game the regulations ex-post.

In this model, a bank selects its portfolio among a wide range of assets, which I interpret as reflecting investments in different granular subsectors (e.g., an asset might represent all loans to small biotechnology companies), subject to regulatory constraints. While these constraints can take many forms, for tractability, I focus on a regulator setting risk-sensitive taxes. These taxes could be interpreted as literal taxes or as shadow costs from quantity-based regulation, such as capital requirements. Each asset has a different level of risk, measured as its beta with respect to a single systematic risk factor, and profitability. Information asymmetry exists because the bank perfectly observes each asset's risk and profitability, while the regulator only receives a noisy signal of risk. The bank and regulator share the objective of increasing profitability and decreasing risk, but bank risk-taking imposes a social externality that the bank does not internalize. Therefore, the regulator wants to reduce the riskiness of the bank's portfolio, but also recognizes the social costs of the bank forgoing profitable investments.

The major contribution of this paper is to develop a tractable and flexible framework that researchers can use to model regulation of bank portfolios under asymmetric information. The framework is rich enough to serve as a baseline for analyzing a wide range of regulatory tools and information asymmetries, but is still simple enough that most results can be solved by hand. I demonstrate how this framework can be used by applying it

[1] This observation comes from the author's personal experiences working in banking supervision.

to analyzing three regulatory tools: (1) not disclosing taxes to banks until after portfolio selection, (2) nonlinear taxes that respond to information contained in banks' portfolio choice, and (3) taxes on banks' realized profits meant to align the incentives of the bank and the regulator.

Before analyzing these tools, I begin by considering a familiar baseline: an asset-specific linear tax, which is similar to the linear risk weights used in most current capital regulation. Assuming a linear social cost to risk exposure, the regulator's optimal policy is to set the tax exactly equal to the expected risk given their signals. Contrary to common intuition, the bank's ex-post gaming of regulations does *not* mean the regulator should set taxes conservatively (i.e., higher than expected risk). The reason is that, in this environment, the costs from setting overly high taxes (i.e., dissuading socially optimal investments) cancel out the costs from setting overly low taxes (i.e., banks taking more risk than is socially optimal). Therefore, bank strategic behavior is not a rationale for conservatism by itself: nonlinearity of social costs or underweighting the benefits of profitable investments are necessary.[2] I then consider how the three regulatory tools can address this asymmetric information problem.

The first tool is strategic nondisclosure. Specifically, I consider a case in which the regulator sets a linear tax on exposure to each asset but does not reveal the magnitude of this tax to the bank until after it has selected its portfolio. Thus, the bank must then select its portfolio based on its best guess of these taxes. If the bank has no information about the regulator's signals, then the bank guesses that taxes will be calibrated correctly on average, which achieves the regulator's first-best outcome. However, even if the bank has imperfect information about the likely size of the tax, nondisclosure can still reduce the bank's ability to game regulation. In practice, nondisclosure is used in stress tests since banks do not fully know the regulator's model. The model supports maintaining opacity in stress testing since it reduces banks' ability to take advantage of weaknesses in the stress testing models, similar to the conclusion in Leitner and Williams (2023). However, nondisclosure could also be applied more broadly, including limiting the disclosure of asset-specific risk weights.

The second tool is a nonlinear tax on risk exposures. The bank investing more in an asset reveals information about its riskiness, which the regulator can incorporate by conditioning the tax rate on the bank's investment. Specifically, the regulator optimally sets the marginal tax equal to the regulator's estimate of risk, conditional on the bank's investment in that asset (along with the regulator's other information). This policy naturally leads to a nonlinear tax. Depending on the model's parameters, this nonlinear

[2]While this paper takes a Bayesian perspective, accounting for model uncertainty by optimizing with respect to worst-case scenarios does not necessarily justify conservatism. Worst-case scenarios could include setting overly high taxes that prevent socially efficient investments.

tax could be increasing or decreasing. For example, if the bank invests more in riskier assets on average, then the regulator will optimally raise the marginal tax rate as the bank invests more. However, in certain cases, the bank might be investing more in an asset because it has lower risk, in which case a regulator would decrease marginal taxes. A nonlinear tax may also not be possible to implement if there is a sufficiently large divergence between the incentives of the bank and the regulator. The most direct policy implication here is that regulators should respond to banks' portfolio choices, whether through formal policies, such as adjusting risk weights based on the concentration of banks' investments, or informal ones, such as calibrating shocks in stress tests based on those same concentrations.[3]

The third tool is a tax on ex-post profits that aligns the bank's risk-taking incentives with the regulator's incentives. The tax reduces the expected return that banks receive for taking a given amount of risk, which worsens the bank's after-tax risk–return tradeoff and effectively makes the bank more risk averse. If perfectly calibrated, this type of tax can perfectly align the incentives of the bank and the regulator. A flat tax on profits is not sufficient: Even though a flat tax reduces banks' after-tax profits, it also reduces their after-tax portfolio risk in a similar way and thus does not change the risk–return tradeoff. One way to alter the risk–return tradeoff is to have a state-dependent tax that is higher during "good times" (i.e., when the stochastic discount factor (SDF) of banks' investors is lower). Since banks' profits are typically higher during booms (which reflect "good times"), a progressive tax on profits could approximate the outcome of a perfectly calibrated tax, similar to the progressive tax proposed by John et al. (1991), to align the incentives of banks' shareholders. Such a policy could reduce risk-taking incentives at banks (or other financial institutions), even when regulators have little insight into those risks.

While this paper is not the first to consider asymmetric information in the context of banking regulation (for example, see Giammarino et al. (1993), Chan et al. (1992), Wu and Zhao (2016), Perotti and Suarez (2018), and Leitner and Williams (2023)), the main high-level difference is its focus on portfolio choice, which considers that banks can invest in many assets and potentially game regulations by tilting their investments towards assets whose risks are underestimated by regulators. Therefore, this paper can address policy questions such as how regulators should specify asset-specific risk weights that take into account information asymmetries. Additionally, this paper considers a substantially different set of tools to address this problem.

This paper is structured as follows. Section 2 lays out the model. Section 3 covers

[3]The informal approach is similar to the proposal in Greenwood et al. (2017) to consider areas of high growth or profitability for banks when calibrating shocks used in stress tests.

regulation based on taxing expected risk exposure, including both linear and nonlinear taxes. Section 4 covers regulation based on nondisclosed taxes, which are not revealed to the bank until after it has selected its portfolio. Section 5 covers regulation based on taxing banks' profits to reduce their effective risk aversion. Section 6 describes the policy applications. Section 7 concludes.

1.1 Related literature

This paper relates most strongly to three broad strands of literature, including papers that study (1) information asymmetries between banks and regulators, (2) the effect of capital regulation on bank portfolio choice, and (3) the effect of taxes on banks.

The most closely related papers address information asymmetries between banks and regulators. Perotti and Suarez (2018) consider optimal regulation when bank illiquidity imposes an externality, similar to how bank risk-taking imposes an externality in this current paper. They consider the use of both Pigovian taxes as well as quantity-based regulation, taking into account regulators' uncertainty with respect to banks' investment opportunities and gambling incentives, in the spirit of Weitzman (1974). One recent strand of literature considers model secrecy in the context of stress tests, including Leitner and Williams (2023) and Kim et al. (2024). At a high level, these papers address questions along the lines of how strictly to calibrate stress tests overall and whether to reveal enough information about the models so that banks can determine whether they are passing on their own. Giammarino et al. (1993), Chan et al. (1992), and Wu and Zhao (2016) are other papers that address asymmetric information between banks and regulators across different contexts. The key differentiating factor of my paper is its focus on calibrating asset-specific taxes or risk weights, including how to limit banks' ability to game those calibrations.

The second strand of related literature addresses optimal capital regulation for banks, and particularly the impacts on portfolio choice. Kim and Santomero (1988) derive the optimal risk weights for a regulator with full information on asset riskiness with the goal of limiting failure probabilities below a certain level. Rochet (1992) similarly derives optimal regulatory policy, both in terms of asset risk weights for capital requirements and pricing of deposit insurance, for a regulator with full information. More recently, Glasserman and Kang (2014) consider the problem of a regulator choosing optimal risk weights, including a case in which the regulator does not know the mean return of each asset. The main difference is that this paper focuses on the problem of a regulator with limited information about the riskiness of individual assets, whereas the regulator in their paper has full knowledge of each asset's riskiness. Greenwood et al. (2017) contains a simple model evaluating how various forms of bank capital regulation affect portfolio choice, and the paper discusses how different types of regulations are subject to gaming by banks. While the broad topic of gaming regulations is closely related to the current paper, their model does not explicitly incorporate asymmetric information.

The third strand of related literature discusses how tax policy affects bank behavior, which is particularly related to this paper's results on taxing banks' ex-post profits. John et al. (1991) is the most relevant, in that they propose setting a progressive tax to mitigate banks' incentives to tax excessive risks. Shackelford et al. (2010) discuss various ways in which taxation may be used to address externalities in the financial sector. They note that information asymmetries make Pigouvian taxation to address these externalities difficult and broadly discuss how financial transactions taxes, taxes on bonuses, and levies on banks may partially mitigate those externalities. Empirically, Celerier et al. (2019) empirically demonstrate the impact of Belgium's adoption of an equity subsidy, which allowed banks to deduct an estimate of the cost of equity from their taxes, on the composition of their portfolio. They show that the equity subsidy led banks to shift toward holding more loans rather than government bonds. And while not directly related to banks, this paper relates to a longstanding literature on the impact of taxation on risk-taking in general, such as in Domar and Musgrave (1944) and Stiglitz (1969).

2 The general framework

This is a single-period model containing a bank and a regulator. There are many assets whose riskiness (which is reflected by each asset's beta with respect to a common risk factor) and expected returns are drawn from known prior distributions. Although the framework is general, I think of a single asset in the model as mapping to an investment in some granular subcategory, such as "loans to small biotechnology companies." The bank observes the betas and expected returns, whereas the regulator only receives noisy signals of each. The regulator can use these signals to establish regulatory constraints, which in general can take many forms, and then the bank selects its investments subject to those constraints.

I typically assume that the bank's starting equity is exogenous, although Section B in the appendix considers the case in which the bank endogenously selects its desired level of equity. Without loss of generality, I normalize the bank's starting equity to one, so that all quantities can be interpreted as relative to the bank's total capital. Large quantities of investments in risky assets correspond to high leverage since banks must issue debt (whether as deposits or other forms) to fund those positions.

There is a continuum of risky assets indexed by $i \in [0, 1]$. The bank selects q_i, its quantity of investment in asset i. The overall expected excess return to investing in asset i is $a_i q_i - \frac{c}{2} q_i^2$. I assume that $c > 0$, so that there are diminishing returns to investment. q_i may be negative, which can be interpreted as the bank taking a short position.[4]

Each asset is exposed to a single systematic factor F with an asset-specific β_i loading.[5] Without loss of generality, I set $E[F] = 0$ and $Var(F) = 1$. Thus, each asset i has a payoff X_i of the form[6]

$$X_i = a_i - \frac{c}{2} q_i + \beta_i F. \tag{1}$$

The bank's objective is to maximize its market value, which is given by

$$E_B[M(\Pi - T)], \tag{2}$$

where $\Pi = \int_0^1 q_i X_i di$ is the bank's pre-tax profit, T represents potential taxes paid to the regulator, M is the investors' stochastic discount factor (SDF), and E_B denotes an expectation with respect to the bank's information set. I assume that the investors' SDF is[7]

$$M = 1 - \gamma F, \tag{3}$$

which implies that the bank maximizes

$$E_B[(\Pi - T)] - \gamma Cov_B((\Pi - T), F) \tag{4}$$

$$= \int_0^1 (aq_i - \frac{c}{2} q_i^2 - \gamma \beta_i q_i) di - E_B[T] - \gamma Cov_B(T, F). \tag{5}$$

The γ term reflects the price of exposure to the systematic factor F, which the bank's investors cannot diversify away. While the γ term could potentially reflect many forces, it is easiest to think of it as representing investors' risk aversion. The bank internalizes

[4]Banks could either literally be taking a short position or achieving similar exposures through other means. For example, a bank might take a short position on a credit exposure through buying a credit default swap.

[5]The portfolio-invariant risk weights in Basel II and III can be justified by the assumption of a single systematic factor, as discussed in Gordy (2003). However, the assumption of a single factor is not as important in this paper's framework. As will be explained in more detail shortly, risk in the context of this paper is based on covariance with the investors' SDF rather than the variance of the bank's portfolio. Under the SDF-based measure of risk, each asset's risk contribution does not depend on the rest of the portfolio even if there are multiple systematic factors.

[6]For simplicity, I assume that there is no idiosyncratic risk. Since there is a continuum of assets, the idiosyncratic risk would be diversified away and would therefore not affect the model's results.

[7]While the SDF can technically become negative for large values of F, this issue is not a concern for my application. This SDF results in a price of γ per unit of exposure to the systematic risk factor, which leads to a more tractable model.

the price of risk exposure and only increases exposure to the systematic factor if justified by the profits.

Here, I emphasize that risk is measured based on the bank's portfolio beta rather than the variance. A negative beta portfolio is considered less risky than a zero-beta portfolio because it provides the bank's investors with insurance by paying out during "bad times" when the marginal value of wealth is high. This linear price of risk exposure matches the approach taken by Froot and Stein (1998).

The bank's risk-taking imposes externalities $\eta > 0$ per unit of portfolio beta $\beta_p = \int_0^1 q_i\beta_i di$. This approach is a reduced-form way of reflecting a wide range of externalities, including government bailouts and bankruptcy costs. The regulator's objective is to maximize social welfare, which is

$$E_R\left[M\Pi - \eta\int_0^1 q_i\beta_i di\right] \tag{6}$$

$$=E_R\left[\int_0^1 (aq_i - \frac{c}{2}q_i^2 - (\gamma+\eta)\beta_i q_i)di\right], \tag{7}$$

where E_R denotes an expectation with respect to the regulator's information set, which will be specified in greater detail shortly.[8] The regulator's objective is identical to the bank's, except with higher effective risk aversion. The regulator is willing to allow the bank to take more risk if the associated profits are sufficiently high, but is more conservative in this tradeoff compared to the bank.

The timing and information structure is as follows. First, the β_i loadings and a_i measures of investment profitability are drawn from a prior distribution known to both the bank and the regulator. Then, the regulator receives a noisy signal $\hat{\beta}_i$ for each asset. These signals reflect the regulator's imperfect information each asset's riskiness. Next, the regulator uses this information to determine regulatory constraints on the bank. There are several types of regulatory constraints, but one example is the regulator selecting a tax k_i that the bank must pay for each unit of exposure to asset i. This cost could be interpreted as either a direct cost, such as a premium for deposit insurance, or an indirect cost, such as the cost of higher capital requirements for a position. These payments represent a private cost to the bank but do not affect social surplus. Finally, the bank optimally selects its portfolio knowing the true β_i and a_i parameters, reflecting superior knowledge of assets' riskiness, subject to paying any costs or following any constraints imposed by the regulator.

[8] In principle, the bank's investments might also produce positive externalities that the regulator takes into account. For simplicity, I'll assume that there are no such externalities, which means that the bank is capturing all social surplus. However, if positive externalities were included in the model, the regulator would additionally include a subsidy that reflects these positive externalities.

In determining optimal regulation, the regulator faces a tradeoff. Since the regulator has a higher effective risk aversion than the bank, the regulator wants to reduce riskiness. But regulations meant to reduce riskiness might lead to socially inefficient investments, which reduce social welfare.

The processes for each asset's true profitability and beta are

$$\beta_i = \bar{\beta} + u_i^b \tag{8}$$

$$a_i = \bar{a} + \chi(\beta_i - \bar{\beta}) + u_i^a, \tag{9}$$

where u_i^a and u_i^b are mean-zero independent normal random variables with variances σ_{ua}^2 and σ_{ub}^2, respectively. Therefore, the draws of a_i and β_i are independent across assets.

χ is an important parameter that determines the strength of the relationship between each asset's true risk and its baseline profitability. Since there is typically an equilibrium relationship between risk and return, it may be tempting to think that a_i should be strongly positively related to β_i (i.e., that χ should be large). However, this relationship is typically between risk and the marginal return. In this model, a_i only reflects the *initial* marginal return starting from zero investment in an asset, whereas $a_i - cq_i$ reflects the equilibrium marginal return. In equilibrium, investment in a highly profitable asset will drive down that asset's marginal return until it exactly compensates for the asset's risk. There is no similar logic for why the initial marginal return, a_i, should inherently be strongly related to β_i. Therefore, I do not restrict the value of χ.

For each asset, the regulator receives a single signal

$$\hat{\beta}_i = \beta_i + e_i, \tag{10}$$

where e_i is a mean-zero normal random variable with variance σ_e^2. e_i are independent across assets.

Based on this signal structure, the regulator's expectation of β_i conditional on observing $\hat{\beta}_i$ is

$$E[\beta_i|\hat{\beta}_i] = \bar{\beta} + \underbrace{\frac{\sigma_{ub}^2}{\sigma_{ub}^2 + \sigma_e^2}}_{\theta_b}(\hat{\beta}_i - \bar{\beta}) \tag{11}$$

based on formulas for Bayesian updating with normally-distributed random variables.

This signal structure is a tractable representation of regulators having less information about asset riskiness than banks. In practice, this information asymmetry could arise for many reasons. Banks could have better access to data, more staff, and a more specialized focus on particular products. This asymmetry could also potentially reflect a timing effect, since risk weights may not be calibrated to the most recent market data. In that case, banks would have an information advantage from using more recent data.

Since my focus is on how to address an information asymmetry, I assume that the signal structure is exogenous. In principle, a regulator might also address this information asymmetry by gathering additional data. Information acquisition could be modeled by allowing the regulator to pay a cost to improve the precision of the $\hat{\beta}$ signal. However, since my focus is not on optimal information acquisition and since including it would greatly complicate the model, I take the information structure as given.

2.1 No regulation

To build intuition for the cases involving regulation that follow, I begin by considering the bank's problem when there is no regulation.

In this case, the bank solves

$$\max_{\{q_i\}} \int_0^1 (aq_i - \frac{c}{2}q_i^2 - \gamma\beta_i q_i)di, \tag{12}$$

which quickly leads to the solution

$$q_i = \frac{1}{c}(a_i - \gamma\beta_i). \tag{13}$$

The bank's investment in an asset increases in a_i and decreases in β_i. All else being equal, banks do not like to take risk and will only do so if they are sufficiently compensated for it. Common intuition is that banks will take advantage of weaknesses in regulation to increase their risk, but this intuition is only true if that risk is sufficiently compensated.

The $a_i - \gamma\beta_i$ term will appear in future results. I call this term the "initial risk-adjusted return." In this model, the bank's marginal return from investing in asset i is $a_i - cq_i$, so a_i represents the marginal return prior to making any investment (i.e., when q_i is zero). Since the bank dislikes beta exposure with a cost of γ, the $-\gamma\beta_i$ term represents a risk adjustment from the bank's point of view.

Whether banks invest more in high-beta assets depends on the relationship between each asset's initial risk-adjusted return and its beta. Specifically, it depends on whether

$$\begin{aligned} Cov(q_i, \beta_i) &= Cov\left(\frac{1}{c}(a_i - \gamma\beta_i), \beta_i\right) && (14)\\ &= \frac{1}{c}Cov\left((\chi - \gamma)u_i^b + u_i^a, u_i^b\right) && (15)\\ &= \frac{1}{c}(\chi - \gamma)\sigma_{ub}^2 && (16) \end{aligned}$$

is positive or negative. This relationship will be positive when $\chi > \gamma$, which means that higher betas are associated with higher initial risk-adjusted returns on average. Since χ could take on any value, this relationship could, in general, take either sign.

Next, I consider what the portfolio beta and expected return look like in this case. The portfolio beta is

$$\beta_p = \int_0^1 q_i \beta_i di. \tag{17}$$

Using the fact that draws of a_i and β_i are independent across assets and applying the law of large numbers implies that the expected portfolio beta is

$$E[\beta_p] = E[q_i\beta_i] = E[q_i]E[\beta_i] + Cov(q_i, \beta_i). \tag{18}$$

This familiar last term will determine how the portfolio beta compares to what would be expected ignoring the link between risk and portfolio choice. If this term is positive, the bank will adjust its portfolio to scale up risky investments. If this term is negative, it will adjust its portfolio to scale down risky investments.

2.2 The first-best solution

Here I consider the first-best outcome, in which the regulator knows the true a_i and β_i parameters and directly selects investment quantities q_i to maximize social welfare. In this case, the regulator maximizes

$$\max_{\{q_i\}} \int_0^1 \left(a_i q_i - \frac{c}{2} q_i^2 - (\gamma + \eta)\beta_i q_i \right) di, \tag{19}$$

where there are no expectations due to the regulator's perfect knowledge of all parameters. I will focus on understanding the behavior of the unconditional expectations of the portfolio beta, portfolio return, and social welfare.

Proposition 1. *In the first-best case, equilibrium investment in asset i is*

$$q_i = \frac{1}{c}(a_i - (\gamma + \eta)\beta_i). \tag{20}$$

The equilibrium expected portfolio beta is

$$E[\beta_p] = \frac{1}{c}\left(E[a_i\beta_i] - (\gamma + \eta)E[\beta_i^2] \right). \tag{21}$$

The equilibrium expected portfolio return is

$$E[\mu_p] = \frac{1}{2c}\left(E[a_i^2] - (\gamma+\eta)^2 E[\beta_i^2]\right). \tag{22}$$

The equilibrium social welfare is

$$E[\mu_p - (\gamma+\eta)\beta_p] = \frac{1}{2c}E[(a_i - (\gamma+\eta)\beta_i)^2]. \tag{23}$$

Proof. See Section D.1. □

The most interesting result is that social welfare depends on the expectation of $a_i - (\gamma+\eta)\beta_i$ squared. This term can be interpreted as the initial risk-adjusted return from the regulator's point of view. Whenever the initial risk-adjusted return differs from zero, the bank can make socially efficient investments. As these differences rise, the amount of socially efficient investments also rise, which increases social welfare. Even investments with negative initial risk-adjusted returns can create social value since banks can take short positions.

The socially optimal investment in asset i matches the case without regulation, except with an effectively higher risk aversion of $\gamma+\eta$ instead of γ.

I also introduce a general result that is useful for calculating expected social welfare under various cases.

Proposition 2. *For all given random variables q_i representing a bank's investment choice, the expected social welfare can be expressed as*

$$\frac{c}{2}\left(E[(q_i^{fb})^2] - E[(q_i - q_i^{fb})^2]\right), \tag{24}$$

where the first-best investment is

$$q_i^{fb} = \frac{1}{c}\left(a_i - (\gamma+\eta)\beta_i\right). \tag{25}$$

Proof. See Section D.2. □

Intuitively, social welfare can be decomposed into one piece that depends on welfare under the first-best case and another piece that depends on deviations away from the first best. These deviations from the first best reflect the impact of socially inefficient investments. A particularly useful application of this result is comparing social welfare between two cases, for which it only becomes necessary to calculate the expected squared deviations from the first-best case.

3 Regulation through taxing estimated risk

Here I consider how a regulator might use taxes on banks' estimated risk to regulate their risk-taking. In interpreting the results, literal taxes are not required but instead something that imposes a direct or indirect cost to the bank. For example, risk-sensitive deposit insurance premiums would be a direct cost, while higher steady-state capital requirements would be an indirect cost. Section A discusses the relationship between setting taxes and capital requirements in further detail. Additionally, this section assumes that the bank's starting level of equity is exogenous. Section B.1 discusses an extension in which the bank selects its desired level of equity.

3.1 Linear tax

Here the regulator picks a tax k_i that the bank must pay for each unit of asset i. The proceeds of this tax do not have any social benefit, so they don't enter the regulator's objective function, but the bank optimizes conditional on the tax. The bank solves

$$\max_{\{q_i\}} \int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - \gamma \beta_i q_i - k_i q_i) di, \tag{26}$$

taking k_i as given. The regulator solves

$$\max_{\{k_i\}} E_R \left[\int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - (\gamma + \eta) \beta_i q_i) di \right], \tag{27}$$

taking into account how the choice of k_i affects the bank's choice of q_i as well as information contained in the signal $\hat{\beta}_i$.

Proposition 3. *The bank optimally selects*

$$q_i = \frac{1}{c}(a_i - \gamma \beta_i - k_i). \tag{28}$$

The regulator optimally selects

$$k_i = \eta E_R[\beta_i]. \tag{29}$$

The equilibrium social welfare relative to the first-best is

$$-\frac{1}{2c} \eta^2 Var(\beta_i - E_R[\beta_i]). \tag{30}$$

Proof. See Section D.3. □

The regulator's optimal policy is to set the tax equal to the expected risk given signals. At first glance this result may seem straightforward, but it runs counter to common intuitions. Financial regulation is often calibrated conservatively based on the argument that banks will take advantage of weaknesses in the regulatory framework. If the regulator underestimates an asset's risk, then banks will overinvest in that asset. Calibrating regulations to be more conservative than the regulator's expectations is then argued as necessary to prevent this type of gaming.

This force exists in this model since banks will overinvest in assets if regulators underestimate their risk. But the optimal solution is not conservative because such conservatism is costly. If regulators overestimate the risk of an asset, then banks will underinvest and, thus, forgo socially-desirable investments. Due to the specification of the risk-taking externality, $\eta\beta_p$, as a linear function, the costs of overinvestment and underinvestment are similar and, therefore, the regulator targets their expectation of optimal investment.

Extensions that allow for a convex risk-taking externality, so that the extra costs of too much risk are substantially more than the benefits of reducing risk, can give rise to conservatism. Although even in these extensions, idiosyncratic uncertainty about the risks of individual assets is not sufficient to generate conservatism since idiosyncratic uncertainty diversifies across the portfolio. Instead, there also needs to be systematic uncertainty. Section C covers the topic in more detail.

3.1.1 Is a linear tax better than command?

Here I consider whether a linear tax outperforms the command case, in which the regulator directly selects the bank's investments. Intuitively, it would seem that the linear tax would always outperform since it controls for the expected externality while still allowing the bank to make use of its private information. However, for certain parameters, it is better for a regulator to select investments directly.

In the command case, the regulator optimizes

$$\max_{\{q_i\}} E_R\left[\int_0^1 (a_i q_i - \frac{c}{2}q_i^2 - \gamma\beta_i q_i - k_i q_i)di\right], \tag{31}$$

which leads to the solution

$$q_i^{com} = \frac{1}{c}\left(E_R[a_i] - (\gamma+\eta)E_R[\beta_i]\right). \tag{32}$$

Proposition 4. *Social welfare under the command case relative to the first-best is*

$$-\frac{1}{2c}Var(\tilde{a}_i - (\gamma+\eta)\tilde{\beta}_i). \tag{33}$$

Social welfare under the command case is higher than in the linear tax case when

$$2(\gamma + \eta)Cov(\tilde{a}_i, \tilde{\beta}_i) > Var(\tilde{a}_i) + \gamma(\gamma + 2\eta)Var(\tilde{\beta}_i), \tag{34}$$

where $\tilde{x} = x - E_R[x]$ is the expectational error of the regulator.

Proof. See Section D.4. □

This expression illustrates that there are some circumstances under which the command case would be preferred to the linear tax.

A necessary, but not sufficient, condition for command to be preferred to a linear tax is for $Cov(\tilde{a}_i, \tilde{\beta}_i) > 0$. In that case, when the regulator underestimates the beta, the regulator also likely underestimates the expected return. The bank would then be more likely to invest more in assets for which the regulator has underestimated the risk.

3.2 Nonlinear tax

Here the problem is similar to before, except that instead of picking a single linear tax governed by k_i, the regulator sets a nonlinear tax schedule $k_i(q_i)$. These nonlinear taxes will implicitly make use of information contained in the bank's choice of q_i. All else being equal, banks prefer to increase q_i when expected returns are high (i.e., a_i is high) or risk is low (i.e., β_i is low), so the bank's choice of q_i is a signal of the combination of these two parameters. Using the nonlinear tax is essentially a way for the regulator to set a tax conditioned not only on the signal $\hat{\beta}_i$, but also on q_i.

More specifically, the bank observes the choice of $k_i(q)$ and then selects its portfolio taking the tax function as given. The bank therefore maximizes over quantities

$$\max_{\{q_i\}} \int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - \gamma \beta_i q_i - k_i(q_i)) di, \tag{35}$$

while the regulator maximizes over taxes

$$\max_{\{k_i(q_i)\}} E\left[\int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - (\gamma + \eta)\beta_i q_i) di\right], \tag{36}$$

taking as given the bank's optimal choice of q_i. Since adding a constant to taxes has no effect on the bank's choice, I select tax schedules that charge banks zero if they make no investments (i.e., $k_i(0) = 0$).

Proposition 5. *The bank's optimal choice of q_i satisfies*

$$q_i = \frac{1}{c}(a_i - \gamma \beta_i - k_i'(q_i)), \tag{37}$$

and the regulator's optimal tax $k_i(q_i)$ satisfies

$$
\begin{aligned}
k_i'(q_i) &= \eta E[\beta_i | q_i, \hat{\beta}_i] && (38)\\
&= \eta E[\beta_i | a_i - \gamma\beta_i, \hat{\beta}_i] && (39)\\
&= \eta \left(\bar{\beta} + \omega_1(\hat{\beta}_i - \bar{\beta}) + \omega_2 \left(\frac{(a_i - \gamma\beta_i) - \bar{a} + \gamma\bar{\beta}}{\chi - \gamma} \right) \right) && (40)\\
&= x + y(a_i - \gamma\beta_i), && (41)
\end{aligned}
$$

where

$$
\begin{aligned}
\omega_1 &= \frac{1/\sigma_e^2}{1/\sigma_{ub}^2 + 1/\sigma_e^2 + (\chi-\gamma)^2/\sigma_{ua}^2} && (42)\\
\omega_2 &= \frac{(\chi-\gamma)^2/\sigma_{ua}^2}{1/\sigma_{ub}^2 + 1/\sigma_e^2 + (\chi-\gamma)^2/\sigma_{ua}^2} && (43)
\end{aligned}
$$

are the updating weights placed on the signals $\hat{\beta}_i$ and $a_i - \gamma\beta_i$ respectively and

$$
\begin{aligned}
x &= \eta \left(\bar{\beta} + \omega_1(\hat{\beta}_i - \bar{\beta}) + \omega_2 \left(\frac{-\bar{a} + \gamma\bar{\beta}}{\chi - \gamma} \right) \right) && (44)\\
y &= \frac{\eta}{\chi - \gamma}\omega_2. && (45)
\end{aligned}
$$

A nonlinear equilibrium only exists for $y < 1$, which can be characterized by $\chi - \gamma < 0$ or $\chi - \gamma > \eta\omega_2$. In this equilibrium, the bank optimally invests

$$
q_i = \frac{1}{c}\left((1-y)(a_i - \gamma\beta_i) - x\right) \tag{46}
$$

and the regulator optimally sets a quadratic tax of the form

$$
k_i(q_i) = \frac{x}{1-y}q_i + \frac{c}{2}\left(\frac{y}{1-y}\right)q_i^2. \tag{47}
$$

Otherwise, the regulator sets a linear tax.

Proof. See Section D.6. □

In the case of a linear tax, the regulator sets the tax equal to their best guess of the beta. In the case of a nonlinear tax, the regulator sets the marginal tax equal to their best guess of the beta, taking into account the information contained in q_i. In this environment, the regulator's expectation of beta is linear in q_i, so the marginal tax is also linear in q_i and the level of the tax is quadratic in q_i.

The regulator observing q_i is equivalent to observing the risk-adjusted return $a_i - \gamma\beta_i$. This equivalence holds because the risk-adjusted return is the only part of the bank's choice of q_i that the regulator does not know. Furthermore, the risk-adjusted return is a noisy signal of β_i, which then affects the regulator's desired marginal tax.

The nonlinear equilibrium only exists for $\chi - \gamma < 0$ or $\chi - \gamma > \eta\omega_2$. Since $\omega_2 \in [0, 1]$, $\chi - \gamma > \eta$ is a sufficient condition. These parameter restrictions correspond to the case in which equilibrium investment q_i increases as the risk-adjusted return $a_i - \gamma\beta_i$ increases. Otherwise, the regulator would want the bank's investment to fall as the risk-adjusted return rises, which cannot be implemented with a nonlinear tax due to the divergence between the incentives of the bank and the regulator.

When $\chi - \gamma < 0$, assets with higher betas have lower risk-adjusted returns, on average, so banks invest less in riskier assets on average. Since the regulator prefers the bank to invest less in riskier assets as well, they are directionally aligned (although the bank and regulator differ on their desired investment magnitudes). In that case, the bank investing more indicates that an asset is less risky, so the regulator reduces the marginal tax rate as the bank invests more.

For the other case, consider the sufficient condition $\chi - \gamma > \eta$. In that case, assets with higher betas have such higher risk-adjusted returns, on average, that the regulator prefers the bank to invest more in risky assets. Since the bank prefers to invest more in risky assets as well, they are also directionally aligned. In that case, the bank investing more indicates that an asset is riskier, so the regulator raises the marginal tax rate as the bank invests more.

The takeaway is that nonlinear taxes can allow the regulator to take advantage of additional information from the bank's investment decisions, but only when both of their incentives are sufficiently aligned. Otherwise, there is not an equilibrium in which the bank reveals its information.

For a straightforward extension of the model in which the prior distributions of a_i and β_i vary across assets, such as by allowing asset-specific χ parameters, the regulator might implement a nonlinear tax for specifically for those assets where incentives are sufficiently aligned and a linear tax for the rest.

4 Effects of nondisclosure

Here I consider the policy of specifying taxes, similar to Section 3, but with the added twist that the regulator does not reveal these taxes to the bank until after they have made their portfolio choice.

4.1 Fully undisclosed linear tax

In this case, the regulator specifies a linear tax, but the bank does not know the value of this tax until after it has selected its portfolio. One interpretation is that this scenario is similar to a stress test in which the bank does not know the regulator's model. While not implemented in practice, another interpretation is that the bank selects its portfolio and the regulator only reveals the associated risk weights after the fact.

In this situation, the bank solves

$$\max_{\{q_i\}} \int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - \gamma \beta_i q_i - E_B[k_i] q_i) di, \tag{48}$$

where the bank's information set does not include the regulator's signals. Meanwhile, the regulator solves

$$\max_{\{k_i\}} E_R \left[\int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - (\gamma + \eta) \beta_i q_i) di \right], \tag{49}$$

taking into account the bank's choice function. I solve for the set of solutions $\{q_i\}$ and $\{k_i\}$ that jointly solve each optimization problem.

Proposition 6. *One solution is for the bank to select*

$$q_i = \frac{1}{c}(a_i - (\gamma + \eta)\beta_i) \tag{50}$$

and the regulator to select

$$k_i = \eta \hat{\beta}_i. \tag{51}$$

This solution achieves the first-best outcome for the regulator.

Proof. See Section D.7. □

To understand why this approach achieves the first-best outcome, note that the bank's expectation of the tax is

$$E_B[k_i] = \eta E_B[\hat{\beta}_i] = \eta E_B[\beta_i + e_i] = \eta \beta_i. \tag{52}$$

Intuitively, the bank expects that the regulator will set the tax correctly on average. Therefore, the bank invests as though the regulator had set all of the taxes correctly.

Recall that $\hat{\beta}_i \neq E_R[\beta_i]$, so the regulator is *not* setting the tax equal to their best guess of the true beta. The rationale is that the regulator wants the tax to respond one-for-one to changes in the true underlying beta. The regulator's best guess of the true beta, $E_R[\beta_i]$, includes some regularization toward $\bar{\beta}$ that typically results in a response that is less than one-for-one.

For practical application, the lesson here is that when regulators are not sharing ex-ante details about taxes, they should respond very strongly to any information they receive about an asset's riskiness. Otherwise, banks will assume that regulators will not adjust taxes sufficiently in response to information.

4.2 Partially disclosed linear tax

In practice, banks may have some sense of deficiencies in regulators' models, even if they may not have full knowledge of those models. For example, banks may learn about some features of regulators' stress testing models. In a dynamic setting, a bank may have learned some information from regulators' prior actions. I model banks' partial knowledge of regulators' models by giving banks noisy signals of the regulator's signals, which banks can use to guess the likely level of tax that regulators will specify.

The set-up is exactly the same as before, except now banks receive a noisy signal of the regulator's signal of the form

$$s_i = \hat{\beta}_i + w_i, \tag{53}$$

where w_i are independent across assets and are normally distributed with mean zero and variance σ_w^2.

Proposition 7. *If the bank receives noisy signals of the regulator's signals, then a regulator's choice of the optimal k_i must satisfy*

$$E_R[E_B[k_i]] = \eta E_R[\beta_i]. \tag{54}$$

One solution satisfying this condition is for the regulator to set

$$k_i = \eta \left(\bar{\beta} + \frac{\theta_b}{\theta_b + \theta_w(1-\theta_b)} (\hat{\beta}_i - \bar{\beta}) \right), \tag{55}$$

where

$$\theta_b = \frac{\sigma_{ub}^2}{\sigma_{ub}^2 + \sigma_e^2} \tag{56}$$

$$\theta_w = \frac{\sigma_w^2}{\sigma_e^2 + \sigma_w^2}. \tag{57}$$

θ_b is the weight that the regulator places on the signal $\hat{\beta}_i$, and θ_w is the weight that the bank places on the signal s_i.

Proof. See Section D.8. □

Intuitively, the condition for the optimality of k_i is that the regulator's best guess of the bank's best guess of the regulator's tax equals the regulator's best guess of the risk externality imposed by the portfolio. Put another way, the regulator wants to calibrate the tax to give the bank the correct incentives on average.

Existing results emerge as special cases of this framework. First, if the bank has a perfect signal of the regulator's information, then $\sigma_w = 0$. This case is equivalent to when the regulator pre-announces the loadings, in which case the result is $k_i = \eta E_R[\beta_i]$. Second, if the bank has no idea of the regulator's information, which is modeled as $\sigma_w \to \infty$, then $\theta_w \to 0$, which matches the earlier result of the regulator setting $k_i = \eta\hat{\beta}_i$.

In the intermediate case, as the bank's signal of the regulator's signal becomes noisier (i.e., the variance of w_i rises), the regulator places greater weight on $\hat{\beta}_i$ than they would in the case of optimal Bayesian updating, but does not place full weight on it as they would in the case of a fully undisclosed tax.

4.3 Nondisclosure with an aversion to idiosyncratic volatility

A potential concern with nondisclosure of taxes is that it exposes banks to additional uncertainty over their future cash flows. In the baseline framework, banks are only averse to systematic risk, so this additional uncertainty does not pose any cost. But, in practice, factors such as capital market imperfections may make banks averse even to idiosyncratic volatility (Froot and Stein, 1998). Therefore, nondisclosure can pose costs to banks that the baseline framework does not capture.

To incorporate these costs, I add a γ_I term reflecting banks' aversion to idiosyncratic volatility resulting from the uncertain taxes. One important question is whether the regulator considers idiosyncratic risk as imposing a social cost or not. To capture a wide range of possibilities, I assume that the regulator recognizes a social cost η_I associated with the bank bearing idiosyncratic volatility. Since the regulator already recognizes indirect effects of volatility affecting banks' portfolio choice, the η_I term reflects only the direct costs of the bank bearing idiosyncratic volatility, even controlling for portfolio choice.

For tractability reasons, I focus on the simpler case in which there is only one asset. In this case, the idiosyncratic risk only comes from the tax on the single asset. In the case with multiple assets, idiosyncratic risk will depend on the volatility of taxes across many assets, in which case the correlation of the errors becomes important. To avoid introducing those complexities, I focus on one asset.

Proposition 8. *Suppose that banks are averse to the volatility of tax payments such that they maximize*

$$\max_q (a - \gamma\beta)q - \frac{c}{2}q^2 - E_B[kq] - \gamma_I \sigma_B(kq), \tag{58}$$

where $\sigma_B(kq)$ is the standard deviation of the taxes paid with respect to the bank's information set.

Additionally, suppose that the regulator recognizes a social cost of η_I from the volatility of tax payments, so that the regulator's objective is to select a tax k to maximize

$$E_R[(a - \gamma\beta)q - \frac{c}{2}q^2 - \eta\beta q - \eta_I \sigma_B(kq)]. \tag{59}$$

The regulator can achieve the first-best outcome by setting

$$k = \eta\hat{\beta} + (\eta_I - \gamma_I) \underbrace{\eta\sigma_e}_{\sigma_B(k)} sgn(q), \tag{60}$$

which results in overall taxes paid of

$$kq = \eta\hat{\beta}q + (\eta_I - \gamma_I)\eta\sigma_e|q|. \tag{61}$$

Proof. See Section D.9. □

The first piece of this expression, $\eta\hat{\beta}$, is the same as in the baseline case without an aversion to idiosyncratic volatility. As before, the bank's best guess of $\hat{\beta}$ is the true β, so nondisclosure by regulators forces the bank to use their information based on the true state of the world.

The second piece of this expression relates to the idiosyncratic volatility. If the private and social costs of idiosyncratic volatility differ (i.e., to the extent that γ_I and η_I differ), the regulator should adjust the taxes to align the bank's incentives. For example, if $\gamma_I = \eta_I$, so that the two are already aligned, there is no need to take action.

But consider the case in which $\gamma_I > 0$ and $\eta_I = 0$, so that there is only a private cost to volatility. For positive q, the regulator should optimally reduce the size of the tax. The reason is that the volatility is already dissuading the bank from investing, so the tax does not need to be as high to achieve the optimal level of investment. In this case, the regulator can compensate for the costs imposed by higher uncertainty in the tax by reducing the average size of the tax. The idea generalizes: The regulator can compensate for the costs imposed by higher regulatory uncertainty by reducing the average tightness of regulations.

5 Taxes on profits

This section explores the potential of using taxes on ex-post bank profits to effectively reduce their risk aversion and, thus, influence their portfolio choice. The broad idea is that the regulator can calibrate a stochastic tax that the bank perceives as having a present value related to the riskiness of the bank's investments. The regulator can then use taxes of this type to align the bank's incentives with the regulator's, even without any knowledge of risk or profitability of individual assets.

5.1 Achieving first best through a tax on expected profits

In the generally infeasible case in which the regulator has perfect knowledge of the bank's expected profits, it's possible for the regulator to achieve a first-best outcome through an appropriately calibrated tax on them. The intuition is that reducing expected profits also reduces the profit per unit of risk the bank takes, which incentivizes the bank to reduce its risk and has an identical effect as reducing the bank's risk aversion.

Proposition 9. *The regulator can achieve the first-best outcome by taxing a fraction* $\frac{\eta}{\gamma+\eta}$ *of the bank's expected profits.*

Proof. In this case, the regulator imposes a tax of

$$T = \frac{\eta}{\gamma + \eta} E_B[\Pi] = \int_0^1 \frac{\eta}{\gamma + \eta}(a_i q_i - \frac{c}{2}q_i^2)di. \tag{62}$$

The bank's objective is now to maximize

$$\max_{\{q_i\}} E_B[M(\Pi - T)] = \max_{\{q_i\}} \int_0^1 \left(\frac{\gamma}{\eta + \gamma} \left(a_i q_i - \frac{c}{2}q_i^2 \right) - \gamma\beta_i q_i \right) di. \tag{63}$$

Multiplying by $(\gamma+\eta)/\gamma$ does not change the optimal solution and yields the maximization problem

$$\max_{\{q_i\}} \int_0^1 (a_i q_i - \frac{c}{2}q_i^2 - (\gamma + \eta)\beta_i q_i)di, \tag{64}$$

which exactly matches the regulator's objective function and therefore leads to a first-best outcome. □

I again emphasize that this outcome is generally not feasible since it relies on the regulator having perfect knowledge of a_i. However, it suggests that feasible strategies that approximate a tax on expected returns might be a fruitful course of action. So I next examine feasible approaches that are in the same spirit.

5.2 Achieving the first best through a tax on ex-post profits

It is possible to achieve the first best through an appropriately-calibrated tax on ex-post profits whose present value, from the bank's point of view, equals a fraction of expected returns.

Proposition 10. *If the regulator sets an ex-post tax of*

$$T = \frac{1}{M}\frac{\eta}{\eta+\gamma}\Pi, \tag{65}$$

then portfolio choice will match the first-best case. In this equation, M and Π are random variables, while η and γ are constants.

Proof. From Proposition 9, a tax that is set to be a fraction $\eta/(\eta+\gamma)$ of the bank's expectation of profits achieves the first best. The regulator can set an ex-post tax T with a present value equal to this tax by selecting T to satisfy

$$E_B[MT] = \frac{\eta}{\eta+\gamma}E_B[\Pi]. \tag{66}$$

Setting T as proposed leads to

$$E_B\left[M\underbrace{\frac{\eta}{\eta+\gamma}\frac{\Pi}{M}}_{T}\right] = \frac{\eta}{\eta+\gamma}E_B[\Pi], \tag{67}$$

so that the present value of the tax to the bank equals a tax on expected profits. □

From before, the regulator would ideally want to tax the bank's expected returns. Unfortunately, a direct tax is infeasible because the regulator does not know asset-specific expected returns. However, the regulator can set a stochastic tax whose present value, from the bank's perspective, equals some fraction of the portfolio's expected return. State-dependent tax rates are required to achieve this result. Since M is high in "bad" times and low in "good" times, the taxes should be lower in "bad" times than in "good."

In practice, the taxes may not necessarily have to explicitly be state-dependent. For example, since bank profits are likely higher during good times than bad, setting a progressive tax rate on profits might achieve a similar outcome. Even a binary approach, such as imposing an extra tax on profits above a particular threshold, may achieve an approximately similar outcome.

If the aim is to tax a constant fraction of expected profits, it may be unclear why it isn't sufficient to tax a constant fraction of realized profits. While this simpler flat tax reduces expected profits, it also reduces risk by the same amount. The overall risk–return tradeoff remains unchanged along with the bank's portfolio choice. The more complicated tax is necessary to alter the expected return without altering the risk.

6 Policy applications

The model yields several broad insights for designing financial regulation that takes into account the information asymmetry between the regulator and the regulated. While the model focuses on specifying optimal taxes, these taxes could be interpreted as reflecting the shadow costs of capital requirements. Since capital requirements are a more common regulatory tool, I will focus on the model's implications for setting risk weights as part of capital regulation.

A first insight is that regulators can benefit by not disclosing information about risk weights to banks. Intuitively, if a bank does not know the regulator's risk weights ahead of time, they will need to choose their investments based on their best guess of the risk weights. If a bank thinks that the regulator will set risk weights correctly, on average, then that bank will invest as though the regulator had set the risk weights correctly. Even if the bank has some information on the regulator's likely risk weights, there is still benefit in disclosing as little information as possible to reduce possibilities to take advantage of misspecification.

A potential concern is that not disclosing risk weights may create regulatory uncertainty that would dissuade socially valuable investment. However, reducing the average level of risk weights could counteract disincentive effects from uncertainty.

The broad approach of not disclosing information to banks is already reflected in the spirit of current stress tests, in which banks provide information on their portfolios and do not know the full details of how regulators will calculate capital requirements. The model supports limiting disclosure of stress test models to limit banks from gaming those models.

Furthermore, similar types of nondisclosure could apply to other parts of the regulatory framework. For example, in the Basel III capital framework, regulators publish prespecified risk weights for each asset. An alternative would be for regulators to ask banks to report data on exposures and then only tell them their aggregate capital requirements, without revealing any asset-specific risk weights. To limit learning, regulators could also recalibrate risk weights more frequently. This approach would limit banks' ability to take advantage of misspecified risk weights.

A second insight is that regulators can account for information contained in the bank's portfolio choice. The most direct application would be nonlinear risk weights that automatically change as banks concentrate investment in particular sectors. While the model is too stylized for the formulas to directly apply to policy settings, regulators can consider what information is revealed when a bank invests more in a particular type of asset. In some cases, concentration in an asset may indicate that the regulator has underestimated risk, and using a tool similar to a nonlinear risk weight may be a way of leaning back against that underestimation.[9] But, in some cases, higher concentration may not be associated with higher risk. One approach to addressing this problem is to calibrate nonlinear risk weights based on the empirical relationship between concentration in particular types of assets and ex-post measures of risk, such as default rates on loans. However, this relationship would change if risk weights change, so multiple recalibrations may be required before converging to an equilibrium (if it exists at all).

Practical application of such nonlinear formulas should also account for bank specialization. Some banks may specialize in particular lines of business and would therefore naturally have higher concentrations. Penalizing specialized banks is likely socially undesirable. For practical implementation, it would likely be preferable to use measures of concentration for the aggregate banking system rather than individual banks. An additional benefit of using measures based on the aggregate banking system is that individual banks may not internalize how their investments would change risk weights, which could reveal more information to regulators about the risks of individual assets.

While formal rules, such as formulas for risk weights, could automatically respond to portfolio choice, a more informal supervisory process could also lead to a similar result. Supervisors could investigate the areas in which banks are investing much more (or less) than expected and consider adjusting risk weights for those areas. Risk weight adjustments could be direct, such as changing Basel III calibrations, or indirect, such as changing the shocks used in stress testing models as proposed by Greenwood et al. (2017).

A third insight is that regulators could implement taxes on ex-post profits. Since the model assumes that banks have an exogenous amount of equity equal to one, the empirical counterpart would be a tax whose rate depends on a bank's return on equity. The intuition is that banks take risk to generate higher returns. Reducing expected returns through a tax thus reduces the bank's incentives to take risk, which can effectively make them

[9] Sector-specific countercyclical capital buffers can provide a similar outcome if adjusted appropriately. But if there are any difficulties in activating such buffers, an automatic nonlinear rule may be able to adjust more quickly.

more risk averse while still using their private information. However, it is important to take into account that taxes on ex-post profits affect both the after-tax risk and expected return, so a flat tax on profits would not be sufficient.

In theory, an optimal solution is a state-dependent tax that is higher during "good times" (i.e., when investors' SDF is lower). The level of this tax would rise with the magnitude of the externality that the bank's risk-taking imposes. An explicitly state-dependent tax could be calibrated based on measures of economic performance, such as unemployment or GDP growth. However, since bank profitability is likely to be highly correlated with economic performance, a progressive tax might also approximate the ideal outcome. For example, a higher marginal tax rate for banks with a return on equity above some predetermined threshold might discourage banks from excessive risk-taking by reducing the after-tax payoff of doing so. Additionally, a tax whose rate rises with a bank's return on equity would incentivize banks to fund themselves with more equity since higher equity would mechanically reduce the return on equity.

Empirically, Meiselman et al. (2018) show that high profitability for banks predicts higher tail risk for both the 2007-08 financial crisis as well as the 1980s savings and loan crisis. Given the empirical link between profits and risk, a well-calibrated tax on profits can effectively be viewed as a tax on risk, without requiring any knowledge on the regulator's part of the riskiness of the bank's portfolio.

One benefit of a tax on ex-post profits is that it could be quickly applied to the shadow banking sector, even if regulators know little about its business model. In contrast, the regulatory framework for traditional banks may not be entirely appropriate for certain shadow banks, and it would likely take time to design a framework that appropriately reflects their activities. Even if the ultimate goal were to apply a more traditional regulatory framework to such institutions, a tax on profits could be useful as an interim measure while regulators prepare the longer-term framework.

7 Conclusion

In this paper, I address the problem of how to regulate bank portfolio choice taking into account the asymmetry of information between banks and regulators. I construct a tractable model that explicitly accounts for this asymmetry. The model is flexible enough to allow for the consideration of a wide range of regulatory tools and information asymmetries, while also being simple enough to be easily solved by hand. Because of its flexibility and simplicity, this model can be adapted by researchers exploring related topics, both within the context of banking and elsewhere.

I then use this model to explore the efficacy of several common regulatory tools and to propose three less common tools: (1) not disclosing taxes to banks until after portfolio selection, (2) nonlinear taxes that respond to information contained in banks' portfolio choice, and (3) state-dependent taxes on banks' realized profits.

While the model is intentionally stylized to aid in communicating intuitions, the broad takeaways could be applied in practice. Regulators could consciously not disclose information from banks to prevent them from gaming regulation, similar to how they already do so for stress tests. Nonlinear taxes (or risk weights) could automatically respond to banks concentrating their investments in a particular sector. And taxes on ex-post profits could incentivize banks to behave more risk averse, even if regulators know little about banks' risks.

Future work can extend the model along several dimensions. One extension is to focus on dynamic interactions, particularly in the case of information nondisclosure. Since banks can learn information about regulators' models over time by observing outcomes, regulators may need to adjust their models or intentionally introduce noise to keep banks from learning too much. Another extension is to ease the single-factor assumption. While a single factor is an implicit assumption underlying much regulation (for example, Gordy (2003) discusses the importance of a single-factor assumption within capital regulation of the banking book), market risk must account for many correlated factors and hedges. Addressing this problem would further aid in regulating bank portfolio choice in the important, but more complicated, setting of banks' trading books.

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A Relationship between taxes and capital requirements

The aim of this section is to demonstrate how approaches based on taxes and capital requirements compare with each other. The exact relationship depends on the assumptions about capital.

In several cases, there is an exact equivalence. In the most straightforward case, the supply of capital is elastic with constant cost relative to other funding sources of r_c. In that case, a capital requirement of m_i per quantity held of asset i has the same effect on the bank's portfolio choice as an asset-specific tax of $k_i = m_i \cdot r_c$.

There is also an exact equivalence between the two approaches when the quantity of capital is exogenous and shocks to a_i and β_i are idiosyncratic (i.e., independent across assets). In that case, setting a capital requirement on a specific asset has the same effect on the bank's portfolio choice as setting a tax equal to the shadow cost of the capital requirement.

Proposition 11. *Suppose that capital is exogenously set to 1 and the regulator sets a capital requirement of the form*

$$\int_0^1 m_i q_i di \leq 1, \tag{68}$$

where m_i indicates the capital requirement for asset i. If there are no aggregate shocks to a_i and β_i (i.e., a_i and β_i are drawn independently across assets), then the Lagrange multiplier on the capital requirement λ is deterministic. The regulator can induce identical portfolio choice by the bank through setting a linear tax k_i according to

$$k_i = \lambda m_i. \tag{69}$$

Proof. See Section D.10. □

However, the two approaches are not equivalent with systematic shocks. In that case, the Lagrange multiplier λ will be stochastic. The difference between capital requirements and taxes will essentially be a choice of price-based versus quantity-based regulation, the considerations of which are discussed most notably by Weitzman (1974). In the baseline case in which there is a known linear social cost to bank risk-taking, a tax is likely the better solution. But for particular forms of nonlinear social cost functions, quantity-based regulation may be preferable.

B Endogenous capital structure

B.1 Linear taxes with endogenous capital structure

The earlier analysis considers the case of linear taxes with an exogenous capital structure (i.e., banks begin with an exogenous amount of capital). I now consider the optimal policy when endogenizing banks' choice of capital structure.

The bank now selects the amount of equity with which it will fund itself. More equity reduces the social cost of bank risk taking for a given level of risk exposure. However, equity may potentially have both social and private costs relative to other sources of funding. Without delving into exactly why these costs exists, I suppose that the social cost of equity is $r_s > 0$ and the private cost of equity is $r_p \geq r_s > 0$.[10] These reflect the change in the overall cost of capital from funding with an additional unit of equity, not the per-unit cost of equity.

For example, if the Modigliani-Miller capital structure irrelevance theorem were to hold, then $r_s = r_p = 0$. I focus on the cases in which both r_s and r_p are positive since these reflect the interesting cases; if equity were socially costless, then the solution would be to hold sufficient equity such that there is no longer any social cost from bank risk-taking.

The social cost of risk exposure is

$$\eta \frac{\int_0^1 \beta_i q_i di}{e}. \tag{70}$$

This social cost is exactly the same as before, except now the portfolio risk is scaled by the amount of equity e.

I consider a case in which the regulator uses two tools. First, the regulator sets a tax that takes the form

$$\frac{\int_0^1 k_i q_i di}{e}, \tag{71}$$

[10]If $r_s = 0$, so that there is no social cost of equity, then the solution is for the bank to fund itself with as much equity as possible. For reasons of tractability, this model allows for potentially unlimited losses on the portfolio, including losses that are larger than the initial investment. Therefore to cover any potential loss would require equity levels approaching infinity. In a model that incorporated limited liability on investments, costless equity would imply that the bank should fund itself with 100% equity.

where k_i sets the magnitude of the asset-specific tax on asset i, but now it is also scaled by the level of equity e since the social cost of risk also depends on the bank's leverage. For this piece, the regulator solves for the optimal k_i.

Second, the regulator could also potentially subsidize equity at a linear rate of s. So the regulator solves for the optimal s as well.

Proposition 12. *If the bank solves*

$$\max_{\{q_i\},e} \int_0^1 \left((a_i - \gamma\beta_i)q_i - \frac{c}{2}q_i^2 - \frac{k_i q_i}{e} \right) di - (r_p - s)e \tag{72}$$

and the regulator solves

$$\max_{\{k_i\},s} E_R \left[\int_0^1 \left((a_i - \gamma\beta_i)q_i - \frac{c}{2}q_i^2 - \eta\frac{\beta_i q_i}{e} \right) di - r_s e \right], \tag{73}$$

taking into account the bank's optimal choices of q_i and e, then the regulator optimally sets

$$k_i = \eta E_R[\beta_i] \tag{74}$$

and

$$s = r_p - r_s. \tag{75}$$

Proof. See Section D.11. □

The first part of this result – that the regulator sets the asset-specific tax k_i based on the asset's expected risk – is intuitively the same as the case with exogenous equity. The second part – that the regulator should set an equity subsidy – is new. Setting the subsidy like this reduces the bank's marginal cost of equity from r_p to r_s, which aligns it with the social cost.

The key difference when introducing endogenous capital structure is that tax policy also affects the bank's incentives to fund itself with equity. When the regulator can align the private and social costs of equity, then the regulator only needs to worry about aligning asset-specific taxes with expected risks.

However, the situation becomes more complicated when the regulator cannot introduce an equity subsidy and there are differences in the social and private costs of equity. In that case, asset-specific taxes can be used to influence the bank's overall selected level of equity. For example, in typical specifications, setting k_i uniformly higher than $\eta E_R[\beta_i]$ can serve as a crude tax on leverage that encourages banks to fund themselves with more equity.

B.2 Tax on profits with endogenous capital structure

Here I consider the regulator taxing ex-post profits, but allowing for the bank to select its level of equity endogenously. This section generalizes Section B.1 by allowing for a general function $\eta(e)$ to characterize how equity affects the social cost of bank risk-taking instead of assuming that it takes the form $\eta(e) = \eta/e$.

In the case with endogenous equity, the social objective is

$$\int_0^1 \left(a_i q_i - \frac{c}{2} q_i^2 - (\gamma + \eta(e))\beta_i q_i \right) di - r_s e, \tag{76}$$

where $\eta(e)$ is the regulator's additional risk aversion, which now depends on the level of equity, and r_s is the social cost of equity funding. $\eta(e)$ is decreasing in e, reflecting that the social cost of bank risk taking declines as the bank becomes better capitalized since the bank is better able to absorb losses.

The regulator can impose a tax T on the bank. Taking this tax into account, the bank solves

$$\max_{e,\{q_i\}} \int_0^1 \left(a_i q_i - \frac{c}{2} q_i^2 - \gamma \beta_i q_i \right) di - r_p e - E_B[MT], \tag{77}$$

subject to $e \geq 0$, where r_p is the bank's private cost of equity funding.

Proposition 13. *The regulator will perfectly align the bank's incentives with its own by setting*

$$T = \frac{\eta(e)}{\gamma} \left(\frac{1}{M} - 1 \right) \Pi - (r_p - r_s)e. \tag{78}$$

Proof. See Section D.12. □

The present value of the first term ultimately simplifies to $\eta(e) \int_0^1 \beta_i q_i di$, so the bank perceives it as a tax on risk. The second term is a subsidy for equity financing to align the private and social costs of equity.

Since $\eta(e)$ is decreasing in e, the magnitude of the tax on profits is also decreasing as e rises. The intuition is that better-capitalized banks will impose fewer externalities, so the magnitude of taxes on ex-post profits required to align their incentives is lower.

C Generalized social cost function with a linear tax

My focus has been on a known linear social cost function of the form $\eta\beta_p$. However, there may be nonlinearities in practice. For example, risk may have a small marginal social cost at low levels if banks are very unlikely to default. But at higher levels of risk, and with higher probabilities of defaulting, the marginal social cost may rise dramatically. Additionally, there may be uncertainty as to the magnitude of the social costs, even for a known level of risk.

To address these concerns, I consider a generalized social cost function that allows for both nonlinearities and uncertainty over costs.

Proposition 14. *Suppose that the social cost to bank risk-taking is a function $S(\beta_p, \eta)$ that is twice-differentiable, increasing in both arguments, and satisfies $\frac{\partial^2 S}{\partial \beta_p^2}(\beta_p, \eta) \geq 0$. $\beta_p = \int_0^1 q_i \beta_i di$ is the bank's portfolio beta and η is an exogenous random variable. Then the regulator's optimal linear tax is*

$$
\begin{aligned}
k_i &= E_R\left[\frac{\partial S}{\partial \beta_p}(\beta_p, \eta)\beta_i\right] && (79)\\
&= E_R\left[\frac{\partial S}{\partial \beta_p}(\beta_p, \eta)\right] E_R\left[\beta_i\right] + Cov_R\left(\frac{\partial S}{\partial \beta_p}(\beta_p, \eta), \beta_i\right). && (80)
\end{aligned}
$$

Proof. See Section D.5. □

One immediate observation is that if the β_i are independent across assets, then the covariance term will be zero. In that case, the regulator's solution is the same as in the case of the known linear social cost, except replacing the marginal social cost of risk with the *expected* marginal social cost, which is $E_R[\frac{\partial S}{\partial \beta_p}(\beta_p, \eta)]$.

If there is a systematic shock to β_i, then the regulator should additionally consider the covariance term. Assets whose riskiness is higher when the social cost of risk is high should receive a higher tax.

D Additional details on proofs

D.1 The first-best solution

The first-order condition for equilibrium q_i is

$$
\begin{aligned}
a_i - cq_i - (\gamma + \eta)\beta_i &= 0\\
\implies q_i &= \frac{1}{c}(a_i - (\gamma + \eta)\beta_i)
\end{aligned}
\tag{81}
$$

as desired. The second derivative, $-c$, is negative, indicating that this solution is a maximum.

The expected portfolio beta is

$$\begin{aligned} E[\beta_p] &= E\left[\int_0^1 q_i\beta_i di\right] \\ &= E[q_i\beta_i] \\ &= \frac{1}{c}\left(E[a_i\beta_i] - (\gamma+\eta)E[\beta_i^2]\right). \end{aligned} \tag{82}$$

The expected portfolio return is

$$\begin{aligned} E[\mu_p] =& E\left[\int_0^1 (a_i q_i - \frac{c}{2}q_i^2) di\right] \\ =& E[a_i q_i] - \frac{c}{2}E[q_i^2] \\ =& \frac{1}{c}(E[a_i^2] - (\gamma+\eta)E[a_i\beta_i]) - \frac{c}{2}\frac{1}{c^2}(E[a_i^2] - 2(\gamma+\eta)E[a_i\beta_i] + (\gamma+\eta)^2E[\beta_i^2]) \\ =& \frac{1}{2c}\left(E[a_i^2] - (\gamma+\eta)^2E[\beta_i^2]\right). \end{aligned} \tag{83}$$

Social welfare, represented by the regulator's objective function, is

$$\begin{aligned} E[\mu_p - (\gamma+\eta)\beta_p] =& \frac{1}{2c}\left(E[a_i^2] - (\gamma+\eta)^2E[\beta_i^2]\right) - \frac{1}{c}(\gamma+\eta)\left(E[a_i\beta_i] - (\gamma+\eta)E[\beta_i^2]\right) \\ =& \frac{1}{c}\left(\frac{1}{2}E[a_i^2] + \frac{1}{2}(\gamma+\eta)^2E[\beta_i^2] - (\gamma+\eta)E[a_i\beta_i]\right) \\ =& \frac{1}{2c}E[(a_i - (\gamma+\eta)\beta_i)^2]. \end{aligned} \tag{84}$$

D.2 Expression of social welfare

By definition, the regulator's expected social welfare conditional on their information set is

$$\begin{aligned} & E_R[a_i q_i - \frac{c}{2}q_i^2 - (\gamma+\eta)\beta_i q_i] \\ =& E_R[(a_i - (\gamma+\eta)\beta)q_i] - \frac{c}{2}E_R[q_i^2]. \end{aligned} \tag{85}$$

Using the fact that $q_i^{fb} = \frac{1}{c}(a_i - (\gamma + \eta)\beta_i)$ and expanding $q_i = q_i^{fb} + (q_i - f_i^{fb})$ leads to

$$\begin{aligned}
&cE_R[q_i^{fb} q_i] - \frac{c}{2}E_R[(q_i^{fb} + (q_i - q_i^{fb}))^2] \\
=&cE_R[q_i^{fb} q_i] - \frac{c}{2}E_R[(q_i^{fb})^2 + 2q_i^{fb}(q_i - q_i^{fb}) + (q_i - q_i^{fb})^2] \\
=&cE_R[q_i^{fb} q_i] - \frac{c}{2}E_R[(q_i^{fb})^2] - cE_R[q_i^{fb} q_i] + cE_R[(q_i^{fb})^2] - \frac{c}{2}E_R[(q_i - q_i^{fb})^2] \\
=&\frac{c}{2}(E_R[(q_i^{fb})^2] - E_R[(q_i - q_i^{fb})^2]).
\end{aligned} \tag{86}$$

Taking unconditional expectations leads to the desired result.

D.3 Linear tax

The first-order condition for the bank with respect to q_i is

$$a_i - cq_i - \gamma\beta_i - k_i = 0 \tag{87}$$

$$\implies q_i = \frac{1}{c}(a_i - \gamma\beta_i - k_i) \tag{88}$$

and this q_i is a maximum since the second derivative, $-c$, is negative.

The first-order condition for the regulator with respect to k_i, taking as given the bank's choice of q_i, is

$$E_R\left[(a_i - cq_i - (\gamma + \eta])\beta_i)\frac{\partial q_i}{\partial k_i}\right] = 0 \tag{89}$$

$$E_R\left[(k_i - \eta\beta_i)(-\frac{1}{c})\right] = 0 \tag{90}$$

$$k_i = \eta E_R[\beta_i]. \tag{91}$$

Using the result from Proposition 2, the difference in social welfare compared to the first-best is

$$\frac{c}{2}E[(q_i - q_i^{fb})^2] \tag{92}$$

$$=\frac{c}{2}\left(\frac{\eta}{c}\right)^2 E[(-E_R[\beta_i] + \beta_i)^2]. \tag{93}$$

I define $\tilde{\beta}_i = \beta_i - E_R[\beta_i]$, which reflects the regulator's expectational error. I can then rewrite the expression as

$$=\frac{1}{2c}\eta^2 Var(\tilde{\beta}_i). \tag{94}$$

D.4 Command vs. linear tax

Using the result from Proposition 2, the social welfare relative to the first-best is

$$-\frac{c}{2}E[(q_i^{com}-q_i^{fb})^2] \tag{95}$$

$$=-\frac{c}{2}\frac{1}{c^2}E[(\tilde{a}_i-(\gamma+\eta)\tilde{\beta}_i)^2] \tag{96}$$

$$=-\frac{1}{2c}Var(\tilde{a}_i-(\gamma+\eta)\tilde{\beta}_i) \tag{97}$$

Using results on the social welfare in the linear tax case from Proposition 3, the difference in social welfare between the command and the linear tax case is

$$-\frac{1}{2c}\left(Var(\tilde{a}_i-(\gamma+\eta)\tilde{\beta}_i)-\eta^2 Var(\tilde{\beta}_i)\right). \tag{98}$$

This quantity is positive (indicating that the command solution provides higher social welfare) when

$$\eta^2 Var(\tilde{\beta}_i) > Var(\tilde{a}_i-(\gamma+\eta)\tilde{\beta}_i) \tag{99}$$

$$\Longrightarrow \eta^2 Var(\tilde{\beta}_i) > Var(\tilde{a}_i)+(\gamma+\eta)^2 Var(\tilde{\beta}_i)-2(\gamma+\eta)Cov(\tilde{a}_i,\tilde{\beta}_i) \tag{100}$$

$$\Longrightarrow 2(\gamma+\eta)Cov(\tilde{a}_i,\tilde{\beta}_i) > Var(\tilde{a}_i)+\gamma(\gamma+2\eta)Var(\tilde{\beta}_i) \tag{101}$$

D.5 Generalized social cost function and a linear tax

The regulator's goal is to maximize social welfare, which is given by

$$E_R\left[\int_0^1\left(aq_i-\frac{c}{2}q_i^2-\gamma\beta_i q_i\right)di - S(\beta_p,\eta)\right], \tag{102}$$

where $\beta_p=\int_0^1 q_i\beta_i di$ is the portfolio beta. The regulator sets constraints subject to the bank's choice of q_i.

First, consider the case in which a regulator sets an asset-specific linear tax k_i. Given a linear tax, the bank maximizes

$$\max_{\{q_i\}} aq_i-\frac{c}{2}q_i^2-\gamma\beta_i q_i-k_i q_i, \tag{103}$$

which leads to the familiar optimal solution

$$q_i=\frac{1}{c}(a_i-\gamma\beta_i-k_i). \tag{104}$$

Next, turn to the regulator's problem of selecting optimal k_i taking the bank's behavior as given. The regulator solves

$$\max_{\{k_i\}} E_R\left[\int_0^1 \left(aq_i - \frac{c}{2}q_i^2 - \gamma\beta_i q_i\right) di - S(\beta_p, \eta)\right], \tag{105}$$

taking into account the effect on the bank's choice of q_i. The regulator's first-order condition for k_i is

$$E_R\left[\left(a_i - \gamma\beta_i - cq_i - \frac{\partial S}{\partial \beta_p}\beta_i\right)\frac{\partial q_i}{\partial k_i}\right] = 0, \tag{106}$$

which leads to an optimal choice of

$$k_i = E_R\left[\frac{\partial S}{\partial \beta_p}\beta_i\right]. \tag{107}$$

The tax should be set to the expected product of the asset beta multiplied by the marginal social cost. This term can be expanded to include a covariance as

$$k_i = E_R\left[\frac{\partial S}{\partial \beta_p}\right] E_R\left[\beta_i\right] + Cov\left(\frac{\partial S}{\partial \beta_p}, \beta_i\right). \tag{108}$$

I then verify that the second-order condition holds with respect to k_i. The first derivative with respect to k_i can be written as

$$E_R\left[\left(k_i - \frac{\partial S}{\partial \beta_p}(\beta_p, \eta)\beta_i\right)\left(-\frac{1}{c}\right)\right], \tag{109}$$

which leads to an expression for the second derivative as

$$E_R\left[\left(1 + \frac{1}{c}\frac{\partial^2 S}{\partial \beta_p^2}(\beta_p, \eta)\beta_i^2\right)\left(-\frac{1}{c}\right)\right], \tag{110}$$

which is negative since $\frac{\partial^2 S}{\partial \beta_p^2}(\beta_p, \eta) \geq 0$.

D.6 Nonlinear tax

From the regulator's point of view, observing q_i carries the same information as observing $a_i - \gamma\beta_i$. The reason is that the bank's first-order condition for selecting q_i, taking as given the tax function $k_i(q_i)$, is

$$cq_i + k_i'(q_i) = a_i - \gamma\beta_i. \tag{111}$$

Since q_i is sufficient for the regulator to pin down the left-hand side, then it is sufficient to pin down the right-hand side.

To simplify the problem, I take a mechanism design approach where the bank truthfully reports $r_i = a_i - \gamma\beta_i$ to the regulator (the "r" stands for "risk-adjusted return"). The regulator then specifies the quantity $q_i(r_i)$ and the tax $k_i(q_i(r_i))$ to maximize social welfare subject to the bank's incentive compatibility constraints. Collapsing the multidimensional types of a_i and β_i into a single $a_i - \gamma\beta_i$ type is similar in spirit to the approach taken in Beshears et al. (2020). Since in this context the problem is separable across each asset i, for ease of readability I drop the i subscript in the rest of this section.

Let r denote the true r and r' the reported one. The change in the bank's objective from modifying its report r' is

$$\frac{d}{dr'}[rq(r') - \frac{c}{2}q(r')^2 - k(q(r'))] = rq'(r') - cq(r')q'(r') - k'(q(r'))q'(r'). \tag{112}$$

The bank does not have an incentive to deviate when this derivative is zero at $r' = r$, which is when

$$(r - cq(r) - k'(q(r))q'(r) = 0. \tag{113}$$

The bank will optimally want to change its quantity invested as the risk-adjusted return $r = a - \gamma\beta$ changes, so I expect that $q'(r) \neq 0$ in this equilibrium. Therefore, I focus on satisfying the condition

$$cq(r) + k'(q(r)) = r. \tag{114}$$

Meanwhile, the regulator maximizes

$$E[(a - \gamma\beta - \eta\beta)q(r) - \frac{c}{2}q(r)^2|\hat{\beta}, r] \tag{115}$$

$$=(r - \eta E[\beta|\hat{\beta}, r])q(r) - \frac{c}{2}q(r)^2 \tag{116}$$

subject to the bank's incentive compatibility constraint. I first substitute

$$q(r) = \frac{1}{c}(r - k'(q(r))) \tag{117}$$

to obtain the regulator's objective solely in terms of $k'(q(r))$ as

$$(r - \eta E[\beta|\hat{\beta}, r])\frac{1}{c}(r - k'(q(r))) - \frac{c}{2}\left(\frac{1}{c}(r - k'(q(r)))\right)^2. \tag{118}$$

I characterize the regulator's solution in terms of $k'(q(r))$ directly. Differentiating by $k'(q(r))$ and rearranging yields the first-order condition

$$k'(q(r)) = \eta E[\beta|\hat{\beta}, r]. \tag{119}$$

Substituting back into the bank's incentive compatibility condition yields

$$q(r) = \frac{1}{c}\left(r - \eta E[\beta|\hat{\beta}, r]\right). \tag{120}$$

Next, I explicitly characterize $E[\beta|\hat{\beta}, r]$. Note that

$$r = a - \gamma\beta = \bar{a} + \chi u_b + u_a - \gamma(\bar{\beta} + u_b) \tag{121}$$

$$= \bar{a} - \gamma\bar{\beta} + (\chi - \gamma)u_b + u_a \tag{122}$$

so that $r = a - \gamma\beta$ can be transformed to obtain a noisy signal of u_b as follows.

$$\frac{r - \bar{a} + \gamma\bar{\beta}}{\chi - \gamma} = u_b + \frac{u_a}{\chi - \gamma}. \tag{123}$$

The noise term is $\frac{u_a}{\chi-\gamma}$ and it is independent of e, so that the error terms in this signal and $\hat{\beta}_i$ are independent. Given the assumptions of normality, the expectation is

$$E[\beta|\hat{\beta}, r] = \bar{\beta} + \omega_1(\hat{\beta} - \bar{\beta}) + \omega_2\left(\frac{r - \bar{a} + \gamma\bar{\beta}}{\chi - \gamma}\right), \tag{124}$$

where

$$\omega_1 = \frac{1/\sigma_e^2}{1/\sigma_{ub}^2 + 1/\sigma_e^2 + (\chi - \gamma)^2/\sigma_{ua}^2} \tag{125}$$

$$\omega_2 = \frac{(\chi - \gamma)^2/\sigma_{ua}^2}{1/\sigma_{ub}^2 + 1/\sigma_e^2 + (\chi - \gamma)^2/\sigma_{ua}^2}. \tag{126}$$

For notational simplicity, I will define x and y such that

$$\eta E[\beta|\hat{\beta}, r] = x + yr, \tag{127}$$

where

$$x = \eta\left(\bar{\beta} + \omega_1(\hat{\beta}_i - \bar{\beta}) + \omega_2\left(\frac{-\bar{a} + \gamma\bar{\beta}}{\chi - \gamma}\right)\right) \tag{128}$$

$$y = \frac{\eta}{\chi - \gamma}\omega_2. \tag{129}$$

Using this notation, straightforward algebra leads to an explicit expression of $q(r)$ as

$$q(r) = \frac{1}{c}\left((1-y)r - x\right). \tag{130}$$

I then solve for the $k(q)$ function explicitly. I first solve for r in terms of $q(r)$. Return to the first-order conditions for the bank and the regulator of

$$k'(q(r)) = r - cq(r) \tag{131}$$
$$k'(q(r)) = \eta E[\beta|\hat{\beta}, r] = x + yr. \tag{132}$$

Setting the right-hand sides of each equation equal to each other yields

$$r - cq(r) = x + yr \tag{133}$$
$$\implies r = \frac{x + cq(r)}{1-y} \tag{134}$$

and then substitute in to the first equation to get

$$k'(q(r)) = \frac{x + cq(r)}{1-y} - cq(r) \tag{135}$$
$$= \frac{x}{1-y} + c\left(\frac{y}{1-y}\right)q(r). \tag{136}$$

Taking the antiderivative with respect to $q(r)$ obtains

$$k(q(r)) = \frac{x}{1-y}q(r) + \frac{c}{2}\left(\frac{y}{1-y}\right)q(r)^2, \tag{137}$$

where I omit the constant since $k(0) = 0$ by assumption.

Now, I check the second-order conditions. The second derivative for the regulator's objective function is $-(1/c)$, so the regulator's choice is a maximum. The second-order conditions for the bank are more complicated. First, I note that

$$k''(q(r)) = c\left(\frac{y}{1-y}\right). \tag{138}$$

The bank's second-order condition is satisfied when

$$k''(q(r)) + c > 0 \tag{139}$$

$$\iff c\left(\frac{y}{1-y} + 1\right) > 0 \tag{140}$$

$$\iff y < 1. \tag{141}$$

Then, plugging in the value of y from earlier, I have

$$\frac{\eta}{\chi - \gamma}\omega_2 < 1, \tag{142}$$

which is equivalent to

$$\chi - \gamma < 0 \text{ or } \chi - \gamma > \eta\omega_2. \tag{143}$$

Note that ω_2 is implicitly a function of $\chi - \gamma$, but since $\omega_2 \in [0,1]$, $\chi - \gamma > \eta$ is a sufficient condition. Therefore, for this range of parameters, the bank's solution is a maximum.

D.7 Fully undisclosed linear tax

To verify that these pair of choices are a solution, I first suppose that the regulator sets $k_i = \eta\hat{\beta}_i$. The bank's first-order condition for q_i leads to

$$q_i = \frac{1}{c}(a_i - \gamma\beta_i - \eta E_B[\hat{\beta}_i]) \tag{144}$$

$$= \frac{1}{c}(a_i - \gamma\beta_i - \eta E[\bar{\beta} + u_i^b + e_i^b | u_i^b]) \tag{145}$$

$$= \frac{1}{c}(a_i - \gamma\beta_i - \eta \underbrace{(\bar{\beta} + u_i^b)}_{\beta_i}) \tag{146}$$

which is the desired solution. Since this quantity matches the first-best quantity from Proposition 1, it also maximizes the regulator's objective function.

Note that this solution is not unique. For all random variables x_i satisfying $E_B[x_i] = \beta_i$, the regulator will achieve first-best by setting $k_i = \eta x_i$. While $\hat{\beta}_i$ is an obvious candidate, $\hat{\beta}_i$ with added noise or the fitted value from regressing β_i on $\hat{a}_i$ are other possibilities.

D.8 Partially disclosed linear tax

First, I'll establish the optimality condition for k_i. Starting with the bank's problem, the bank maximizes given knowledge over the distribution of k_i. The bank solves

$$\max_{\{q_i\}} \int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - \gamma \beta_i q_i - E_B[k_i] q_i) di, \tag{147}$$

which leads to the first-order condition

$$q_i = \frac{1}{c}(a_i - \gamma \beta_i - E_B[k_i]). \tag{148}$$

The second-order condition is satisfied since $-c < 0$, which ensures that the solution is a maximum.

Given this choice of q_i for the bank, the regulator solves

$$\max_{\{k_i\}} E_R \left[\int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - (\gamma + \eta) \beta_i q_i) di \right], \tag{149}$$

which leads to a first-order condition of

$$E_R \left[(a_i - c q_i - (\gamma + \eta) \beta_i) \underbrace{\frac{\partial q_i}{\partial k_i}}_{= -(1/c)} \right] = 0 \tag{150}$$

$$\implies E_R[E_B[k_i]] = \eta E_R[\beta_i] \tag{151}$$

as desired.

Next, I show that the proposed solution satisfies this condition. I conjecture that the regulator sets a linear tax according to

$$k_i = \eta(\bar{\beta} + y(\hat{\beta}_i - \bar{\beta})), \tag{152}$$

where y is an unknown constant. I then solve for the y that satisfies the previous first-order condition.

First, I begin by computing the left-hand side of the first-order condition, $E_R[E_B[k_i]]$. Start by taking the expectation of the tax with respect to the bank's information set as

$$E_B[k_i] = \eta(\bar{\beta} + y E_B[\hat{\beta}_i - \bar{\beta}]) \tag{153}$$

$$= \eta(\bar{\beta} + y(u_i + E_B[e_i])). \tag{154}$$

Recall that the bank receives a noisy signal $s_i = \hat{\beta}_i + w_i$. Since the bank perfectly observes β_i, then $s_i - \beta_i = e_i + w_i$ is a noisy signal of e_i. Applying standard formulas for Bayesian updating with normally distributed variables yields

$$E_B[e_i] = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_w^2}(s_i - \beta_i) \tag{155}$$

$$= \underbrace{\frac{\sigma_e^2}{\sigma_e^2 + \sigma_w^2}}_{\theta_w}(e_i + w_i). \tag{156}$$

Thus, the bank's expectation of the tax is

$$E_B[k_i] = \eta(\bar{\beta} + y(u_i + \theta_w(e_i + w_i))). \tag{157}$$

Now take the regulator's expectation of this quantity. $E_R[w_i] = 0$, so the key pieces are $E_R[e_i]$ and $E_R[u_i]$. Using the relationship

$$\hat{\beta}_i = \beta_i + e_i \tag{158}$$

$$\implies \hat{\beta}_i = E_R[\beta_i] + E_R[e_i] \tag{159}$$

and recalling that $E_R[\beta_i] = \bar{\beta} + \theta_b(\hat{\beta}_i - \bar{\beta})$ yields the relationships

$$E_R[e_i] = (1 - \theta_b)(\hat{\beta}_i - \bar{\beta}) \tag{160}$$

$$E_R[u_i] = \theta_b(\hat{\beta}_i - \bar{\beta}_i). \tag{161}$$

Substituting these leads to the expression

$$E_R[E_B[k_i]] = \eta(\bar{\beta} + y(E_R[u_i] + \theta_w E_R[e_i])) \tag{162}$$

$$= \eta(\bar{\beta} + y(\theta_b + \theta_w(1 - \theta_b))(\hat{\beta}_i - \bar{\beta})). \tag{163}$$

Now I solve for a value of y that satisfies $E_R[E_B[k_i]] = \eta E_R[\beta_i]$. y must ensure that

$$\eta(\bar{\beta} + y(\theta_b + \theta_w(1 - \theta_b))(\hat{\beta}_i - \bar{\beta})) = \eta(\bar{\beta} + \theta_b(\hat{\beta}_i - \bar{\beta})) \tag{164}$$

$$\implies y(\theta_b + \theta_w(1 - \theta_b)) = \theta_b, \tag{165}$$

which leads to

$$y = \frac{\theta_b}{\theta_b + \theta_w(1 - \theta_b)} \tag{166}$$

as desired.

D.9 Nondisclosure with an aversion to idiosyncratic volatility

If the regulator sets the tax in this manner, then the bank's objective function transforms into the regulator's, which immediately leads to the first best. Setting the two objectives equal, the bank and regulator have perfectly-aligned incentives when

$$E_B[k]q + \gamma_I \sigma_B(k)|q| = \eta\beta q + \eta_I \sigma_B(k)|q| \tag{167}$$

$$\iff E_B[k] + \gamma_I \sigma_B(k) sgn(q) = \eta\beta + \eta_I \sigma_B(k) sgn(q). \tag{168}$$

Recall that $\hat{\beta} = \beta + e$ and that the bank knows β, but has no information about e. Therefore $E_B[\hat{\beta}] = \beta$ and $\sigma_B(\hat{\beta}) = \sigma_e$. Using this fact yields

$$E_B[k] = (\eta_I - \gamma_I)\eta\sigma_e sgn(q) + \eta\beta \tag{169}$$

$$\sigma_B(k) = \eta\sigma_e. \tag{170}$$

Therefore it follows that the equation

$$E_B[k] + \gamma_I \sigma_B(k) sgn(q) = \eta\beta + \eta_I \sigma_B(k) sgn(q) \tag{171}$$

$$(\eta_I - \gamma_I)\eta\sigma_e sgn(q) + \eta\beta + \gamma_I \eta\sigma_e sgn(q) = \eta\beta + \eta_I \eta\sigma_e sgn(q) \tag{172}$$

$$\eta_I \eta\sigma_e sgn(q) + \eta\beta = \eta_I \eta\sigma_e sgn(q) + \eta\beta \tag{173}$$

is satisfied and thus the incentives of the regulator and the bank are aligned.

D.10 Equivalence between taxes and capital requirements with exogenous capital and idiosyncratic shocks

If the regulator has set a capital requirement, then the bank solves

$$\max_{\{q_i\}} \int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - \gamma\beta_i q_i) di \tag{174}$$

subject to the constraint that

$$\int_0^1 m_i q_i di \leq 1. \tag{175}$$

Setting up the Lagrangian, the problem becomes

$$\max_{\{q_i\}} \int_0^1 (a_i q_i - \frac{c}{2} q_i^2 - \gamma\beta_i q_i) di - \lambda \left(\int_0^1 m_i q_i di - 1 \right), \tag{176}$$

which leads to the first-order condition for q_i of

$$a_i - cq_i - \gamma\beta_i - \lambda m_i = 0 \tag{177}$$

$$\Longrightarrow q_i = \frac{1}{c}(a_i - \gamma\beta_i - \lambda m_i). \tag{178}$$

Compare this first-order condition to the case in which the regulator sets an asset-specific tax k_i, in which case the bank selects

$$q_i = \frac{1}{c}(a_i - \gamma\beta_i - k_i). \tag{179}$$

The regulator can replicate the outcome from the capital requirement by setting a tax $k_i = \lambda m_i$. However, this replication is possible in this case because λ is deterministic, which I will show next.

If the capital requirement is not binding, then $\lambda = 0$. Otherwise, multiplying the first-order condition for q_i by m_i and then integrating over i yields

$$\int_0^1 m_i q_i di = \frac{1}{c}\int_0^1 (m_i(a_i - \gamma\beta_i) - \lambda m_i^2)di = 1 \tag{180}$$

In the case in which there is only idiosyncratic uncertainty in a_i and β_i, the integral almost certainly equals the expectation, so that

$$\frac{1}{c}(E[m_i(a_i - \gamma\beta_i)] - \lambda E[m_i^2]) = 1 \tag{181}$$

$$\Longrightarrow \lambda = \frac{E[m_i(a_i - \gamma\beta_i)] - c}{E[m_i^2]}, \tag{182}$$

which is deterministic. Intuitively, the idiosyncratic shocks to a_i and β_i diversify away in the aggregate so that the regulator understands how tightly the capital requirements will bind overall.

D.11 Linear taxes with endogenous capital structure

I will show that the proposed solution of $s = r_p - r_s$ and $k_i = \eta E_R[\beta_i]$ satisfies the conditions for optimal s and k_i.

I start with the bank's problem, taking s and k_i as given. The bank solves

$$\max_{\{q_i\},e} \int_0^1 \left((a_i - \gamma\beta_i)q_i - \frac{c}{2}q_i^2 - \frac{k_i q_i}{e}\right) di - (r_p - s)e. \tag{183}$$

The first-order conditions yield

$$q_i = \frac{1}{c}\left(a_i - \gamma\beta_i - \frac{k_i}{e}\right) \tag{184}$$

$$e = \sqrt{\frac{k_p}{r_p - s}}. \tag{185}$$

The second derivatives are

$$\text{Second of } q_i : -c \tag{186}$$

$$\text{Second of } e : -2\frac{k_p}{e^3}. \tag{187}$$

At $e = \sqrt{\frac{k_p}{r_p - s}}$ and $s = r_p - r_s$, the second derivative with respect to e is

$$e = -2\frac{k_p}{e^2}\frac{1}{e} = -2\frac{r_s}{e}, \tag{188}$$

which is negative since both $r_s > 0$ and $e > 0$. Thus, the second-order conditions for a maximum are satisfied.

Next, switch to the regulator's problem. The regulator solves

$$\max_{\{k_i\},s} E_R\left[\int_0^1 \left((a_i - \gamma\beta_i)q_i - \frac{c}{2}q_i^2 - \eta\frac{\beta_i q_i}{e}\right)di - r_s e\right], \tag{189}$$

where q_i and e follow the bank's strategy from before. The first-order conditions with respect to k_i and s yield

$$E_R\left[\int_0^1 \left(a_i - \gamma\beta_i - cq_i - \eta\frac{\beta_i}{e}\right)\frac{\partial q_i}{\partial k_i}di + \left(\eta\frac{\int_0^1 \beta_i q_i di}{e^2} - r_s\right)\frac{\partial e}{\partial k_i}\right] = 0. \tag{190}$$

Substitute the expressions for cq_i and for e^2 to obtain

$$E_R\left[\int_0^1 \left(\frac{k_i - \eta\beta_i}{e}\right)\frac{\partial q_i}{\partial k_i}di + \left(\eta\frac{\int_0^1 \beta_i q_i di}{\int_0^1 k_i q_i di}(r_p - s) - r_s\right)\frac{\partial e}{\partial k_i}\right] = 0. \tag{191}$$

To verify that this first-order condition is satisfied for $s = r_p - r_s$ and $k_i = \eta E_R[\beta_i]$, substitute in these values and apply the law of iterated expectations to obtain

$$E_R\left[E\left[\int_0^1 \left(\frac{\eta E_R[\beta_i] - \eta\beta_i}{e}\right)\frac{\partial q_i}{\partial k_i}di + \frac{r_s}{\int_0^1 k_i q_i di}\int_0^1 (\eta\beta_i - \eta E_R[\beta_i])\, q_i\frac{\partial e}{\partial k_i}di\,\middle|\, e, \{q_i\}, \{\hat{\beta}_i\}, \{\hat{a}_i\}\right]\right] = 0, \tag{192}$$

which follows since $E_R[\beta_i - E_R[\beta]] = 0$. Showing that the first-order condition with respect to s is satisfied is identical, except that $\frac{\partial q_i}{\partial k_i}$ and $\frac{\partial e}{\partial k_i}$ are replaced with $\frac{\partial q_i}{\partial s}$ and $\frac{\partial e}{\partial s}$.

D.12 Taxes on profits with endogenous capital structure

Social welfare equals the bank's objective function when

$$\int_0^1 \left(a_i q_i - \frac{c}{2} q_i^2 - (\gamma + \eta(e))\beta_i q_i\right) di - r_s e = \int_0^1 \left(a_i q_i - \frac{c}{2} q_i^2 - \gamma \beta_i q_i\right) di - r_p e - E_B[MT], \tag{193}$$

which simplifies to

$$E_B[MT] = \eta(e) \int_0^1 \beta_i q_i di - (r_p - r_s)e. \tag{194}$$

The next step verifies that the specified T leads to this expression for $E_B[MT]$. Recall that $M = 1 - \gamma F$, where F is normalized such that $E[F] = 0$ and $Var(F) = 1$. The $E_B[MT]$ term becomes

$$\begin{aligned}
E[MT] &= \frac{\eta(e)}{\gamma}(E_B[\Pi] - E_B[M\Pi]) - E_B[M(r_p - r_s)e] && (195)\\
&= -\frac{\eta(e)}{\gamma} Cov_B(M, \Pi) - (r_p - r_s)e && (196)\\
&= -\frac{\eta(e)}{\gamma} Cov_B\left(1 - \gamma F, \int_0^1 \beta_i q_i F\right) - (r_p - r_s)e && (197)\\
&= \frac{\eta(e)}{\gamma}\gamma \int_0^1 \beta_i q_i di - (r_p - r_s)e && (198)\\
&= \eta(e) \int_0^1 \beta_i q_i di - (r_p - r_s)e, && (199)
\end{aligned}$$

as desired. This algebra uses the decomposition $E[M\Pi] = E[M]E[\Pi] + Cov(M, \Pi)$.