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Is the Rent Too High? Aggregate Implications of Local Land-Use Regulation

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Abstract

Highly productive U.S. cities are characterized by high housing prices, low housing stock growth, and restrictive land-use regulations (e.g., San Francisco). While new residents would benefit from housing stock growth in cities with highly productive firms, existing residents justify strict local land-use regulations on the grounds of congestion and other costs of further development. This paper assesses the welfare implications of these local regulations for income, congestion, and urban sprawl within a general-equilibrium model with endogenous regulation. In the model, households choose from locations that vary exogenously by productivity and endogenously according to local externalities of congestion and sharing. Existing residents address these externalities by voting for regulations that limit local housing density. In equilibrium, these regulations bind and house prices compensate for differences across locations. Relative to the planner's optimum, the decentralized model generates spatial misallocation whereby high-productivity locations are settled at too-low densities. The model admits a straightforward calibration based on observed population density, expenditure shares on consumption and local services, and local incomes. Welfare and output would be 1.4% and 2.1% higher, respectively, under the planner's allocation. Abolishing zoning regulations entirely would increase GDP by 6%, but lower welfare by 5.9% because of greater congestion.

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1 Introduction

Neighborhoods in productive, high-rent regions have very strict controls on housing development and very limited new housing construction. Home to Silicon Valley, the San Francisco Bay Area is the most productive and most expensive metropolitan region in the country, and yet new housing construction has been very slow, especially in contrast to less-productive large cities like Houston, Texas.[1] The evidence suggests that this slow-growth environment results from locally determined regulatory constraints.[2] Existing residents justify these constraints by appealing to the costs of new development, including increased vehicle traffic and other types of congestion, and claim that they see few, if any, of the benefits from new development.[3] However, the effects of local regulation extend beyond the local regulating authorities: regions with highly regulated municipalities experience less-elastic housing supply (Glaeser et al., 2006; Saiz, 2010).[4]

This paper takes seriously their concerns while assessing quantitatively the aggregate welfare costs of local land-use regulation. To do so, I build a spatial equilibrium model of housing, location choice, and endogenous regulation. To motivate households' preference for zoning, I introduce local externalities of agglomeration (e.g., residents can share fixed-cost infrastructure or a diversity of restaurants) and congestion (e.g., traffic or limited on-street parking) to a standard model featuring locations with heterogeneous productivity. In line with the externalities under study, I identify these locations with *neighborhoods*.[5] Existing residents address these externalities by establishing *zoning laws* that limit development within their location. The tradeoff between agglomeration and congestion ensures that households prefer a positive but limited number of co-residents. Compared with the utilitarian planner's allocation, high-productivity locations are developed less intensively, low-productivity loca-

[1] The gross domestic product per capita in the Bay Area of over \$92,000 for 2014 is higher than that of all countries outside of the tiny banking center Luxembourg. Even for workers in non-tech sectors, wages in the region are higher than elsewhere in the United States: median earnings for those with just a high school diploma are 12% higher.

As of the 2014 American Community Survey one-year estimates, the Metropolitan Statistical Areas centered on San Jose, San Francisco, and Santa Cruz have median home values of \$735,400, \$657,300, and \$615,200, respectively.

The region of 6.1 million people permitted just 19,174 new housing units in 2015, consistent with a housing-stock growth rate of 0.79% annually. For comparison, similar-sized but lower-wage Houston and Atlanta grew at 2.3% and 1.4%, respectively. These comparisons are visualized in Figure 3 in the appendix.

[2] This point is made by Glaeser and Gyourko (2002, 2003) and Glaeser et al. (2006), among others.

[3] For example, the group Livable Boulder supports a ballot initiative that it argues will "ensure that City levels of service are not diminished by new development" due to concerns about "huge buildings, blocked views of the mountains, more congestion, proposals to change the unique character of many Boulder neighborhoods" (`http://livableboulder.org/initiatives-proposed-by-livable-boulder/initiative-development-shall-pay-its-own-way/` and `http://livableboulder.org/`).

[4] That regulations are determined locally—at the level of a neighborhood, city district, or municipality—is argued by Hills and Schleicher (2011, 2014), Schleicher (2013), and Monkkonen and Quigley (2008).

[5] Neighborhood productivity differences within and across cities arise from differential access to employment opportunities, transportation networks, or productive amenities.

tions are developed more intensively, and too many (low-productivity) locations are opened in the local-zoning equilibrium. Implementing the planner's allocation would increase gross domestic product (GDP) by 2.1% and welfare by 1.4%.

The key model components—fixed costs, congestion, and endogenous local regulation—are grounded in empirical evidence. The fixed cost of infrastructure and local services can explain the sharp edge of development visible in metropolitan areas, in contrast with the predictions of the standard monocentric city model of a smooth gradient approaching zero density at the fringe.[6] Local congestion resulting from fixed quantities of street parking, from limited space for children's play, and from concerns about sunshine, shadows, and wind permeate antidevelopment discourse.[7] Finally, zoning laws in U.S. cities are the results of neighborhood or municipal (and not metropolitan) political processes (Hills and Schleicher, 2014; Monkkonen and Quigley, 2008).[8]

I model a large set of locations analogous to the neighborhoods or municipalities of the national economy. Opening a location to settlement necessitates payment of a fixed cost that can be shared more broadly by increasing the number of residents: agglomeration. Households in a location experience congestion as disutility from the addition of further co-residents. The forces of agglomeration and congestion constitute the *endogenous amenities* provided by a location. In light of those forces, local residents choose zoning laws that restrict the maximum number of housing units that can be built by a competitive construction sector. Distributions of house prices, zoning laws, and household location choices are jointly determined in equilibrium. Households are *ex ante* homogeneous and fully mobile, and so house prices adjust to ensure that households are indifferent between all occupied locations. In equilibrium, productive locations have binding zoning restrictions that cause house prices to become significantly higher than the marginal cost of construction, an empirical finding documented in Glaeser and Gyourko (2002, 2003), Glaeser et al. (2005a,b), Cheshire and Hilber (2008), and Koster et al. (2012).

I use this model to analyze the welfare implications of local zoning laws by comparing the equilibrium allocation with the optimal allocation chosen by a constrained planner.[9] When choosing the intensity of development within a location, the planner considers the local forces of agglomeration and congestion as well as the value of placing the marginal household in a location of higher or lower productivity. When passing zoning laws, households consider only the local amenity forces and thus restrict development too heavily in productive locations.

[6]For a graphic example, see Figure 4 in the appendix.

[7]A recent example from a zoning debate in Santa Monica, California is described at `http://smdp.com/santa-monica-beach-town-dingbat-city/147819`.

[8]Relatedly, Aura and Davidoff (2008) argue that housing and land demand elasticities imply that any particular municipality would be unable to increase supply sufficiently to move down housing demand curves to lower prices. Conversely, municipal (or neighborhood) governments are limited in their ability to act as monopolists and raise prices by simply restricting supply. That they restrict development anyway suggests they are motivated by concerns like local externalities.

[9]The planner is *constrained* in that it maximizes welfare subject to a spatial equilibrium constraint: identical households must receive identical utility, regardless of location.

This underdevelopment induces *sprawl*: the opening of too many unproductive locations, consistent with, e.g., Fischel (1999). I further show that the planner's allocation can be implemented through a zoning reform whereby residents choose all zoning laws at the national level.

The model enables a straightforward calibration. I calibrate the utility function to the share of household expediture going to non-local goods excluding housing and local services. For the local sharing parameter, I calibrate to the share of spending on local services. The congestion cost curvature is chosen to match the average population density of urban census tracts.[10] For location productivity, I use census-tract income data, adjusted for average household observable characteristics (Hsieh and Moretti, 2015). I calibrate construction-sector productivity to match the ratio of price to construction costs in high-price locations like New York and San Francisco where the effects of zoning laws have been well studied (see Glaeser et al., 2005b).

The model provides a unified framework for addressing the costs and benefits of reforming local zoning policy by implementing the optimal allocation. Avent (2011) and Hsieh and Moretti (2015) argue that these local growth patterns have been quite costly for national growth, while incumbent residents argue that new development would itself be costly. My main exercise is to quantify these costs while also accounting for the changes to endogenous amenities resulting from the planner's allocation. The planner's allocation raises welfare and GDP by 1.4% and 2.1%, respectively. The more intensive development of productive locations necessary to increase GDP would also increase the congestion disamenity received by the residents of these regions. The median resident experiences an increase in density of 3.6%; increases of as much as 10–15% are experienced by the most productive locations. More intensive development reduces the total number of locations opened to development by 3%. The planner's allocation thus features less *sprawl*, with less fixed-cost expenditure and commensurately higher consumption.

A second exercise offers a quantitative assessment of an alternative policy: zoning abolition, wherein developers are free to build housing without constraint.[11] This policy results in overbuilding in high-productivity areas, as construction firms do not internalize congestion effects. Model GDP increases by 6%, but increased congestion causes welfare to decline by 5.9%. The additional output arises because with the constant-returns production technology, productive regions like San Francisco see population inflows of 50% or more. The policy implication is that zoning should be relaxed but not abolished in productive locations.

Treating zoning as endogenous enables the inference of welfare losses from zoning in the absence of detailed data on the wedges between housing prices and marginal costs of construction at the location level. Were such data available the wedges could be treated as

[10] While the parameter governs the shape of the congestion cost function, in equilibrium it has a first-order effect on the optimal zoning laws and thus on average density. The approach is similar to the tradition of using wage shares to calibrate labor and capital exponents in a production function: in equilibrium, these shape parameters have first-order effects on levels.

[11] This option is implicitly studied by Hsieh and Moretti, who find that output would increase by 13.5%.

exogenous and the welfare costs of zoning would be the costs of moving the wedges from their observed to their optimal level. In principle, observed density could be used to proxy for underlying regulations. The relationship between productivity and zoned density, however, is confounded in the data by factors like exogenous amenities—which make land more expensive and increase density—and long-lived investments, both in building stock and in infrastructure such as subway networks. Instead of attempting to account precisely for these myriad forces, I model zoning as an equilibrium object, infer the unobserved wedge from the model, and calculate the loss in welfare accordingly. The key qualitative prediction of this model is that, in high-productivity areas, density does not rise fast enough relative to productivity.

Endogenous zoning has a further benefit: it offers predictions on the likely outcomes of different zoning and housing policy reforms. For example, policies that ignore the endogeneity of zoning may be unable to meet their stated goals. The federal government provides large housing subsidies throughout the income distribution, and increased subsidies may appear to offer an outlet for reducing the burden of high costs in expensive cities. Within the specified model, a housing subsidy would increase household willingness to pay in expensive areas. However, endogenous zoning driven by existing residents will not respond to these subsidies by creating more supply, and so the policy will not have the intended effect of making housing more affordable, nor will it stimulate additional supply in zoning-constrained locations.

Additionally, the model provides guidance for the outcomes of direct zoning reforms. Can a single location allow more development—increasing housing supply—and hope to bring down prices? Consider an infinitesimal neighborhood in the model that relaxes its zoning restriction and allows more development. The outside option—the value of locating elsewhere—will be unaffected, as a single location is insufficient to move the price gradient. Instead, house prices in the neighborhood will decline only to the extent that additional development makes congestion worse. This reasoning echoes the complaints of homeowners who fear new development will hurt their house values.[12]

Finally, could a successful *upzoning*, whereby a set of productive locations increase the zoned capacity or even abolish zoning restrictions, harm landowners in less productive locales? Suppose that an entire productive region—perhaps a city or a state—chooses to allow more intensive development. Many new households will flow in, abandoning the less productive locations they once occupied. For the least productive locations, the initial population outflows will make the fixed costs too burdensome, and all households will choose to leave. If profits from the development of a location are tied to that location, perhaps via homeownership, then residents of less productive locations may lose out from this policy reform.

[12]For example, this claim is offered by an advocate for a moratorium on development in the Mission District of San Francisco here: `https://medium.com/@danancona/putting-market-fundamentalism-on-hold-432ecf1aab3c`.

1.1 Related Literature

This paper draws on work that has established a strong relationship between zoning and the elasticity of housing supply (Fischel, 1999; Mayer and Somerville, 2000; Glaeser and Gyourko, 2002; Glaeser et al., 2005a,b; Saiz, 2010; Koster et al., 2012; Turner et al., 2014).[13] This literature has shown that zoning laws restrict the supply response to high house prices, and that cities that do zone restrictively have systematically higher housing prices. Following this literature, zoning laws in this paper shape local amenities (cf. Turner et al., 2014) and regional housing supply (cf. Fischel, 1999; Mayer and Somerville, 2000).

Several authors in diverse contexts have modeled zoning as an endogenous outcome of political processes (Hamilton, 1975; Fischel, 1987; Monkkonen and Quigley, 2008; Fischel, 2009; Hilber and Robert-Nicoud, 2013; Ortalo-Magné and Prat, 2014; Hills and Schleicher, 2011; Schleicher, 2013; Hills and Schleicher, 2014; Fischel, 2015). I implement the findings of, e.g., Fischel (2009) and Hills and Schleicher (2011) that zoning laws are actively determined by highly engaged utility-maximizing households at the local level—municipalities, neighborhoods, or even direct neighbors.[14] In some contexts, zoning enables households to implement the planner's allocation, usually by resolving problems of free-riding on local expenditure. I complement this literature by embedding a set of heterogeneous locations into a general-equilibrium model so that zoning is locally optimal but has aggregate external costs. Incorporating the endogeneity of local zoning enables a welfare calculation in the absence of detailed local data and also reflects the consensus of the literature.

My work builds on Hsieh and Moretti (2015), who also link strict zoning and changes in aggregate output. They study the productivity effects of wage dispersion across metropolitan areas in a Rosen-Roback model and attribute the increase in this dispersion (and resulting loss in output) to a decrease in the elasticity of housing supply. From this basis, I introduce local externalities and calculate welfare losses from zoning relative to the planner's optimum, in addition to output losses from spatial misallocation. My paper differs in terms of the economic interpretation of zoning: here, it is an endogenous limit on neighborhood development instead of a change in the elasticity of metropolitan housing supply. As noted, this endogeneity serves two purposes: it enables me to infer the costs of welfare in the absence of location-specific data on marginal costs and housing prices, and it allows me to study the likely local outcomes of different policy reforms.

This paper is related to others that study the conditions under which zoning laws and other local government interventions may be theoretically optimal (Stull, 1974; Hamilton, 1975; Henderson, 1991; Hochman et al., 1995; Rossi-Hansberg, 2004; Calabrese et al., 2007; Chen and Lai, 2008; Allen et al., 2015). These papers focus on a variety of externalities to motivate zoning, including nuisance, production, and free-riding interactions from which I abstract. Of course, the optimal zoning scheme depends on the nature of the externalities

[13]Gyourko and Molloy (2014) present an in-depth review of the economics of housing regulation.

[14]Lawsuits seeking to halt development in the name of various ills are increasingly common (Ganong and Shoag, 2013).

under consideration. Similarly to some of these papers, I characterize the zoning regime consistent with maximizing welfare given the externalities studied. This paper also addresses the optimal distribution of population between communities that share local public goods (Flatters et al., 1974; Arnott and Stiglitz, 1979; Hochman et al., 1995). Like Hochman et al. (1995), I find that jurisdictions that are well positioned to deal with one externality may not be optimal in the presence of other inter-location effects.

More broadly, this paper relates to others that study the spatial determinants of aggregate productivity and welfare (Albouy, 2012; Desmet and Rossi-Hansberg, 2013; Allen and Arkolakis, 2014; Behrens et al., 2014; Eeckhout and Guner, 2015; Morales et al., 2015; Hsieh and Moretti, 2015). I also account for the joint spatial distributions of household location choices, incomes, and housing prices while incorporating the effects of zoning regulations. As such, the paper relates to Glaeser et al. (2005a), Van Nieuwerburgh and Weill (2010), Gyourko et al. (2013), and Diamond (2016), among others.

The rest of the paper is organized as follows. Section 2 presents the model and the analytic results. Section 3 calibrates a quantitative version of the model to match features of the data. Section 4 quantitatively analyzes the welfare costs of local zoning and the no-zoning counterfactual relative to the planner's optimal allocation. Section 5 concludes.

2 Model

This section describes the model. I begin by outlining the environment. I then study the determination of equilibrium in three steps. First, I study the *local equilibrium* that results from the actions of households and firms, which take as given the local zoning rules as well as the endogenous outside option and level of profit. Second, I study the *local zoning choice* of initial residents. The zoning law restricts the level of housing production available to construction firms. Third, I study the economy-wide *general spatial equilibrium* in which the aggregate variables are determined: the endogenous outside-option level of utility and aggregate profits of construction firms.

Environment The spatial environment consists of a mass of locations of measure 1 with productivity indexed by $x \in [0, 1]$. These locations represent the set of potential neighborhoods in the entire economy. The neighborhood with $x = 1$ is the most productive in the country, e.g., downtown San Francisco. Locations with x less than but close to 1 include less productive locations in the same area—for instance, Oakland, a train ride from downtown San Francisco. A similar x could also correspond to the best locations in less productive regions, like downtown Denver. Locations with x close to zero would be at the far fringes of metro areas: either a short commute to an unproductive job or a very long commute to a better job.[15]

Production takes place at each location according to a linear production function. Productivity is given by a continuous function $y(x)$ with $y(0) = \underline{y}$, $y'(x) > 0$, and $y(1) = \bar{y}$. Each location begins empty and available for settlement by a measure 1 of households, whose preferences are described later. Opening a location to urban settlement requires a fixed cost F. The set of locations that are open in equilibrium is an endogenous outcome.[16]

I now turn to the description of agents and their maximization problems.

2.1 Local Equilibrium

2.1.1 Household and Construction Firm Problems

The economy consists of three types of agents. First, there is a measure 1 of households that choose location and consumption to maximize utility, taking as given location-specific rents, population, and the endogenous outside option (i.e., the maximum attainable utility from making the optimal choice of location).

Second, there is a representative construction firm that chooses the housing production level to maximize profits at each location, taking as given rents and the local zoning law. I represent a zoning law as a maximum allowable number of housing units.[17]

Third, some locations begin with a measure 0 of *initial residents*. The initial residents choose local zoning laws and consumption to maximize utility while taking as given a pre-fixed initial rent.[18] Initial residents have rational expectations about the equilibrium choices that

[15] Unlike the static model of this paper, cities are dynamic. However, the locations of business districts, transportation networks, natural geographic features, and ethnic enclaves evolve on the scale of decades or longer. New York's Second Avenue subway opened in 2017, decades after planning began in the 1930s. San Francisco's Chinatown has remained a destination for Chinese immigrants since the 1800s, and Downtown Boston sits on the original city site from 1630. The model takes as fixed medium- or long-term factors such as these that may affect productivity or other parameters.

[16] As a technical matter, it is possible that fixed and construction costs are sufficiently large that there is no feasible distribution of households under which total output is sufficient to cover these costs. In that case, the economy can be thought of as *agricultural*: no fixed costs are paid, and each location is occupied by a solitary household that produces $y_A < \underline{y}$. In the urban equilibrium, fewer locations will be opened. A final case is that the endogenous outside option of the urban economy is equal to the utility earned from agricultural production, in which case some locations would remain agricultural. Restricting y_A to be sufficiently small effectively rules out this equilibrium.

[17] As each household consumes a single housing unit, the choice of zoning law restricts both the number of housing units and the number of households.

[18] The initial rent can be thought of as the fixed mortgage payment of an existing homeowner, which does not fluctuate based on local rental conditions. It can also be thought of as rent control, where the price cannot legally be raised on existing tenants. Rent control is a common policy tool in expensive rental markets like San Francisco.

households and firms will make given the zoning law. In locations with no initial residents, construction firms maximize profits without constraint.

I proceed by defining the problems of the household and the construction firm and then the neighborhood equilibrium given zoning laws. Then I define the problem of the initial residents and their equilibrium choices of zoning law.

Household Problem A household in location x has preferences summarized by the utility function

$$u(c(x)) - v(n(x)).$$

The function $u(\cdot)$ represents preferences over consumption, $c(x)$. The function $v(\cdot)$ represents a congestion externality that depends on the neighborhood population, $n(x)$. Utility from consumption is twice continuously differentiable and concave: $u'(c) > 0$ and $u''(c) \leq 0$. The congestion externality $v(\cdot)$ is bounded below, increasing, twice continuously differentiable, and convex: $v'(n) > 0$ and $v''(n) \geq 0$. Without loss of generality, I assume that $v(0) = 0$.

The household faces budget constraint

$$c(x) = y(x) + \Pi - \frac{F}{n(x)} - p(x),$$

where $y(x)$ and $p(x)$ represent the location-specific income and house price. Consumption is written as $c(x)$. Households own a diversified portfolio of construction firms in all locations, and Π is the location-independent transfer of construction-sector profits from the national economy. The fixed cost F is shared equally among all residents.

Households take as given local prices $p(x)$, total profits Π, and their endogenous outside option $\bar{u}$. Households are freely mobile and choose location x and consumption $c(x)$ to maximize utility subject to the budget constraint.

Construction Firm Problem Construction firms choose the development intensity $n^f(x)$ to maximize profits at each location subject to a local zoning constraint $\bar{n}(x)$. There is no pre-existing housing stock in any location. Taking the price $p(x)$ as given, the firm problem in location x is

$$\max_{n^f(x)} \Pi(x) = p(x) n^f(x) - \left(\frac{n^f(x)}{Z} \right)^2$$

subject to zoning constraint

$$n^f(x) \leq \bar{n}(x).$$

It is natural to think that construction costs within a location are convex due to the costs associated with building taller and denser structures. Construction firms earn positive profits from locations with positive construction. Profits are aggregated across all locations and redistributed equally to all households.[19]

2.1.2 Local Spatial Equilibrium

Consider a fixed $\bar{n}(x)$, a fixed $\bar{u}$, and a fixed Π. I define a *local spatial equilibrium* for location x to be a house price $p^*(x)$ and local population $n^*(x)$ such that households and firms solve their respective problems with households consuming their budget, the local housing market clears, and spatial equilibrium must hold. That is,

$$u\left(y(x) + \Pi - \frac{F}{n^*(x)} - p^*(x)\right) - v\left(n^*(x)\right) \leq \bar{u}, \tag{1}$$

with equality if $n^*(x) > 0$. Housing market clearing implies $n^f(x) = n^*(x)$. Using this result, the firm's problem yields a complementary slackness condition that states that either the zoning constraint binds or price is equal to the marginal cost of construction:

$$\left(p^*(x) - 2n^*(x)/Z^2\right)\left[\bar{n}(x) - n^*(x)\right] = 0. \tag{2}$$

The firm must also abide by the zoning constraint:

$$n^*(x) \leq \bar{n}(x). \tag{3}$$

A *local spatial equilibrium* in location x is a pair $\{n^*(x), p^*(x)\}$ satisfying Equations (1)), (2)), and (3).

Housing Equilibrium Selection The existence of a fixed cost creates complementarities in household location decisions: if some households go to a location, the fixed cost can be shared more broadly and the location becomes more attractive to other households. This complementarity implies that there can be multiple equilibria. Finally, the congestion externality implies that eventually an additional household will lower the utility of existing households as the congestion costs outweigh the sharing benefits. I show that there are up to three pairs of equilibrium population $n^*(x)$ and price $p^*(x)$ that may satisfy the definition of an equilibrium. Figure 1 characterizes graphically the set of potential equilibria. The *optimal location* curve, labeled $OL(n, x)$, shows the set of (n, p) points consistent with the household's maximizing choices and is defined as follows:

$$OL\left(n, x\right) = \begin{cases} y(x) + \Pi - \frac{F}{n} - u^{-1}\left(v\left(n\right) + \bar{u}\right) & \text{if } n > 0 \\ [0, \infty] & \text{if } n = 0. \end{cases} \tag{4}$$

[19] The cost function is equivalent to a housing production function that is Cobb-Douglas in building materials and land, with a coefficient of $\frac{1}{2}$ on each term. Under this alternative production function, the profit redistribution outlined here corresponds to an assumption that each household owns a diversified portfolio of land. This share is within the range of estimates provided by Albouy and Ehrlich (2012).

The curve labeled $MC\,(n)$ corresponds to the marginal cost of construction and is independent of location:

$$MC\,(n) = 2\;n/Z^2. \tag{5}$$

Proposition 1. *The portion of the $OL\,(n, x)$ curve with $n(x) > 0$ is strictly concave, is hump shaped, and intersects the $MC\,(n)$ curve twice, not at all, or once at a point of tangency.*

Proof. See the appendix. □

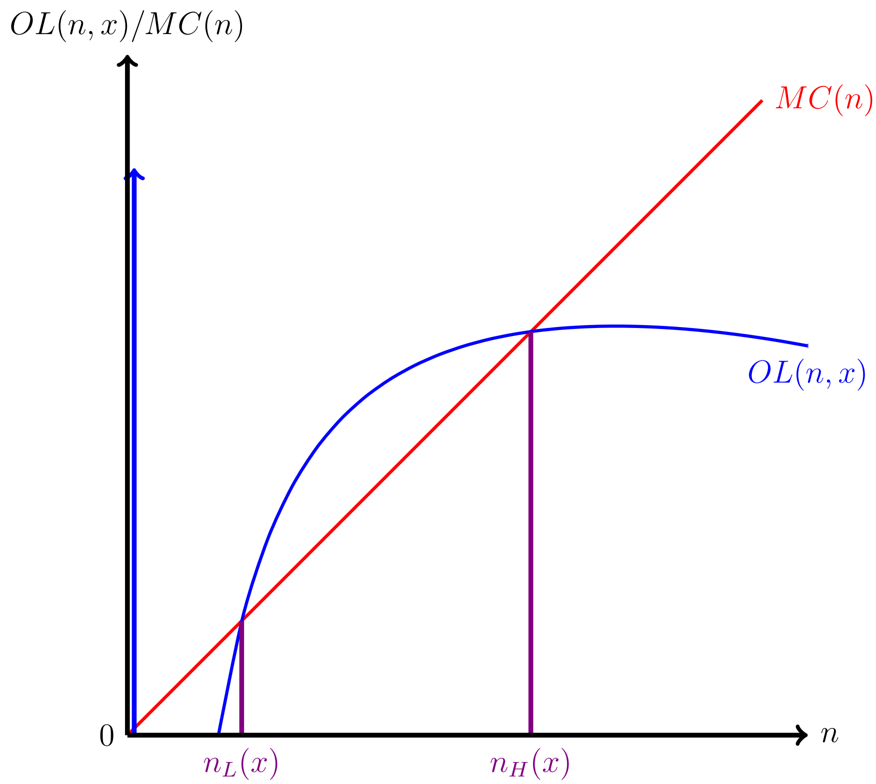

Figure 1: Marginal cost ($MC(n)$) and indifference ($OL(n, x)$) curves. Note that the $OL(n, x)$ curve includes the entire vertical axis above 0. The set of points potentially consistent with equilibrium, for which $OL(n, x) > MC(n)$, is the closed interval $(n_L(x), n_H(x))$ and the point 0.

If the zoning constraint lies to the right of $n_H(x)$—the upper intersection of $MC(n)$ and $OL(n, x)$—then both $n_H(x)$ and $n_L(x)$ are consistent with equilibrium. If the zoning constraint lies within the interval $(n_L(x), n_H(x))$, then both $n_L(x)$—the lower intersection—and $\bar{n}(x)$ are consistent with equilibrium. If the zoning constraint lies to the left of $n_L(x)$, then the origin is the sole equilibrium. There is always an equilibrium at the origin: households and firms may expect a $p^*(x) = 0$ and $n^*(x) = 0$ and see these expectations fulfilled.

If there are multiple equilibria, I select the one with the largest population.[20] This equilibrium selection has two virtues. First, it abstracts from coordination failures where both households and firms expect the location to be empty, and so it remains empty. Second, it is *stable* in the sense of economic geography models, i.e., a small deviation of population does not induce population movements that lead the location to a different equilibrium (see Krugman, 1991). This selection criterion rules out $n_L(x)$ as an equilibrium population, as it would be unstable.[21] Concretely, both households and construction firms expect that all locations will be settled at the highest-population equilibrium.

The equilibrium population $n^*(x)$ and price $p^*(x)$ consistent with this selection criterion for a given zoning law $\bar{n}(x)$ are

$$(6) \qquad \{n^*(x), p^*(x)\} = \begin{cases} \{0, 0\} & \text{if } MC\,(n) > OL\,(n, x)\,\forall\ n \in (0, \bar{n}(x)] \\ \{\bar{n}(x), OL\,(n, x)\} & \text{if } \bar{n}(x) \in [n_L(x), n_H(x)] \\ \{n_H(x), MC\,(n)\} & \text{if } \bar{n}(x) > n_H(x). \end{cases}$$

In the first case, there is no intensity of development $n(x)$ that abides by the zoning constraint and that delivers utility of at least $\bar{u}$, which could happen for a given x if $\bar{n}(x) < n_L(x)$ or if $OL\,(n, x) < MC\,(n)$ for every $n(x) > 0$. In the second case, the zoning constraint binds, and the equilibrium price is consistent with household spatial equilibrium. In the third case, the zoning constraint does not bind, and the equilibrium price is consistent with both household spatial equilibrium and (unconstrained) profit maximization.

2.1.3 Locally Optimal Zoning Choice

This section describes the determination of zoning laws within a location.

I assume that some locations have a measure 0 of *initial residents* who choose the zoning regulation, while some locations have no initial residents and are thus unregulated. The set of inhabited locations will be endogenized later. Formally, they can choose the zoning constraint $\bar{n}(x)$; as Equation (6) makes clear, the choice of zoning law affects the local equilibrium.

For the subset of locations with no initial residents, the zoning law is effectively infinite: $\bar{n}(x) = \infty$. The equilibrium population in such locations is given by Equation (6): it will be $\{n_H(x), MC\,(n)\}$ if the $OL(n, x)$ and $MC(n)$ curves intersect and $\{0, 0\}$ if the $OL(n, x)$ curve is strictly lower than the $MC(n)$ curve for all positive populations.

[20] Note that this supremum is taken over a closed and bounded set. By continuity of the utility and construction cost functions, the supremum of the set of points consistent with equilibrium is, itself, a member of this set.

[21] In the case where $n_L(x) = n_H(x)$, $n_L(x)$ satisfies the equilibrium selection process and is stable. See the appendix for details on equilibrium stability.

Locations with initial residents The measure 0 of initial residents have identical preferences and productivity as the households described earlier, but they differ in two respects. First, the initial residents of location x face a fixed rent $p_0(x)$. I take this rent as given for now and will endogenize it later. Second, they choose the zoning constraint $\bar{n}(x)$ that limits the maximum level of development within their location. The initial residents have rational expectations about the consequences of their zoning choice. Namely, they anticipate that a zoning constraint $\bar{n}(x)$ will induce the local spatial equilibrium of Equation (6). Like the households described above, the initial residents take as given the endogenous outside option $\bar{u}$ and the level of aggregate profits Π.

Let $\theta(\bar{n}(x); \bar{u}, \Pi) = n^*(x)$ denote the population $n^*(x)$ from Equation (6) given a zoning constraint $\bar{n}(x)$ and aggregate variables $\bar{u}$ and Π. The initial residents solve the following maximization problem:

$$\max_{\{c_0(x), \bar{n}(x)\}} u(c_0(x)) - v(\theta(\bar{n}(x); \bar{u}, \Pi)).$$

subject to

$$c_0(x) = y(x) + \Pi - \frac{F}{\theta(\bar{n}(x); \bar{u}, \Pi)} - p_0(x)$$

We can rewrite the maximization problem in a more convenient form. Initial residents in location x act as if they were directly choosing the local population given $\bar{u}$ and Π, subject to a household participation constraint.

Proposition 2. *Consider an arbitrary x for which there exists n with $OL(n, x) \geq MC(n)$. For this x, the locally optimal zoning constraint solves*

$$\max_{\{c_0(x), \bar{n}(x)\}} u(c_0(x)) - v(\bar{n}(x)) \tag{7}$$

subject to the budget constraint

$$p_0(x) + c_0(x) + \frac{F}{\bar{n}(x)} = y(x) + \Pi \tag{8}$$

and the participation constraint

$$u\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - 2\bar{n}(x)/Z^2\right) - v(\bar{n}(x)) \geq \bar{u}. \tag{9}$$

Proof. See the appendix. □

Note that the participation constraint is a function of the marginal cost of housing rather than its equilibrium price. The participation constraint is satisfied if and only if $\bar{n}(x) \in [n_L(x), n_H(x)]$ as defined above. If the participation constraint is non-binding, Equation

(6) states that equilibrium population $n^*(x)$ will equal the initial resident choice $\bar{n}(x)$. The logic of the constraint is as follows: for $\bar{n}(x) < n_L(x)$, the equilibrium population will be 0. However, the initial residents strictly prefer to share the fixed cost F with a positive population, and so they will not choose $\bar{n}(x) < n_L(x)$. If they choose $\bar{n}(x) > n_H(x)$, the equilibrium population will be $n_H(x)$, and so limiting the initial resident choice to being below $n_H(x)$ does not restrict their potential payoffs.

In general, the participation constraint may or may not bind. If the participation constraint does not bind, then the zoning constraint will bind in the local equilibrium: the equilibrium house price is above the firm's marginal cost. If the participation constraint does bind, then the zoning constraint does not bind: the house price is equal to marginal cost.

When the participation constraint does not bind, the initial resident choice of zoning choice is the $\bar{n}(x)$ that solves

$$u'\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - p_0(x)\right)\frac{F}{\bar{n}(x)^2} - v'\left(\bar{n}(x)\right) = 0. \tag{10}$$

The first term is the marginal benefit of sharing the fixed cost F more broadly.[22] The second term is the marginal cost of congestion. As u is concave and v convex, the expression is strictly decreasing and the $\bar{n}(x)$ that solves the equation is unique.[23]

When the participation constraint binds, the initial resident's optimal zoning choice is, by definition, not in the interior of the set of points consistent with the participation constraint. By concavity of the initial resident problem, the zoning choice in these cases is given by the endpoint of the set that offers greater utility to the initial residents:

$$\underset{\bar{n}(x)\in\{n_L(x),n_H(x)\}}{\arg\max}\; u\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - p_0(x)\right) - v\left(\bar{n}(x)\right). \tag{11}$$

2.2 General Spatial Equilibrium

Define a *stable general spatial equilibrium* in this economy to be an endogenous outside option $\bar{u}$, a level of profit Π, a set of open locations $\mathcal{X}$, local populations $n^*(x)$, local prices $p^*(x)$, and zoning laws $\bar{n}(x)$ such that the zoning choices $\bar{n}(x)$ and local equilibrium outcomes

[22] The constraint states that the utility delivered by the location *with house price equal to marginal cost* is greater than or equal to $\bar{u}$. If the constraint is non-binding, then Equation (6) states that the equilibrium house price will be greater than marginal cost.

[23] As there is a measure 0 of initial residents, an inflow of $\bar{n}(x)$ households ensures that the new households will be the majority of the community. This feature of the model raises the question of whether they would seek *ex post* to hold a new vote and modify the zoning law. However, the new residents would not have a strong preference to modify the law. Given an outside option $\bar{u}$, households expect that rents will adjust to make them indifferent across locations regardless of their vote. As such, households do not strictly prefer any alternative zoning law for their location. The pre-fixed rent of the initial residents eliminates this feedback mechanism.

$\{p^*(x), n^*(x)\}$ solve household, firm, and initial resident problems and are consistent with the population constraint

$$\int_0^1 n^*(x)dx = 1, \tag{12}$$

with the definition of profits

$$\Pi = \int_0^1 \left[p^*(x)n^*(x) - \left(\frac{n^*(x)}{Z} \right)^2 \right] dx, \tag{13}$$

with the definition of $\mathcal{X}$

$$x \in \mathcal{X} \iff n^*(x) > 0, \tag{14}$$

and with the stability condition

$$p^*(x) = p_0(x) \ \forall \ x \in [0, 1]\,. \tag{15}$$

The stability condition imposes restrictions on the prices faced by initial residents. This natural restriction is informed by a notion of dynamic stability and enables the static model to approximate the steady state of a corresponding dynamic overlapping generations model, wherein a subset of agents inherit a fixed rent from the previous period's equilibrium.

2.2.1 Characterization of General Equilibrium

I proceed by characterizing the triplet $\{\bar{u}, \Pi, \mathcal{X}\}$ as well as optimal zoning $\bar{n}(x)$, local population $n^*(x)$, and price gradient $p^*(x)$ consistent with general equilibrium. Focusing on the *stable equilibrium* described earlier, I consider the natural restriction that $p_0(x) = p(x)$. Two preliminary results will be useful in characterizing the general equilibrium.

Proposition 3. *The set of occupied locations $\mathcal{X}$ is $[\underline{x}, 1]$ for some threshold location $\underline{x}$. For the threshold location $\underline{x}$, the $OL(n, \underline{x})$ and $MC(n)$ curves are tangent at a unique level of population that depends on $\bar{u}$.*

Proof. See the appendix. □

This tangency condition proves useful for pinning down the price and population gradients. The intuition for tangency is as follows: from Proposition 1, the two curves cross twice, once with tangency, or not at all. For a given location x, if they do not cross, then the location must be unoccupied: no level of population can deliver utility $\bar{u}$. If they do cross or meet with tangency, then $n^*(x)$ must be strictly positive. Otherwise, there would be no initial residents and thus no zoning constraint, and so developers would find it optimal to build (and households to settle) to the local equilibrium population of $n_H(x)$. Thus, x must be in $\mathcal{X}$ for all locations where the $OL(n, x)$ curve meets the $MC(n)$.

The tangency of $OL(n, \underline{x})$ and $MC(n)$ at the threshold location $\underline{x}$ also implies equality between $OL(n, \underline{x})$ and $MC(n)$. Thus, Proposition 1 offers two additional conditions that will be used to pin down the price and population gradients. First, tangency implies

$$\frac{F}{n^*(\underline{x})^2} - \left[u^{-1}\right]' \left(v\left(n^*(\underline{x})\right) + \bar{u}\right) \times v'\left(n^*(\underline{x})\right) = 2/Z^2. \tag{16}$$

Second, equality implies

$$p(\underline{x}) = 2n(\underline{x})/Z^2. \tag{17}$$

The first condition pins down the threshold population $n^*(\underline{x})$ as a function of $\bar{u}$. The second condition pins down the price at the threshold as a function of this population.

Lemma 1. *Define $\bar{n}$ to be the population that maximizes the $OL\,(n, x)$ curve. Given outside option $\bar{u}$, the level of population $\bar{n}$ is unique and independent of location.*

Proof. See the appendix. □

Lemma 2. *Define location $\hat{x}\,(\bar{u}, \Pi)$ as follows:*

$$\hat{x}\,(\bar{u}, \Pi) = \begin{cases} x \text{ s.t. } OL\,(\mathcal{N}(\bar{u}), \hat{x}) = MC\,(\mathcal{N}(\bar{u})) & \text{if } OL\,(\mathcal{N}(\bar{u}), 1) \geq MC\,(\mathcal{N}(\bar{u})) \\ 1 & \text{otherwise.} \end{cases}$$

Then $\hat{x}\,(\bar{u}, \Pi)$ is unique and in the range $[\underline{x}, 1]$.

Proof. See the appendix. □

Proposition 4. *In a stable general equilibrium, the following characterizes the local equilibrium populations and optimal zoning. For all locations $x > \hat{x}$, $n^*(x) = \bar{n}(x) = \mathcal{N}(\bar{u})$. For all locations $x \in [\underline{x}, \hat{x}]$, the population $n^*(x)$ and optimal zoning $\bar{n}(x)$ are given by the upper intersection of the $OL\,(n, x)$ and $MC\,(n)$ curves. For all $x \geq \underline{x}$, $p^*(x)$ is given by the value of the $OL\,(n, x)$ curve evaluated at $n^*(x)$.*

Proof. See the appendix. □

The $OL\,(n, x)$ curve reflects the willingness to pay for a location as a function of its population: if a mix of congestion and sharing offers a higher level of utility, households are willing to pay more. As $p_0(x) = p(x)$ in the stable equilibrium, initial resident utility is maximized where the willingness to pay is maximized: at the peak of the $OL(n, x)$ curve. As the productivity $y(x)$ shifts the $OL\,(n, x)$ curve only vertically, the preferred zoning choice is independent of location. Hence, for locations with $x > \hat{x}$, zoning laws bind in equilibrium at the population $\mathcal{N}(\bar{u})$.

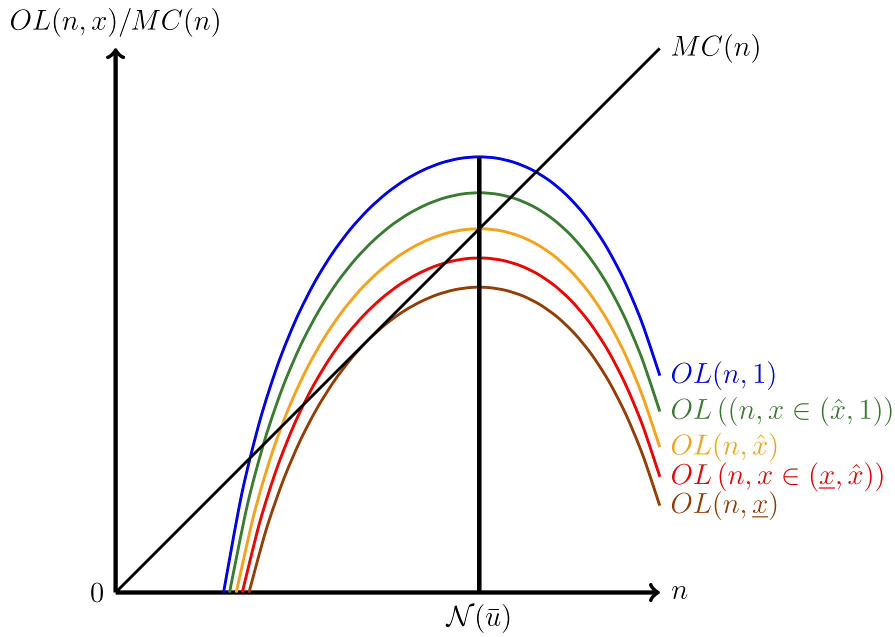

Figure 2: Optimal location $OL(n,x)$ curves for threshold occupied location $\underline{x}$, $\hat{x}$, and other locations with the marginal cost $MC(n)$ curve. The lowest curve, $OL(n,\underline{x})$, is the threshold location and its equilibrium population will be given by the point of tangency with the $MC(n)$ curve. For the next curve, the equilibrium population is given by the intersection on the right. For the top three curves, the initial resident zoning choice and equilibrium population is given by $\mathcal{N}(\bar{u})$. Note that all of the $OL(n,x)$ curves reach their peak at $\mathcal{N}(\bar{u})$; the equilibrium population at each location is as close to this as possible subject to ensuring that $OL(n,x) \geq MC(n)$.

For locations $x \leq \hat{x}$, the level of population $\mathcal{N}(\bar{u})$ preferred by the initial residents is inconsistent with local equilibrium: the price that would induce households to settle at population $\mathcal{N}(\bar{u})$ is too low to induce construction firms to build $\mathcal{N}(\bar{u})$ units of housing. Vice versa, any price high enough to induce firms to build $\mathcal{N}(\bar{u})$ will be higher than households are willing to pay, given local wages $y(x)$. In this case, the equilibrium population adjusts downward: the house price falls, marginal costs fall, and so long as $x \geq \underline{x}$, the location settles at a smaller equilibrium population. Because the initial resident problem is concave, the participation constraint binds and the local equilibrium population (and optimal zoning law) is the upper intersection of the $OL\,(n, x)$ and $MC\,(n)$ curves. This case is shown in Figure 2.

Characterization of aggregate variables: $\bar{u}$ and Π. As noted, the equilibrium levels of population $n^*(x)$ and prices $p^*(x)$ described in Proposition 4 are consistent with the local equilibrium, optimal zoning, and stable equilibrium conditions for a given pair $\{\bar{u}, \Pi\}$. The general equilibrium $\bar{u}$ and Π are the pair for which the local equilibria $n^*(x)$ and $p^*(x)$ and locations $\underline{x}$ and $\hat{x}$ described earlier are consistent with the population constraint and profit definition.

The threshold conditions from Proposition 3 implicitly define the threshold location $\underline{x}$ as a function of $\bar{u}$ and Π: $\underline{x}(\bar{u}, \Pi)$. The profit definitions states

$$\Pi = \int_{\mathcal{X}} \left(p^*(x) n^*(x) - \left(\frac{n^*(x)}{Z} \right)^2 \right) dx.$$

Substituting the equilibrium conditions, this equation becomes

(18)

$$\Pi = \int_{\underline{x}(\bar{u},\Pi)}^{\hat{x}(\bar{u},\Pi)} \left(\frac{n_H(x)}{Z} \right)^2 dx + \left(\int_{\hat{x}(\bar{u},\Pi)}^{1} y(x)dx + \Pi - \frac{F}{\mathcal{N}\,(\bar{u})} - u^{-1}\left(v(\mathcal{N}\,(\bar{u})) + \bar{u} \right) \right) \mathcal{N}\,(\bar{u}) - \left(\frac{\mathcal{N}\,(\bar{u})}{Z} \right)^2.$$

From above, $n_H(x)$ is the upper intersection of the $OL(n, x)$ and $MC(n)$ curves and is pinned down as a function of Π and $\bar{u}$. From the population constraint,

$$1 = \int_{\underline{x}(\bar{u},\Pi)}^{\hat{x}(\bar{u},\Pi)} n_H(x)dx + \int_{\hat{x}(\bar{u},\Pi)}^{1} \mathcal{N}\,(\bar{u})\,dx. \tag{19}$$

The characterization of a stable general equilibrium is completed by an outside option $\bar{u}$ and a profit Π that solve Equations (18) and (19). I do not yet have a formal existence proof. However, given functional forms for $u(c)$ and $v(n)$, it is straightforward to solve the equations and characterize the equilibrium numerically.

2.2.2 Qualitative Predictions

The model can be used to generate a set of empirical predictions regarding density, house prices, and marginal costs. Regarding density, the model predicts that the equilibrium density will be uncorrelated with local productivity for locations where zoning is binding. For all tracts, it should be positive. For the set of urban census tracts, the correlation between income and density is -0.02. Adjusting for skill differences, in a process to be detailed later in the paper, moves the correlation to 0.02. After adjusting for county-level averages—which may result from common investments in infrastructure, or exogenous amenities—the correlation moves to 0.07. These findings are consistent with the model.

The model predicts that house prices will perfectly offset productivity differences in locations with binding zoning. Of course, housing prices respond to many factors not included in the model, so the the correlation is unlikely to be perfect. To test this prediction, I compute a residual house price by regressing tract-level housing costs on the same observable characteristics that I use to adjust income. I then correlate this measure with tract-level income, adjusted for observable characteristics. The correlation is 0.42 for owner-occupied housing costs and 0.38 for rental costs. These correlations are not inconsistent with the model. In addition to the aforementioned exogenous amenities, unmodeled forces that may affect this relationship include the quality of the housing stock, length of ownership for homeowners, and housing subsidies or rent-control laws.

Finally, the model predicts that marginal costs will be constant across locations with binding zoning. Together, these last two predictions imply that house prices in the most expensive locations will be well above marginal costs of construction. This prediction is well supported by the literature (e.g., Glaeser et al., 2005a).

2.3 Optimality: The Constrained Planner's Problem

To provide a welfare benchmark with which to contrast the zoning equilibrium, this section describes the problem of a constrained planner. The planner chooses the set $\mathcal{X}$ of locations to open, allocates population $n(x)$ among these locations, and allocates consumption $c(x)$ to households in order to maximize welfare. The planner is free to transfer output across locations. The planner faces standard population and aggregate resource constraints as well as a spatial equilibrium constraint. This final constraint restricts the planner to choosing allocations that deliver identical utility to each household regardless of location; in this sense, the planner is *constrained*. The spatial equilibrium condition implies that the welfare criterion to be maximized is simply $\bar{u}$. The set of locations to open can be simplified to the choice of a threshold location $\underline{x}$.[24]

The planner's problem is thus given by the following:

$$\max_{\{\bar{u},c(x),n(x),\underline{x}\}} \bar{u}$$

[24] The planner will never choose to open location x_1 if location $x_2 > x_1$ has not already been opened. Hence, choosing the set $\mathcal{X}$ from within the set of locations $[0, 1]$ amounts to choosing the lowest-productivity location to open: $\underline{x}$.

subject to the spatial equilibrium constraint

$$u\left(c(x)\right) - v\left(n(x)\right) = \bar{u} \;\forall\; x \in \left[\underline{x}, 1\right],$$

the population constraint

$$\int_{x=\underline{x}}^{1} n(x)dx = 1,$$

and the aggregate resource constraint

$$\int_{x=\underline{x}}^{1} \left[\left(y(x) - c(x)\right) n(x) - F - \left(\frac{n(x)}{Z}\right)^2\right] dx = 0.$$

The population $n(x)$ can be thought of as the *intensive* margin of development, while $\underline{x}$ is the *extensive* margin of development. Note that each opened location is subject to a spatial equilibrium condition. Given the choice of intensive margin $n(x)$, the location-specific spatial equilibrium constraints ensure the planner engages in transfers of output such that household consumption offsets the level of congestion and each household receives utility $\bar{u}$.

The Lagrangian associated with the problem is as follows:

$$\begin{aligned}\mathcal{L} = \bar{u} &+ \int_{x=\underline{x}}^{1} \lambda(x)\left[u\left(c(x)\right) - v\left(n(x)\right) - \bar{u}\right] dx + \mu\left[1 - \int_{x=\underline{x}}^{1} n(x)dx\right] \\ &+ \Lambda \int_{x=\underline{x}}^{1} \left[\left(y(x) - c(x)\right) n(x) - F - \left(\frac{n(x)}{Z}\right)^2\right] dx.\end{aligned} \tag{20}$$

Here, $\lambda(x)$ is the Lagrange multiplier on the spatial equilibrium constraint at location x, μ is the multiplier on the population constraint, and Λ is the multiplier on the resource constraint. Taking first-order conditions for $n(x)$ and $c(x)$ and rearranging, the planner weighs the following objects against one another when choosing the intensive margin of development $n(x)$:

$$\Lambda\left[y(x) - c(x) - 2n(x)/Z^2\right] - \Lambda n(x)\frac{v'(n(x))}{u'(c(x))} - \mu = 0. \tag{21}$$

The first term is the shadow resource value of the marginal household; the second term is the shadow cost of congestion, in terms of resources; and the third term is the shadow cost of having one fewer household to allocate elsewhere. The marginal household in a location adds output—net of consumption and the construction cost—but also increases congestion for the $n(x)$ households already there.

To recall, the first-order condition for the choice of local zoning was

$$\frac{F}{n(x)} - n(x)\frac{v'(n(x))}{u'(c(x))} = 0.$$

Both the planner and the initial residents consider the role of the congestion externality. While the initial residents weigh this externality against the value of sharing the local fixed cost more broadly, the planner weighs it against the resource value of allocating a marginal household to location x and the shadow value of the binding population constraint. The first term in Equation (21) represents the net output of allocating a marginal household to location x. Under the flat gradient that arises from the local zoning equilibrium, the marginal value would be higher in productive locations. The planner will thus allocate more households to such locations. This is the key intensive-margin wedge between the optimal solution and the allocation with local zoning.

While initial residents ignore aggregate effects, they do consider the marginal value of sharing the fixed cost more broadly. The planner ignores this margin: the fixed cost is paid when the location was opened and should not affect the intensive margin. This is the second wedge between the optimal solution and the allocation with local zoning. In short, the initial residents ignore the aggregate effects of their choice to restrict housing supply.[25]

The planner instead considers the fixed cost F at the extensive margin. As shown in Equation (22), the planner weighs the net output of opening the marginal location $\underline{x}$ against the shadow value of assigning $n(\underline{x})$ households to this location.

$$\Lambda\left[n(\underline{x})\left(y(\underline{x})-c(\underline{x})\right)-\left(\frac{n(\underline{x})}{Z}\right)^2-F\right]-n(\underline{x})\mu=0. \tag{22}$$

Recall that in the local zoning allocation outlined previously, the location in which the $OL(n,x)$ and $MC(n)$ curves are tangent becomes the threshold. Restating, this condition was met where

$$\frac{F}{n^*(\underline{x})^2}-\left[u^{-1}\right]'\left(v\left(n^*(\underline{x})\right)+\bar{u}\right)\times v'\left(n^*(\underline{x})\right)=2/Z^2.$$

This location arises through the general spatial equilibrium rather than as the explicit choice of any agent. In choosing the optimal threshold, the planner weighs the value and costs of opening a marginal location. The differential outcomes for the threshold $\underline{x}$ highlight an additional externality generated by the initial resident choice of local zoning: restrictive zoning ensures that too many locations are opened in equilibrium, necessitating the payment of fixed costs to open locations that would not be paid under the optimal allocation.

[25] As noted previously, the model takes as fixed medium- or long-term factors that may affect productivity or other parameters. To the extent that implementation of the planner's allocation may shape, for instance, medium-term investments in transportation infrastructure, this counterfactual may underestimate the gains from zoning reform. Similarly, if metropolitan agglomeration involves positive externalities that effect the *growth* as well as the *level* of metropolitan productivity—due for example to externalities of human capital (Moretti, 2004), matching (Overman and Puga, 2010), or entrepreneurship (Bunten et al., 2015)—then the static model will again underestimate the gains.

2.3.1 Decentralization: Socially Optimal Zoning

The planner's allocation can be decentralized through the adoption of an alternative zoning regime. In this decentralization, the full measure 1 of households votes on the set $\mathcal{X}$ of locations to open and the gradient of zoning laws $\bar{n}(x)$ for each location x in $\mathcal{X}$. In so doing, households have rational expectations over the spatial equilibrium induced by the choices they make. Households therefore choose zoning laws $\bar{n}(x)$ and the set of opened locations $\mathcal{X}$ to maximize the endogenous outside option $\bar{u}$. Households choose these laws subject to the population constraint and to the local and general spatial equilibria conditions.

Proposition 5. *The utility-maximizing choices for the set of opened locations $\mathcal{X}$ and the set of zoning laws $\bar{n}(x)$ are identical to those chosen by the planner. The equilibrium price gradient and aggregate profits implement the planner's choice of consumption. This set of instruments allows households to fully implement the planner's allocation.*

Proof. See the appendix. □

Intuitively, households have the option of choosing the same zoning as the planner and will do so. Prices will adjust such that households are indifferent across all locations. With identical population across locations as the planner's allocation, total output and aggregate construction and fixed costs are also identical. By resource balance, aggregate consumption must also be identical. By spatial equilibrium, consumption must make households indifferent between locations, and therefore equilibrium consumption is identical to the planner's allocation. The zoning laws and set of opened locations chosen by households in this problem can therefore be described as *socially optimal zoning*, as opposed to the *locally optimal zoning* described previously.

The profits paid to households are a crucial mechanism for ensuring that this spatial equilibrium will also be consistent with construction firm behavior. Note that household spatial equilibrium pins down relative prices, but not levels. Because profits are distributed to households regardless of location, an increase of ε in the absolute level of prices at each location leads to an increase in household profits by ε. This increase leaves household consumption identical. This mechanism ensures that there exists a price gradient consistent with local equilibrium from the firm perspective.

3 Calibration

3.1 Data

Four key model components must be calibrated: the magnitude of the local fixed cost, the shape of utility from consumption, the disutility of congestion, and the distribution of location-specific productivity. This section describes the moments in the data that the calibration seeks to match. In matching the moments, I take the set of urban U.S. census tracts to be the empirical counterpart of model locations.

For the utility of consumption, I consider $u(c) = c$. Linear utility follows related papers (Van Nieuwerburgh and Weill, 2010; Davis and Dingel, 2014).

For the disutility of congestion, I consider $v(n) = \gamma n^\eta$. To calibrate γ, I match the fraction of income spent on consumption goods as measured in the Consumer Expenditure Survey of the Bureau of Labor Statistics. As γ controls the relative utility weights of consumption and congestion, a higher γ will induce households to open more locations and spend more on housing and fixed costs rather than consumption. To ensure the model concept is well matched to its empirical counterpart, I calibrate the consumption share of income to spending on tradable goods. These spending shares range from 52 to 61% for different income deciles and average 58% overall.

As noted previously, an alternative interpretation of the local fixed cost has residents gaining utility from a greater diversity of monopolistically competitive local services, each of which is subject to a fixed cost.[26] Using this interpretation, I calibrate the fixed cost F to match the share of consumer spending on local services. In particular, I use the Consumer Expenditure Survey categories for food away from home, personal services, medical services, and fees and admissions. These spending shares range from 7 to 10% for different income deciles and average 8.5% overall.

I calibrate the congestion shape parameter η to match the average population density of urban census tracts, as identified by the Census Bureau for 2010. Given the functional form assumptions, the local equilibrium population for locations with binding zoning is given by

$$n = \left(\frac{F}{\gamma\,\eta}\right)^{1/(1+\eta)}.$$

As the zoning law will bind for a substantial fraction of locations, the shape parameter η has a first-order effect on average population density.[27] Intuitively, too low a congestion cost would cause households to live at too-high densities in productive locations, compared with the data.[28]

I calibrate construction firm productivity to match the ratio of price per square foot to cost per square foot in the most productive places: Manhattan, San Francisco, or Silicon Valley. For this exercise, I recorded the average price per square foot from Zillow and the average

[26] In a local services model, expenditure will be precisely $F/n(x)$ if the elasticity of substitution between local service firms is 1 or if aggregated local services enter the utility function linearly, such that $u(c, S) = c + S$, where S is a constant elasticity of substitution aggregator of local services. More generally, local expenditure will be $F/n(x)^\sigma$, where σ is the elasticity of substitution between local services.

[27] The intuition is similar in spirit to using wage shares to calibrate the labor and capital exponents in a production function: in equilibrium, the parameters can have first-order effects on levels.

[28] Note that it is not necessary to take a stand on the precise productivity of *non-urban* locations, whose measured incomes may not be a good guide to those earned by new households. It is sufficient to assume that non-urban households have lower productivity than those occupied at urban population levels.

Param	Value	Description	Model Target	Data	Model
γ	1/215	Congestion weight	Consumption sh. of expend	52%–61%	55%
F	0.1265	Fixed cost	Local services sh. of expend	7%-10%	7.8%
η	7	Congestion curvature	Ave. urban density	5,100/sq mi	5,900/sq mi
Z	2.7	Construction prod.	p/MC in best locations	5.2–5.8	5.6

Table 1: The classification of tracts as *urban* follows the Census Bureau's 2010 classification. The consumption share of expenditure is taken from the Consumer Expenditure Survey for 2012, and is equal to consumption net of spending on housing and local services. The price-to-marginal cost ratio follows the approach of Glaeser et al. (2005a).

cost per square foot from RS Means. The prices and costs are reported at the level of counties and, in some instances, for subcounty regions.

The distribution of income is taken from census-tract median incomes, adjusted for observables in a process that follows Hsieh and Moretti (2015). I take individual-level data from the Integrated Public Use Microdata Series on income, education, race, and gender (Ruggles et al., 2015). I then run a regression using the following equation:

$$y_i = X_i\beta + \varepsilon_i,$$

where y_i is the income of individual i and X_i is the vector of observable characteristics. I then take the estimate of β and calculate the residual income $\tilde{y}$ of each tract ℓ:

$$\tilde{y}_\ell = y_\ell - X_\ell\beta,$$

where y_ℓ is the measured average income per worker and X_ℓ is the fraction of the tract with the given observable characteristics.

Table 1 summarizes the calibration targets and the model fit. For three of the four parameters, the calibrated parameter is within the range from the data. The search of the parameter space failed to find a set of values for which the density was closer to the data while maintaining consistency with the other parameters. At the same time, it is plausible that households have reasons beyond productivity for choosing locations.

4 Quantitative Results

The key quantitative question this paper addresses is, what are the welfare costs of locally determined zoning laws? In the baseline calibration, implementing the planner's allocation would increase welfare by 1.4%. Consumption would increase by 2.4% and GDP by 2.1%, but increased congestion would mitigate these gains.[29] The planner would close the 3% lowest-productivity locations, and population density at the 95th percentile location would

[29] Recall that utility is quasilinear, so the welfare and consumption figures can be compared meaningfully.

Relative Outcome	Planner	Market	Upzoning
Welfare	1.4%	-5.9%	-2.0%
GDP	2.1%	6.0%	2.0%
Consumption	2.4%	5.6%	2.1%
Median Rent	2.9%	-16%	-1.8%

Table 2: All results are relative to the main zoning results. The planner results give the gains from moving to the optimal allocation. The market results give the counterfactual with no zoning laws. The upzoning results give the counterfactual with no zoning laws in the 5% highest-productivity locations.

increase by 18%. The density at the median location would increase by 2%. These results are summarized in Table 2, along with results from two additional counterfactuals.

The first counterfactual, labeled *Market* in the table, corresponds to an allocation with no zoning in which construction firms are free to build without constraint. In the language of the model, the equilibrium for each location is the upper intersection of the $OL(n, x)$ and $MC(x)$ curves. This counterfactual corresponds to that studied by Hsieh and Moretti (2015), and I find an increase in GDP of 6%, half of the 13.5% that they report. Part of the difference may be due to the presence in this model of congestion externalities, which limit the willingness of households to crowd into productive locations. Moreover, the estimates are of the same order of magnitude despite using different methodologies.

While the GDP increase from zoning abolition is substantial, the increase in congestion is sizable as well. The median location sees an increase in population density of 15%, and the increase is almost 50% at the 95th percentile. Accordingly, welfare declines by almost 6%, despite the large increase in consumption. This result suggests that the productivity gains to zoning abolition put forth by Hsieh and Moretti (2015) will not, in fact, increase welfare.

The second counterfactual, labeled *Upzoning* in the table, corresponds to an allocation with no zoning in locations with the productivity greater than the 95th percentile. As census tracts average about 4,000 inhabitants, these locations are home to approximately 10 million residents. For scale, the San Francisco Bay Area is home to 5 million urban residents. Under this allocation, GDP increases by 2% while welfare declines by 2%. This result contrasts with that of Hsieh and Moretti (2015), who find large productivity gains of 9.5% from increasing the population of the best cities.

The median rent, also in Table 2, provides insight into the changing tradeoffs with each reform. Three key forces determine the median rent: the productivity of the median resident, the congestion faced by the median resident, and the outside option $\bar{u}$. Under the decentralized version of the planner's allocation, house prices *increase* by 3% for the median household. Each location is a little more crowded—and so rents for each location fall, between 1% and 5%—but the median household is now in a more productive location, and thus its rent increases. The market equilibrium also sees the median household in a more productive location, but this location is now much more crowded. Correspondingly, rents fall by 16%. Finally, in the upzoning case, the median household is in a more productive location with the same level of congestion. However, it is slightly worse off, and so it is unwilling to bid as much for housing, and rents fall.

Given the calibrated parameters, the main policy implication of the model is to allow zoning laws to be chosen at a higher geographic level—preferably, the national level. This policy change would ensure that the productivity effects of zoning laws are internalized and would enable the implementation of the planner's optimal allocation.[30]

The fundamental role of congestion in shaping preferences and outcomes points toward a second policy implication beyond implementing the planner's regime: introducing reforms that could lower the cost of congestion to the neighborhood. Understanding the specific component of congestion that drives externalities will identify the appropriate form of mitigation. If neighborhood congestion is driven by vehicle traffic, then perhaps rapid transit would enable more-dense development by reducing the costs of congestion imposed by new development. Congestion-mitigation efforts like transit can be quite costly, and within a context of spatial equilibrium, the benefits would be felt widely. As with local zoning, the current transit-planning regime may not take into consideration the general equilibrium effects of transit.

Distributional Outcomes Abolishing zoning—whether nationally or just in the highest-productivity locations—cannot improve welfare under this calibration. This fact is driven in part by the curvature to the cost of congestion: increasing the intensity of development in productive places is simply too costly in terms of welfare. A second factor that plays an important role is the assumption that profits are shared equally among all households. When restrictive zoning drives up house prices in productive locations, the profits earned are redistributed across all locations. To understand the role played by this assumption, I now introduce a second calibration, wherein construction-sector profits are not returned to households. This calibration mirrors the traditional urban economics assumption of *absentee landlords*, who collect rent but do not otherwise interact with households.[31]

Table 3 shows the analogous results under this model.[32] Here, the gains to GDP are similar to those before, and the losses to welfare of zoning abolition are even greater. However, the change to welfare discounting profits—i.e., the welfare of the typical household—is positive. This exercise highlights the key role played by construction profits: residents of productive

[30] It is worth noting that under a national choice of housing supply, a home-owning household could see an incentive to restrict housing supply in order to raise the value of its asset—akin to the choice studied by Ortalo-Magné and Prat (2014).

[31] Eeckhout and Guner (2015) perform a similar exercise and identify absentee landlords with the concentrated landholdings of the 1% wealthiest households. They suppose that the planner values a fraction of housing sector profits that is distributed to households, but not the fraction distributed to absentee landlords.

[32] For these results, I have recalibrated the model so that the new model-defined moments are consistent with the data. The consumption share is 54%, the local service share is 7.7%, the construction-sector markup is 7.4%, and 88% of locations in the data are opened.

Relative Outcome	Planner	Market	Upzoning
Welfare	1.5%	-10%	-3.2%
Welfare ex profits	7.3%	23%	3.7%
GDP	2.6%	6.3%	1.9%
Consumption	9.3%	50%	12%
Median Rent	-5.3%	-42%	-2.1%

Table 3: All results are relative to the main zoning results, and all results ignore the welfare derived from profits—this allows the welfare gains *discounting profits* from the market allocation to exceed those from the planner's. The planner results give the gains from moving to the optimal allocation. The market results give the counterfactual with no zoning laws. The upzoning results give the counterfactual with no zoning laws in the 5% highest-productivity locations.

places do not actually enjoy the high productivity—they simply pay higher rents. When these profits are shared broadly, these rents are redistributed so that all households, regardless of location, share the output of high-productivity locations. When these profits are not shared broadly, households in less productive locations have more to gain from zoning abolition because moving into a productive location earns them significantly more consumption.

4.1 Empirical Implications and Extensions

The model focuses on just two of potentially many local externalities. Numerous facets of the model lend themselves to further empirical validation. Similarly, the simple model can be extended with greater heterogeneity, including of household productivity, of locational or household preferences over congestion costs and fixed costs, and of locational amenities.

Within the model, existing residents choose zoning regulations to maximize their amenity mix, trading sharing externalities against congestion. In doing so, they also maximize the rents that new households are willing to pay: maximizing amenities and maximizing land values are identical. However, local ownership of land (e.g., in the form of homeownership) may break this felicitous duality as landowners could seek to restrict the supply of new housing, a close substitute for their own asset. Empirically, it may be possible to distinguish between these intents by using evidence from surveys or by identifying preferences over regulations that would increase supply but have a positive effect on local amenities (or vice versa).

Local variation in model parameters could be tested to examine the strength of their relationship to equilibrium outcomes. For instance, the fixed costs of development may be higher in regions that are more arid, or prone to extreme weather, due to the costs of necessary infrastructure. New development in the arid fringe of Los Angeles is quite dense by the standards of new development in rainier eastern cities like Atlanta. All else being equal, do locations with higher fixed costs see higher-density development, as the model would predict?

Similarly, some locations have seen dramatic changes to their productivity, but homeowners—like the initial residents of the model—do not see any immediate change in their costs. Outside of general equilibrium, locations with low initial prices $p_0(x)$ relative to productivity $y(x)$ should see lower densities and higher house prices than locations with similar initial prices but lower productivity. Anecdotally, Palo Alto—a key center of innovation in the San Francisco area—has grown from a population of 55,225 in 1980 to 64,403 in 2010, over a time when house prices (but not quantities) have increased substantially. Part of this stagnation of quantities could be explained by the effects of durable capital where the value of an existing house limits the net returns of new construction (see Siodla, 2015), but as many newly purchased homes are torn down or remodeled, there may be a role for zoning constraints as well.

In Boulder, Colorado, a measure on the ballot in the fall of 2015 would have empowered dozens of local neighborhood groups with an easy path to halting unwanted development, within a city of just 100,000 residents. The outcome and voting patterns in the election could help to test model predictions. In particular, the model assumes locations are sufficiently small that residents can ignore the aggregate effects of their zoning decisions. Regulatory changes at higher levels of government are more likely to affect aggregate variables, such as the threshold occupied location. Residents who stand to benefit more from changes to aggregate variables would be more likely to oppose the measure. Renters, the young, and new arrivals may be more likely to benefit from aggregate changes within Boulder, relative to longtime homeowners with substantial equity dependent on the status quo.

4.1.1 Potential Extensions

This paper focuses on the links between productivity, house prices, and population density. This section briefly presents a set of potential extensions that would complicate these relationships and an overview of their likely effects on model outcomes. I consider heterogeneity of the congestion parameter η and the construction productivity Z, a consumption amenity, and worker skill heterogeneity.

Congestion costs Long-lived investments in transportation networks and other aspects of the built environment have drastically lowered the congestion cost of the average resident in some cities.[33] The types of investments that seem to matter most are well- known: every U.S. census tract with a population density greater than 100,000 people per square mile sits atop a heavy-rail rapid transit line. Such neighborhoods are confined to five metropolitan areas: New York, San Francisco, Los Angeles, Boston, and Chicago. Among census tracts with density above 50,000, almost all are in the aforementioned cities, others with heavy-rail mass transit like Philadelphia and Washington, or the beach cities of Miami and Honolulu.

[33] Boustan et al. (2013) provide more background on historical investments and urbanization in U.S. cities.

The remainder are single tracts adjacent to universities, where students are expected (or choose, to save money) to share rooms.

Incorporating heterogeneity of the congestion parameter η would yield a model wherein some locations—presumably through historical processes left unmodeled—could have a much greater equilibrium population than other locations. At the cost of simplicity, this extension would yield some benefits. First, it would incorporate local population density data, currently unused. Accordingly, such a calibration would pin down the set of η_i more precisely than is possible with the current national average approach. The counterfactual would thus be improved by taking account of the differential potential of, e.g., Silicon Valley and New York City to add population, given the current realities of long-lived transportation infrastructure.

Construction productivity The cost of construction varies extensively throughout the country, even for a single building typology (R.S. Means Company, 2014). In general, per unit costs of a given building type appear higher in more productive and more expensive locations. Accounting for these differences would shrink the welfare costs of local zoning by increasing the marginal cost of reallocating households to more-productive locations.

Consumption amenity As noted earlier, Miami and Honolulu are represented among the set of cities featuring high-density census tracts. Of course, cities with amenable climates are well-known as being among the most expensive, a fact at the heart of the Rosen (1979) and Roback (1982) model. These facts can be reconciled by adapting their model to this environment by including a local consumption amenity $a(x)$ in the utility function such that utility is given as

$$a(x) \times c(x) - \gamma n(x)^{\eta}.$$

In this case, equilibrium prices and populations will be greater in high-amenity locations. If utility is multiplicative across both terms or additive, then equilibrium prices will be higher in high-amenity locations but population will be unaffected by the amenity.[34] As with heterogeneous congestion costs, heterogeneity of amenities would enable a calibrated model to incorporate more data and to produce a more subtle counterfactual. Doing so would, however, introduce an additional assumption: any new households introduced to a location value the amenity identically to those who were already there. In the case of income, it is straightforward to control for observable characteristics. For amenities, this assumption would be somewhat stronger.

[34] An amenity that is multiplicative on the congestion term alone will also increase equilibrium prices and populations in high-amenity locations. This kind of amenity would be equivalent to introducing heterogeneous costs of congestion $\gamma(x)$, with the interpretation that high-amenity locations are those with low costs of congestion—although in this case, the curvature of the congestion function η would be identical across locations.

Skill heterogeneity As noted, the income measure used in the calibration is the residual income after controlling for variation in observable characteristics, including educational attainment, race, and gender. Another extension of the model would account for unobservable variation in types by introducing a distribution of worker types and calibrating that distribution to the differential gradients of income and prices across locations, as in Van Nieuwerburgh and Weill (2010). Assuming complementarities between households and locations, this extension would tend to reduce the estimated losses of local zoning: when the planner moves an additional household to a high-productivity location, that household sees a smaller boost to its productivity than a naive calculation would imply.

Similarly, the model could be extended to include differential preferences over congestion: perhaps some households are more or less bothered by adding more neighbors. For instance, households with children might be more sensitive to congestion, even conditioning on income and dwelling size. This extension could refine the estimates of welfare gains from adding density to productive locations.

5 Conclusion

This paper provides a unified framework to address the local and aggregate welfare effects of local land-use regulation. It provides empirically grounded externalities that incent households to pass restrictive zoning laws that prevent new housing development at the neighborhood level. In their endogenous choice of zoning, households rationally ignore the aggregate implications that arise from location heterogeneity. Households thus bid up the price of artificially scarce housing in productive locations so that prices exceed the marginal costs of construction.

The endogeneity of zoning plays a critical role in overcoming the lack of sufficient data to identify the wedge between house price and marginal cost at the neighborhood level. A calibrated version of the model is used to perform a welfare calculation: how large are the aggregate losses to welfare from local zoning relative to the planner's optimum? The model provides an interpretation of these welfare costs without measuring these wedges directly. In the preferred calibration, welfare could be raised 1.4% by implementing the planner's allocation. Aggregate output would be raised 2.1%: one-third of the gains to output are negated by the increased congestion felt by residents of productive locations.

References

Albouy, D. (2012). Evaluating the efficiency and equity of federal fiscal equalization. *Journal of Public Economics*, 96(9):824–839.

Albouy, D. and Ehrlich, G. (2012). Metropolitan land values and housing productivity. Unpublished.

Allen, T. and Arkolakis, C. (2014). Trade and the topography of the spatial economy. *The Quarterly Journal of Economics*, 1085:1139.

Allen, T., Arkolakis, C., and Li, X. (2015). Optimal City Structure. Unpublished.

Arnott, R. J. and Stiglitz, J. E. (1979). Aggregate land rents, expenditure on public goods, and optimal city size. *The Quarterly Journal of Economics*, pages 471–500.

Aura, S. and Davidoff, T. (2008). Supply Constraints and Housing Prices. *Economics Letters*, 99(2):275–277.

Avent, R. (2011). *The Gated City*. Amazon Digital Services.

Behrens, K., Mion, G., Murata, Y., and Südekum, J. (2014). Spatial frictions. Technical report, Düsseldorf Institute for Competition Economics.

Boustan, L. P., Bunten, D., and Hearey, O. (2013). Urbanization in the united states, 1800-2000. Technical report, National Bureau of Economic Research.

Bunten, D., Weiler, S., Thompson, E., and Zahran, S. (2015). Entrepreneurship, Information, and Growth. *Journal of Regional Science*, 55(4):560–584.

Calabrese, S., Epple, D., and Romano, R. (2007). On the Political Economy of Zoning. *Journal of Public Economics*, 91(1):25–49.

Chen, C.-S. and Lai, F.-C. (2008). Location choice and optimal zoning under cournot competition. *Regional Science and Urban Economics*, 38(2):119–126.

Cheshire, P. C. and Hilber, C. A. (2008). Office Space Supply Restrictions in Britain: The Political Economy of Market Revenge. *The Economic Journal*, 118(529):F185–F221.

Davis, D. R. and Dingel, J. I. (2014). The Comparative Advantage of Cities. Technical report, National Bureau of Economic Research.

Desmet, K. and Rossi-Hansberg, E. (2013). Urban accounting and welfare. *American Economic Review*, 103(6):2296–2327.

Diamond, R. (2016). The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980–2000. *American Economic Review*, 106(3):479–524.

Eeckhout, J. and Guner, N. (2015). Optimal Spatial Taxation: Are Big Cities Too Small? Technical report, Institute for the Study of Labor.

Fischel, W. A. (1987). *The Economics of Zoning Laws: A Property Rights Approach to American Land Use Controls*. Johns Hopkins University Press.

Fischel, W. A. (1999). Does the American Way of Zoning Cause the Suburbs of Metropolitan Areas to be Too Spread Out? *Governance and Opportunity in Metropolitan America*, 15:1–19.

Fischel, W. A. (2009). *The Homevoter Hypothesis*. Harvard University Press.

Fischel, W. A. (2015). *Zoning Rules!* Lincoln Institute of Land Policy, Cambridge, Massachusetts.

Flatters, F., Henderson, V., and Mieszkowski, P. (1974). Public goods, efficiency, and regional fiscal equalization. *Journal of Public Economics*, 3(2):99–112.

Ganong, P. and Shoag, D. (2013). Why has Regional Income Convergence in the US Declined. *Manuscript, Harvard*.

Glaeser, E. L. and Gyourko, J. (2002). Zoning's Steep Price. *Regulation*, 25:24.

Glaeser, E. L. and Gyourko, J. (2003). The Impact of Building Restrictions on Housing Affordability. *Federal Reserve Bank of New York. Economic Policy Review*, 9(2):21.

Glaeser, E. L., Gyourko, J., and Saks, R. E. (2005a). Why Have Housing Prices Gone Up? *The American Economic Review*, 95(2):pp. 329–333.

Glaeser, E. L., Gyourko, J., and Saks, R. E. (2005b). Why Is Manhattan So Expensive? Regulation and the Rise in Housing Prices. *Journal of Law and Economics*, 48(2):331–369.

Glaeser, E. L., Gyourko, J., and Saks, R. E. (2006). Urban Growth and Housing Supply. *Journal of Economic Geography*, 6(1):71–89.

Gyourko, J., Mayer, C., and Sinai, T. (2013). Superstar Cities. *American Economic Journal: Economic Policy*, 5(4):167–199.

Gyourko, J. and Molloy, R. (2014). Regulation and Housing Supply. Technical report, National Bureau of Economic Research.

Hamilton, B. W. (1975). Zoning and Property Taxation in a System of Local Governments. *Urban Studies*, 12(2):205–211.

Henderson, J. V. (1991). Optimal regulation of land development through price and fiscal controls. *Journal of Urban Economics*, 30(1):64–82.

Hilber, C. A. and Robert-Nicoud, F. (2013). On the Origins of Land Use Regulations: Theory and Evidence from US Metro Areas. *Journal of Urban Economics*, 75:29–43.

Hills, R. and Schleicher, D. (2011). Balancing the "Zoning Budget". *Case Western Reserve Law Review*, 42(81):112–118.

Hills, R. M. and Schleicher, D. (2014). City Replanning. *George Mason Law & Economics Research Paper*, (14-32).

Hochman, O., Pines, D., and Thisse, J.-F. (1995). On the optimal structure of local governments. *The American Economic Review*, pages 1224–1240.

Hsieh, C.-T. and Moretti, E. (2015). Why Do Cities Matter? Local Growth and Aggregate Growth. Technical report, National Bureau of Economic Research.

Koster, H. R., van Ommeren, J., and Rietveld, P. (2012). Bombs, Boundaries and Buildings: a Regression-Discontinuity Approach to Measure Costs of Housing Supply Restrictions. *Regional Science and Urban Economics*, 42(4):631–641.

Krugman, P. (1991). Increasing Returns and Economic Geography. *The Journal of Political Economy*, 99(3):483–499.

Mayer, C. J. and Somerville, C. T. (2000). Land Use Regulation and New Construction. *Regional Science and Urban Economics*, 30(6):639–662.

Minnesota Population Center (2015). National historical geographic information system: Version 11.0.

Monkkonen, P. and Quigley, J. M. (2008). The Spatial Consequences of Autarky in Land-Use Regulation: Strategic Interaction or Parallelism? *Berkeley Program on Housing and Urban Policy*.

Morales, E., Serrato, J. C., and Zidar, O. (2015). State Taxes and Spatial Misallocation. Working Paper.

Moretti, E. (2004). Estimating the social return to higher education: evidence from longitudinal and repeated cross-sectional data. *Journal of Econometrics*, 121:175–212.

Ortalo-Magné, F. and Prat, A. (2014). On the Political Economy of Urban Growth: Homeownership versus Affordability. *American Economic Journal: Microeconomics*, 6(1):154–81.

Overman, H. G. and Puga, D. (2010). Labor pooling as a source of agglomeration: An empirical investigation. In Glaeser, E. L., editor, *Agglomeration Economics*, pages 133–150. University of Chicago Press.

Roback, J. (1982). Wages, Rents, and the Quality of Life. *The Journal of Political Economy*, 90(6):1257–12T8.

Rosen, S. (1979). Wage-Based Indexes of Urban Quality of Life. In Mieszkowski, P. M. and Straszheim, M. R., editors, *Current Issues in Urban Economics*, volume 3. Johns Hopkins University Press.

Rossi-Hansberg, E. (2004). Optimal Urban Land Use and Zoning. *Review of Economic Dynamics*, 7(1):69–106.

R.S. Means Company (2014). *Square Foot Costs*. R.S. Means Co, Norwell, MA.

Ruggles, S., Genadek, K., Goeken, R., Grover, J., and Sobek, M. (2015). Integrated public use microdata series: Version 6.0.

Saiz, A. (2010). The Geographic Determinants of Housing Supply. *The Quarterly Journal of Economics*, 125(3):1253–1296.

Schleicher, D. (2013). City unplanning. *Yale Law Journal*, 122(7):1670–1737.

Siodla, J. (2015). Razing San Francisco: The 1906 disaster as a natural experiment in urban redevelopment. *Journal of Urban Economics*, 89:48 – 61.

Stull, W. J. (1974). Land Use and Zoning in an Urban Economy. *American Economic Review*, pages 337–347.

Turner, M. A., Haughwout, A., and van der Klaauw, W. (2014). Land Use Regulation and Welfare. *Econometrica*, 82(4):1341–1403.

Van Nieuwerburgh, S. and Weill, P.-O. (2010). Why Has House Price Dispersion Gone Up? *The Review of Economic Studies*, 77(4):1567–1606.

Yglesias, M. (2012). *The Rent Is Too Damn High: What To Do About It, And Why It Matters More Than You Think*. Simon and Schuster.

Zillow Group (2015). Zillow data.

A Additional Figures

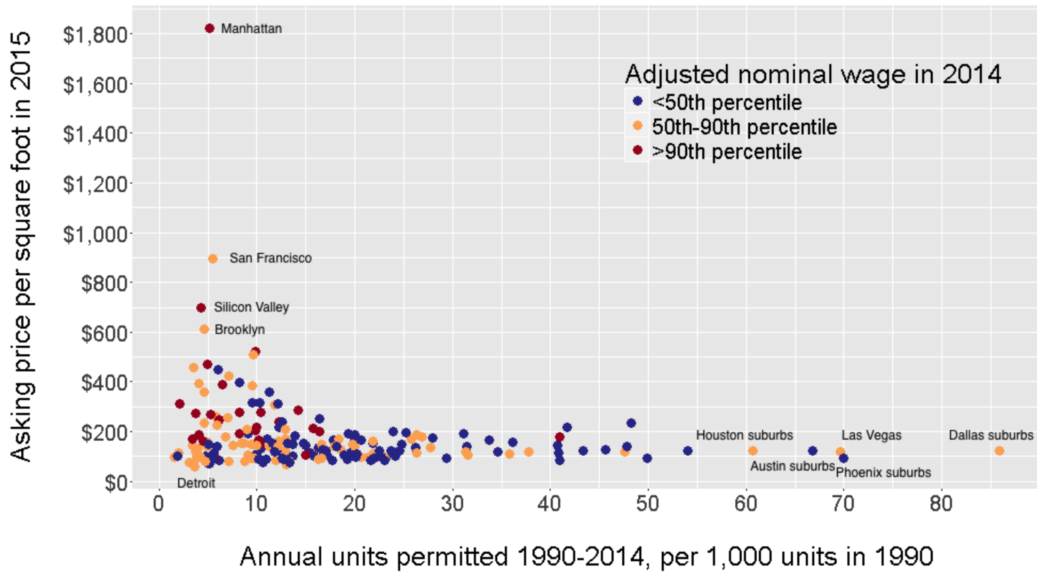

Figure 3: Plot of housing prices and increases in the quantity of housing units in selected counties. In descending order, the top five counties are Manhattan, San Francisco, San Mateo, Brooklyn, and Santa Clara: New York and Silicon Valley. Asking price data is from Zillow Group (2015), `https://www.zillow.com/research/data/`; units permitted from the U.S. Census Bureau, Building Permits Survey, `https://www.census.gov/construction/bps/`; data on units in 1990 from the U.S. Census Bureau obtained via Minnesota Population Center (2015), `http://www.nhgis.org`; and data on nominal wages from the U.S. Census Bureau, *American Community Survey (ACS)*, `https://www.census.gov/programs-surveys/acs`.

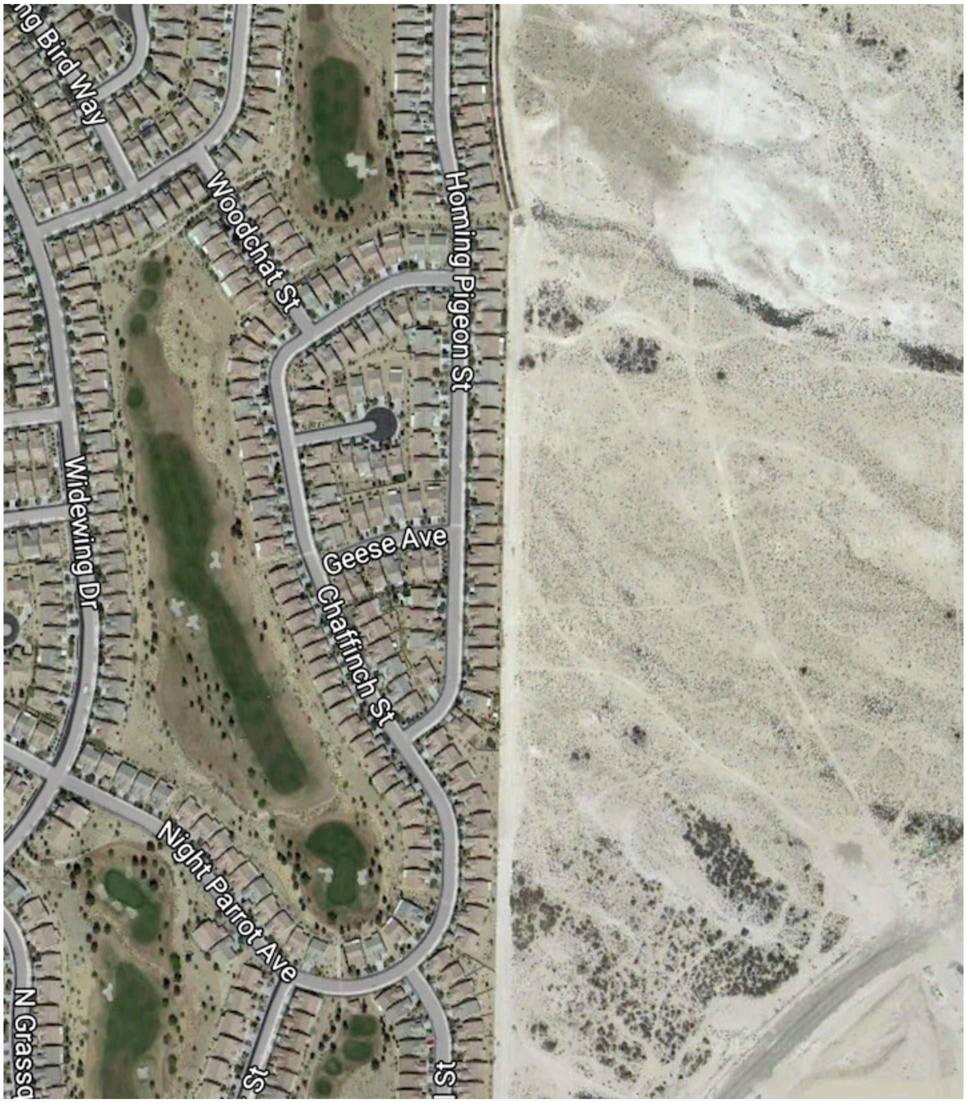

Figure 4: A Google Earth satellite image of the built-up edge of Las Vegas. Note that the city edge is obvious: on the left, population densities are in the upper single-digit thousands while the right is undeveloped desert. While this desert is itself likely to fill in, the pattern of a sharp edge will likely remain. While desert development may entail unusually large fixed costs, similar patterns hold in regions with friendlier climates.

B Proofs

Proposition 1. *For a given x, the portion of the $OL\,(n,x)$ curve with $n>0$ is strictly concave, hump-shaped, and intersects the $MC\,(n)$ curve twice, not at all, or once at a point of tangency.*

Proof. Consider an arbitrary $x>0$. Recall that $OL((n,x)$ for $n>0$ is defined as

$$OL\,(n,x) = y(x) + \Pi - \frac{F}{n} - u^{-1}\left(v(n) + \bar{u}\right).$$

As $F>0$, $-\frac{F}{n}$ is strictly concave. As $u(c)$ is concave and strictly monotone, $u^{-1}(\cdot)$ is convex and strictly monotone. Similarly, $v(n)+\bar{u}$ is convex and strictly monotone for a given $\bar{u}$. Their composition is thus convex as well, implying that $-u^{-1}\left(v(n)+\bar{u}\right)$ is concave. The first two terms do not depend on n, and so $OL((n,x)$ is strictly concave in n.

By assumption, $v(0)$ is bounded below. Thus we have:

$$lim_{n\to 0} OL\,(n,x) = -\infty.$$

From above, $-u^{-1}\left(v(n)+\bar{u}\right)$ is concave and decreasing in n, so that

$$lim_{n\to \infty} OL\,(n,x) = -\infty.$$

Hence $OL\,(n,x)$ is hump-shaped.

Finally, consider the relation between $OL\,(n,x)$ and $MC(n)$. As marginal cost is linear in n, the hump shape of $OL\,(n,x)$ implies that for a given x they must intersect twice, once with tangency, or not at all. □

Proposition 2. *Consider an arbitrary x for which there exists n with $OL(n,x) \geq MC(n)$. For this x, the locally optimal zoning constraint solves:*

$$\max_{\{c_0(x),\bar{n}(x)\}} u(c_0(x)) - v(\bar{n}(x)) \tag{23}$$

subject to the budget constraint

$$p_0(x) + c_0(x) + \frac{F}{\bar{n}(x)} = y(x) + \Pi \tag{24}$$

and the participation constraint

$$u\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - 2\bar{n}(x)/Z^2\right) - v\left(\bar{n}(x)\right) \geq \bar{u}. \tag{25}$$

Proof. Note that the participation constraint can be rewritten as

$$OL\left(\bar{n}(x), x\right) \geq MC\left(\bar{n}\right).$$

By Equation (6), satisfying the participation constraint implies $n^*(x) = \bar{n}(x)$: the zoning law chosen by the initial residents will become the equilibrium population. For $\bar{n}(x)$ not satisfying the participation, the equilibrium population will be $n^*(x) = 0$ if $\bar{n}(x)$ is to the left of the intersection of the $OL\left(n, x\right)$ and $MC(n)$ curves or $n^*(x) = n_H(x)$, the rightmost intersection of $OL\left(n, x\right))$ and $MC(n)$, if $\bar{n}(x)$ is to the right. The first case cannot maximize the initial resident problem: due to the assumption that F is large, they will always prefer to have a positive population. In the second case, the eventual equilibrium population will be $n_H(x)$, which satisfies the participation constraint. That is, choosing $\bar{n}(x) > n_H(x)$ cannot improve upon simply choosing $\bar{n}(x) = n_H(x)$. Therefore there exists a zoning law $\bar{n}(x)$ that satisfies the participation constraint and that maximizes the underlying initial resident problem:

$$\max_{\{c_0(x), \bar{n}(x)\}} u\left(c_0(x)\right) - v\left(\theta\left(\bar{n}(x); \bar{u}, \Pi\right)\right)$$

subject to

$$c_0(x) = y(x) + \Pi - \frac{F}{\theta\left(\bar{n}(x); \bar{u}, \Pi\right)} - p_0(x)$$

It remains to see that the second-order conditions for a maximum are satisfied. Note that $u(\cdot)$ is concave and utility is separable in c and n. Writing as $\lambda_0(x)$ and $\mu_0(x)$ the Lagrange multipliers for the budget and participation constraints, the first order condition for $\bar{n}(x)$ is

$$\begin{aligned} (26) \qquad & -v'\left(\bar{n}(x)\right) + \lambda_0(x)\frac{F}{\bar{n}(x)^2} \\ & + \mu_0(x)\left[u'\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - 2\bar{n}(x)/Z^2\right)\left(\frac{F}{\bar{n}(x)^2} - 2/Z^2\right) - v'\left(\bar{n}(x)\right)\right] = 0. \end{aligned}$$

As $v(\cdot)$ is convex, the second order condition for a maximum is satisfied as long as

$$\begin{aligned} (27) \qquad & -v''\left(\bar{n}(x)\right) - 2\lambda_0(x)\frac{F}{\bar{n}(x)^3} + \mu_0(x)u''\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - 2\bar{n}(x)/Z^2\right)\frac{F}{\bar{n}(x)^2} \\ (28) \qquad & -\mu_0(x)\left[u'\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - 2\bar{n}(x)/Z^2\right)\left(\frac{2F}{\bar{n}(x)^3} + 2/Z^2\right) + v''\left(\bar{n}(x)\right)\right] \\ (29) \qquad & < -2/Z^2\mu_0(x)u''\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - 2\bar{n}(x)/Z^2\right). \end{aligned}$$

Every term on the left hand side of Equation (29) is negative. This equation is easy to check for particular functional forms. For example, it is satisfied whenever

$$\frac{F}{\bar{n}(x)^2} > 2/Z^2.$$

□

Proposition 3. *The set of occupied locations $\mathcal{X}$ is $[\underline{x}, 1]$ for some threshold location $\underline{x}$. For the threshold location $\underline{x}$, the $OL(n, \underline{x})$ and $MC(n)$ curves are tangent at a unique level of population that depends on $\bar{u}$.*

Proof. Consider an occupied location $\tilde{x}$. As $y(x)$ is strictly increasing in x, $OL(n, x)$ must also be strictly increasing in x. For all locations $x > \tilde{x}$, this implies that there exists a positive n such that $OL(n, x) > MC(n)$. As initial residents prefer a strictly positive population, they will choose such an n and, by Equation (1), that location will be occupied.

Recall that by assumption, the productivity and fixed cost parameters are such that a *proper* subset of locations are occupied. By Equation (6) and Proposition 2, all locations with n such that $OL(n, x) \geq MC(n)$ will be occupied in equilibrium. By the continuity of $y(x)$, this set is closed. Thus there exists a threshold $\underline{x}$ such that $OL(n, \underline{x})$ and $MC(n)$ are tangent.

Tangency implies that

$$\frac{F}{n^2} - \left[u^{-1}\right]' (v(n) + \bar{u})\, v'(n) = 2/Z^2. \tag{30}$$

The left-hand side is strictly decreasing in n and satisfies $\lim_{n\to 0} = \infty$ and $\lim_{n\to\infty} = -\infty$, and thus has a unique solution that depends solely on $\bar{u}$ and parameters. □

Lemma 1. *Define $\bar{n}$ to be the population that maximizes the $OL(n, x)$ curve. Given outside option $\bar{u}$, the level of population $\bar{n}$ is unique and independent of location.*

Proof. From Proposition 1, the $OL(n, x)$ curve is strictly concave and twice continuously differentiable. The portion of the $OL(n, x)$ curve with $n(x) > 0$ is given:

$$OL(n, x) = y(x) + \Pi - \frac{F}{n} - u^{-1}(v(n) + \bar{u}).$$

The following first order condition is thus necessary and sufficient for the maximum:

$$\frac{F}{n^2} - \left[u^{-1}\right]' (v(n) + \bar{u})\, v'(n) = 0. \tag{31}$$

Call the population level that solves this $\bar{n}$. Given $\bar{u}$, the value of $\bar{n}$ is constant and independent of location. Write as $\mathcal{N}(\bar{u})$ the strictly decreasing function that gives the level of population that solves Equation 31 as a function of $\bar{u}$. □

Lemma 2. *Define location $\hat{x}(\bar{u}, \Pi)$ as follows:*

$$\hat{x}(\bar{u}, \Pi) = \begin{cases} x \text{ s.t. } OL(\mathcal{N}(\bar{u}), \hat{x}) = MC(\mathcal{N}(\bar{u})) & \text{if } OL(\mathcal{N}(\bar{u}), 1) \geq MC(\mathcal{N}(\bar{u})) \\ 1 & \text{otherwise.} \end{cases}$$

Then $\hat{x}(\bar{u}, \Pi)$ is unique and in the range $[\underline{x}, 1]$.

Proof. In the first case, the highest-productivity location has an $OL(n, 1)$ curve with maximum value that exceeds the marginal cost at the corresponding level of population $\bar{n}$. At the threshold occupied location $\underline{x}$, the $OL(n, \underline{x})$ and $MC(n)$ curves meet with tangency. The marginal cost curve has positive slope, and so this tangency must occur to the left of the maximum of the $OL(n, \underline{x})$ curve. Evaluated at $\mathcal{N}(\bar{u})$, $OL(\mathcal{N}(\bar{u}), x)$ is continuous and strictly increasing in x. Therefore there exists a unique $\hat{x}(\bar{u}, \Pi) \in [\underline{x}, 1]$ such that $OL(\mathcal{N}(\bar{u}), \hat{x}(\bar{u}, \Pi)) = MC(\mathcal{N}(\bar{u}))$.

In the second case, $OL(\mathcal{N}(\bar{u}), x) < MC(\mathcal{N}(\bar{u}))$ for all locations. Then $\hat{x}(\bar{u}, \Pi)$ is uniquely defined as 1 and is within the specified range. □

Proposition 4. *In a stable general equilibrium, the following characterizes the local equilibrium populations and optimal zoning. For all locations $x > \hat{x}$, $n^*(x) = \bar{n}(x) = \mathcal{N}(\bar{u})$. For all locations $x \in [\underline{x}, \hat{x}]$, the population $n^*(x)$ and optimal zoning $\bar{n}(x)$ are given by the upper intersection of the $OL(n, x)$ and $MC(n)$ curves. For all $x \geq \underline{x}$, $p^*(x)$ is given by the value of the $OL(n, x)$ curve evaluated at $n^*(x)$.*

Proof. For locations $x > \hat{x}$, the $OL(n, x)$ curve is strictly greater than the $MC(n)$ curve, and therefore the equilibrium population can only be given by $\bar{n}$ if the initial residents choose the zoning law $\bar{n}(x) = \bar{n}$. I will show that this is the case. Note $OL(n, x) > MC(n)$ implies that the participation constraint will not bind, and so the solution to the initial resident problem is given by the first order condition

$$u'\left(y(x) + \Pi - \frac{F}{n} - p_0(x)\right)\frac{F}{n^2} = v'(n).$$

And in stable equilibrium, $p_0(x) = p(x)$ so substituting from the household spatial equilibrium condition gives

$$u'\left(u^{-1}(v(n) + \bar{u})\right)\frac{F}{n^2} = v'(n).$$

From above, the maximum of the $OL(n, x)$ curve is given by the n that solves

$$\left[u^{-1}\right]'(v(n) + \bar{u})\, v'(n) = \frac{F}{n^2}.$$

By the definition of the derivative of the inverse, these are equivalent: for locations with non-binding participation constraints, the level of population $\mathcal{N}(\bar{u})$ that gives the maximum of the $OL(n, x)$ curve is identical to that which maximizes initial resident utility (given the stability condition that $p_0(x) = p(x)$). For all locations $x > \hat{x}(\bar{u}, \Pi)$, $\mathcal{N}(\bar{u})$ is consistent with the participation constraint and hence $n^*(x) = \bar{n}(x) = \mathcal{N}(\bar{u})$ for such locations. By Equation (6), the equilibrium price for such locations is given by $OL(\mathcal{N}(\bar{u}), x)$.

Consider location $x \in [\underline{x}, \hat{x}]$. From above, with $p_0(x) = p(x)$ the unconstrained choice of zoning that maximizes initial resident utility is the level of population that maximizes the $OL(n, x)$ curve. By definition, for locations with $x < \hat{x}(\bar{u}, \Pi)$ the maximum of the

$OL\,(n,x)$ curve is below the $MC\,(n)$ curve and hence infeasible. By concavity of the initial resident problem, the participation constraint will bind and the local equilibrium population (and optimal zoning law) is given by the upper intersection of the $OL\,(n,x)$ and $MC\,(n)$ curves. □

Proposition 5. *The utility-maximizing choices for the set of opened locations $\mathcal{X}$ and the set of zoning laws $\bar{n}(x)$ are identical to those chosen by the constrained planner. The equilibrium price gradient and aggregate profits implement the planner's choice of consumption. This set of instruments allows households to fully implement the planner's allocation.*

Proof. Under this voting regime, the measure 1 of households cast the decisive vote. As the vote takes place before development, households are identical and so the "median vote" is simply the representative household. For any set of zoning choices, the representative household rationally expects prices and consumption bundles to adjust to equalize utility across all occupied locations.

The representative household problem thus implies choosing the set of open locations and the set of zoning laws in order to maximize the level of utility $\bar{u}$ consistent with rational expectations about spatial equilibrium across all locations. Equivalently, the representative household can also choose consumption $c(x)$ for each location subject to the location-specific household budget constraint.

In making these choices, households are constrained by the spatial equilibrium conditions for all open locations, the budget constraints of households in all open locations, the definition of aggregate profits, the population constraint, and to a participation constraint.[35] The participation constraint implies that the price paid by households is consistent with firm construction decisions at each location.

As before, define $\theta\,(\bar{n}(x);\bar{u},\Pi)$ as the equilibrium population for a location as a function of the zoning law for particular values of $\bar{u}$ and Π. With this notation, the representative household solves

$$\max_{\{\mathcal{X},\bar{n}(x),c(x),\bar{u}\}} \bar{u}$$

subject to the spatial equilibrium condition

$$u\,(c(x)) - v\,(\theta\,(\bar{n}(x);\bar{u},\Pi)) \leq \bar{u}$$

with equality if $\theta\,(\bar{n}(x);\bar{u},\Pi) > 0$ and for all $x \in \mathcal{X}$, subject to the household budget constraints

$$y(x) + \Pi - \frac{F}{\theta\,(\bar{n}(x);\bar{u},\Pi)} - p(x) - c(x) = 0,$$

[35] It could theoretically be possible that the utility-maximizing choice is to leave some households out of the urban economy, but by assumption the income of the agricultural sector is low enough that this is not the case.

subject to the definition of profits

$$\Pi = \int_{\mathcal{X}} p(x)\theta\left(\bar{n}(x);\bar{u},\Pi\right) - \left(\theta\left(\bar{n}(x);\bar{u},\Pi\right)/Z\right)^2 dx,$$

subject to the population constraint

$$1 = \int_{\mathcal{X}} \theta\left(\bar{n}(x);\bar{u},\Pi\right) dx,$$

and subject to the participation constraint

$$p(x) \geq 2\theta\left(\bar{n}(x);\bar{u},\Pi\right)/Z^2.$$

Now, note that adding a constant increment Δ to the price for every location raises aggregate profits by a total of Δ. Further, if the spatial equilibrium condition holds a particular set of prices and profits, it will also hold when after adding Δ to the price in each location. Spatial equilibrium only pins down the *relative* price gradient. If it is the case that zoning laws bind in all locations, this further implies that any price gradient and incumbent definition of aggregate profits that satisfies spatial equilibrium will not distort location choices. As before, if the participation constraint is not binding then $\theta\left(\bar{n}(x);\bar{u},\Pi\right) = \bar{n}(x)$. As any price gradient can be shifted upward by a sufficient increment such that the participation constraint does not bind and the remaining equations are unaffected, the representative agent's problem can thus be rewritten as a direct choice of the local population with no participation constraint.

Rewriting the budget constraint yields

$$p(x) = y(x) + \Pi - \frac{F}{n(x)} - c(x).$$

Substituting the full set of household budget constraints directly into the profit definition yields

$$\Pi = \int_{\mathcal{X}} \bar{n}(x)\left(y(x) + \Pi - \frac{F}{\bar{n}(x)} - c(x)\right) - \left(\frac{\bar{n}(x)}{Z}\right)^2 dx.$$

Rewriting, we have

$$0 = \int_{\mathcal{X}} \left[\left(y(x) - c(x)\right)\bar{n}(x) - F - \left(\frac{\bar{n}(x)}{Z}\right)^2\right] dx.$$

This is simply the aggregate resource constraint face by a constrained planner. Choosing a local zoning law $\bar{n}(x)$, consumption level $c(x)$, and outside option $\bar{u}$ for all locations x that satisfies the population and aggregate resource constraint implicitly defines a set of prices and level of profit consistent with the definition of aggregate profits.

Finally, the set $\mathcal{X}$ consists of the set $[\underline{x}, 1]$ for some $\underline{x}$. If not, there would exist an unoccupied location $\tilde{x}$ for which some $x < \tilde{x}$ were occupied. In this case, the representative household could loosen the aggregate resource constraint by changing the zoning in any occupied location $x < \tilde{x}$ from $\bar{n}(x)$ to 0 and in $\tilde{x}$ from 0 to $\bar{n}(x)$. Loosening the binding aggregate resource constraint must increase the achievable level of utility $\bar{u}$, and so the representative household will always make this choice.

With these considerations, the restated problem is now

$$\max_{\{n(x),c(x),\bar{u},\underline{x}\}} \bar{u}$$

subject to spatial equilibrium conditions

$$u\left(c(x)\right) - v\left(n(x)\right) = \bar{u},$$

the aggregate resource constraint

$$0 = \int_{\underline{x}}^{1} \left[\left(y(x) - c(x)\right) n(x) - F - \left(\frac{n(x)}{Z}\right)^2\right] dx,$$

and the population constraint

$$1 = \int_{\underline{x}}^{1} n(x) dx.$$

This problem is identical in objective and constraints to that of the constrained planner and so must share a solution. $\square$